Quantum information processing with trapped electrons

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Motivations

- Quantized external (motional) and internal (spin) degrees of freedom;
- Lighter mass brings to higher trapping frequencies;
- Microwave and radiofrequency radiation;
- Ground state cooling of cyclotron motion;
- Electron spin states are very long lived;
- Extremely good isolation from the environment (negligible damping and reduced thermal fluctuations);
- High-fidelity detection of the quantum state (spin and cyclotron) via observation of quantum jumps.
Overview

• Basic properties of Penning traps
• Qubit encoding and manipulation
• Scalability
• New trap geometries
• Coupling qubits
• Outlook and applications
How Penning traps work


**STATIC electric and magnetic FIELDS:**

Homogeneous magnetic field $\rightarrow$ radial confinement
(cyclotron motion)

Quadrupole potential $\rightarrow$ axial confinement and slow circular drift
in the radial plane (magnetron motion)

$$V(x, y, z) = V_0 \frac{x^2 + y^2 - 2z^2}{2d^2}$$
Illustration of electron motion in a Penning trap

Fast radial oscillation: modified cyclotron motion
Slow radial oscillation: magnetron motion
Red: axial oscillation
Electron motion inside the trap

\[ \omega_+ \equiv \frac{\omega_c + \sqrt{\omega_c^2 - 2\omega_z^2}}{2} \]  
\[ \omega_- \equiv \frac{\omega_c - \sqrt{\omega_c^2 - 2\omega_z^2}}{2} \]  
\[ \omega_z \equiv \sqrt{\frac{2eV_0}{md^2}} \]  

with \( \omega_c \equiv \frac{|e|B}{m} \) being the free cyclotron frequency

For a trap size of 1 cm, a voltage \( V_0 \sim 10 \) V, a magnetic field \( B \sim 3 \) T:

• \( \omega_+/(2\pi) \sim 100 \) GHz modified cyclotron frequency,
• \( \omega_z/(2\pi) \sim 100 \) MHz axial frequency,
• \( \omega_-/(2\pi) \sim 10 \) kHz magnetron frequency.
Energy level structure

\[ E = -\hbar \omega_0 \left( l + \frac{1}{2} \right) + \hbar \omega_z \left( k + \frac{1}{2} \right) + \hbar \omega_+ \left( n + \frac{1}{2} \right) + \frac{\hbar \omega_s}{2} s \]

with \( l, k, n = 0, 1, 2, \ldots \)

\( s = \pm 1 \)

\[ \omega_s \equiv \frac{g |e| B}{2m} \]
The DiVincenzo’s criteria


- A scalable physical system with well defined qubits;
- The ability to initialize the state of the qubits;
- A universal set of quantum gates;
- Decoherence times much longer than the gate operation time;
- A qubit-specific measurement;
- Interconverting stationary and flying qubits;
- Transmitting qubits over long distances.
Observing the Quantum Limit of an Electron Cyclotron: QND Measurements of Quantum Jumps between Fock States

S. Peil and G. Gabrielse
Department of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 18 March 1999)

Quantum jumps between Fock states of a one-electron oscillator reveal the quantum limit of a cyclotron. With a surrounding cavity inhibiting synchrotron radiation 140-fold, the jumps show a 13 s Fock state lifetime and a cyclotron in thermal equilibrium with 1.6 to 4.2 K blackbody photons. These disappear by 80 mK, a temperature 50 times lower than previously achieved with an isolated elementary particle. The cyclotron stays in its ground state until a resonant photon is injected. A quantum cyclotron offers a new route to measuring the electron magnetic moment and the fine structure constant.

PACS numbers: 03.65.–w, 42.50.Ct

FIG. 2. Quantum jumps between the lowest states of the one-electron cyclotron oscillator decrease in frequency as the cavity temperature is lowered.

