Galilei's "Discorsi e Dimostrazioni Matematiche" for Modern Readers

UNIVERSITÀ DEGLI STUDI DI PADOVA

French edition: cover not yet available

Italian edition: Codice (Apr 21)

English edition: Springer (Oct 21)

French edition: EDP (Jan 22)

Portuguese edition: ?? 22

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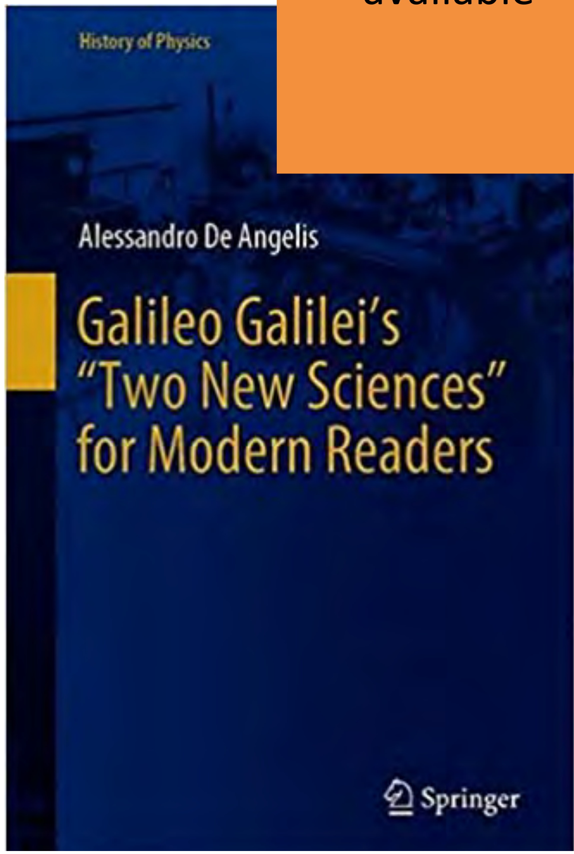
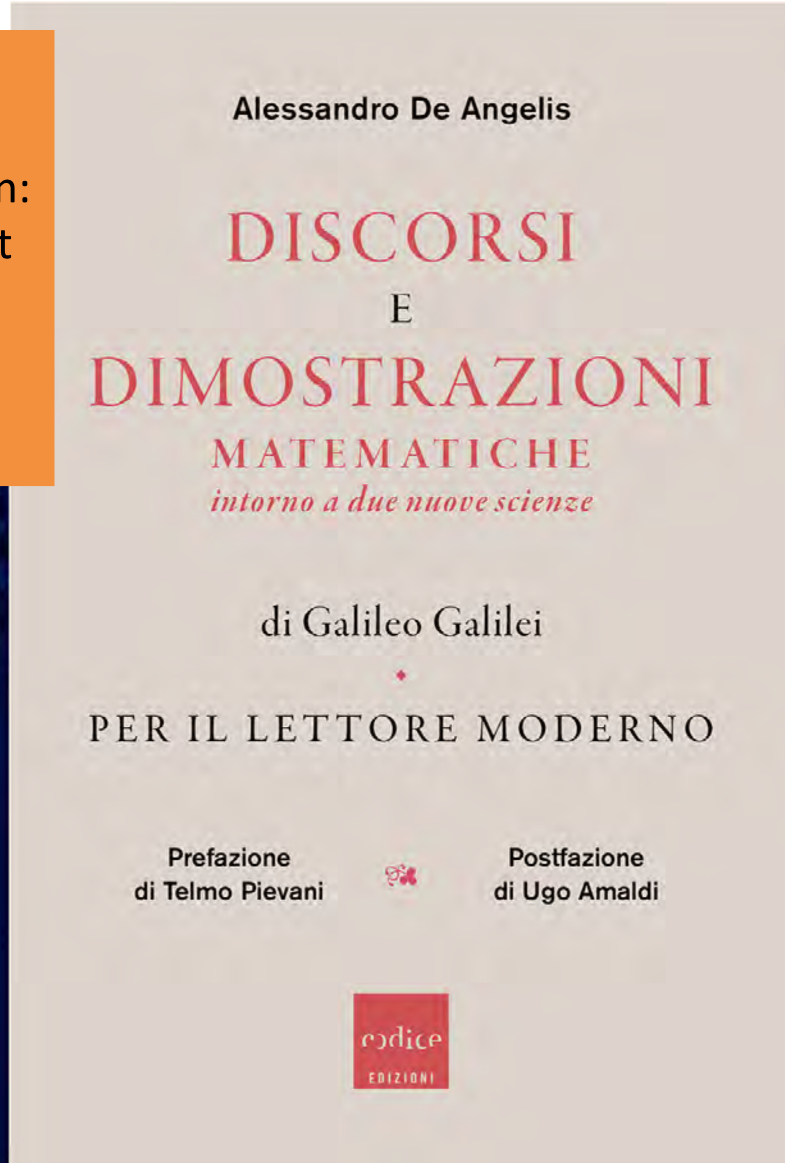




Dipartimento di Fisica e Astronomia Galileo Galilei   INFN   LIP   INAF

Galilei's last book: *Discorsi e Dimostrazioni*

- Galileo Galilei (1564-1642) is the father of modern science (as recognized also by Einstein), and a paradigm of freedom in science
- According to (the generous) Newton, he first proposed the first and of the second law of mechanics
- He established previously unknown mathematical truths, discovered that gravitational acceleration is independent of mass, revolutionized kinematics and the science of constructions

- But don't expect to see in Galilei things such as $x=vt$, $F=ma$, $a=g$ or $y=v_{oy}t - gt^2/2$: Galilei (like the early Newton) just did not use formulas
- He used instead classical (Euclidean) geometry, and long discussions: this is why he is hard to understand for non-professional, non-classically trained readers
- This is why I wrote a "translation" of his mathematical demonstrations

Galilei, the scientist (in short)

