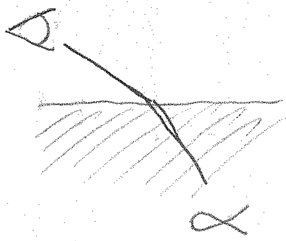
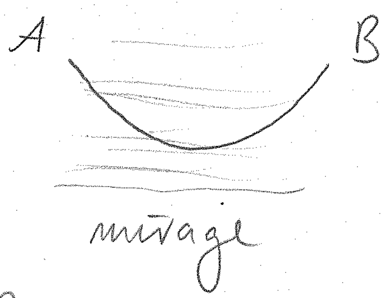
TRANSFORMATION OPTICS





- Fermat's Principle 1662

extremum $= \int_A^B n\,dl$

$$= \int_A^B n\sqrt{dx^2+dy^2+dz^2} = \int_A^B \sqrt{g_{ij}\,dx^i dx^j}$$

sum convention

$g_{ij} = n^2 \delta_{ij}$ media = geometry

- Maxwell's equations $\nabla\cdot\vec{E} = 0,\ \nabla\times\vec{E} = -\partial_t\vec{B}$
  $\nabla\cdot\vec{B} = 0,\ \nabla\times\vec{B} = \frac{1}{c^2}\partial_t\vec{E}$

$\nabla\cdot\vec{E} = \frac{1}{\sqrt{g}}\partial_i\sqrt{g}\,g^{ij}E_j,$ $\boxed{g = \det(g_{ij}),\ g^{ij} = (g_{ij})^{-1}}$

$\nabla\times\vec{E} = \pm\frac{[ijk]}{\sqrt{g}}\partial_j E_k = -g^{ij}\partial_t B_j$ $\boxed{\pm \text{ right/left-handedness}}$

- $\partial_i D^i = 0,\ [ijk]\partial_j E_k = -\partial_t B^i$
  $\partial_i B^i = 0,\ [ijk]\partial_j H_k = \partial_t D^i$

$$D^i = \varepsilon_0\underbrace{\left(\pm\sqrt{g}\,g^{ij}\right)}_{\varepsilon^{ij}}E_j,\quad B^i = \mu_0\underbrace{\left(\pm\sqrt{g}\,g^{ij}\right)}_{\mu^{ij}}H_j$$

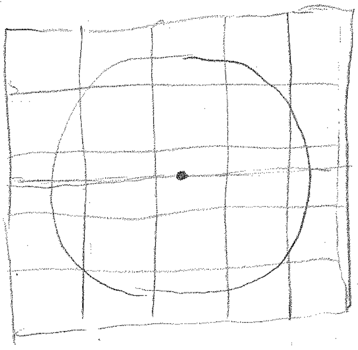
$\varepsilon^{ij} = \mu^{ij} = \pm\sqrt{g}\,g^{ij}$ impedance matching

$\det\varepsilon = g^{3/2}\cdot\frac{1}{g} = \sqrt{g} \Rightarrow g^{ij} = \frac{\varepsilon^{ij}}{\det\varepsilon}$

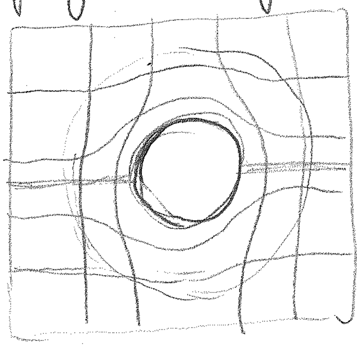
impedance-matched media = spatial geometry

