Spin orbit interaction in semiconductors

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Lecture 1

• Motivation

• Spin & orbit: an early experiment

• Spin-orbit interaction: a simple picture

• Sommerfeld model + spin orbit: spin splitting

• New experiment: interband s-o coupling (2D)

• ‘Cross-dressed’ atoms as cool spins
  (energy scales, Zitterbewegung)

• Summary
Some (early) motivation

Conventional electronics: charge plays a fundamental role in devices [integrated circuits: resistors, transistors (billions), etc.]

Field Effect Transistor (FET)

“spin: factor of 2”

‘Rashba effect’

Datta and Das (APL 56, 1990)

Emmanuel Rashba

Supriyo Datta
Motivation
Spin FET (Semiconductor spintronics, etc.)

- Most popular proposed spintronic device (Datta and Das ’90);
- Coherent *electric* control of *magnetic* degrees of freedom;
- Low energy to rotate/flip spins; faster (?) (Hall & Flatté, APL/2006);
- More control & various modes of operation. Logic gates with “dynamic” functions: reprogrammable on the fly; Multifunctional: logic, storage, processing, communication, etc, in a single chip?
- Design flexibility (hybrid devices, geometry, etc. etc.)
- General: fertile ground for novel spin-dependent phenomena. Interplay of many-body effects (tunability) & quantum confinement; spin-flip mechanisms, spin injection and spin polarized transport, non-local quantum correlations (EPR), etc.
Topological insulators

‘Two-faced’ solid: an insulator from the inside... a conductor from the outside...

Spin-orbit interaction: define the ‘topology’ of the bands

Majorana fermions

Exotic (half) fermion: ‘particle=antiparticle’

• Spin-orbit interaction + quantum wire + s-wave SC: proximity eff. → unconventional $p$-wave pairing in the wire

• Zeeman (or exchange) gap: topological superconducting wire (Kitaev: spinless $p$-wave SC chain → Majorana end modes)

Synthetic gauge potentials: cold atoms in optical lattices
Spin-orbit: an early experiment
Richardson (1908);
Einstein & de Haas (1915)
Angular momentum → "spin orbit"

$L_{\text{tot}} = 0 \Rightarrow \text{magnet rotates}$

Compass: difficulties
- Ships (steel)
- Submarines
- Proximity to poles

Patent office (Bern):
- Navigation system using gyroscopes

Sci. Am., October/2004
Basic idea:

1) Orbital motion (loop) $\Rightarrow$ magnetic moment.

2) Alignment of moments (via a field $B$) $\Rightarrow$ change in angular momentum of bar.

3) Angular momentum conservation $\Rightarrow$ Fe bar must rotate ("s-o coupling").

4) The experiment worked beautifully!

5) However (factor of 2): the ferromagnetism of Fe is due to the intrinsic electron spin (rather than the orbital one).

More mathematically:

$$\vec{L}_{\text{bar}}^{\text{loops}} + \vec{L}_{\text{bar}}^{\text{mech}} = 0,$$

$$\vec{M}_{\text{bar}}^{\text{loops}} = \gamma \vec{L}_{\text{bar}}^{\text{loops}}$$

($\gamma = \frac{M_{\text{bar}}^{\text{loops}}}{L_{\text{bar}}^{\text{mech}}}$ (since', $\vec{\mu} = \gamma \vec{S}$)

C. Kittel, Phys. Rev. 76, 743 (1949)
Solids: basics
Sommerfeld electrons

**Sommerfeld model**: free & non-interacting electrons in a box with periodic BC. ('Fermi gas')

\[ H \phi = \varepsilon \phi, \quad H = \frac{\hat{p}^2}{2m_0} \]

\[ \Rightarrow \phi_{k,\sigma} = \frac{1}{\sqrt{V}} \exp(i \vec{k} \cdot \vec{r}) |\sigma\rangle \quad \text{orbital} \]

\[ \varepsilon_{k,\sigma} = \frac{\hbar^2 k^2}{2m_0} \quad \text{spin} \]

\[ e^{ik_x x} = e^{ik_x (x+L)} \Rightarrow e^{ik_x L} = 1 \]

Volume V, N electrons

Fermi sea

(degenerate gas – spin)

Parabolic band

“Potential”

Plane waves: delocalized

k space: localized

Density of states: (DOS)

\[ \rho(\varepsilon) \propto \sqrt{\varepsilon} \]

Carlos Egues, Varenna 2012
Rashba spin orbit: basics
(spin orbit interaction in two-dimensional electron gases)

Electric manipulation of intrinsic magnetic degrees of freedom.