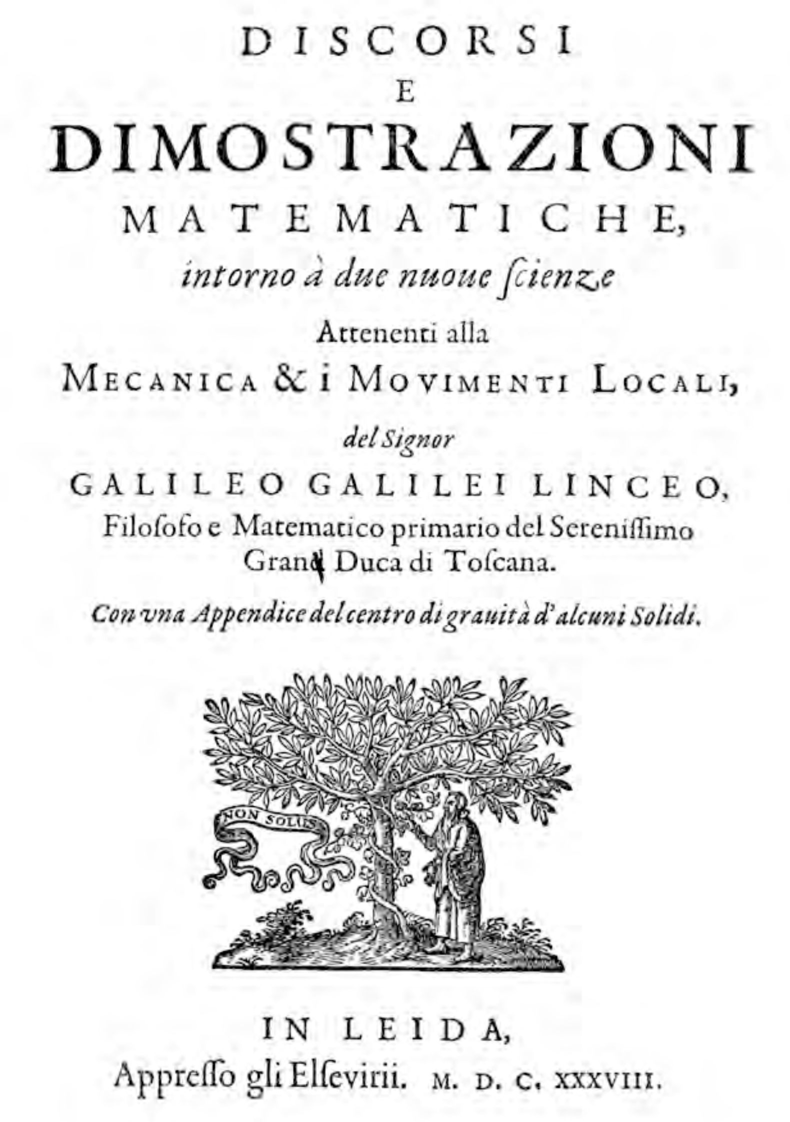
- Studied Mathematics in Pisa (without graduating), where he started lecturing
- In 1592, after an unsuccessful competition in Bologna, was selected for the chair in Padova, at the age of 28
  - Fundamental experiments in mechanics
  - Definition of new mathematical and astronomical concepts (important discussions with Paolo Sarpi and Johannes Kepler)
- In 1608, started collecting his mathematical demontrations on mechanics and motion
- In 1609, his interests are absorbed by astronomy: he perfects the telescope, and writes his first book: *Sidereus Nuncius* (in Latin). He explains nebulae, the structure of the Moon’s surface, discovers Jupiter’s satellites, …
- In 1610, gets a better proposal from Firenze and leaves Padova; this ends “the best 18 years” of his life

Galilei, the scientist, in short (cont'd)

- A personal triumph follows, celebrated in Roma and Firenze; but the Roman Church starts critically analyzing his positions, in particular about the Copernican question (he exchanges letters about it, in particular re-examining the Augustinian opinion of the separation between Science and Faith)
- In 1623 he publishes his second book, *Il Saggiatore* (in Italian)
- In 1632 he publishes his third book, *Dialogo sopra i due massimi sistemi...* (in Italian). His cosmological *summa*, it's a dialog between Sagredo, Salviati and Simplicio, each with a role
- In 1633, is judged "vehemently suspect of heresy" by the Inquisition, sentenced to indefinite imprisonment; his cosmology is banned
- At home arrest near Firenze, he concludes his *summa* about mathematics and mechanics: *Discorsi e dimostrazioni matematiche…* (in a mix of Italian and Latin), published in 1638. His fourth and last book.
- Dies in 1642 in Arcetri

The *"Discourses and Mathematical Demonstrations…"*

- Published in 1638 in Leiden by Elzevir (!). Not easy to find an editor… Galilei is 74 years old, and will die in 4 years
- Should include 5 days of discussion, but Galilei concludes only 4, and keeps the 5th day for later
- The plot: 3 friends meet for 4 (5) days and discuss a Latin treatise by a Paduan Academician (Galilei himself), the friend of one of them
    - A stratagem: probably to recycle his old Paduan notes, certainly to separate firm demonstrations (in Latin) and motivated speculations (in Italian)


DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue ſcienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filoſofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.
Con una Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.

The *Discourses and Mathematical Demonstrations* (cont'd)

- Same 3 characters as in the *Dialog*
  - Two of them are real characters, friends of Galilei: the Florentine Salviati, a Linceo like Galilei, and the Venetian Sagredo, in Padua a pupil of Galilei. The third is Simplicio, a fictional character whose name (that implies a certain scientific simplicity) is that of an ancient commentator (VI century a.D.) of Aristotle. Often Simplicio plays the role of an Aristotelian professor, not too critical. But in this book, sometimes Simplicio's arguments represent the opinions of the young Galilei; Sagredo represents his middle age, and Salviati is the author in his mature age
- Days 1 and 2: the science of materials. What keeps matter together and gives strengths to solids? But also questions on infinity, the speed of light, acoustics and the pendulum…
- Days 3 and 4: kinematics, gravitation, friction and viscosity. Inertia.
- Last day: what changes the state of rest or uniform motion: the force of percussion (will appear only in the 3rd edition, Firenze 1718)
  - Simplicio is replaced by Paolo Aproino, a pupil from Treviso in the Paduan period, who collaborated with Galileo and Daniele Antonini from Udine to most experiments

Critical reception of the "Two New Sciences"

- Difficult to read, when compared to the Dialogue
- Mersenne attempts a translation into "modern" mathematics
- Descartes says that many demonstrations are far-fetched
- Newton says it's the basis of mechanics
- Hawking places it among the 5 most important works in the history of Physics and Astronomy (together with Copernicus', Kepler's, Newton's, Einstein's)
- Rényi says it's the most important work in Mathematics in 2000 years

- Difficult to read both due to language and mathematical structure; after Mach(Germany 1883) and Favaro (Italy 1898), re-discovered in the XX century thanks mostly to Vilasante & Isnardi (Argentina 1945), Geymonat (Italy 1958), Clavelin (France 1970), Drake (US/Can 1974)

Why is this a difficult book?

