Optimum persistent currents for ultracold bosons stirred on a ring

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Abstract
Experimental advances in the manipulation of low-dimensional ultracold atomic gases and the creation of traps with ring topology [1] have stimulated the interest in fundamental properties of one-dimensional (1D) Bose liquids, and in particular 1D superfluidity. We consider a 1D ring of interacting bosons, in the presence of a moving barrier. We study the current response of the system, which depends on the interplay between the barrier strength and the interparticle interaction strength. Interestingly, we find a maximum of the current amplitude at intermediate interactions.

The system
Gas of interacting bosons confined on a 1D mesoscopic ring at T=0. The ring contains a localized barrier adiabatically moving with velocity \( \Omega \). \( I(\Omega) \) particle current induced in the system by the barrier.

\[
I(\Omega) = \frac{1}{2\pi h} \frac{\partial E(\Omega)}{\partial \Omega}
\]

\[
H = \sum_{i=1}^{N} \left[ \left( -i \frac{\partial}{\partial \theta} - \Omega \right)^{2} + \lambda \delta(\theta_{i} + \pi) + \hat{g} \sum_{j=1}^{N} \delta(\theta_{i} - \theta_{j}) \right]
\]

\[
\hat{g} = g(mL^{2}/2\pi^{2}h^{2})
\]

Current response
Define \( \alpha \) amplitude of \( I(\Omega) \).
No barrier \( \Rightarrow \alpha=1 \) for any \( g \) [2]. Weak barrier \( \Rightarrow \) rounded sawtooth [3]. Strong barrier \( \Rightarrow \) sinussoid.

Adimensional interaction parameter: \( \gamma = mgL^{2}h^{2}/\hbar^{2}n \)

Bibliography

Gross-Pitaevskii
Small \( \gamma \): Mean-Field approximation \( \rightarrow \)
Gross-Pitaevskii (GP) equation.

\[
\left[ -i \frac{\partial}{\partial \theta} - \Omega \right]^{2} \psi(\theta) + \gamma \frac{N^{2}}{\pi} \psi(\theta)^{2} \psi(\theta) = 0
\]

Large \( \gamma \): Numerical simulation on a discretized lattice (Bose-Hubbard model), with small occupation per site \( \langle n_{\text{max}} \rangle \).
Method: variational optimization à la Density Matrix Renormalization Group (DMRG) of a Matrix Product State (MPS) trial wavefunction with \( L_{\text{dim}}^{d} \) parameters [6].

DMRG
Large - Intermediate \( \gamma \): Numerical simulation on a discretized lattice (Bose-Hubbard model), with small occupation per site \( \langle n_{\text{max}} \rangle \).
Method: variational optimization à la Density Matrix Renormalization Group (DMRG) of a Matrix Product State (MPS) trial wavefunction with \( L_{\text{dim}}^{d} \) parameters [6].

\[
\psi_{\text{DMRG}}(\theta) \rightarrow \text{mps} \left( \mathbf{P}_{1}^{d} \ldots \mathbf{P}_{m}^{d} \right)
\]

\[
d = n_{\text{max}} + 1 \quad m \leq 30
\]

Periodic Boundary Conditions are implemented via a factorization procedure for long products of MPS matrices [7].

Luttinger liquid
Large \( \gamma \): Low-energy and low-momentum approximation \( \rightarrow \)
mode expansion of the density and phase bosonic fluctuation fields. Perturbative analysis of the weak and strong barrier regimes.

- Weak barrier: \( \lambda_{\text{eff}} = \frac{\lambda}{2\pi} \left( \frac{K}{N} \right)^{K} \) Luttinger parameter

\[
\Rightarrow I(\Omega) \frac{N}{\lambda} = \delta \Omega \left( 1 - \frac{1}{2} \sqrt{\Omega^{2} + \lambda_{\text{eff}}^{2}} \right)
\]

- Strong barrier: \( \lambda_{\text{eff}} \propto \lambda \)

\[
\Rightarrow I(\Omega) \frac{N}{\lambda} = -t \left( \frac{K}{N} \right)^{1/K} \sin(2\Omega)
\]

Tonks-Girardeau
\( \gamma = \infty \): Infinitely interacting limit, the many-body wave function vanishes at contact \( \rightarrow \) exact solution by mapping onto non-interacting fermions [5].

Friedel oscillations of the density:

\[ N=18 \quad \Omega=0.4 \]

Conclusion
By tuning the interparticle interaction strength at fixed barrier, it is possible to reach an “optimum current” response, which has the maximum amplitude and is the closest to superfluidity.

Results
Non-monotonous behavior of current amplitude \( \alpha \) vs. interaction strength. Signatures of a current maximum at intermediate interaction strength.

Different curves for different values of \( \lambda \).

Color legend: NI, TG, GP, LL, DMRG.

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