Energy driven fault growth in a layered medium

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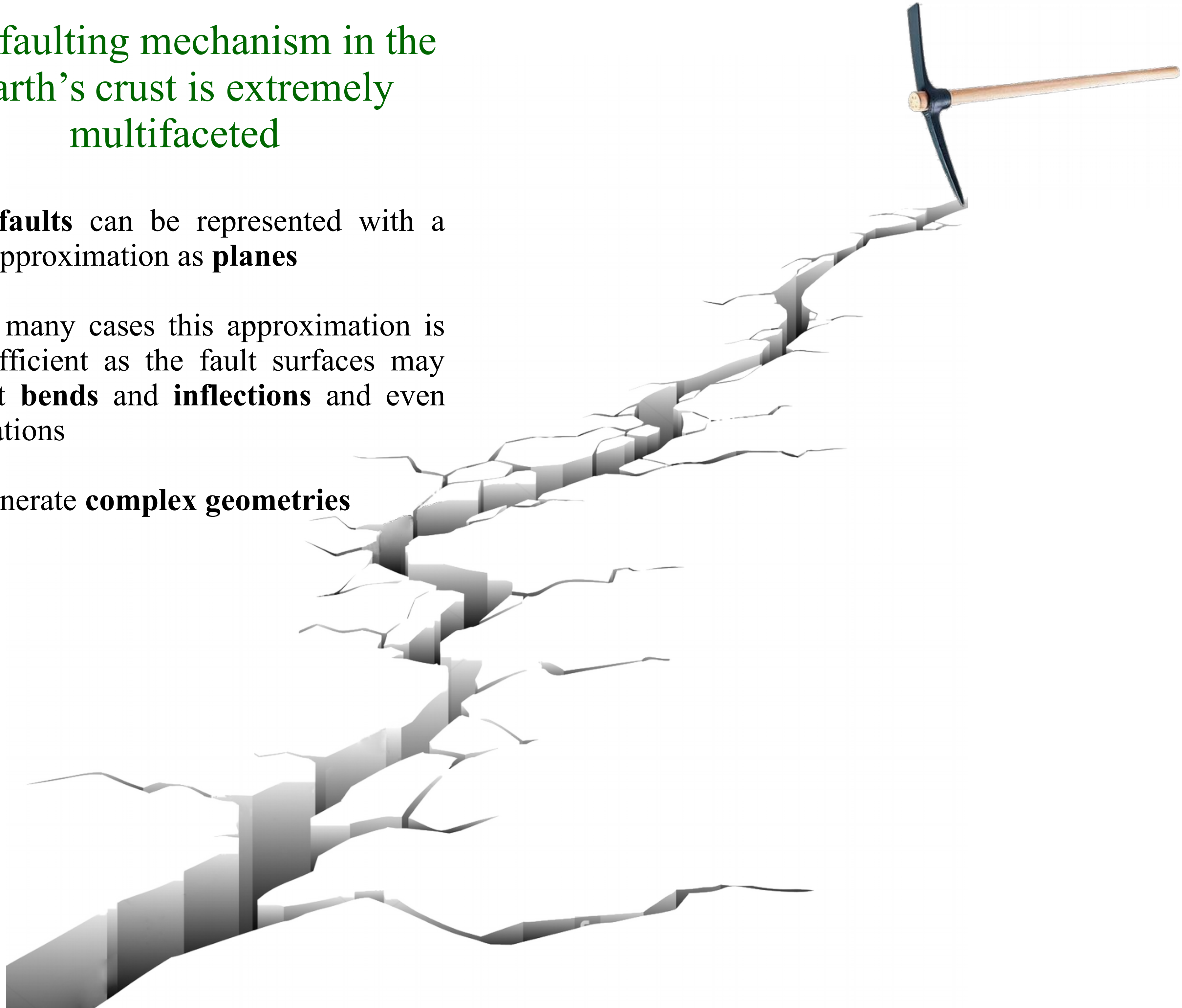
106° CONGRESSO NAZIONALE
SOCIETÀ ITALIANA DI FISICA
14-18 Settembre 2020

The faulting mechanism in the Earth's crust is extremely multifaceted

some **faults** can be represented with a good approximation as **planes**

but in many cases this approximation is not sufficient as the fault surfaces may present **bends** and **inflections** and even bifurcations

that generate **complex geometries**



The Anderson's theory

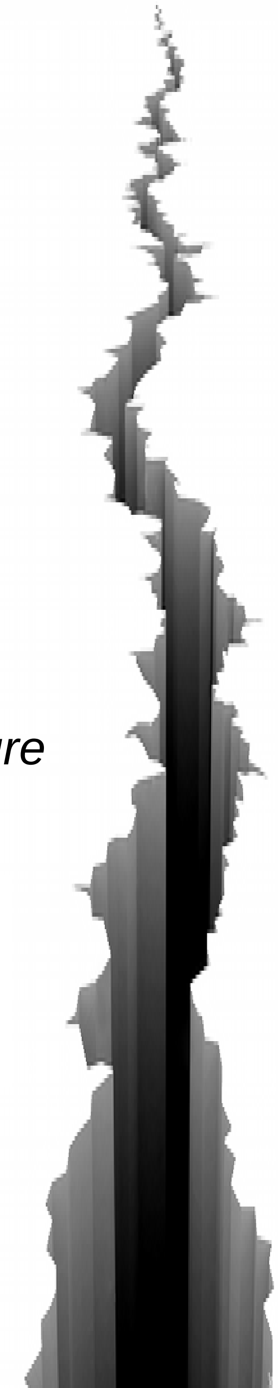
assumes that

- **normal** and **reverse faults** (extensive and compressive)
- **optimally oriented fault planes** where the modified **Coulomb's fracture criterion** is first fulfilled

Static friction coefficient *Pore pressure*

$$|\tau| = \begin{cases} -f_s\left(\sigma_n + p\right), & \text{if } \sigma_n < 0 \\ 0, & \text{if } \sigma_n \geq 0 \end{cases}$$