(coherent spin control)
Rashba spin-orbit interaction
(“poor man’s derivation”)

Atomic case:

In its rest frame, the electron feels:

$$B_{\text{eff}} = \frac{1}{2c} \mathbf{v} \times \mathbf{E}_{\text{nucleus}}$$

Spin-orbit interaction:

$$\Delta E_{s-o} = -\mu_{\text{spin}} \mathbf{B}_{\text{eff}} = \frac{ge}{2mc^2} \mathbf{S} \cdot (\mathbf{v} \times \mathbf{E})$$

Corresponding Hamiltonian:

$$H_{s-o} = \frac{ge}{2mc^2} \frac{\sigma}{2} \cdot (\mathbf{p} \times \nabla V)$$

$$H_{s-o} \propto \frac{dV}{dr} \hat{r} \cdot (\sigma \times \mathbf{p}), \quad \nabla V = \hat{r} \frac{dV}{dr}$$

Heterojunction:

Potential profile: $V(z)$

Note structural inversion asymmetry (SIA)

Can control slope of $V(z)$ via gate!

In analogy with the atomic case: $\nabla V = \hat{z} \frac{dV}{dz}$

Hence, the Rashba s-o term is:

$$H_{s-o}^R \propto \frac{dV}{dz} \hat{z} \cdot (\sigma \times \mathbf{p})$$

“Electric Field”

Defining a s-o coupling constant $\alpha$:

$$H_{s-o}^R = \frac{\alpha}{\hbar} (\sigma \times \mathbf{p})_z = \frac{\alpha}{\hbar} (\sigma_x p_y - \sigma_y p_x)$$

Nitta et al. PRL ‘97
Engels et al. PRB ‘97
Bernardes et al. PRL 99, 076603 (2007)
Main ingredients: \(2 \text{DEG} + \text{spin orbit} + \text{“FM” emitter/collector}\)

Hamiltonian:

\[
H_{2\text{DEG}} = \frac{1}{2m} \left( p_x^2 + p_y^2 \right) + \frac{\alpha}{\hbar} \left( \sigma_x p_y - \sigma_y p_x \right)
\]

“Time evolution”: rotation about the y axis

\[
U_R = e^{i\alpha \sigma_x \tau / \hbar} = \begin{pmatrix} \cos(\theta_R / 2) & \sin(\theta_R / 2) \\ -\sin(\theta_R / 2) & \cos(\theta_R / 2) \end{pmatrix}
\]

\[
\theta_R \equiv 2m\alpha L / \hbar^2 \\
L = v_F \tau = (\hbar k_F / m)\tau
\]

\[
H_{1\text{DEG}} = \frac{1}{2m} p_x^2 - \frac{\alpha}{\hbar} \sigma_y p_x
\]

\[
U_R \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \cos(\theta_R / 2) \\ -\sin(\theta_R / 2) \end{pmatrix}
\]

\[
I_{\uparrow, \downarrow} \propto 1 \pm \cos \theta_R
\]
Sommerfeld 2D + Rashba: quasi-1D case

\[ H = \frac{1}{2m} \left( p_x^2 + p_y^2 \right) + i\alpha \left( \sigma_y \partial_x - \sigma_x \partial_y \right) + V(y) \]

Zeroth-order solution (no Rashba): quantum wire subbands

\[ H^0 = \frac{1}{2m} \left( p_x^2 + p_y^2 \right) + V(y) \]

\[ \varepsilon_n,\sigma \left( k_x \right) = \frac{\hbar^2 k_x^2}{2m} + \varepsilon_n, \quad n = a, b, \ldots \]

\[ \phi_{k,n,\sigma} \propto e^{ik_x x} \phi_n(y) |\sigma\rangle, \quad \sigma = \uparrow, \downarrow \]