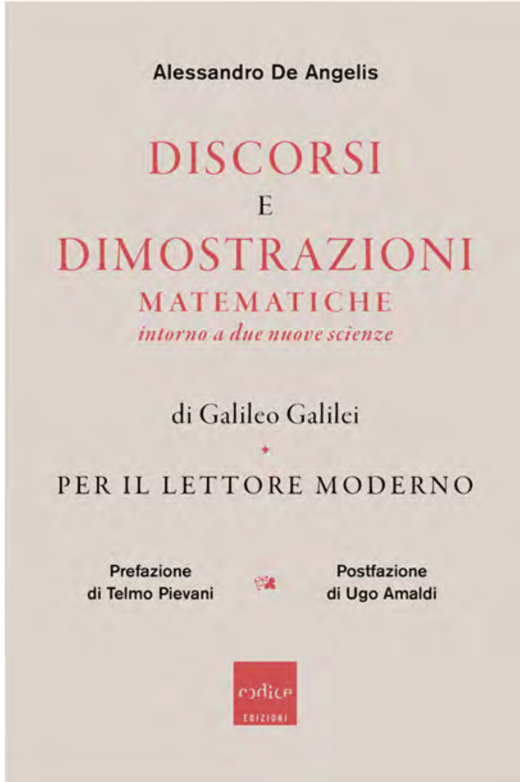
- Language: a mixture of complex Italian ad of Latin
- The verb “demonstrate” is ambiguous: sometimes it is used in its mathematical form, sometimes in the form that Cicero attributed to the term (“to show”)
- Full of citations, mostly from Greeks, sometimes from “that guy…”; sometimes not in the form accepted today

What is the added value from my study?

- I translated classical geometry into mathematics
  - Something similar to what done by Chandrasekhar with Newton's Principia, but I decided to stay on the mathematics known at Galilei's times

- I simplified the language
  - Latin made uniform to Italian. Notice that Galilei's Latin is structurally simpler than his Italian (SM)
  - Collaboration with the Liceo "Duca d'Aosta" di Padova

- The original plots by Galilei (and, in the latest times, Viviani), have been digitally restored in collaboration with a team from the Italian National Library in Firenze and with Codice Edizioni
  - The excellent work by Favaro in ~1900 had some limitations due to the techniques available at his times



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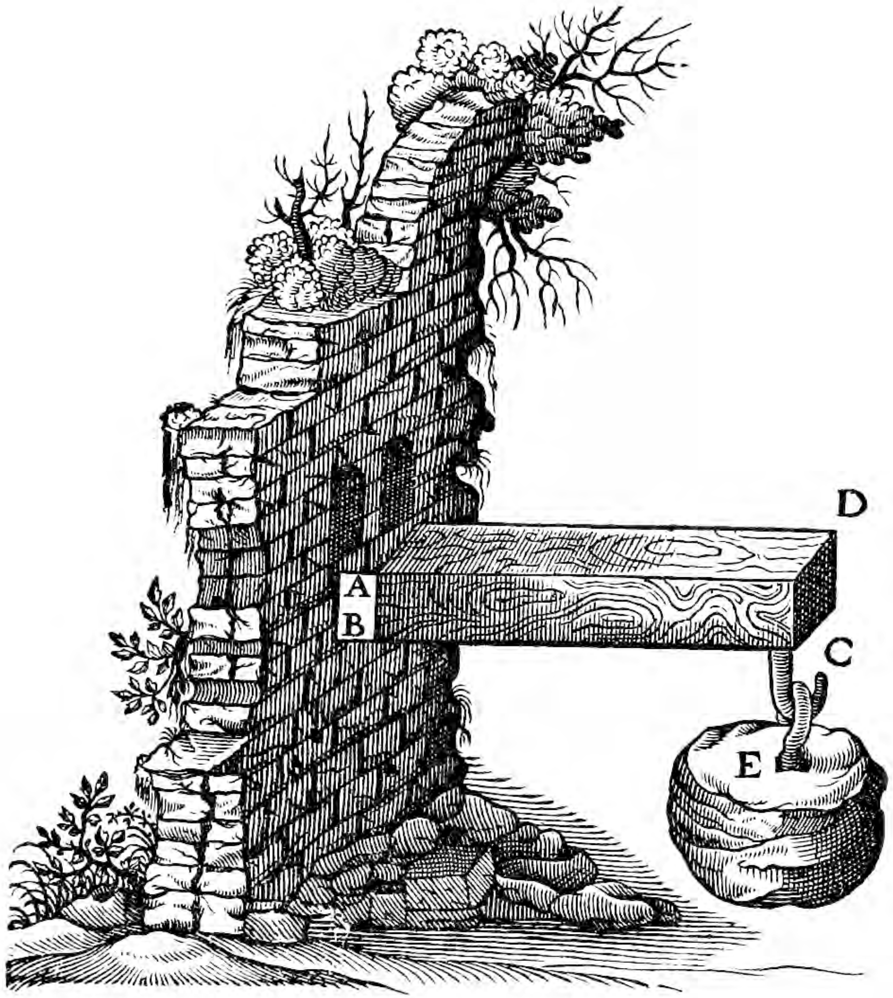
More on the Paduan (Venetian) context

Galileo in Padua: a strong, young person

- Venice at the top of its power
  - Mail system, editing, hydrogeology
- In Padua, 1500 students mostly foreigners, from 23 nations