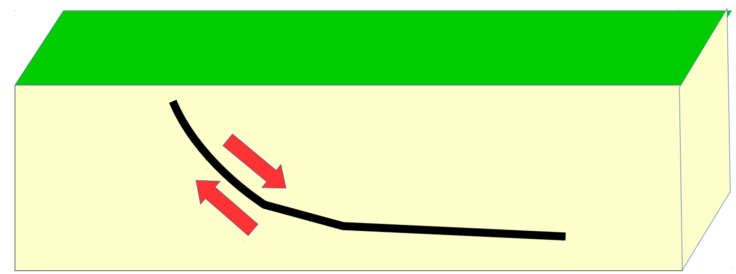
Shear stress *Normal stress*



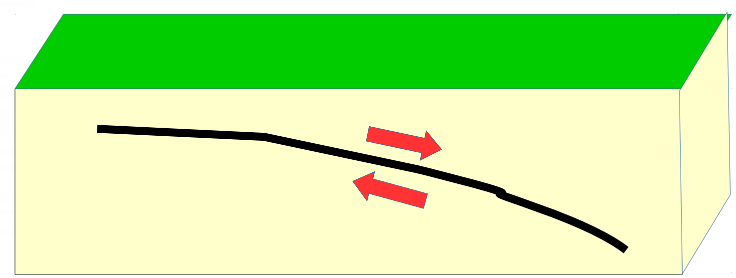
Most faults in the Earth's crust are in rough accordance with the Anderson's theory: **normal** and **reverse faults** have dip angles greater and smaller than **45°**, respectively

even if a considerable number of faults have **non-Andersonian geometries**

- Listric



- Detachment



- Low angle normal faults
- High angle reverse fault

Open questions:

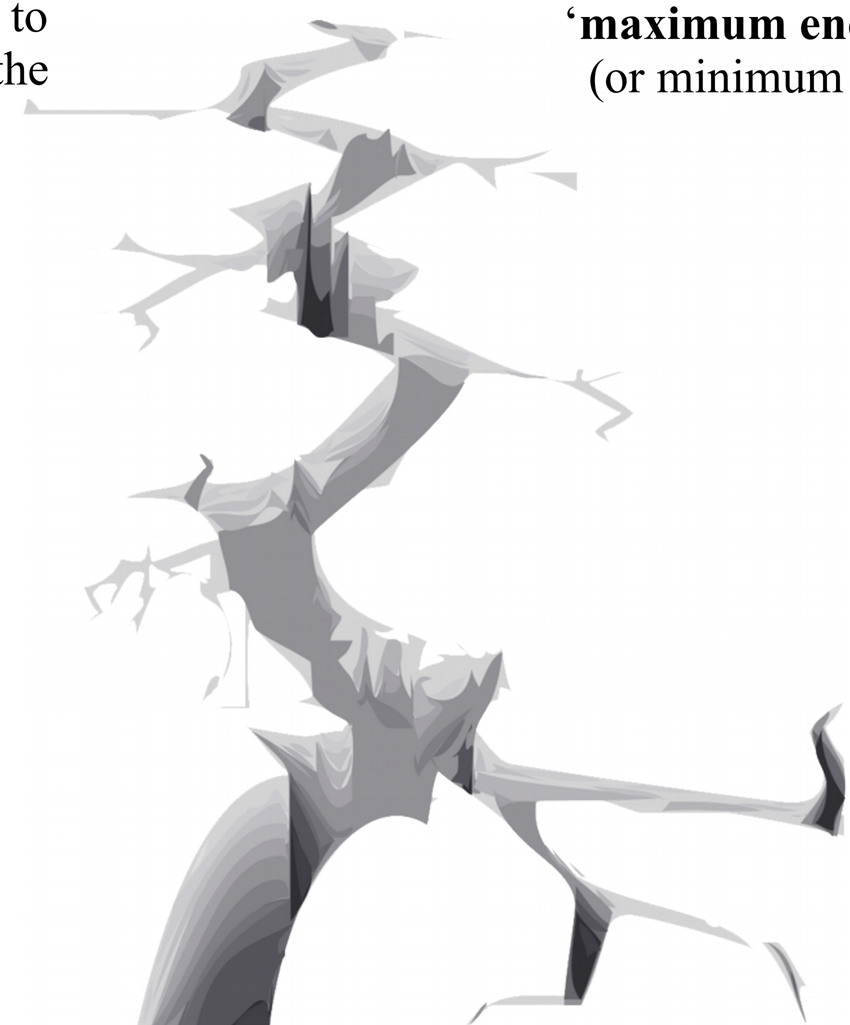
- Can **non-Andersonian fault** geometries be favoured by the presence of **rigidity contrasts** within the crust?

- Is it possible to devise a method to **predict the growth direction** on the basis of elastic parameters?

Modeling approach

- **2-D crack** model (plane strain) for **quasi-static fault growth** in a **two layer** medium following the criterion of

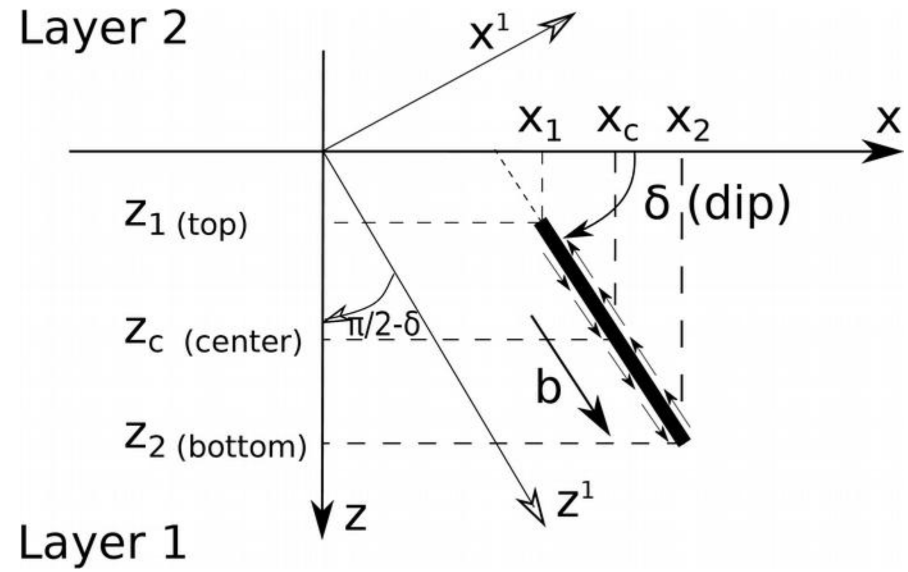
‘**maximum energy release**’
(or minimum final energy)



Starting from a single dislocation model

Initial stress field

$$S^0_{zz} = -\rho g z;\; S^0_{xx} = -\rho g z + \Delta\sigma;\; S^0_{xz} = 0$$



b = Burger's vector

Anderson's theory: fault slip occurs over a plane where $|\Delta\sigma|$ is minimum

$$\frac{d\Delta\sigma}{d\delta} = 0 \rightarrow \tan\, 2\delta = \pm\frac{1}{f_s}$$

Energetic criterion: the best fault plane is the one maximizing the energy release.

$$\Delta E = \frac{1}{2}\int_\Sigma \left(\tau^0 + \tau^1\right) b d\Sigma$$

Initial shear stress *Final shear stress*

$$\frac{d\Delta E}{d\delta} = 0$$

$$\sin^2\delta = \frac{f_d^2\left(\rho-\rho_f\right)^2 g^2 W^2 - 24\left(\rho-\rho_f\right) g\Delta\sigma f_d^2 z_C - 12\Delta\sigma^2}{-24\Delta\sigma^2\left(1+f_d^2\right)}$$

Fault width

Anderson's theory

$$\tan\ 2\delta = \pm\frac{1}{f_s}$$

- Pre-existing faults with **all possible orientations** are present before failure (f_s is a property of the surfaces)
- The dip angle **does not** depend on $\Delta\sigma$
- **St**atic friction coefficient f_s.