Effective quasi-1D Rashba Hamiltonian: two lowest subbands \( a \) and \( b \)

\[ k_x \rightarrow k \]

\[
H = \begin{bmatrix}
\varepsilon_{a,\uparrow}(k) & i\alpha k & 0 & -i\alpha d \\
-i\alpha k & \varepsilon_{a,\downarrow}(k) & -i\alpha d & 0 \\
0 & i\alpha d & \varepsilon_{b,\uparrow}(k) & i\alpha k \\
i\alpha d & 0 & -i\alpha k & \varepsilon_{b,\downarrow}(k)
\end{bmatrix}
\]

\[ d \equiv \langle \phi_a | d / dy | \phi_b \rangle \]

(“interband mixing”)

Moroz & Barnes, ‘99
Mireles & Kirczenow, ’01
Egues, Burkard, Loss,' 02
Sommerfeld 2D + Rashba: quasi-1D case

\[ H = \frac{1}{2m} \left( p_x^2 + p_y^2 \right) + i\alpha \left( \sigma_y \partial_x - \sigma_x \partial_y \right) + V(y) \]

Zeroth-order solution (no Rashba): quantum wire subbands

\[ H^0 = \frac{1}{2m} \left( p_x^2 + p_y^2 \right) + V(y) \]

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\[ \phi_{k,n,\sigma} \propto e^{ik_x x} \varphi_n(y) |\sigma\rangle, \quad \sigma = \uparrow, \downarrow \]

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  -i\alpha k & \varepsilon_{a,\downarrow}(k) & -i\alpha d & 0 \\
  0 & i\alpha d & \varepsilon_{b,\uparrow}(k) & i\alpha k \\
  i\alpha d & 0 & -i\alpha k & \varepsilon_{b,\downarrow}(k)
\end{bmatrix} \]

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Zeroth-order solution (no Rashba): quantum wire subbands

$$H^0 = \frac{1}{2m} \left( p_x^2 + p_y^2 \right) + V(y)$$

$$\varepsilon_{n,\sigma}(k_x) = \frac{\hbar^2 k_x^2}{2m} + \varepsilon_n, \quad n = a, b,...$$

$$\phi_{k,n,\sigma} \propto e^{ik_x x} \varphi_n(y)|\sigma\rangle, \quad \sigma = \uparrow, \downarrow$$

Effective quasi-1D Rashba Hamiltonian: two lowest subbands $a$ and $b$

$$k_x \rightarrow k$$

$$H = \begin{bmatrix}
\varepsilon_{a,\uparrow}(k) & i\alpha k \\
-i\alpha k & \varepsilon_{a,\downarrow}(k)
\end{bmatrix} \begin{bmatrix}
\varepsilon_{b,\uparrow}(k) & i\alpha k \\
-i\alpha k & \varepsilon_{b,\downarrow}(k)
\end{bmatrix}$$

$$d \equiv \langle \phi_a | d / dy | \phi_b \rangle$$

("interband mixing")

Moroz & Barnes, '99
Mireles & Kirczenow, '01
Egues, Burkard, Loss, '02
Strictly 1D Case: $d=0$

Eigenvalues:

$$\epsilon_s(k) = \frac{\hbar^2}{2m} (k - sk_R)^2 - \frac{\hbar^2 k_R^2}{2m} \frac{m\alpha}{\epsilon_R}, \quad s = \pm$$

$$k_R = \frac{m\alpha}{\hbar^2}$$

Eigenvectors (spin):

$$|+\rangle = |\uparrow\rangle_y = \frac{1}{\sqrt{2}} \left[ |\uparrow\rangle + i |\downarrow\rangle \right]$$

$$|-\rangle = |\downarrow\rangle_y = \frac{1}{\sqrt{2}} \left[ |\uparrow\rangle - i |\downarrow\rangle \right]$$

Neglects interband mixing (lowest subband)

No Rashba

Rashba (gate induced)