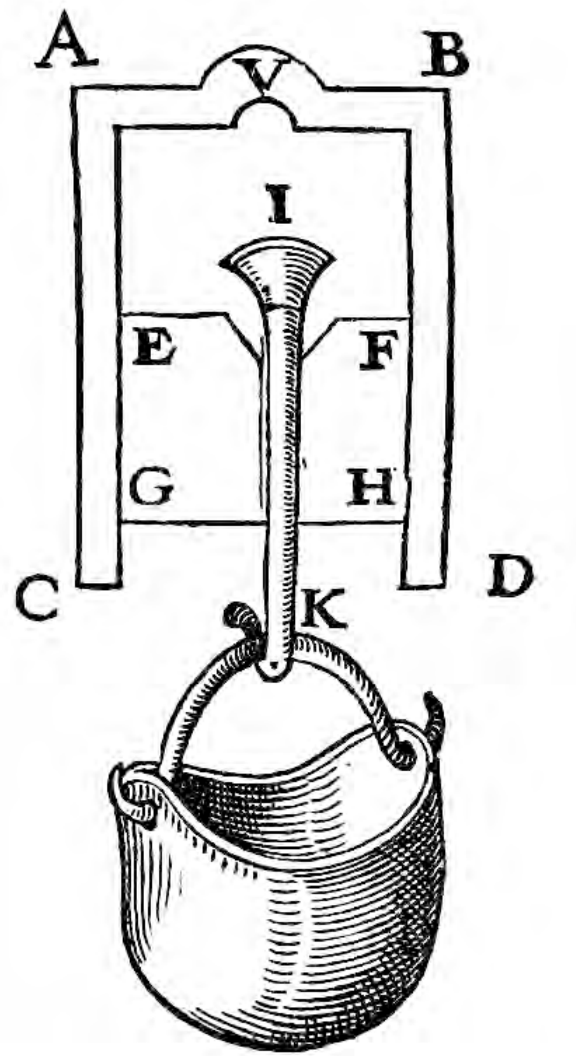
- Friendship with Sagredo
- Three children with Marina Gamba
- Professional friendship with Campanella, Sarpi, Keplero
- A few “riots”, accuses of plagiarism, astrology

Some examples of beauty and marvel (could find many more)



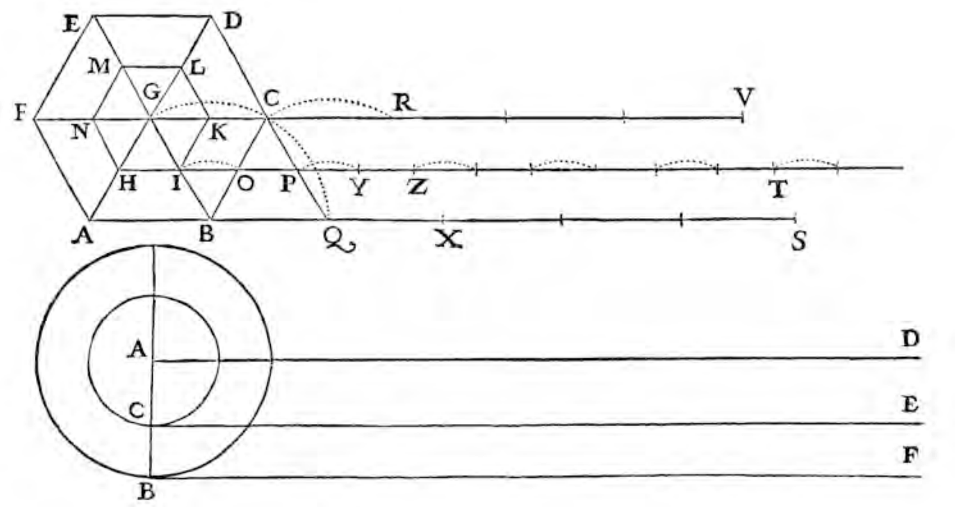
Day 1: Can the fear of the vacuum explain the cohesion of solids?

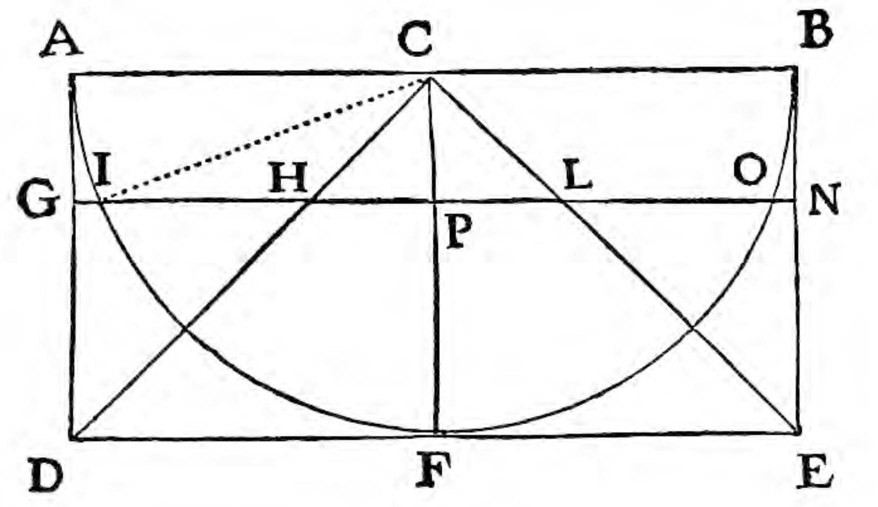
- What keeps solids together? Is it the fear of the vacuum?
- Galilei thinks of a machine to measure the strength of a column of water. This can be due only to the fear of the vacuum. The maximum height is 18 *braccia* (10.3 m), but a column of metal can be higher. Thus the strength of what is on top of the 10.3 m must be caused by a new strength
    - Newton will call this interaction "strong"
    - A possible new definition of the *braccio* (see the *bar dei osei* in Piazza dei Frutti)



Day 1: The (different) infinites and the one

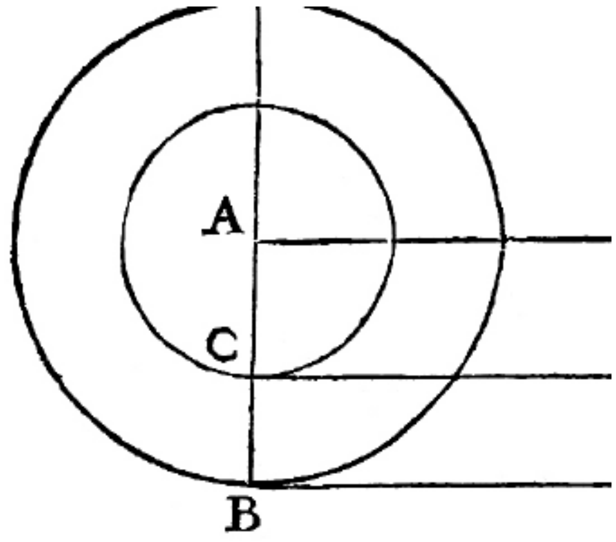
- A polygon of a thousand sides, having completed a rotation, draws a line equal to its perimeter; at the same time, a minor polygon similar to the larger one describes a line approximately equal, but composed of small portions equal to its thousand sides with the interposition of a thousand spaces that, compared to the lines that correspond to the sides of the polygon, we can call vacua
- Concentric circles will draw the same lines
- The figure on the right: the solids coming from the rotation of AGI and CHP have the same value, but one ends in a point, the other ends in a circumference