Energetic approach

$$\sin^2\delta = \frac{f_d^2\left(\rho-\rho_f\right)^2 g^2 W^2 - 24\left(\rho-\rho_f\right) g \Delta\sigma f_d^2 z_C - 12\Delta\sigma^2}{-24\Delta\sigma^2\left(1+f_d^2\right)}$$

- Creation of **new fault surface.** (No reactivation of **pre-existing fault** planes.)
- The dip angle **depends** on $\Delta\sigma$
- **Dynamic** friction coefficient f_d.

Energy budget for a single dislocation

We assume that the **energy release** (per unit of length) ΔE must be greater than the sum of the **work** E_f done against friction and the **fracture energy** E_T

$$\Delta E > E_f + E_T$$

where

$$E_f = -f_d(\sigma_n + p)Wb$$ *(Released as thermal energy)*

$$E_T = 2(1-\nu^2)\gamma_s$$ *(Energy required to generate fault surface)*

Poisson modulus

Specific fracture energy
(Here assumed equal to 1 J/m²)

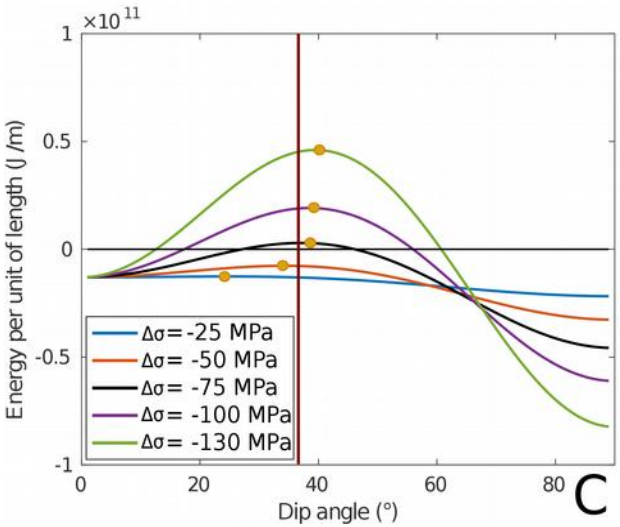
ΔE

Fracture energy

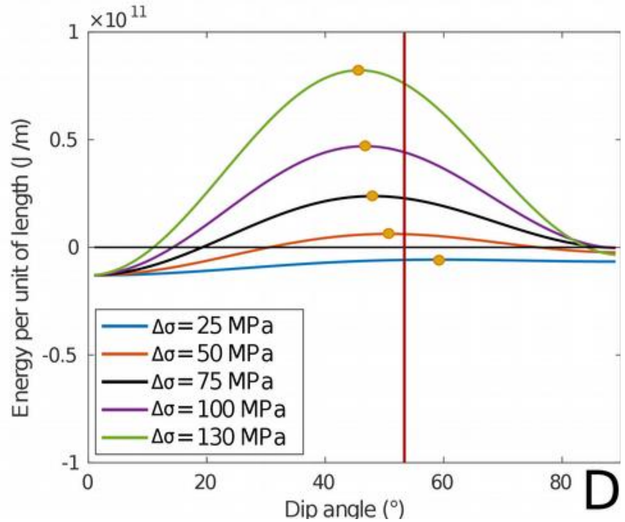
Heat

Seismic waves

Reverse faults



Normal faults



Energy release ΔE

as a **function of the dip angle**.

The vertical **red lines** represent the **Anderson's solution** for the dip angle computed with a $f_s = f_d$

Fault depth = 5 km W = 1 km $f_s=f_d=0.3$

- The energetic criterion provides, respectively, **greater** and **smaller** dip angles for reverse and normal faults **with respect to the Anderson's** condition.

- The same solution is obtained only if $\Delta E \rightarrow 0$ with f_d replacing f_s, **but (!!!)**

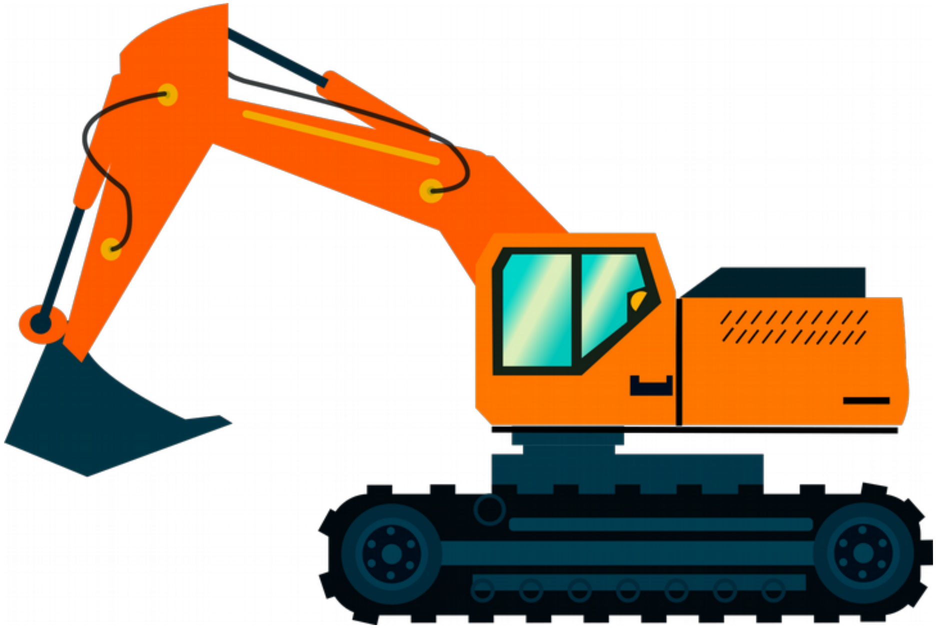
  If $\Delta E \rightarrow 0$ then $E_f > \Delta E$

Building the crack growth model

Step 1: Representing dislocations in two welded half-spaces

Step 2: Implementing the boundary element model (BEM) – crack model

Step 3: Let the crack grows



Model – Step 1

Representing dislocations in two welded half-spaces

- **2-D fault,** arbitrarily placed in a medium consisting of **two welded half-spaces**.