Quasi 1D
A recent experiment
Direct Observation of Interband Spin-Orbit Coupling in a Two-Dimensional Electron System

Hendrik Bentmann, Samir Abdelouahed, Mattia Mulazzi, Jürgen Henk, and Friedrich Reinert

(a) Energy vs Wave vector

Rashba $\alpha_1$, $\alpha_2$

Interband $\Delta$

(b) $\Delta = 42$ meV

(c) $\Delta = 31$ meV

$\Delta = 14$ meV

Tunable $\Delta$: # of ML

$\alpha_1$, $\alpha_2$: const. in this range

($\alpha_1$, $\alpha_2 \approx 10\Delta$)
Direct Observation of Interband Spin-Orbit Coupling in a Two-Dimensional Electron System

Hendrik Bentmann, Samir Abdelouahed, Mattia Mulazzi, Jürgen Henk, and Friedrich Reinert

[9] or ultrafast demagnetization [10]. Note that spin-mixing is expected to gain increasingly in importance for materials with high atomic number and thus enhanced SO interaction. It is therefore of fundamental importance to explore whether interband SO coupling effects due to spin-mixing are present in heavy-element 2DEGs and how they modify the spin-split electronic structure. Indeed, recent theoretical reports predict interband SO coupling phenomena in 2DEGs formed in zinc blende quantum wells [11,12] and on high-Z metal surfaces [13] as a result of higher-order perturbation theory corrections in the SO interaction. Yet, to the best of our knowledge, these effects have not been addressed in experiment so far.
Spin-Orbit Interaction in Symmetric Wells with Two Subbands

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We investigate the spin-orbit (SO) interaction in two-dimensional electron gases in quantum wells with two subbands. From the $8 \times 8$ Kane model, we derive a new intersubband-induced SO term which resembles the functional form of the Rashba SO but is nonzero even in symmetric structures. This follows from the distinct parity of the confined states (even or odd) which obliterates the need for asymmetric potentials. We self-consistently calculate the new SO coupling strength for realistic wells and find it comparable to the usual Rashba constant. Our new SO term gives rise to a nonzero ballistic spin–Hall conductivity, which changes sign as a function of the Fermi energy ($e_F$) and can induce an unusual Zitterbewegung with cycloidal trajectories without magnetic fields.

$$\mathcal{H} = \left( \frac{\vec{p}^2}{2m^*_e} + \epsilon_+ \right) \mathbf{1} \otimes \mathbf{1} - \epsilon_- \tau^z \otimes \mathbf{1}$$

$$\eta = \left( \frac{1}{E_g^2} - \frac{1}{(E_g + \Delta)^2} \right) \frac{P^2}{3} \langle \epsilon | \partial_z V(z) | o \rangle$$

Intersubband coupling strength
Intersubband-induced spin-orbit interaction in quantum wells

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(Received 10 July 2008; published 16 October 2008)

Recently, we have found an additional spin-orbit (SO) interaction in quantum wells with two subbands [Bernardes et al., Phys. Rev. Lett. 99, 076603 (2007)]. This new SO term is nonzero even in symmetric geometries, as it arises from the intersubband coupling between confined states of distinct parities, and its strength is comparable to that of the ordinary Rashba. Starting from the 8 × 8 Kane model, here we present a detailed derivation of this new SO Hamiltonian and the corresponding SO coupling. In addition, within the self-consistent Hartree approximation, we calculate the strength of this new SO coupling for realistic symmetric modulation-doped wells with two subbands. We consider gated structures with either a constant areal electron density or a constant chemical potential. In the parameter range studied, both models give similar results. By considering the effects of an external applied bias, which breaks the structural inversion symmetry of the wells, we also calculate the strength of the resulting induced Rashba couplings within each subband. Interestingly, we find that for double wells the Rashba couplings for the first and second subbands interchange signs abruptly across the zero bias, while the intersubband SO coupling exhibits a resonant behavior near this symmetric configuration. For completeness we also determine the strength of the Dresselhaus couplings and find them essentially constant as function of the applied bias.