FIG. 3. Excitations to excited Fock states which are stimulated by 4.2 K blackbody photons in (a) and (b), and by an externally applied microwave field in (c) and (d).
Qubits in the electron spin and cyclotron motion


\{ |00\rangle, |01\rangle, |10\rangle, |11\rangle \}
Single qubit operations

Spin: resonant pulses at the spin transition frequency $\omega_s$

$$b(t) = b_0 [\cos(\omega t) \hat{x} + \sin(\omega t) \hat{y}]$$

$$H_{\text{int}} = -\mathbf{\mu} \cdot \mathbf{b}(t) = \frac{g}{2} \mu_B \mathbf{\sigma} \cdot \mathbf{b}(t) = \frac{\hbar \Omega_s}{2} (\sigma_x \cos \omega t + \sigma_y \sin \omega t)$$

$$|0\rangle = |0\rangle \cos \left( \frac{\Omega_s t}{2} \right) - i |1\rangle \sin \left( \frac{\Omega_s t}{2} \right)$$

$$|1\rangle = |1\rangle \cos \left( \frac{\Omega_s t}{2} \right) - i |0\rangle \sin \left( \frac{\Omega_s t}{2} \right)$$
Cyclotron qubit manipulation

\[ I(t) = I \cos(\omega_d t + \phi) \]

\[ b(t) = b_1 p(t) \cos(\omega_d t + \phi) \]

\[ b_1 = \frac{1}{c} \frac{\pi a^2 d}{(a^2 + d^2/4)^{5/2}} \]

\[ H_{\text{int}} = \frac{g\mu_B}{2} \left[ \sigma_x x(t) + \sigma_y y(t) \right] b_1 \cos(\omega_d t + \phi) \]

in RWA

\[ H_{\text{int}}^{(IP)} \approx \frac{g\mu_B b_1}{2} \sqrt{\frac{\hbar}{2m\tilde{\omega}_c}} \left( \sigma_+ a_c e^{-i\phi} + \sigma_- a_c^+ e^{i\phi} \right) \]

\[ \tilde{\omega}_c \equiv \sqrt{\omega_c^2 - 2\omega_z^2} \]

\[ |11\rangle \]

\[ |10\rangle \quad \omega_s - \omega_c \]

\[ |01\rangle \]

\[ |00\rangle \]
Unitary evolution operator

\[ U(t) = \exp \left( -\frac{iH^{(ip)}_{\text{int}} t}{\hbar} \right) \]

where \( \theta \equiv g \mu_B b_1 \sqrt{\frac{\hbar}{2m \tilde{\omega}_c}} t \)

\[ M(\theta, \phi) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & A & -B & 0 & 0 \\
0 & B^* & A & 0 & 0 \\
0 & 0 & 0 & C & -D \\
0 & 0 & 0 & D^* & C
\end{pmatrix} \]

\[ A \equiv \cos \left( \frac{\theta}{2} \right) \]
\[ B \equiv i e^{i\phi} \sin \left( \frac{\theta}{2} \right) \]
\[ C \equiv \cos \left( \frac{\theta}{\sqrt{2}} \right) \]
\[ D \equiv i e^{-i\phi} \sin \left( \frac{\theta}{\sqrt{2}} \right) \]
Composite pulse technique

A. M. Childs and I. L. Chuang, PRA 63, 012306 (2001);

Cyclotron:

1) Swap the cyclotron and spin qubits

\[
\begin{pmatrix}
M\left(\frac{\pi}{\sqrt{2}}, 0\right) & M\left(\frac{2\pi}{\sqrt{2}}, \phi_s\right) & M\left(\frac{\pi}{\sqrt{2}}, 0\right)
\end{pmatrix}
\]

\[\text{SWAPPING}\]

\[
\phi_s = \arccos \left[ \cot^2 \left(\frac{\pi}{\sqrt{2}}\right) \right]
\]