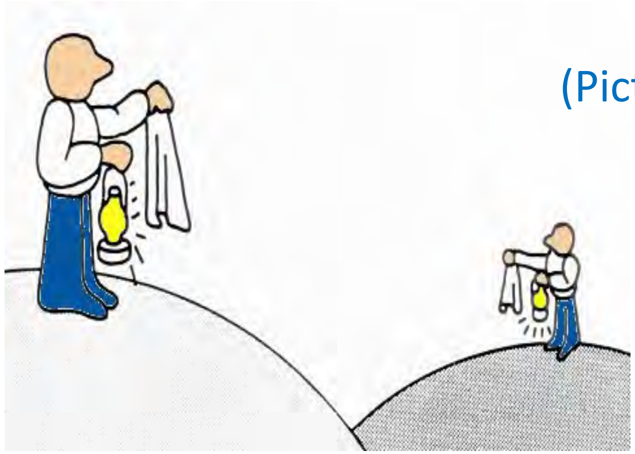


The (different) infinites and the one (cont'd)

- Conclusions: one cannot compare infinites
- Same for integer numbers: the squares are in the same number as the numbers, but they are a subset
- But numbers are something in between the finite and the infinite of a line
- Since numbers are equivalent to their squares, their cubes, etc., but the number of squares, cubes, etc., becomes smaller and smaller in proportion as we approach infinity, the only number that has the property of infinity is the unit, which coincides with its square, its cube, etc.



Day 1: Actual infinites in nature? Measuring the speed of light

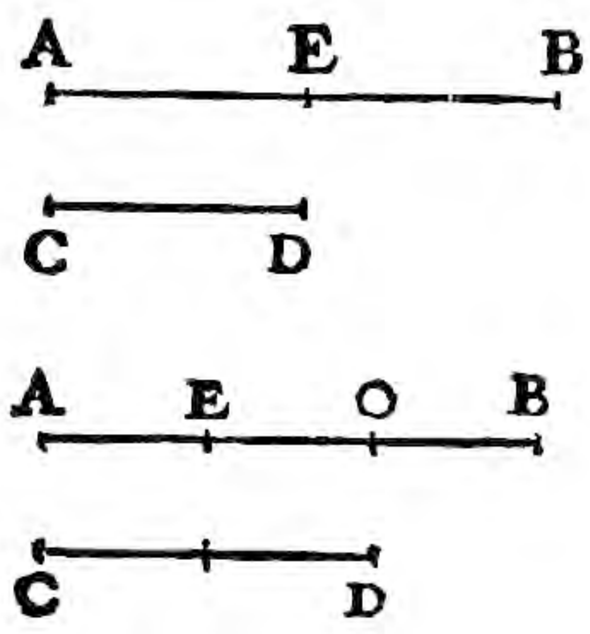


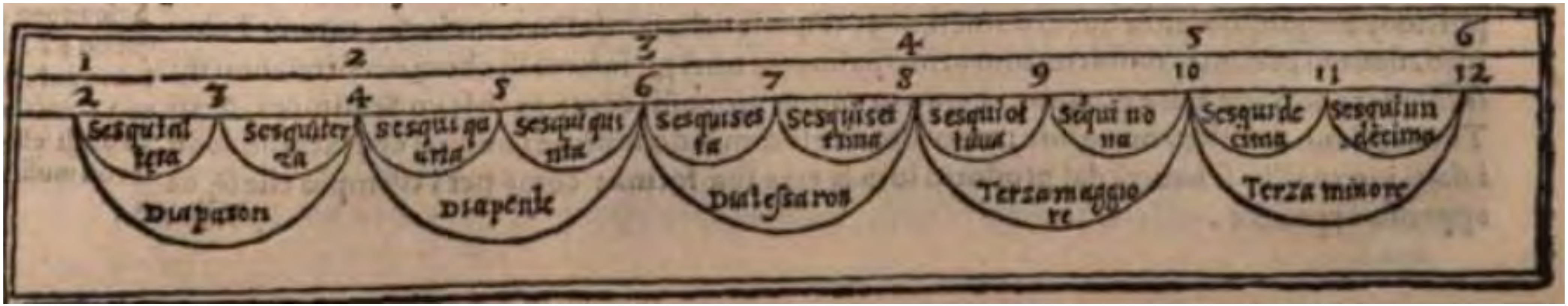
(Picture not from Galilei!)

- Each of two persons take a light contained in a lantern, and can shut o or admit the light to the vision of the other
- They stand opposite each other at a distance of a few cubits and practice until they acquire such skill in uncovering and occulting their lights that the moment one sees the light of his companion he will uncover his own. Measure of 0
- Take up positions separated by a distance of 2 or 3 miles and perform the same experiment at night; do the exposures and occultations occur in the same time as at short distances? If they do, the propagation of light is compatible with being instantaneous; if not, one can measure the speed of light
- If the experiment is to be made at still greater distances, say eight or ten miles, telescopes may be employed
- Unfortunately, the test was done only from a small distance, less than a mile, and could not discern if the appearance of the opposite light was really immediate or not. *Anyhow it is at least very fast, and I would compare it to the movement of lightning which we see in the clouds far tens of miles. This seems to me an argument to conclude that propagation takes at least some (short) time*