- **Galerkin** components

(Bonafede & Rivalta, 1999; Rivalta et al., 2002)

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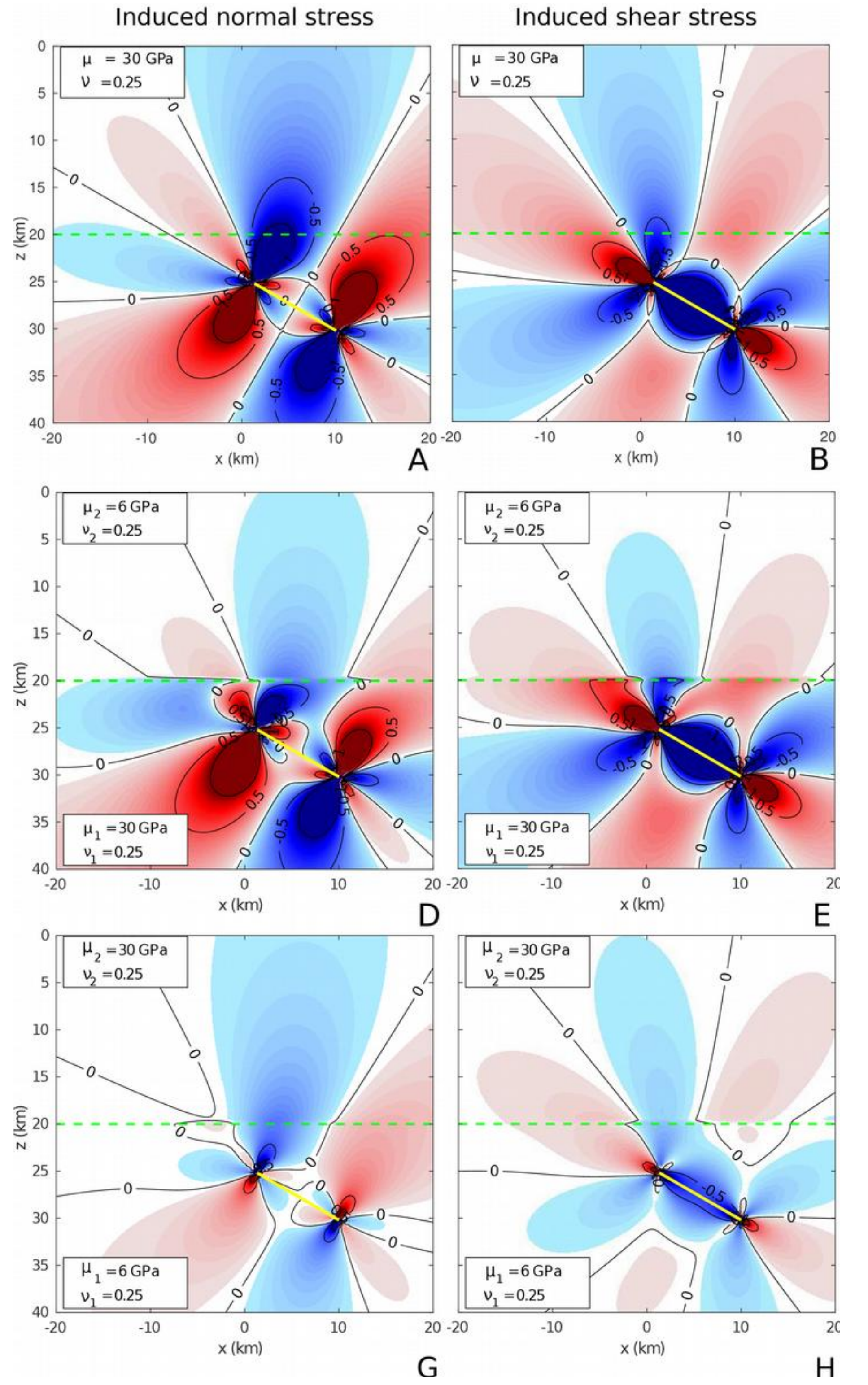
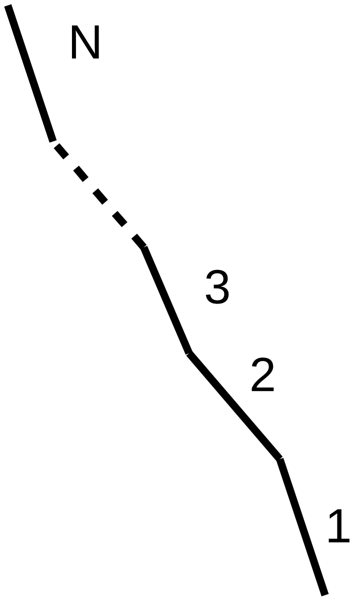


Model – Step 2

Implementing the boundary element model (BEM) – crack model

$$\begin{cases} \sum_{k=1}^{N} b_k \left(I_{1k} + f_d Y_{1k}\right) = -\left[\tau_1^0 + f_d(\sigma_{n1} + p_1)\right] \\ \cdots \\ \cdots \\ \sum_{k=1}^{N} b_k \left(I_{Nk} + f_d Y_{Nk}\right) = -\left[\tau_N^0 + f_d(\sigma_{nN} + p_N)\right] \end{cases}$$



$\sigma_{nm}, p_m \ (m = 1,..N)$:

Environmental **normal stress components** and **pore pressure** on the m-th dislocation

I_{mk} :

Shear stresses computed at the midpoint of the *m*th dislocation due to the *k*th dislocation with unitary Burger's vector

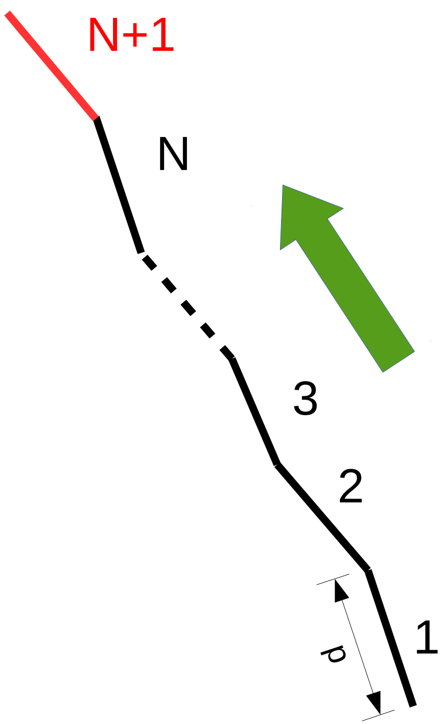
Y_{mk} :

Normal stresses computed at the midpoint of the *m*th dislocation element due to the *k*th dislocation element with unitary Burger's vector

Model – Step 3
Let the crack grows

The **crack growth** is simulated by:

- **adding a dislocation** element beyond the **tip** of the **crack**
- **recomputing the new equilibrium** using the boundary element technique with N+1 dislocation elements.