2) Operate on the spin qubit

3) Swap back the cyclotron and spin qubits

\[
\begin{pmatrix}
M\left(\frac{\pi}{\sqrt{2}}, \pi\right) & M\left(\frac{2\pi}{\sqrt{2}}, \pi + \phi_s\right) & M\left(\frac{\pi}{\sqrt{2}}, \pi\right)
\end{pmatrix}
\]

\[(\text{SWAPPING})^{-1}\]
Two-qubit gate: Conditional phase shift

\[ C \text{-PS (} \phi = \pi ) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \]

\[ M(\pi,0) \times M\left(\frac{\pi}{\sqrt{2}}, \frac{\pi}{2}\right) \times M(\pi,0) \times M\left(\frac{\pi}{\sqrt{2}}, \frac{\pi}{2}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \]
Towards electron-electron entanglement

L. Lamata et al., PRA 81, 022301 (2010)

Two electrons in a Penning trap with a weak rotating wall potential

\[ \tilde{V}(x_i) = \frac{1}{2} m \omega_{\rho}^2 \rho_i^2 + \frac{1}{2} m \omega_z^2 z_i^2 - \frac{1}{2} m \omega_{\rho}^2 \delta(x_i^2 - y_i^2) \]

with \( \omega_{\rho}^2 = (\omega_c - \omega)\omega - \frac{1}{2} \omega_z^2 \)

Equilibrium position at \( x = \pm x_0/2 \) with

\[ x_0 = \left[ \frac{e^2}{2\pi\epsilon_0 m (\omega_{\rho}^2 - \omega_z^2 \delta)} \right]^{1/3} \]
Two-electron gate

Axial drive at the anomaly frequency + magnetic bottle

\[ z_{CM} = z_0 \cos(\omega_v + \Delta)t \]
\[ \Delta B(x) = \beta_2 \left[ \left( z^2 - \frac{\rho^2}{2} \right) \hat{z} - z \rho \right] \]

Hamiltonian in interaction picture

\[ H_{\text{gate}}^I = \hbar \Omega \sum_{i=1,2} \left( \sigma^+_i a_{CM,c} e^{-i\Delta t} + \sigma^-_i a^+_{CM,c} e^{i\Delta t} \right) \]
\[ \Omega = \frac{g \mu_B \beta_2 z_0}{2 \sqrt{4m\hbar}} \frac{1}{\left( \omega_c^2 - 2\omega_z^2 \right)^{1/4}} \]

For \( z_0 \sim 100 \, \mu m \), \( \Omega/(2\pi) \sim 10 \) Hz
Applications

Spin-spin entanglement

\[ \left| \downarrow \downarrow \right\rangle \left| 0 \right\rangle_{CM,c} \Rightarrow \left| \downarrow \downarrow \right\rangle \left| 1 \right\rangle_{CM,c} \quad \text{weak resonant cyclotron drive} \]

\[ \left| \downarrow \downarrow \right\rangle \left| 1 \right\rangle_{CM,c} \Rightarrow \frac{1}{\sqrt{2}} \left( \left| \uparrow \uparrow \right\rangle + \left| \downarrow \uparrow \right\rangle \right) \left| 0 \right\rangle_{CM,c} \pi \text{- pulse at } \omega_a \]

Two-qubit gates (off-resonance regime \( \Delta \gg \Omega \))

\[
H_{I}^{\text{off}} = \frac{\hbar \Omega^2}{\Delta} \sum_{i,j} \left[ -(n+1)\sigma_i^+ \sigma_j^- + n \sigma_i^- \sigma_j^+ \right]
\]

Entanglement of the spin and motional degrees of freedom (spin-cyclotron GHZ states useful for quantum metrology)

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} \left( \left| \uparrow \uparrow \right\rangle \left| 0 \right\rangle_{CM,c} + \left| \downarrow \downarrow \right\rangle \left| 2 \right\rangle_{CM,c} \right) \]
Hyperbolical trap
From a cylindrical Penning trap…


- Well defined cavity modes;
- Quality factor $Q = 5 \times 10^4$
- Cavity-induced suppression of spontaneous emission by synchrotron radiation.
... to a planar Penning trap