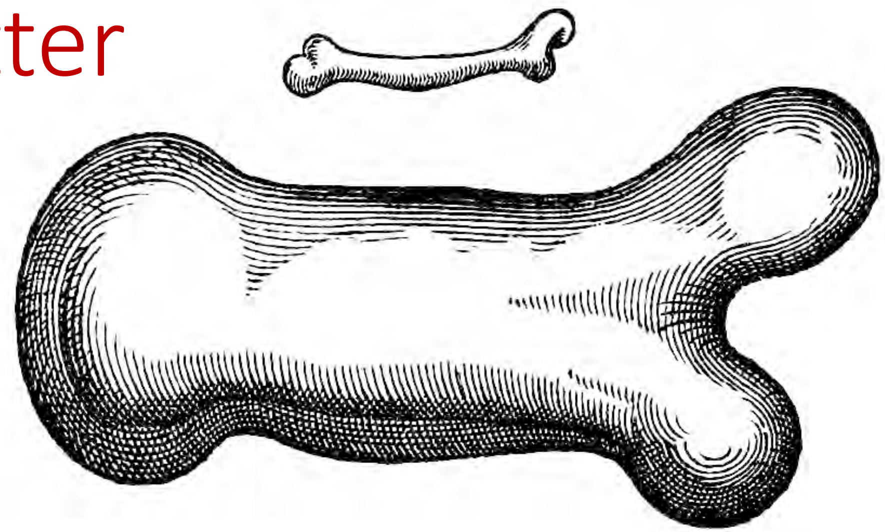
Day 1: oscillations & harmony

- A Galileo piace «ogni tipo di strumento musicale» e presta molta attenzione all'armonia
- Nota emessa da una corda vibrante e tensione, lunghezza, spessore
- Le onde viaggiano nell'aria portando al timpano uno stimolo che la mente traduce in suono
- Esperimenti per rendere visibile il suono
- Le consonanze e le dissonanze dipendono dall'eccitazione dei timpani: le consonanze gradevoli sono coppie di note che colpiscono l'orecchio con una certa regolarità: gli impulsi emessi dai due toni, nello stesso intervallo di tempo, devono essere commensurabili.





Day 1 and 2: Smaller is better



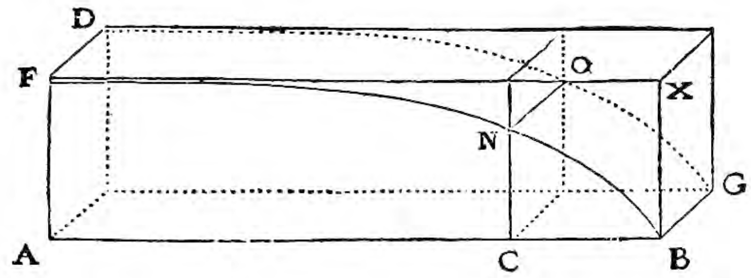
- In the Arsenal of Venice one can learn many things… In particular, smaller structures are proportionally stronger than larger ones
- Also for animals: a horse falling from 3 or 4 cubits will break its bones while a dog that falls from the same height or a cat from a height of eight or ten cubits will not. The fall of a grasshopper from a tower or the fall of an ant from the distance of the Moon would be equally harmless
- Also for plants
- Whales can support themselves since they live in water
- Measurement of the weight of air

$$S \propto L^2 \; ; \; V \propto L^3$$

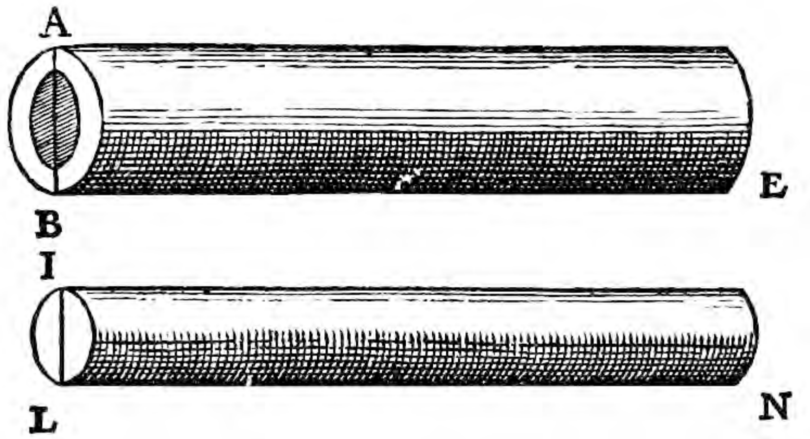
$$\frac{V}{S} \propto L \, .$$

Day 2: Optimal profile of beams; bird’s bones

- What is the optimal profile of a beam”? It is parabolic
- Integration of the parabola: a calculation different from Archimedes’

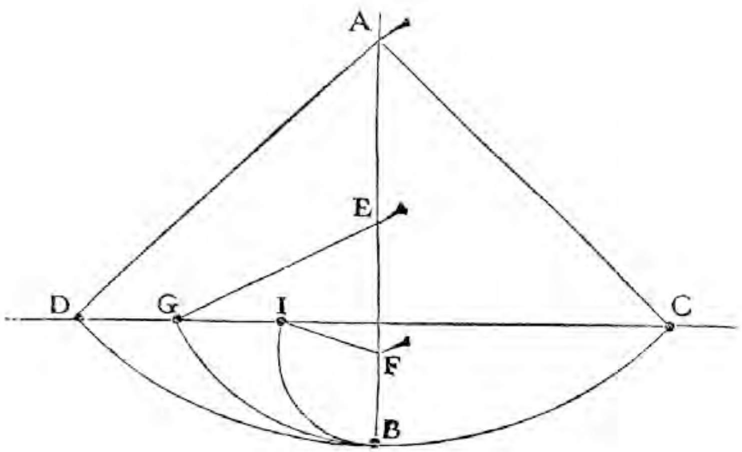


- To build stronger beams with the same weight, make them hollow!
    - Nature is magister: birds’ bones



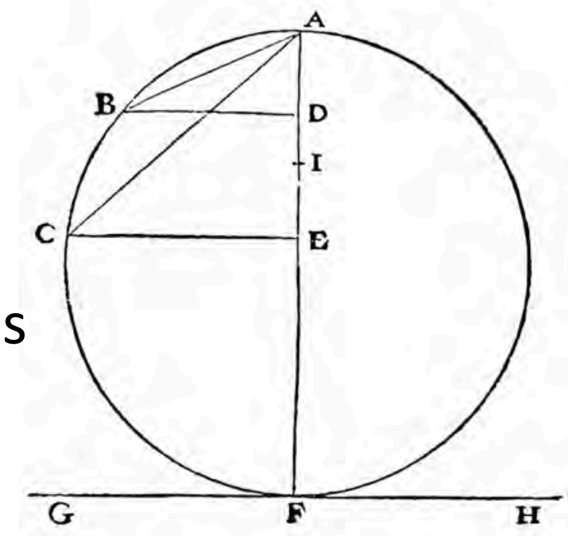
Day 3: Il pendolo interrotto (conservazione dell'energia nel moto dei gravi)