The crack can grow only if the new configuration is energetically possible

$$\Delta E(W+d) > E_f(W+d) + E_T(W+d)$$

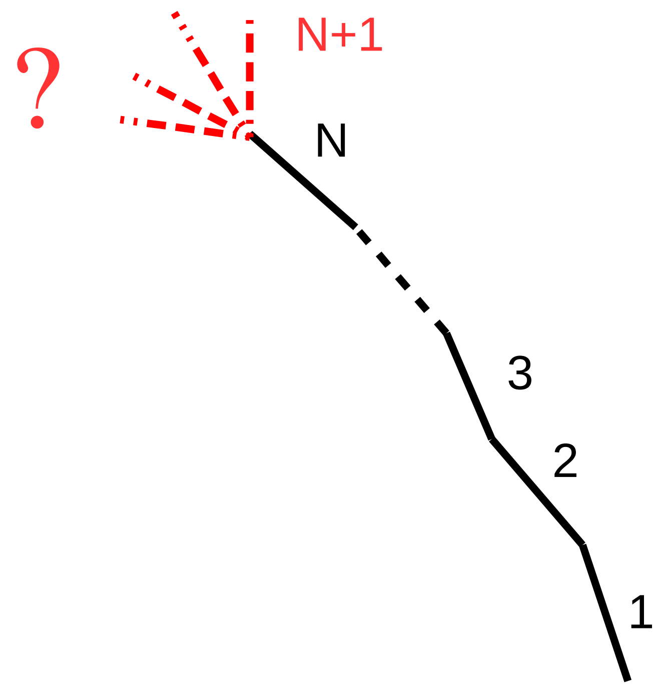
otherwise it stops

Model – Step 3

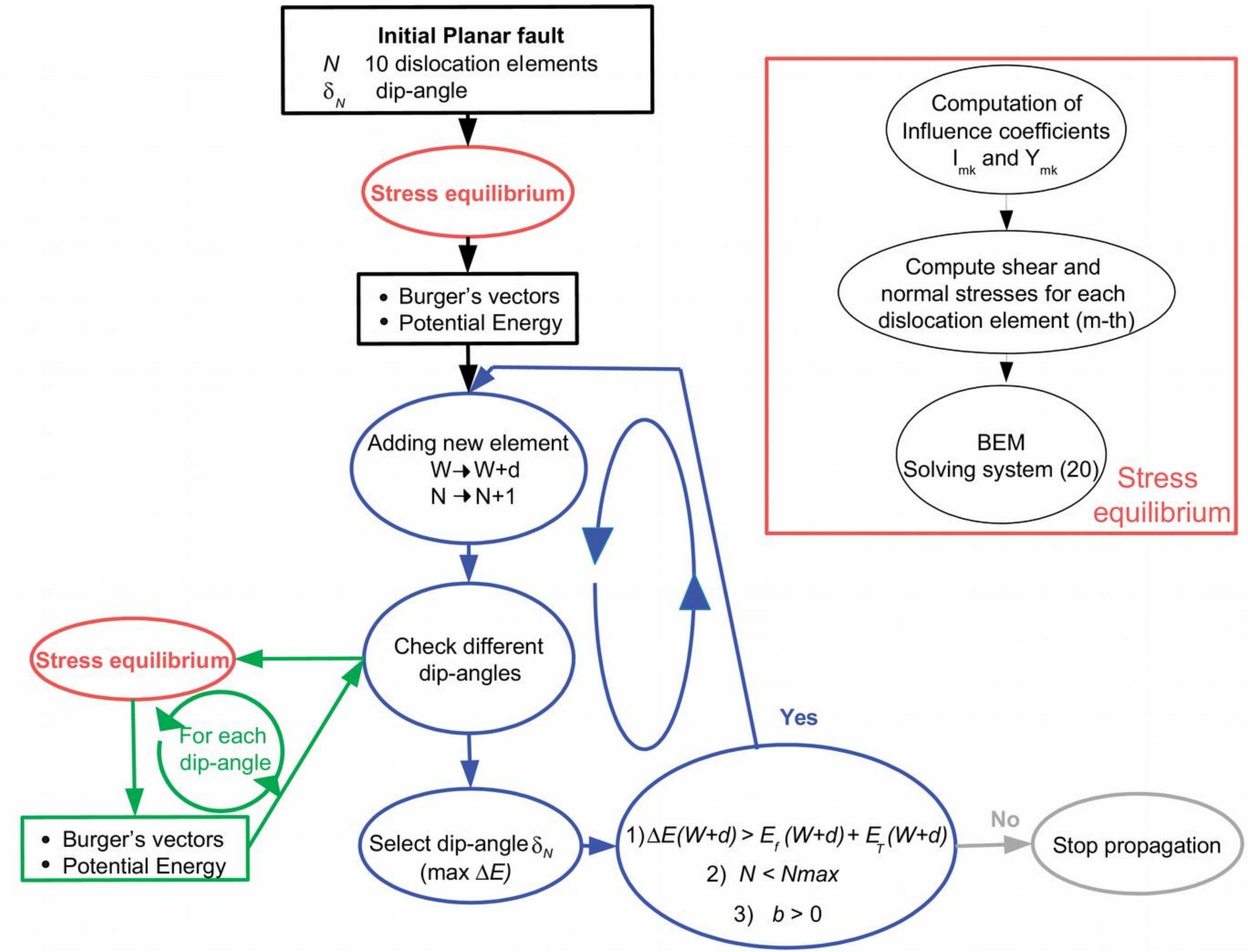
Let the crack grows

Variable direction of crack growth: the **dip angle** of the additional dislocation element (δ_{N+1}) is the one that **maximizes the energy release**