Electrodes printed on an insulating substrate (Al₂O₃):
• easily produced and miniaturized (thick- and thin-film technology)
• open geometry for easy access with radiation
• more traps on the same substrate to form a two-dimensional array
Silver plated $\text{Al}_2\text{O}_3$ ceramic disk with electrodes of $R_0=2.5$ mm, $R_1=5.8$ mm, $R_2=9.1$ mm and $d=3$ mm
Axial potential

\begin{align*}
R_0 &= 300 \, \mu m, \quad R_1 = 600 \, \mu m, \quad R_2 = 900 \, \mu m \\
U_0 &= 0 \, V, \quad U_1 = 0.5 \, V, \quad U_2 = 0 \, V \text{ (compensation voltage -0.417 V)}
\end{align*}

\[ \frac{\omega z}{2\pi} = 89.9 \, MHz \quad (99.0 \, MHz \text{ solid line}) \]

(a) \quad \phi(z) [eV]

(b) \quad \text{equ. dist. [mm]}

radius $R_0$ [mm]
Operation of \textit{mm}-sized planar traps


Planar trap with D = 4.8 cm, operated at room temperature

Optimized parameters:
\[ U_0=0 \text{ V, } U_1=-13.6 \text{ V, } U_2=33.6 \text{ V} \]
\[ \omega_z/2\pi \sim 35 \text{ MHz} \]

Planar trap with D = 2.0 cm,
\[ U_0=0 \text{ V, } U_1=0.5 \text{ V, } U_2=-0.417 \text{ V} \]
\[ \omega_z/2\pi \sim 100 \text{ MHz} \]

Microscopic planar Penning trap with multiple ring electrodes

Adjusting the control voltages:

from a large trap size with $R_{0/1/2} = 1.5 / 3 / 4.5 \text{ mm}$

to a small trap with $R_{0/1/2} = 50 / 100 / 150 \mu\text{m}$

Change the distance of the electron to the electrode surface from 6 mm to 50 $\mu\text{m}$
Microscopic planar Penning trap with multiple ring electrodes

Adjusting the control voltages: variable size of the effective central disc and ring electrodes

Change the distance of the electron to the electrode surface from 6 mm to 50 μm
Microscopic planar Penning trap with multiple ring electrodes

Investigation of decoherence effects depending on the electron-surface distance
Microscopic planar Penning trap with multiple ring electrodes

Larger trap volume for easier loading
Varying the coupling between spin state and axial motion

Place a small nickel piece under the central electrode

**Large distance:**
Homogeneous B field for long coherence and single qubit operations

**Short distance:**
Inhomogeneous B field for analysis and two-qubit operations
Planar electron micro-trap arrays

Interconnected micro traps with
- \( R_0 = 300 \, \mu\text{m} \)
- \( R_1 = 600 \, \mu\text{m} \)
- \( R_2 = 900 \, \mu\text{m} \)

Flexible variation of the connections using bonding pads on the gold surface

Drawing courtesy of Michael Hellwig (University of Ulm/Mainz)
QUELE experimental setup
Microfabricated trap with multiple ring electrodes

The effective electrode size can vary from $R_{0/1/2} = 1500/3000/4500 \ \mu m$ down to $R_{0/1/2} = 50/100/150 \ \mu m$
Prototype planar trap (Univ. Mainz)
Onion trap (Univ. Ulm/Mainz)
Pixel trap (Univ. Ulm/Mainz)
Pixel trap
Mirror-image planar Penning trap

Coplanar-waveguide Penning trap

Figure 1 from J. Verdú 2011 New J. Phys. 13 113029
Linear array of traps

F. Mintert and Ch. Wunderlich, Phys. Rev. Lett. 87, 257904 (2001)

A planar trap at a distance $x_0$ from the center of the substrate

Inhomogeneous magnetic field
(linear gradient)

Direct Coulomb interaction

$$B_1 = b \left( z \mathbf{k} - \frac{x}{2} \mathbf{i} - \frac{y}{2} \mathbf{j} \right)$$

$$H_{i,j}^C = \frac{e^2}{4\pi\varepsilon_0 \left| \mathbf{r}_i - \mathbf{r}_j \right|}$$
The magnetic field gradient couples each electron internal (spin) and external (motional) degrees of freedom.