\[
H = \begin{pmatrix}
E_{k||0} & -i\alpha_0 k_- & 0 & -i\eta k_-
\alpha_0 k_+ & E_{k||0} & i\eta k_+ & 0
0 & i\eta k_- & E_{k||1} & -i\alpha_1 k_-
0 & -i\eta k_- & i\eta k_+ & \alpha_1 k_+ & E_{k||1}
\end{pmatrix},
\]

Eigenvalues

\[
E_{k||,\lambda_1,\lambda_2} = E_{k||+} + \lambda_2\alpha_+ k|| + \lambda_1 \sqrt{(\eta k||)^2 + (E_{k||-} + \lambda_2\alpha_- k||)^2},
\]

Eigenvectors

\[
\langle r||k||,\lambda_1,\lambda_2\rangle = \sqrt{1 + \lambda_1} \frac{\epsilon_{\lambda_2}(0)}{\epsilon_{\lambda_2}(\eta)} \frac{\lambda_1\lambda_2\eta k||}{\epsilon_{\lambda_2}(\eta) + \lambda_1\epsilon_{\lambda_2}(0) - i\lambda_2 e^{-i\theta}} \frac{e^{ik||r||}}{4\pi}
\]

\[
\lambda_1, \lambda_2 = \pm 1
\]

\[
\epsilon_{\pm}(\eta) = \sqrt{(\eta k||)^2 + (E_{k||-} + \alpha_- k||)^2}, \quad e^{\pm i\theta} = \frac{k_+}{k||}.
\]
Intersubband-induced spin-orbit interaction in quantum wells

Rafael S. Calsaverini,1 Esmerindo Bernardes,1,* J. Carlos Egues,1,† and Daniel Loss2

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Crossings

Dirac cones

Eigenvalues

$$\lambda_1, \lambda_2 = \pm 1$$

$$\epsilon_\pm(\eta) = \sqrt{\left(\eta k_\parallel\right)^2 + (\mathcal{E}_{k_\parallel 1} \pm \alpha k_\parallel)^2}, \quad e^{\pm i\theta} = \frac{k_\pm}{k_\parallel}.$$
Intersubband-induced spin-orbit interaction in quantum wells

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Department of Physics, University of Basel, CH-4056 Basel, Switzerland

(Received 10 July 2008; published 16 October 2008)

Eigenvectors

Anticrossings

Dirac cones

Eigenvalues

$$
\begin{pmatrix}
\mathcal{E}_{k_{\parallel}^0} \\
-\alpha_0 k_+ \\
0 \\
i\eta k_+ & 0 & \mathcal{E}_{k_{\parallel}^1} \\
\end{pmatrix}
$$

$$
\mathcal{E}_{k_{\parallel},\lambda_1,\lambda_2} = \mathcal{E}_{k_{\parallel}} + \lambda_2 \alpha_+ k_+ + \lambda_1 \sqrt{(\eta k_{\parallel})^2 + (\mathcal{E}_{k_{\parallel}} + \lambda_2 \alpha_+ k_{\parallel})^2},
$$

$$
\lambda_1, \lambda_2 = \pm 1
$$

$$
\epsilon_{\pm}(\eta) = \sqrt{(\eta k_{\parallel})^2 + (\mathcal{E}_{k_{\parallel}} \pm \alpha k_{\parallel})^2}, \quad e^{+i\theta} = \frac{k_ \pm}{k_{\parallel}}.
$$

two subbands in symmetric arities, and its we present a on, within the listic symmet-constant areal s give similar esion symmetry each subband. ds interchange avior near this couplings and
H Bentmann, PRL 108, 196801 (2012)

(ab initio)

J. Fu, M. Hachiya, P. H. Penteado, & CE

\[
\eta = 0.1\alpha_2 \\
\alpha_2 = -4\alpha_1 \\
\beta_1 = \beta_2 \\
\gamma = 0.1\beta_2
\]

\[
\langle \sigma_x \rangle
\]
Important interplay of Rashba and Dresselhaus!
Unconventional spin topology in surface alloys with Rashba-type spin splitting

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(Received 4 May 2009; published 24 June 2009)