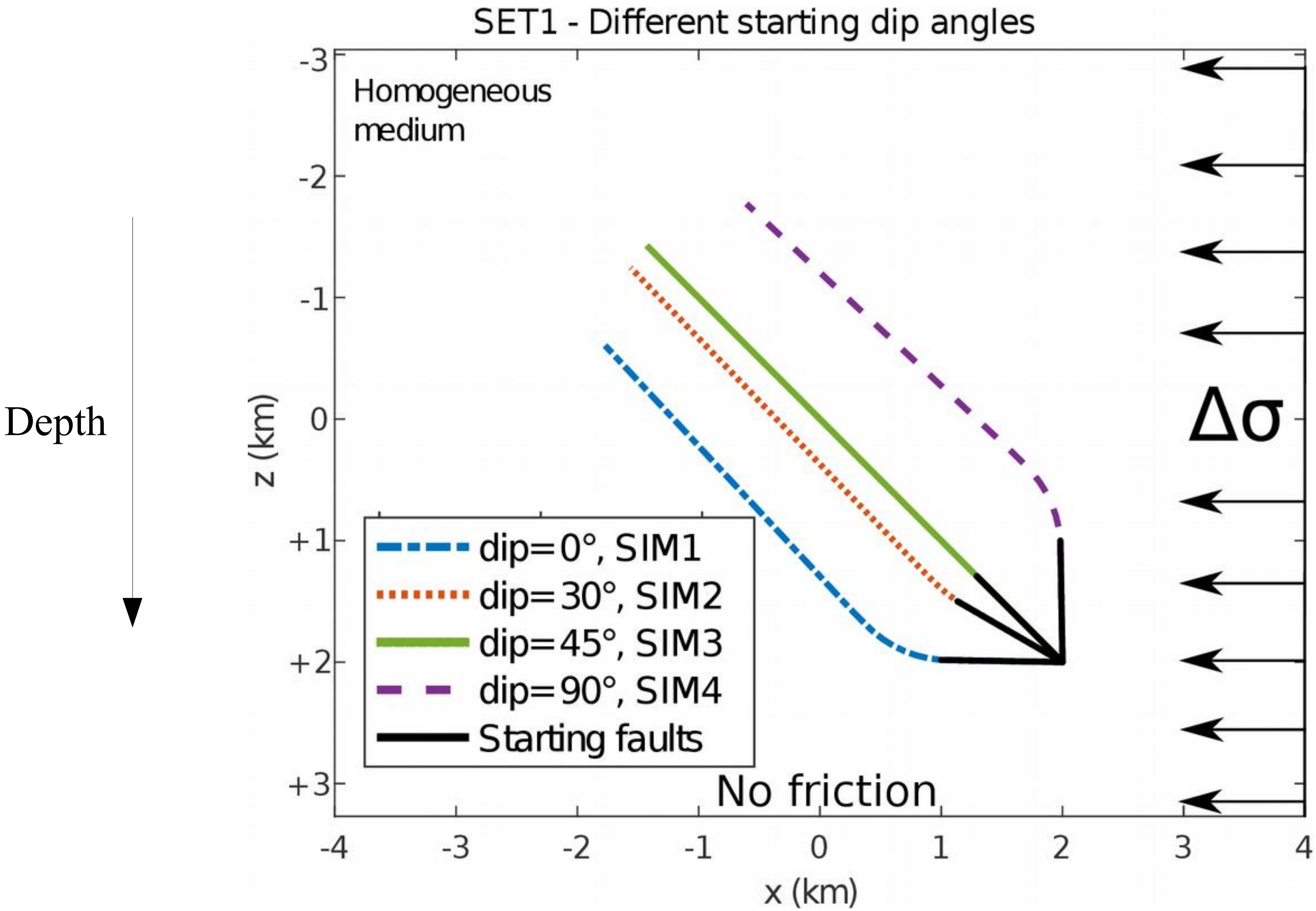
and it is chosen **exploring different configurations** with one degree dip-angle variations with respect to the dip, δ_N , of the adjacent dislocation element.