\[
H_i^{NC} \approx -\hbar \omega_m a_{m,i}^+ a_{m,i} + \hbar \omega_c a_{c,i}^+ a_{c,i} + \hbar \omega_c a_{z,i}^+ a_{z,i} + \frac{\hbar}{2} \omega_s \sigma_i^z
\]

\[
+ \frac{g}{4} \hbar \omega_z \varepsilon \left( a_{z,i}^+ + a_{z,i} \right) \sigma_i^z - \frac{g}{4} \hbar \omega_z \varepsilon \sqrt{\frac{\omega_z}{\omega_c}} \left( \sigma_i^{(-)} a_{c,i}^+ + \sigma_i^{(+)} a_{c,i} \right)
\]

\[
\varepsilon = \frac{|e| b \Delta z}{m \omega_z} = \frac{|e| b}{m \omega_z} \sqrt{\frac{\hbar}{2m \omega_z}}
\]

Size of the ground state
The Coulomb interaction couples axial and cyclotron motion of different electrons

\[ H_{i,j}^C \approx \hbar \xi_{i,j} \left( a_{z,i}^+ + a_{z,i} \right) \left( a_{z,j}^+ + a_{z,j} \right) \]

\[ -\hbar \xi_{i,j} \frac{\omega_z}{\tilde{\omega}_c} \left( a_{c,i} a_{c,j}^+ + a_{c,i}^+ a_{c,j} \right) \]

\[ \xi_{i,j} = \frac{e^2}{8\pi \varepsilon_0 m \omega_z d_{i,j}^3} = \frac{1}{\hbar} \frac{e^2}{4\pi \varepsilon_0 d_{i,j}} \left( \frac{\Delta z}{d_{i,j}} \right)^2 \]
Remove the coupling between internal and external degrees of freedom with a canonical transformation

\[ H \rightarrow e^S He^{-S} \]

\[
S = \frac{g}{4} \varepsilon \sum_{i=1}^{N} \left[ \sigma_i^z \left( a_{z,i}^+ - a_{z,i}^- \right) + \frac{\omega_z}{\omega_a} \sqrt{\frac{\omega_z}{\tilde{\omega}_c}} \left( \sigma_i^{(-)} a_{c,i}^- + - \sigma_i^{(+)} a_{c,i}^+ \right) \right]
\]

\[ \omega_a \equiv \omega_s - \omega_c \quad \text{anomaly frequency} \]
Effective spin-spin Hamiltonian


\[ H_s \approx \sum_{i=1}^{N} \frac{\hbar}{2} \omega_{s,i} \sigma_i^z + \frac{\hbar}{2} \sum_{i<j}^{N} \left[ 2J_{i,j}^z \sigma_i^z \sigma_j^z - J_{i,j}^{xy} \left( \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \right] \]

\[ J_{i,j}^z = \left( \frac{g}{2} \right)^2 \xi_{i,j} \varepsilon^2 = \left( \frac{g}{2} \right)^2 \frac{\hbar e^4}{16\pi\varepsilon_0 m^4} \frac{b^2}{\omega_z^4 d_{i,j}^3} \]

\[ J_{i,j}^{xy} = 10^6 \left( \frac{g}{4} \right)^2 \xi_{i,j} \varepsilon^2 \left( \frac{\omega_z}{\omega_c} \right)^4 = 10^6 \left( \frac{g}{2} \right)^2 \frac{\hbar e^4}{64\pi\varepsilon_0 m^4} \frac{b^2}{\omega_c^4 d_{i,j}^3} \]

Dipolar decay
Array of trapped electrons as an NMR molecule


\[ H'_s \approx \sum_{i=1}^{N} \frac{\hbar}{2} \omega_{s0,i} \sigma_{z,i} + \sum_{i>j}^{N} \frac{\hbar}{2} \pi J_{i,j} \sigma_{z,i} \sigma_{z,j} \]

\[ \omega_{s0,i} = \frac{geB_0}{2m} \left( 1 + \frac{b^2 x_{i,0}^2}{8B_0^2} \right) \]

\[ J_{i,j} = \frac{g^2}{2\pi} \frac{\xi_{i,j} \varepsilon^2}{\omega_z^4 d_{i,j}^3} \approx \frac{b^2}{\omega_z^2} \]

