**Transformation Optics**

- **Fermat's Principle**: 1662

  \[
  \rho = \sum_{A} \int_{B} S \sqrt{-dx'^2 + dy'^2 + dz'^2} = \int_{A}^{B} \sqrt{g_{ij} dx^i dx^j}.
  \]

  \[g_{ij} = n^2 \delta_{ij} \quad \text{media} = \text{geometry}\]

- **Maxwell's equations**

  \[\nabla \cdot \vec{E} = 0, \quad \nabla \times \vec{E} = -\frac{1}{c^2} \frac{\partial}{\partial t} \vec{B} \]

  \[\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} \]

  \[\nabla \cdot \vec{E} = \frac{1}{\sqrt{g}} \partial_i \left( \sqrt{g} g^{ij} \vec{E}_j \right) \]

  \[\nabla \times \vec{E} = \pm \frac{1}{\sqrt{g}} \partial_j \left( \sqrt{g} g^{ij} \vec{E}_j \right) \]

  \[\partial_i \vec{B} = 0, \quad [\partial_j \nabla \partial_i \vec{E}_j]_t = -\partial_t \vec{B}_i \]

  \[\partial_i \vec{B} = 0, \quad [\partial_j \nabla \partial_i \vec{B}_j]_t = \partial_t \vec{D}_i \]

  \[\vec{D}_i = \varepsilon_0 \left( \pm \sqrt{g} g^{ij} \right) \vec{E}_j \quad \vec{B}_i = \mu_0 \left( \pm \sqrt{g} g^{ij} \right) \vec{H}_j \]

  \[\varepsilon_0 = \varepsilon_0^0 = \pm \sqrt{g} g^{ij} \text{ impedance matching} \]

  \[\det \varepsilon = g^{3/2} \quad \frac{1}{g} = \sqrt{g} \quad g^{ij} = \varepsilon_0 \frac{\varepsilon_0^0}{\det \varepsilon} \]

  \[\text{impedance-matched media} = \text{spatial geometry} \]
- Minimizing chaos in virtual space

- Physical space

- Problem: Unfriable phase velocity

- Perfect imaging with negative refractivity

\[ ds^2 = dx'^2 + dy'^2 + dz'^2 = (\frac{dx}{d\lambda})^2 dx^2 + dy^2 + dz^2 = dx^2 + dy^2 + dz^2 \]

\[ g_{ij} = S_{ij}, \text{ but left-handed } \Rightarrow \varepsilon = \mu = -1 \]

- Perfect imaging with positive refractivit

\[ n = \frac{2m_e}{1 + (v/a)^2} \]

Complete timelike minimal = discover nucleid.
- proper time: \( \tau = \int \sqrt{g_{\mu\nu} \frac{dx^\mu}{c} \frac{dx^\nu}{c}} \), \( g_{\alpha\beta} \partial_\alpha \partial_\beta \)

- \( \mathbf{E} = \mathbf{E} + \frac{\mathbf{w} \times \mathbf{H}}{c}, \quad \mathbf{B} = \mu_0 \mathbf{H} - \frac{\mathbf{w}}{c} \times \mathbf{E} \)

- \( \mathbf{E} = \mathbf{E}_0 + \mathbf{E}' \), \( \mathbf{B} = \mathbf{B}_0 + \mathbf{B}' \), \( \omega = \frac{\mathbf{E}_0}{\mathbf{B}_0} \)

- optical analogue of the event horizon

\[
\begin{array}{c|c}
\infty & 0 \\
\frac{c/m}{c/n} & \frac{d}{d'} \\
1/u &= \frac{c}{n}
\end{array}
\]

- virtual space-time

\[
\begin{align*}
t'_\pm &= t \mp \frac{x}{c} \\
t'_\pm &= \frac{c}{2} (t_+ + t_-), \quad x' = \frac{c}{2} (t - t_+)
\end{align*}
\]

- horizon: \( V_+ \sim \alpha x, \quad t_\pm = t - \frac{\ln|x'|}{\alpha} \)

- light pulse: \( m = m_0 + dm \)

- with negative frequencies

- Hawking radiation

- black-body laws