Precise time scales and navigation systems

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Metrology: from physics fundamentals to quality of life
July 2016
In 1612 Galileo wrote a letter to the King of Spain, proposing the use of the Jupiter satellites for navigation.

“….these stars display conjunctions, separations, eclipses, and other precise configurations…more than 100 a year…and they are so unique, identifiable, and so exact that nobody, of medium intelligence, is not able to use them to estimate the longitude and the position of the ship based on the ephemerids I have computed for the next years to come.”
Sept 7th, 1612
Navigation by “moon” observation
In spherical navigation,

currently is estimated by measuring distance from 3 known fixed points.

The distance measurement are measurement of the flight time of an electromagnetic signal.
Where are we?

Electromagnetic signals cover 1 meter in 3 nanoseconds
We therefore need:

**good** clocks (on Ground and in Space)

**good** clock synchronisation system

**good** reference time scale

**good** algorithms for clock evaluation

in timekeeping and navigation
GNSS: Where are the clocks and why?

Clocks on board: nav message, pseudorange

Ground stations, pseudorange

Control station, reference time

User clocks estimated as additional unknown

Time dissemination

Universal Time Coordinated by the Bureau International des Poids et Mesures
Galileo clocks: space

Space Rubidium Atomic Frequency Standard (RAFS)

Frequency Stability RAFS1-R2 EQM5
December 2002

Averaging Time (\(\tau\)), sec.

Allan Dev., Sigma y (\(\tau\))

RAFS Galileo Specification

measured data
**FIRST Space H-Maser Atomic Clocks**

**FREQUENCY STABILITY**

Allan Variance - medium term

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**Allan Deviation, σ(τ)**

- **Averaging Time, τ, Second**

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**INRiM**

Istituto Nazionale di Ricerca Metrologica
Ground Clocks inside the Control Centres

2 Active H maser

4 Cesium beam clocks
Cryogenic cesium Fountain INRIM ITCsF2

Relative accuracy $2 \times 10^{-16}$

- Laser cooling 1 µK;
- Cryogenic structure 89 K;
- Italian realization of the definition of the second
Miniaturized atomic clocks

New conception of miniaturized clocks is leading towards atomic clocks of a few mm dimension

They could be implemented in GNSS receivers to improve the positioning (altitude estimate), reduce the noise of the phase measures, allowing holdover navigation …

http://www.nist.gov/pml/div688/grp90/index.cfm

Close-up on the physics package of the Swiss Miniature Atomic Clock, 24x24 mm²
Having a good space clock is not enough…

Relativity effects

With atomic clock on board
relativistic effect are common routine

Travelling clock are slowing down
Clock at high altitude are going faster

The relative effect is $10^{-13}$ / km
3 microsec / year for one km in altitude

On board GPS/Galileo at 20000 km
The effect is about $4 \times 10^{-10}$ which means
40 microseconds in a day

Corresponding to about 10 km error in positioning in one day
The clock signal is emitted from space and then measured on ground

**GNSS Code measurement**

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Distance = Speed of Light $\times$ Time Difference

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Courtesy of P. Defraigne, ORB
range equations

$$P_{rec,1}^{sat} = c \left[ (t_{rec} - t_{sat1}) \right] = \left| \mathbf{x}_{sat1} - \mathbf{x}_{rec} \right|$$

$$P_{rec,2}^{sat} = c \left[ (t_{rec} - t_{sat2}) \right] = \left| \mathbf{x}_{sat2} - \mathbf{x}_{rec} \right|$$

$$P_{rec,3}^{sat} = c \left[ (t_{rec} - t_{sat3}) \right] = \left| \mathbf{x}_{sat3} - \mathbf{x}_{rec} \right|$$

$x_{rec} =$ receiver position  \hspace{1cm} $t_{rec} =$ receiver clock

$x_{sat} =$ satellite position  \hspace{1cm} $t_{sat} =$ satellite clock
From space clock to ground receiver