The spin (qubit) frequency depends on the trap position

The coupling constant depends on the magnetic gradient, the axial frequency, and the inter-particle distance
Estimate of the spin-spin coupling strength $J$

The axial frequency $\omega_z/(2\pi)$ is 100 MHz

<table>
<thead>
<tr>
<th>$b$ = 50 T/m</th>
<th>$d$ = 100 μm</th>
<th>$d$ = 50 μm</th>
<th>$d$ = 10 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3 Hz</td>
<td>18 Hz</td>
<td>2300 Hz</td>
<td></td>
</tr>
<tr>
<td>$b$ = 500 T/m</td>
<td>0.23 kHz</td>
<td>1.85 kHz</td>
<td>230 kHz</td>
</tr>
</tbody>
</table>
A channel for quantum communication


\[
H_s = \sum_{i=1}^{N} \frac{\hbar}{2} \omega_s \sigma_i^z - \hbar \sum_{i<j}^{N} \left[ 2J_{i,j}^z \sigma_i^z \sigma_j^z - J_{i,j}^{xy} \left( \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \right]
\]

with \( J_{i,j}^{xy} = J_{i,j}^z \). Transmission fidelity, up to 20 spins, larger than 90%!
The transfer time scales as the cube of the chain length.
Coupling the axial qubits with wires

The oscillating image charges of two electrons are coupled to each other.

Coherent swapping of excitation between the traps.

\[ |10\rangle = |n_1 = 1, n_2 = 0\rangle \rightarrow |01\rangle = |n_1 = 0, n_2 = 1\rangle \]

J. Zurita-Sánchez and C. Henkel, PRA 73, 063825 (2006)
Coherent wire coupling

\[ H = \hbar \Omega_{12} \left( a_{z,1}^+ a_{z,2}^+ + a_{z,1} a_{z,2} \right) \]

\[ \Omega_{12} = \frac{e^2}{2mR_0^2 \omega_z} \left( \frac{R_0^2}{h^2 + R_0^2} \right)^3 \frac{1}{2C_0 + C_w} \]

where \( R_0 \) is the central electrode radius and \( h \) is the height of the electron from the trap surface.

\( C_0 \sim \pi \varepsilon_0 R_0 \) intrinsic electrode capacitance
\( C_w \) capacitance of the wire

The shorter the wire and the smaller the trap, the stronger the coherent coupling.
Coupling strength

<table>
<thead>
<tr>
<th></th>
<th>d =100 mm</th>
<th>10 mm</th>
<th>1 mm</th>
<th>100 μm</th>
<th>10 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₀ =1 mm</td>
<td>0.12 Hz</td>
<td>1.0 Hz</td>
<td>3.2 Hz</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td>10 μm</td>
<td>1.3 kHz</td>
<td>13 kHz</td>
<td>130 kHz</td>
<td>990 kHz</td>
<td>3.2 MHz</td>
</tr>
</tbody>
</table>

\[ \frac{\omega_z}{2\pi} = 100 \text{ MHz} \]
\[ C_0 = R_0 \times 100 \text{ fF mm}^{-1} \text{ intrinsic electrode capacitance} \]
\[ C_w = d \times 66 \text{ fF mm}^{-1} \text{ capacitance of the wire} \]
Summary & future perspectives

- Universal quantum gates with trapped electrons;
- Scalable planar Penning traps of variable size and geometry;
- Coupling distant electrons;
- Design of an effective spin-spin interaction;
- Short distance quantum communication;
- Entanglement generation;
- Investigation of decoherence effects;
- Applications to quantum metrology;
- Integration into circuit QED.
IOTA
ION TRAPS FOR TOMORROWS APPLICATIONS

• Giacomo Ciaramicoli

• Paolo Tombesi