Model – Flowchart

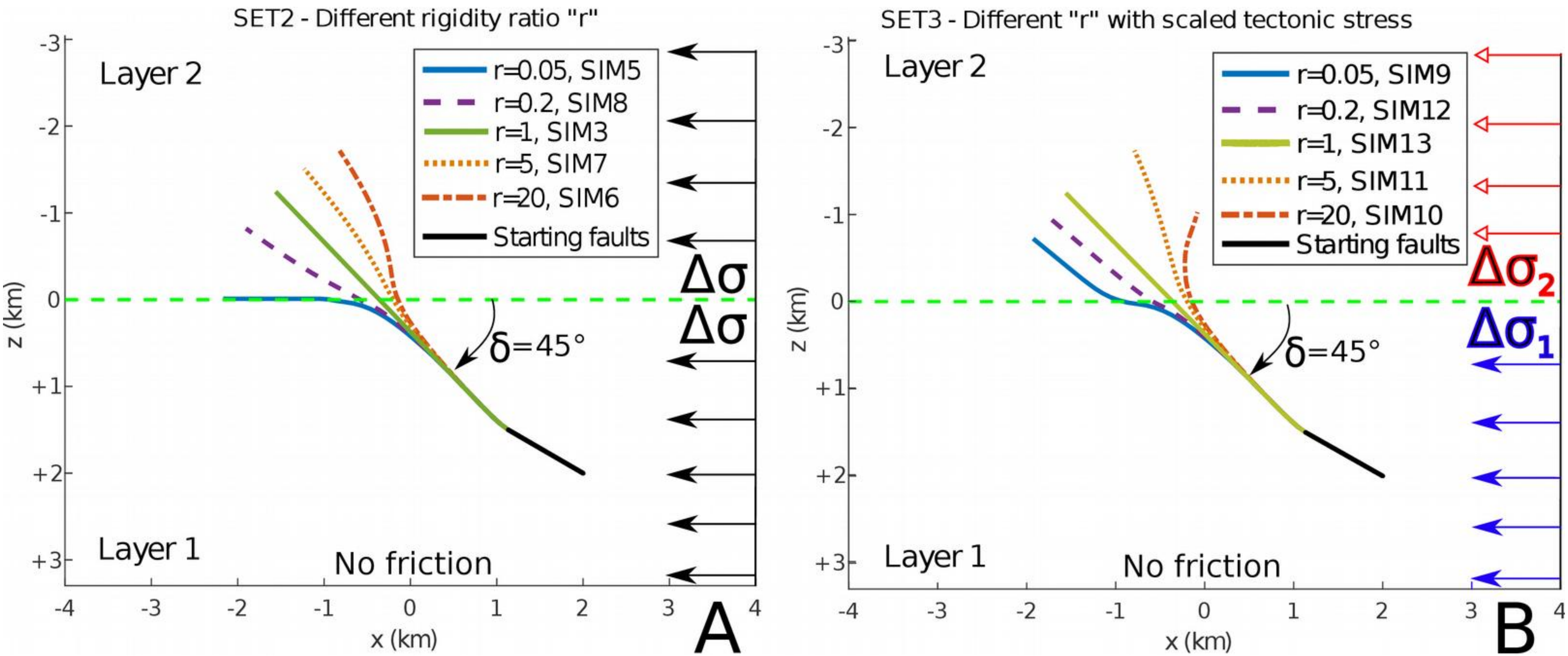


Results - SET1



- **Homogeneous elastic medium**
- **No friction**
- **Different dip angles**
- $\Delta\sigma$ = 100 MPa is assumed **uniform** along depth.