- Satellite clock
- Satellite orbit
- Atmosphere (Ionosphere + troposphere)
- multipath
- Receiver clock

Courtesy of P. Defraigne, ORB

Ground displacements
Observation equations

Range:

\[ P_{rec,1}^{sat} = c \left[ (t_{rec} - t_{sat1}) \right] = | \mathbf{x}_{sat1} - \mathbf{x}_{rec} | \]

Information from satellite

Unkowns to be estimated

Pseudorange:

\[ P_{rec,1}^{sat} = c \left[ (t_{rec} - t_{sat}) \right] = | \mathbf{x}_{sat} - \mathbf{x}_{rec} | + c \left[ (t_{rec} - t_{ref}) - (t_{sat} - t_{ref}) \right] + I_{rec,1} + Tr + \delta_{rec,1} + \epsilon_{rec,1} \]

Clock error versus \( t_{ref} \)

iono
troppo
Hardware delay

\( x_{rec} = \) receiver position
\( x_{sat} = \) satellite position
\( t_{rec} = \) receiver clock
\( t_{sat} = \) satellite clock
\( t_{ref} = \) reference time
The ionosphere is due to solar radiation, which ionizes the atoms and molecules in the upper atmosphere, producing a sea of ions and free electrons.

**Ionosphere**

Layer of electrons and electrically charged atoms and molecules that surrounds the Earth, 50 km → ~1000 km.

Effects on GNSS signal:
- 0-15 m at high elevation
- Up to 45 m at low elevation

Not same iono for different azimuth-elevation
Solar flares shape ionosphere

GPS significantly impacted by powerful solar radio burst

NOAA NEWS RELEASE
Posted: April 4, 2007

During an unprecedented solar eruption last December, researchers at Cornell University confirmed solar radio bursts can have a serious impact on the Global Positioning System (GPS) and other communication technologies using radio waves. The findings were announced Wednesday in Washington, D.C., at the first Space Weather Enterprise Forum—an assembly of academic, government and private sector scientists focused on examining the Earth’s ever-increasing vulnerability to space weather impacts.

A satellite image of the Dec. 5, 2005, solar flare that caused the following day’s intense radio burst that affected GPS systems. Credit: NOAA

Solar flare effect on quartz clocks

**Neutron Damage**

A fast neutron can displace about 50 to 100 atoms before it comes to rest. Most of the damage is done by the recoiling atoms. Net result is that each neutron can cause numerous vacancies and interstitials.

J. Vig, PTTI 2004, Tutorial, http://www.umbc.edu/photonics/Menyuk/Phase-Noise/Vig-tutorial_8.5.2.2.pdf
\[ x_i = TA(t) - h_i(t) = \sum_{j=1}^{N} w_j \left[ h_j'(t) + x_j(t) \right] \]

\[ w_i = \frac{1}{\sum_{k=1}^{N} \frac{1}{\langle \epsilon^2_k(\tau) \rangle}} \cdot \frac{1}{\langle \epsilon^2_i(\tau) \rangle} \]

\[
\begin{cases}
  x_i(t) = \sum_{j=1}^{N} w_j \left[ \hat{x}_i(t) + x_j(t) \right] \\
  x_{ij} = x_j(t) - x_i(t)
\end{cases}
\]
1. $t_{\text{ref}}$ is the System Reference Time to be defined from the ensemble of space/ground clocks (as any national ref time scale)
   - GPS time is a paper time scale estimated with a Kalman filter and steered versus UTC(USNO),
   - Galileo System Time is a weighted average of the ground clocks steered versus UTC

2. The offset $t_{\text{sat}} - t_{\text{ref}}$ is estimated by a complex algorithm using the same pseudorange measures and estimating orbits and clocks (Kalman filter in case of GPS, Batch least square in case of Galileo, …)

3. The real time offset $t_{\text{sat}} - t_{\text{ref}}$ transmitted to the user is a prediction based on previous measures
Predicted clock offsets (and predicted orbits) are estimated on ground and uploaded to the satellite. They are transmitted to the user as part of the navigation message and should be valid for a certain period in the future (GPS needs one day validity, Galileo plans 100 minutes validity, ...).