- invisibility cloaking

virtual space

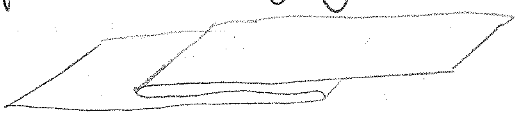


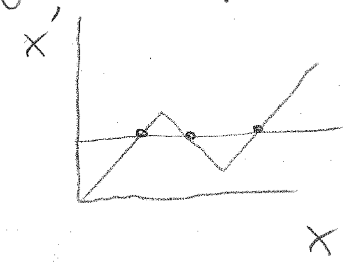
physical space



problem: infinite phase velocity

- perfect imaging with negative refraction



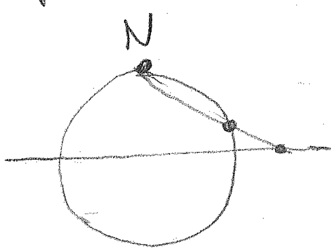


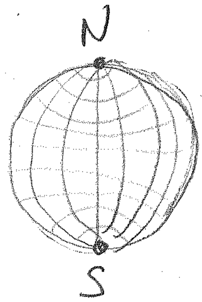
$$ds^2 = dx'^2 + dy'^2 + dz'^2$$

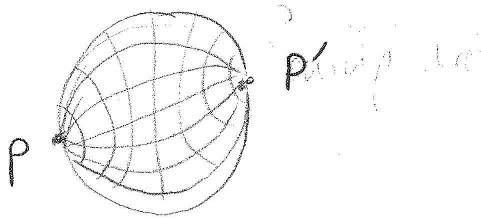
$$= \left(\frac{dx'}{dx}\right)^2 dx^2 + dy^2 + dz^2 = dx^2 + dy^2 + dz^2$$

$g_{ij} = \delta_{ij}$, but left-handed $\Rightarrow$ $\varepsilon = \mu = -1$

- perfect imaging with positive refraction



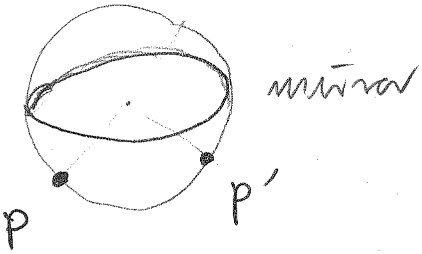




sphere

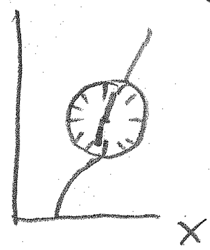
$$n = \frac{2n_1}{1+(r/a)^2}$$

trick:



complete time reversal = detector needed.

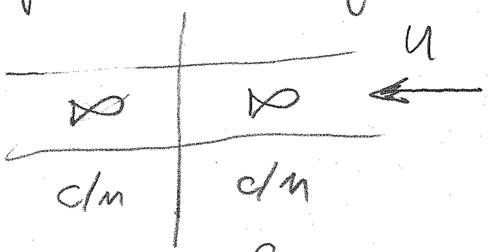
- space-time geometries $ct = x^0$, $g_{\alpha\beta}$



$\vec{D} = \varepsilon_0 \varepsilon \vec{E} + \frac{\vec{w}}{c} \times \vec{H}$, $\vec{B} = \mu_0 \mu \vec{H} - \frac{\vec{w}}{c} \times \vec{E}$

$\varepsilon = \mu = \mp \frac{\sqrt{-g}}{g_{00}} g^{ij}$, $w_i = \frac{g_{0i}}{g_{00}}$ moving media

- optical analogue of the event horizon



$|u| = \frac{c}{n}$

virtual space-time

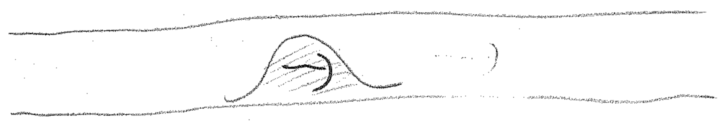
$t'_\pm = t' \mp \frac{x'}{c}$

$ct' = \frac{c}{2}(t_- + t_+)$, $x' = \frac{c}{2}(t_- - t_+)$

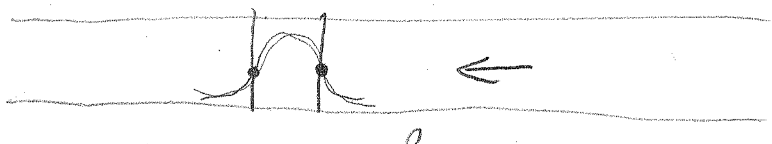
$$t_\pm = t - \int \frac{dx}{v_\pm}, \quad v_\pm = \frac{u \pm c/n}{1 \pm \frac{u}{cn}}$$

horizon: $v_+ \sim \alpha x$, $t_\pm = t - \frac{\ln|x|}{\alpha}$

- experiment



light pulse $n = n_0 + \delta n$



in moving frame

- light with negative frequencies

Hawking radiation

black-hole lasers