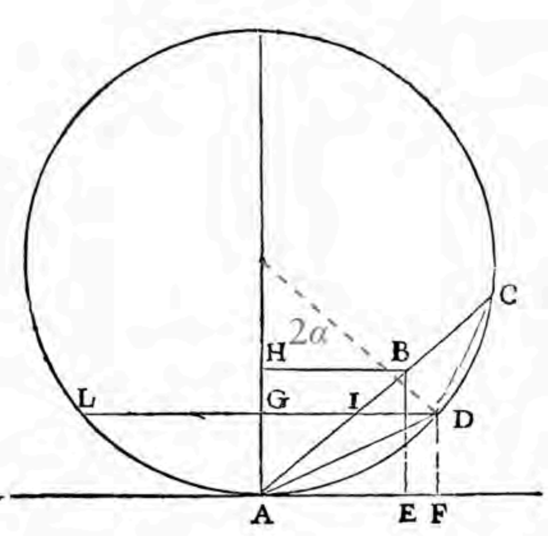
- Cadendo da una certa quota, indipendentemente dal percorso, si ottiene la spinta necessaria per risalire alla stessa quota

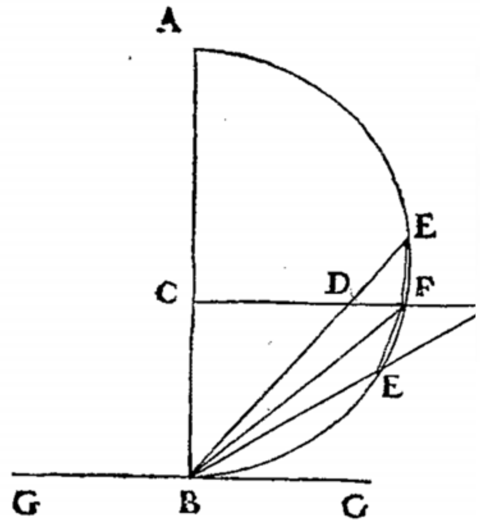


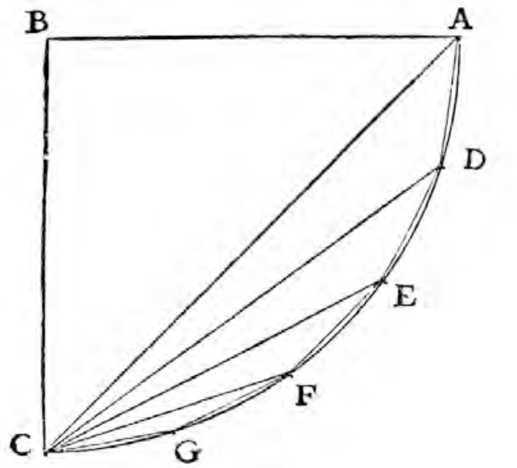
Day 3: Motion along the chords of a circumference

- Motion along chords is isochronous (very elegant demonstration)
  - Corollary: pendulum oscillations are isochronous (well, more or less…)
- Motion along two subsequent chords is faster than along a single chord (incredible, but true!)
  - Corollary: the arc of a vertical circumference is the fastest path between two points, i.e., the brachistochrone (false)









Day 3: The law of free fall

- $v=gt \Rightarrow s[0,1] = 1 \times g/2$; $s[1,2] = 1 \times 3g/2$; $s[2,3] = 1 \times 5g/2$
- If we consider time intervals of equal length from the beginning of the movement, the spaces traveled in successive intervals will stay between them in the same relationship as the succession of odd numbers: 1, 3, 5, ...

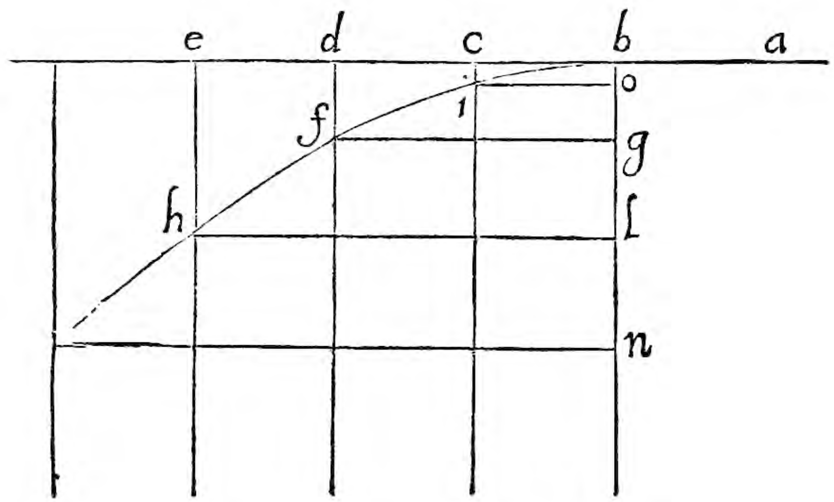
- Expressing the odd numbers starting from the unit as $(2j - 1)$, with $j = 1, 2, \ldots$ (the sequence of natural numbers), we have

$$\begin{aligned} 1 &= 1^2 \\ 1+3 &= 2^2 \\ 1+3+5 &= 3^2 \\ &\cdots \\ 1+3+\ldots+(2n-1) &= n^2 . \end{aligned}$$

$$\Rightarrow s = \tfrac{1}{2} g t^2$$

Day 4: A movement composed of a uniform horizontal rectilinear motion and a vertical fall is parabolic

- A lot of discussion on the problem: can one treat the horizonal and vertical motion independently?
  - Oresme 1377



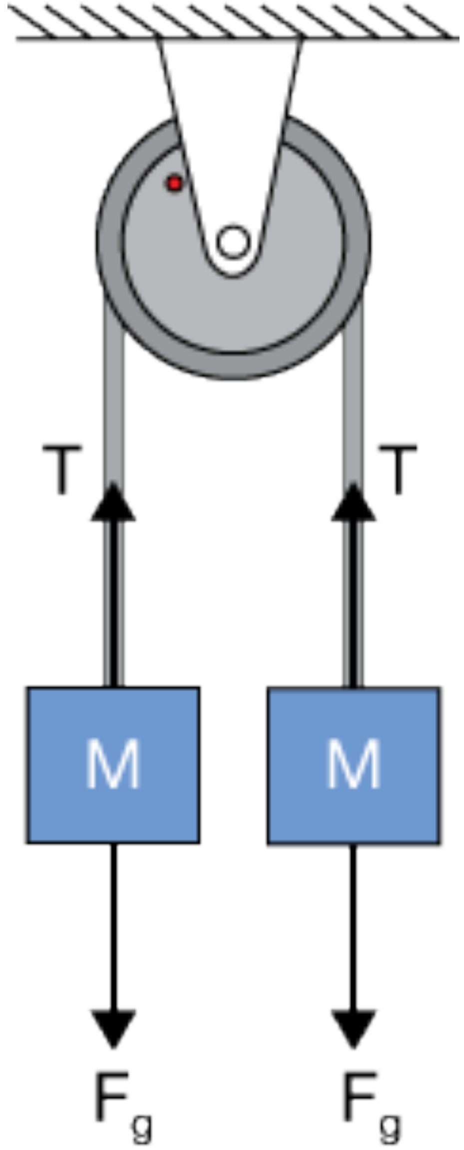
- Descartes will criticize a lot Galilei’s demonstration, but will say the resul is correct