The uploaded navigation message contains predictions orbit prediction, clock prediction...
After synchronisation, any clock accumulate an error

Galileo requirement for maximum offset = 1.5 ns

Rubidium

Cesium clock

Ultra Stable Oscillator (quartz)
The clock signal affected by White and Random Walk frequency noises plus deterministic drifts can be handled exactly with

**stochastic differential equations.**

Iterative solution useful for simulations, filter, ...

\[
\begin{align*}
X_1(t_{k+1}) &= X_1(t_k) + X_2(t_k)\tau + a\frac{\tau^2}{2} + \sigma_1 W_{1,k}(\tau) + \sigma_2 \int_{t_k}^{t_{k+1}} W_2(s)ds \\
X_2(t_{k+1}) &= X_2(t_k) + a\tau + \sigma_2 W_{2,k}(\tau)
\end{align*}
\]

with initial conditions
\[
\begin{align*}
X_1(0) &= x_0 \\
X_2(0) &= y_0
\end{align*}
\]
Stochastic processes helps the clock prediction

Example: White frequency noise

Random walk of phase \( x(t) \)

At time \( t \) after synchronisation, the time error \( x(t) \) is described by a Gaussian probability density with

\[
E[x(t)] = 0
\]

\[
\text{Variance}[x(t)] = q_1 t
\]

Diffusion coefficient linked to Allan Deviation

\[ q_1 t = \text{AVAR}(t) \cdot t^2 \]
Clock data

Time Offset [ns]

Day Of the Year [days]

good clocks:
G25 vs GPS Time Normalized Frequency Offset
September 2012

G10 vs GPS Time Normalized Frequency Offset
February 2008

L. Galleani, P. Tavella IEEE FCS/ EFTF 2013 Prague
GNSS Reference time dissemination
\[
P_{\text{rec,1}}^{\text{sat}} = c \left[ (t_{\text{rec}} - t_{\text{sat}}) \right] = \\
|x_{\text{sat}} - x_{\text{rec}}| + c \left[ (t_{\text{rec}} - t_{\text{ref}}) - (t_{\text{sat}} - t_{\text{ref}}) \right] + I_{\text{rec,1}} + Tr + \delta_{\text{rec,1}} + \varepsilon_{\text{rec,1}},
\]

The User clock error =offset versus the Ref time is estimated by the receiver

- GNSS receiver and its antenna
  - Timing receiver
  - Fixed (and known) location
- External frequency reference for receiver
  - Atomic clocks (H-maser, Cesium...)
  - Physical time scale

the difference Local clock - GPS time = \((t_{\text{rec}} - t_{\text{ref}})\) is estimated

Signals-in-space (SIS) transmitted by the GNSS satellites contains also a prediction of \((\text{UTC}_{\text{SIS}} - t_{\text{ref}})\) as for example the predicted \((\text{UTC(USNO)} - \text{GPStime})_{\text{SIS}}\)

\((t_{\text{rec}} - t_{\text{ref}}) - (\text{UTC}_{\text{SIS}} - t_{\text{ref}})\) allows to estimate Local clock - UTC_{\text{SIS}}

The timing user can get UTC_{\text{SIS}} from GNSS
A navigation system is also a mean for

UTC time dissemination

What is the Universal Time Coordinated?
For centuries

The time was given by the rotating Earth
on which we set the clock

From 1967

The time is given by atomic clock
used to study Earth rotation
Along centuries...

- day and night are the “natural” time unit
- it was observed that during the year the length of day changes but the “Mean Solar Day” was deemed constant and Universal
- Universal Second = 1/86400 of rotational day (Mean Solar Time)
- 1884 Greenwich reference meridian
- 1925 International Astronomical Union fixes the beginning of the mean solar day at h. 00 and defines the Universal Time
Universal Time

the rotation rate is constant?
Polar motion

Suspected around 1850 from astronomers

That’s odd! The Polar seems lower!
The Polar is higher!