The spins of a pair of spin-orbit split surface states at a metal surface are usually antiparallelly aligned, in accord with the Rashba model for a two-dimensional electron gas. By first-principles calculations and two-photon photoemission experiments we provide evidence that in the surface alloy Bi/Cu(111) the spins of an unoccupied pair of surface states are parallelly aligned. This unconventional spin polarization, which is not consistent with that imposed by the Rashba model, is explained by hybridization of surface states with different orbital character and is attributed to the spin-orbit interaction. Since hybridization is a fundamental effect our findings are relevant for spin electronics in general.
Quasi 1D channels
Rashba Spin-Orbit Interaction and Shot Noise for Spin-Polarized and Entangled Electrons

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(Received 1 May 2002; published 4 October 2002)

Datta–Das transistor with enhanced spin control

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(Received 1 October 2002; accepted 5 February 2003)
An interesting realization of the spin orbit interaction
‘Cross-dressed’ atoms as cool spins
Optical traps & lattices

- Lasers + magnetic fields $\rightarrow$ trap & cool atoms
- Control interaction among atoms & BECs
- Simulate canonical condensed matter models
- Light induced gauge fields (Synthetic E’s & B’s)
Atoms as two-level (spin) systems

1) Hyperfine ground and excited states of alkali atoms, e.g., F=1 & 2 in $^{87}\text{Rb}$;  
2) Dipole trap;  
3) Intersecting Raman lasers detuned from the resonance $5S_{1/2} \rightarrow 5P_{1/2, 3/2}$;  
4) Zeeman fields – linear and quadratic: state selection.

\[ H = \begin{pmatrix} \varepsilon_\uparrow & \Omega_R \\ \Omega_R & \varepsilon_\downarrow \end{pmatrix} \]

\[ H = \varepsilon_\uparrow \langle \uparrow | \uparrow \rangle + \varepsilon_\downarrow \langle \downarrow | \downarrow \rangle + \Omega_R \langle \uparrow | \downarrow \rangle + \Omega_R \langle \downarrow | \uparrow \rangle \]
Optical lattice

Crossed lasers

Carlos Egues, Varenna 2012
Spin-orbit with ultra cold atoms

- Main ingredients
- Physical picture
‘Ultracold’ spin orbit: physics

1) Crossed (perpendicular) Raman lasers (+ dipole trap)
2) Optical lattice (counterpropagating beams)
‘Ultracold’ spin orbit: physics

Crossed Raman lasers impart momentum to the atom while exciting it.
In ‘spinish’
(spin language)

Crossed lasers **impair momentum** to the atom **while flipping its** (pseudo) **spin**!

\[ p_f = p_i + \Delta k \]

:: Momentum and spin are coupled!
Equal Rashba & Dresselhaus couplings (1D)

\[ \hat{H} = \frac{\hbar^2 k^2}{2m} \hat{I} - \left( B + B_{SO}(k) \right) \cdot \mathbf{\mu} = \frac{\hbar^2 k^2}{2m} \hat{I} + \frac{\Lambda}{2} \hat{\sigma}_z + \frac{\delta}{2} \hat{\sigma}_y + 2 \hat{\sigma}_x \hat{\sigma}_y \]
Hamiltonian for the atoms:

$$H = -\frac{\hbar^2\nabla^2}{2m} \mathbf{1}_{2x2} + F(\mathbf{r})$$

Periodic spin-dependent potential:

$$F(\mathbf{r}) = \Omega_R^{(y)} \sin(G_0 y) \sigma_y - \Omega_R^{(x)} \sin(G_0 x) \sigma_x + \left[ \Omega_R^{(x)} \cos(G_0 x) + \Omega_R^{(y)} \cos(G_0 y) \right] \sigma_z$$

k space Schroedinger equation:

$$\left[ \left( k^2 - \varepsilon^{(n)}_k \right) \mathbf{1} + \frac{\delta}{2} \sigma_z - \sum_{G \neq 0} \frac{F(-\mathbf{G})F(\mathbf{G})}{|k + \mathbf{G}|^2 - \varepsilon^{(n)}_k} \right] C^{(n)}_k = 0$$