Day 4: A conjecture on the speed of planets (the wrong virial theorem, or a trivial version of it)

- Plato thought that God, after having created the heavenly bodies, made them start from rest and move over definite distances under a natural and rectilinear acceleration such as governs the motion of terrestrial bodies. He added that once these bodies had gained their proper and permanent speed, their rectilinear motion was converted into a circular one, the only motion capable of maintaining uniformity, a motion in which the body revolves without either receding from or approaching the center.
- Since astronomical science provides us such complete information concerning the size of the planetary orbits, the distances of these bodies from their centers of revolution, and their velocities, our Author had some curiosity to discover whether or not a definite height might be assigned to each planet, such that, if it were to start from rest at this particular height and to fall with naturally accelerated motion along a straight line, and were later to change the speed thus acquired into uniform motion, the size of its orbit and its period of revolution would be those actually observed.
- He once made the computation and found a satisfactory correspondence with observation. But he did not wish to speak of it, because of the hate that his many new discoveries had already brought upon him, this might be adding fuel to the fire.

Additional day: "Atwood" machine and the principle of inertia

- (Follows day 3, where the principle of inertia is discussed for inclined planes)
- Two equal weights, hanging at the ends of the rope, will be at rest when balanced, and if one is given a downward speed, it will always maintain it
  - External and accidental impediments must be removed, such as the roughness and heaviness of the rope and of the pulleys, the friction in the rotation of these around their axis, and any other resistance
- Complementary to the addition of speeds (Day 4)
- Clarifies that a famous argument by Koyré, that Galileo's inertia is circular, is just wrong (maybe he did not read carefully)



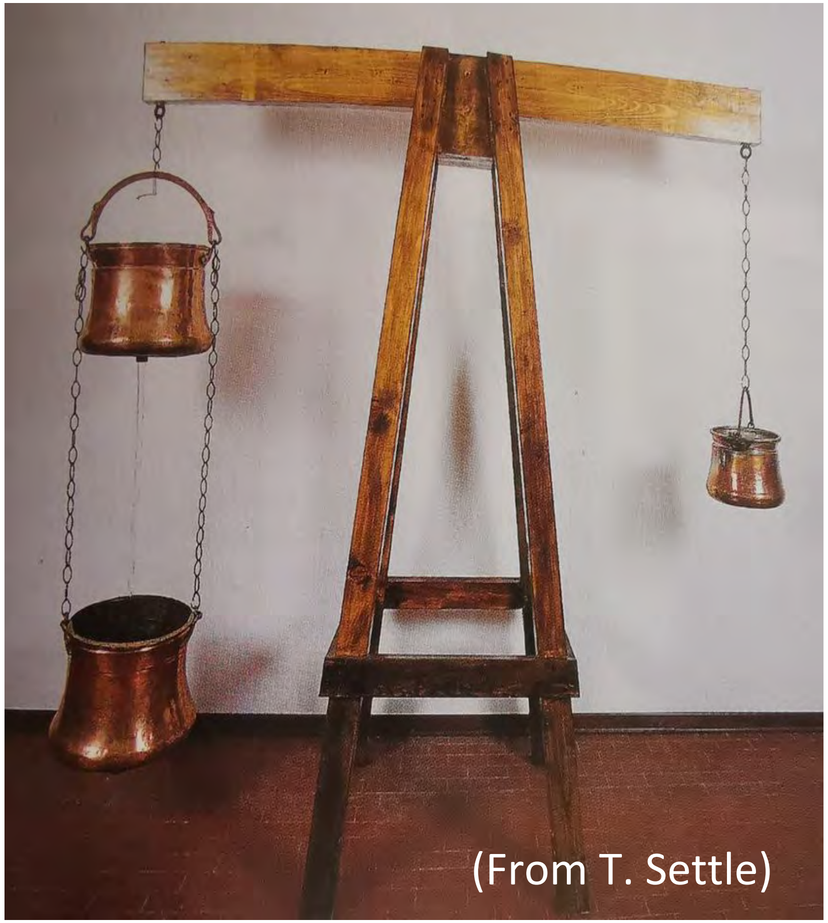
(Not Galilei's picture!)

Additional day (this one I leave for you):
Piling of wooden poles for foundations (or for *bricole*)

- Big weights used to push strong poles into the ground to build, e.g., a foundation, using the percussion caused by the fall from a certain height on such poles
  - Let the weight of a pile driver be of 100 pounds, and let the height from which it falls be 4 cubits; and assume that the pole, when pushed from a single blow, will sink four inches into a hard soil
- Now suppose we want to achieve the same pressure and thus the same four-inch collapse without using the impact, and we find that this can be done with a weight of 1000 pounds, which we can call “dead weight”
- Does the force of this speed mean as much as the pressure of 900 pounds of dead weight?
- Then we bring the same pile driver to the same height, so that it falls a second time on the same pole, but now the pole sinks by only two inches.
- Can we assume that the dead weight of 1000 pounds would cause the same effect?

Additional day (this is also for you): A question water falling from a bucket hanging on a balance arm onto another one suspended to the same arm

- With the scale in equilibrium, the hole in the upper bucket was opened so that water could slide, and quickly descend to hit the lower bucket.
- *Our first conjecture was that this impact would have added some momentum on that side, so that to restore equilibrium more weight had to be added to the other arm. This addition would evidently have compensated for the strength of the impact of water, so that the force of impact of the water would have been equivalent to the weight of the ten or twelve pounds that it would have been necessary, we imagined, to add to the counterweight*
- What happened in reality?



(From T. Settle)