Polar motion can be measured but is not predictable
Polar motion, 1995-1998
Solid line: mean pole displacement,

About 10 m
Seasonal variation: in summer we spin faster

- A. Scheibe, 1936 in Berlin
- N. Stoyko, 1936 in Paris (BIH)

with crystal clock the day was measured shorter of about 1.2 ms
Variations in the duration of the day

http://www.iers.org
Secular slowing down

LENGTH OF DAY exceeding 86400 s

Advance of one second per year

Delay of one second per year

Years
The Universal Time was improved

UT = Universal Time scale

UT1 = Universal Time corrected by polar motion

UT2 = Universal Time scale corrected by seasonal variations

…. (UT not GMT!)
...in 1960

- the “revolution” of the Earth around the Sun is constant.
- Measuring the longitude of the Sun and using the equation of the apparent Sun orbit
- The new time scale: Ephemeris Time starts from h. 0 UT of January 1st, 1900.

- Time unit is the Ephemeris Second = 1/31 556 925.9747 of the tropical year on day January 0, 1900
- any new definition of the Second has to be in agreement with the previous one. For continuity with UT, this is the duration of the second in 1900.

in 1960 this duration was already shorter than 1/86400 of the Mean Solar Day
• Atomic Second = 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cs 133 atom

• First comes the second, then the time scale: in 1971: Temps Atomique International TAI, International Atomic Time

• TAI starts from h. 0 UT of January 1st, 1958.

• The length of the atomic second is in agreement with the Ephemeris second therefore shorter than 1/86400 of the Mean Solar Day
So far we have learnt that the Atomic Second is, by definition, shorter than the **current** Rotational Second (Universal Time) because it was defined in agreement with the duration of the Rotational Second in 1900 and the Earth is (slowly!) slowing down.
The International Astronomical Union recommends time scales and reference frames for the different applications in Geocentric or Solar System Barycentric frames. On the Earth or in the vicinity (50000 km) the reference time scale (1991) is the **Terrestrial Time**

The Terrestrial Time is a coordinate time scale defined in a geocentric reference frame (centered at the centre of the Earth), with scale unit the SI second as realised on the rotating geoid, i.e. differing by a constant rate with respect to a geocentric clock.
is an optimal realisation of the Terrestrial Time other realisation are for example TT(BIPM), some TA(k)

But which is now the angular position of the EARTH?
Some users need to know the relationship between the Universal Time UT1 (rotational) and the Atomic Time TAI. The **Universal Coordinated Time (UTC)** is a trade-off defined with the same time unit as TAI but with insertion of additional leap second.

\[
\text{TAI-UTC} = n \text{ seconds} \quad n = 0, \pm 1, \pm 2, \ldots
\]

| UT1-UTC | < 0.9 s
Universal Coordinated Time and leap seconds

UTC - TAI

Leap Seconds (1 s steps)

January 1, 1958
TAI = UT1

January 1, 1972
UTC - TAI = -10 s
Beginning of the Leap Seconds

Last leap second
June 30, 2015

Today
TAI - UTC = 36 s
Universal Time Coordinated

is computed at the BIPM in different steps:

**EAL**

= Echelle Atomique Libre = weighted mean of all atomic clocks in the world

**TAI**

= Temps Atomique International = EAL plus frequency steering to maintain the TAI second in agreement with the definition of the second of the International System (SI). This is obtained through the evaluation of primary frequency standards

**UTC**

= Universal Time Coordinated = TAI with the addition of leap second to remain close to the rotating Earth
The Universal Time Coordinated is the ultimate time reference (also with a «rapid» version) and it is available in deferred time.

Local time scale UTC(k) are realised by national laboratories in real-time.
Leap seconds are useful or annoying?