Lowest bands (k→0):

$$\left[ \left( k^2 - \mu - \varepsilon^{(n)}_k \right) \mathbf{1} + \frac{\delta}{2} \sigma_z + \frac{\Omega_R^2}{2E_F G_0} \left( k_y \sigma_x - k_x \sigma_y \right) \right] C^{(n)}_k = 0$$
Rashba Interaction + Zeeman

\[ H = \frac{p^2}{2m} + \frac{\Delta}{2} \sigma_z + \frac{\alpha}{\hbar} (p_x \sigma_y - p_y \sigma_x) \quad \Leftrightarrow \quad H = \begin{pmatrix} \frac{p^2}{2m} + \frac{\Delta}{2} & -i \frac{\alpha}{\hbar} p_- \\ i \frac{\alpha}{\hbar} p_+ & \frac{p^2}{2m} - \frac{\Delta}{2} \end{pmatrix} \]

Eigenvalues & eigenvectors: \[ H \varphi = \varepsilon \varphi \]

\[ \varepsilon_+(k) = \frac{\hbar^2 k^2}{2m} + \frac{1}{2} \sqrt{\Delta^2 + 4\alpha^2 k^2} \]
\[ \varphi_+(r) = \begin{pmatrix} e^{-i\phi} \sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix} e^{ik \cdot r} \]

\[ \varepsilon_-(k) = \frac{\hbar^2 k^2}{2m} - \frac{1}{2} \sqrt{\Delta^2 + 4\alpha^2 k^2} \]
\[ \varphi_-(r) = \begin{pmatrix} -e^{-i\phi} \cos(\theta/2) \\ \sin(\theta/2) \end{pmatrix} e^{ik \cdot r} \]

with: \[ e^{i\phi} = i \frac{k_+}{k} \quad \text{and} \quad \cos(\theta) = 1/\sqrt{1 + 4\alpha^2 k^2 / \Delta^2} \]
Rashba dispersions: 1D Case

\[ \varepsilon_{\pm}(k) = \frac{\hbar^2 k^2}{2m} \pm \frac{1}{2} \sqrt{\Delta^2 + 4\alpha^2 k^2} \]

\( \Delta = 0 \& \alpha = 0 \) (no Zeeman; no Rashba)

\( \Delta \neq 0 \& \alpha \neq 0 \) (Zeeman+Rashba)

\( k_R = \frac{m\alpha}{\hbar^2} \)

No orbital effects included. Not an approximation!!!

(cold atoms)

No orbital effects included. Not an approximation!!!

(cold atoms)
What can we do with this?

Relativistic QM on a chip: Zitterbewegung
Zitterbewegung: basic idea

“Orthodox” zitterbewegung: Free Dirac equation (beating: E>0 & E<0 eigenstates)

Semiconductor zitterbewegung: Rashba Hamiltonian (Kane model + V(z) + “FW”) (Schliemann et al. PRL/2005, Bernardes et. al. PRL/2007)

Rashba spin-orbit: total angular momentum $J_z = L_z + S_z$ is conserved.

$$[J_z, H_R] = 0 \Rightarrow [L_z, H_R] = -[S_z, H_R] \Rightarrow \frac{dS_z}{dt} = -\frac{dL_z}{dt}$$

($S_z$ and $L_z$: constrained dynamics)

$S_z$ precesses $\Rightarrow L_z = x p_y - y p_x$ oscillates $\Rightarrow$ jittery motion of x & y

Spin-orbit coupling $\Rightarrow$ zitterbewegung
Summary – Lec. 1

• Early motivation: spin FET & coherent control

• Spin orbit: atomic physics $\rightarrow$ semiconductors

• Intersubband s-o coupling: wells with 2 subbands
  (details in lecture 3)
  ➢ generalized Rashba model: two Dirac cones
    (Bernardes et al. PRL/2007; Calsaverini et al. PRB/2008)
  ➢ anticrossings due to intersubband coupling
    (recent experiment – Bentmann, PRL/2012)
  ➢ unusual spin textures

• Spin orbit in optical lattices: physical picture
  ➢ ‘cross-dressed’ atoms as cool spins