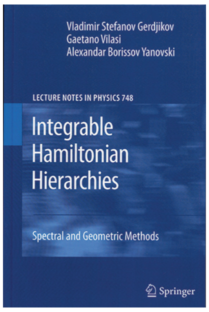
RECENSIONI



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Integrable Hamiltonian Hierarchies
Spectral and Geometric Methods

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The book under review provides a wide overview on the so-called soliton equations, a topic that has been developing intensively in the past decades and became an important subject influencing other fields in Mathematics, such as Analysis, Algebra and Geometry. It covers various approaches existing in the literature and explains the relations among them. The basic concept of the book is the Recursion Operator, a subject in which the editors and authors have made some pioneer contributions. The book is very interesting and quite unique in its main idea – to discuss the soliton equations from completely different points of view and using different tools and methods. The two main groups of methods are those from the Spectral Theory of Operators, belonging to the Analysis and those from the theory of the Hamiltonian Systems, belonging to the Differential Geometry. Using these different points of view simultaneously provides the reader with much deeper understanding of the theory of soliton equations comparing with the situation of using just one method. Because of this approach the book is naturally divided into two parts: Part I, Spectral Theory of the Recursion Operators and Part II, Geometry of the Recursion Operators. As we mentioned, the combination of these two parts provides the reader with a deeper understanding of the subject. However, the book is organized in such a way that one can also read these two parts independently, which, in case one is familiar with one of the approaches and wants to learn about the other, is a definite advantage.

Part I deals with the Inverse Scattering Method (ISM) for the evolution equations related to the Zakharov-Shabat system (ZS system) . Chapter 1 provides a historical overview. It also gives an idea about the inverse scattering method and information about the contents of the first part. In chapters 2 to 4, some important issues related to the soliton equations are discussed, things that are needed for the development of the theory: the Lax representation, the direct and inverse scattering method for the ZS system, etc., as well as and some techniques for finding exact solutions to the nonlinear evolution equations (NLEEs) associated with the ZS system. In chapter 5 the idea of generalized Fourier transform is introduced which is central to the classical Ablowitz-Kaup-Newell-Segur (AKNS) approach. In it the passage from a given nonlinear evolution equation related to the ZS system to the linear evolution equation of the scattering data is interpreted as a generalized Fourier transform defined by expansion over some functions constructed from the fundamental solutions of the ZS system. The Recursion Operator which has been introduced earlier, appears here as the operator for which those functions are eigenfunctions. It plays an important role for the study of the hierarchy of the NLEEs. By means of the Recursion Operator one finds the hierarchies of NLEEs related to the ZS system, their hierarchies of conservation laws and Hamiltonian structures. Chapters 6, 7 are devoted to the description of these ideas. In chapter 8 the concept of gauge equivalent NLEEs is introduced and how the AKNS approach can be developed in gauge covariant way is discussed. The last chapter discusses r-matrix related to the ZS system together with the gauge covariant property.

Part II in a very clear manner discusses the Recursion Operators from symplectic geometry and Poisson geometry viewpoint. Chapter 10 is an introduction, in it a short review about the integrable NLEEs needed for the subsequent geometric picture is given so that the reader gets the essential concepts related to them. Chapter 11 introduces the basic geometric concepts and tools from the theory of the differentiable manifolds. Chapter 12 introduces Hamiltonian dynamics and the geometric structures through which it is defined – symplectic forms and Poisson bivector fields. Their restrictions to submanifolds are discussed as well as some related topics as for example the Dirac brackets. Furthermore, section 12.2 introduces the concept of complex Hamiltonian systems, an issue that is becoming an important area in Geometry, and discusses the relationship between real and complex dynamics, with a concrete and meaningful example, Toda chain model. In section 12.4, the reader is led to the idea of the multi-Hamiltonian formulation of integrable systems and then naturally to the notion of the Nijenhuis tensor arising from the hierarchy of symplectic forms. The idea of a Nijenhuis tensor and multi-Hamiltonian dynamics is nice and very important for the theory of the integrable Hamiltonian systems. Chapter 13 explains the basic properties of the Nijenhuis tensors. One sees that these properties are behind the existence of the hierarchies of symmetries for soliton equations. In chapter 14, the concepts of integrability and the Nijenhuis tensors are linked, and the criterion of complete integrability is proposed in terms of Nijenhuis tensors. The concept of compatible Poisson structure and that of Poisson-Nijenhuis manifold (P-N manifold) are given. This leads immediately to a hierarchy of Poisson structures on a P-N manifold. The idea of the P-N manifold is crucial for the treatment of the integrable Hamiltonian systems, they should be understood as fundamental fields of the P-N structure. In chapter 15, the Recursion Operator of the generalized Zakharov-Shabat system in the pole gauge is identified with the conjugate of the Nijenhuis tensor of some P-N manifold of potentials. The geometric picture is given for the following P-N manifolds: i) the manifold of potentials in canonical gauge, ii) the manifold of the Jost solutions in canonical gauge and iii) the manifold of potentials in pole gauge. Chapter 16 explains the way the P-N manifold structure arises from compatible Poisson tensors and discusses two important examples of Nijenhuis tensors that arise using such construction, namely, the Nijenhuis tensor related to the hierarchy of the $O(3)$ chiral system and the hierarchy of the Landau-Lifshitz equation.

In conclusion, the book contains many topics that experts will find interesting, it is well organized and is an excellent research monograph on soliton equations. At the same time it is self-contained in the sense that it introduces the basic concepts that are used and does not require a high preliminary level of knowledge which makes it beginner-friendly. Because of this, it is accessible to a wide range of readers, from the experts in the field, to researchers who are interested in nonlinear evolution equations and PhD students who would like to learn more about this field.

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