Idea first raised in public in 1999

Source: GPS World
Nov 1999
Global Positioning System: navigation and timing services

GPS time was set in agreement with UTC on h. 00 Jan 6, 1980

The accumulate time difference between UTC and GPS time is now of 17 seconds. GPS time is ahead 17 s
Leap seconds in Global Navigation Satellite System time scales

GNSS prefer not to apply leap seconds (except GLONASS), their time scale is easily available all over the world inside the navigation message, reference time scales differ from seconds, source of CONFUSION!!!
Timing from a MOTOROLA GPS receiver

UTC or GPS time can be chosen as reference time.
Time: 06:50:00 UTC

Time: 06:50:16 GPS
Leap seconds are useful or annoying? The current proliferation of time scales is generating confusion and possible danger.

Several international organisations created working groups to evaluate this issue. In November 2015 ITU General Assembly decided not to change till 2022. ITU would continue to be responsible for the dissemination of time signals via radiocommunication and BIPM for establishing and maintaining the second of the International System of Units (SI) and its dissemination through the reference time scale.
The proposed redefinition of Coordinated Universal Time, UTC

Today, leap seconds keep UTC, a time scale based on atomic clocks, in phase with the slightly variable rotation of the Earth.

The possibility of dropping the leap seconds in UTC has created misconceptions in the popular press as to what is at stake.

There are an increasing number of users of precise timing for whom the leap second causes serious technical challenges.
The proposed redefinition of Coordinated Universal Time, UTC

There is a need to set out clearly the reasons for the change and what is involved. This is the purpose of what follows:

The international character of the world's time scale

The measure of time and its unit the second are matters of international cooperation. Up until the middle of the 20th century, time scales were based on astronomical observations of the rotation of the Earth and the movement of the Earth in its orbit round the Sun. These had been within the purview of astronomers for centuries and, since the 1920s, had been the concern of the International Astronomical Union (IAU). With the invention of the atomic clock in 1955, however, everything began to change. By then, the irregular rate of rotation of the Earth and the practical difficulties in the realization of ephemeris time, based on the period of the Earth's orbit round the Sun, made it necessary to move to a time scale based on the atomic clock. .................

UTC for the 21st century

November 2011 at The Royal Society at Chicheley Hall,

Organised by Dr Terry Quinn and Dr Felicitas Arias


Metrologia

Special issue on “modern time scales” 48 (2011) S121–S124

Time and navigation will return in space?

Pulsar: a rotating star

A clock in space?
Pulsar

- Neutron star
- 20 km as diameter
- 1.4 time the solar mass
- in our Galaxy, thousand light years apart
- some spinning with millisecond period
- emitting radiowave as a lighthouse
Pulsar

- The rotation is highly stable
- Every millisecond we see a radio pulse

Is it a “clock”?
Many different difficulties:

- The rotation period is slowing down
- The pulse has to cross $10^{16}$ km of interstellar region
- The Earth is a rotating observatory

But...
**Pulsar**

Nobel Prize to Taylor and Hulse for gravitational wave detection (1993)

some ideas on the long term instabilities of atomic clocks

Pulsar is the hot topic:
new decades of observations are now available,
tens of millisec pulsars will be discovered by the Square Kilometer Array

G. Petit, Astronomy and Astrophysics 308, April 1996.

Pulsars for extraterrestrial space navigation?
an ESA study

The position is estimated by comparing measured pulse arrival times with respect to expected pulse arrival times using information from a pulsar database

The letter to the King of Spain in 1612 still alive…

These stars display conjunctions, separations, eclipses, and other configurations…more than 100 a year…and they are so unique, identifiable, and so exact that nobody, of medium intelligence, is not able to use them to identify the longitude and the position of the ship based on the ephemerids I have computed for the next years to come.
The 2015/16 IEEE UFFC Distinguished Lecturer Program support is kindly acknowledged.

Thanks for your attention!