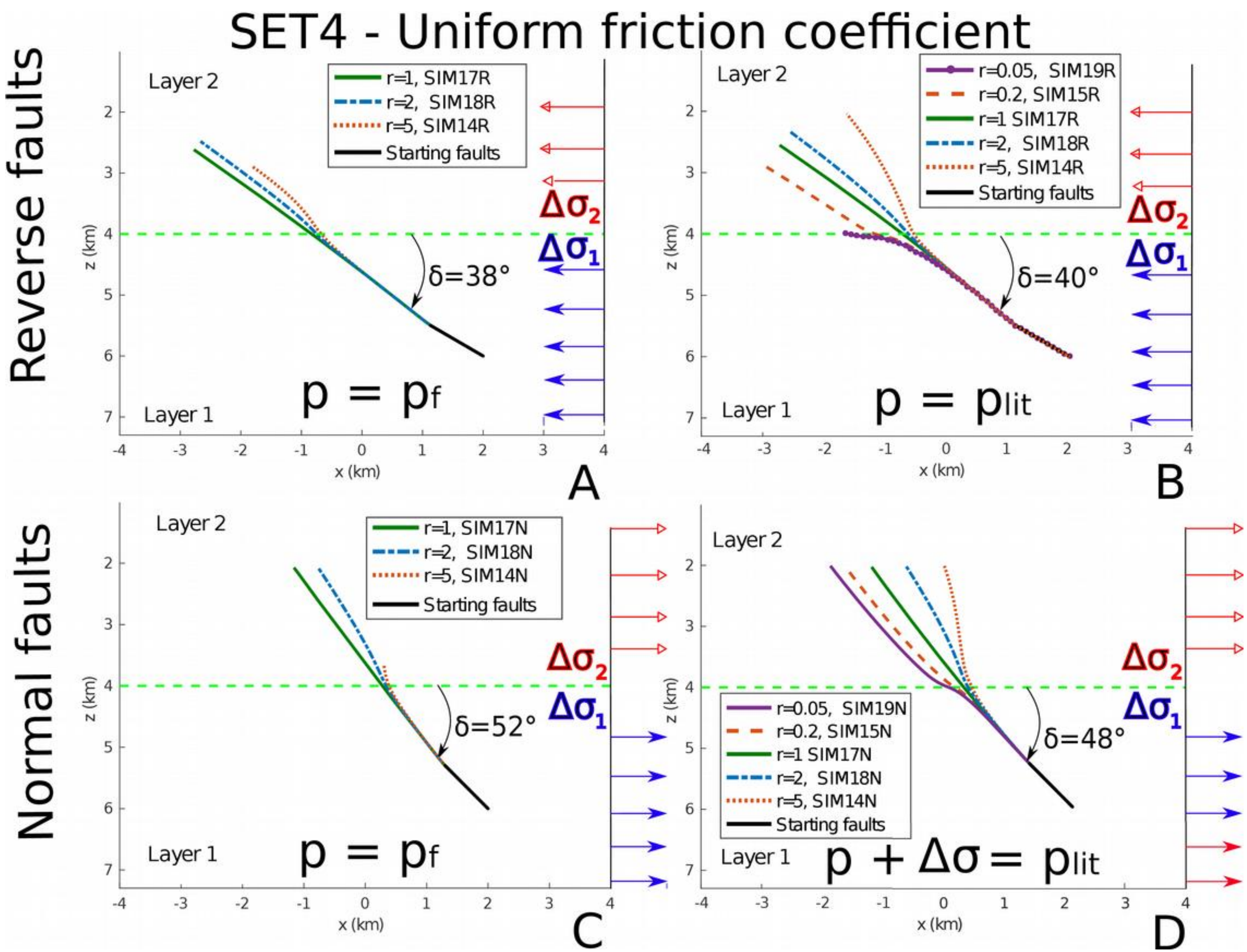
Results – SET2 and SET3



$$r = \mu_1 / \mu_2$$

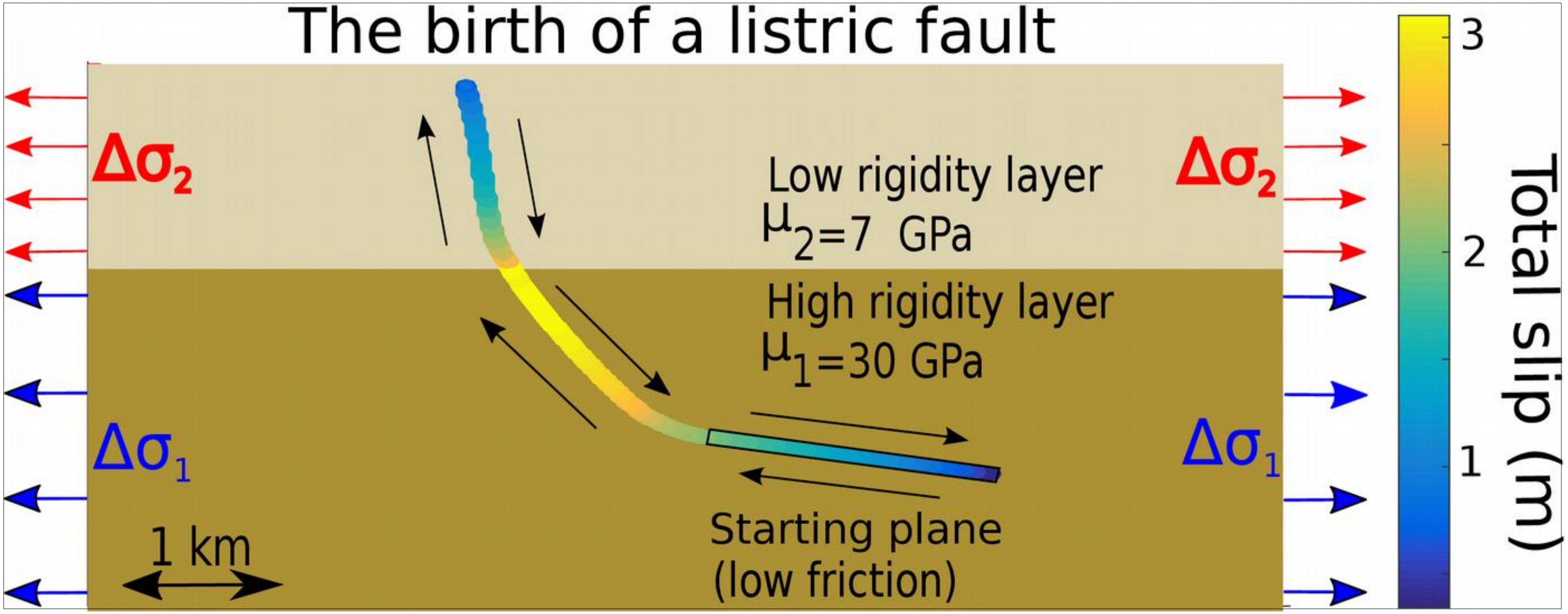
- **Heterogeneous elastic medium**
- **No friction**
- The **tectonic stress** $\Delta\sigma$ = 100 MPa is assumed **uniform** over depth in SET2 (A) while in SET3 (B) it is **rescaled** according to the rigidity ratio r .

Results – SET4



- **Heterogeneous elastic medium**
- **Friction**
- **Pore pressure**
- The tectonic stress $\Delta\sigma = 100$ MPa is **rescaled** according to the rigidity ratio r **.**

Results – Listric fault



- **Heterogeneous elastic medium**
- The pore pressure is assumed as **hydrostatic**.
- The **dynamic friction coefficient** is $f_d = 0.3$ all over the fault surface apart from the starting low dip segment (enclosed within a black rectangle) that has lower friction coefficient ($f_d = 0.05$).

Discussion and conclusions

(with respect to the Anderson's theory)

- The **maximum energy release criterion**

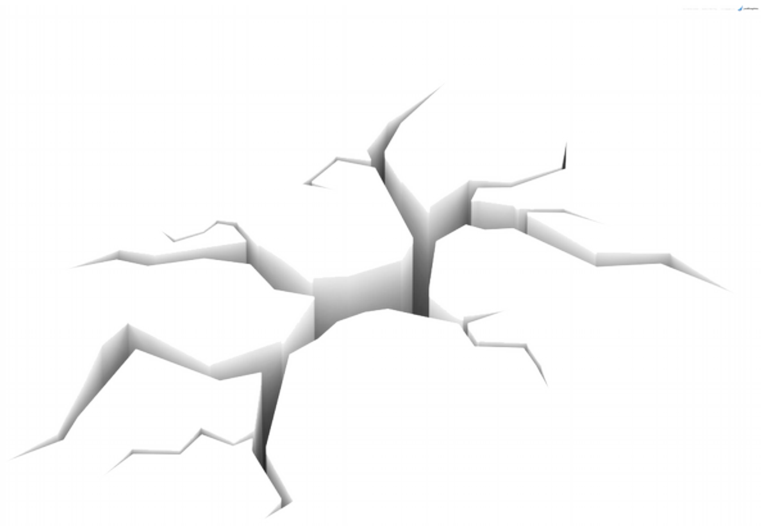
lower dip-angle for **normal** faults

greater dip-angle for **reverse** faults

- When faults cross a **rigidity contrast** interface, they are affected by a **dip angle change** that increases in magnitude for larger rigidity contrasts

***r* > 1** dip-angle increases rising towards the interface

***r* < 1** dip-angle decreases rising towards the interface



Discussion and conclusions

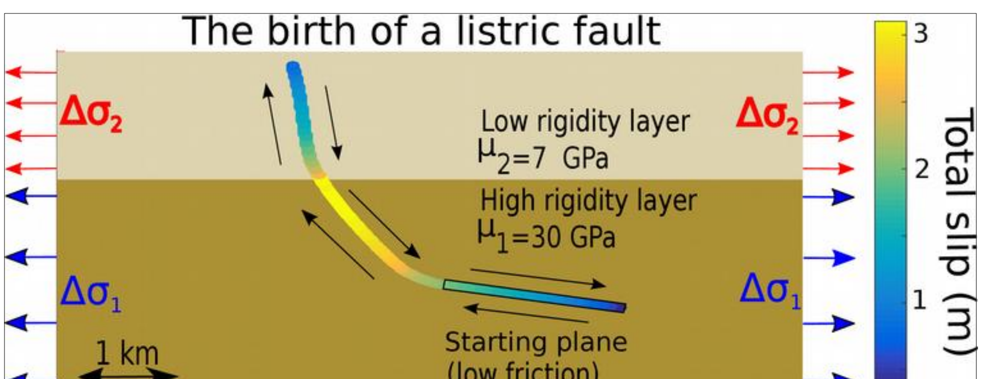
- **Listric Faults**

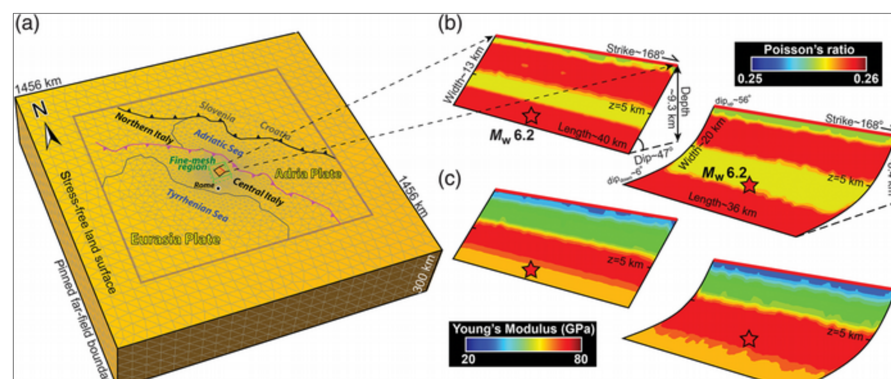
According to our model, listric geometries can be obtained only if ***r* > 1**, that is when **the fault meets an interface above which the medium is softer**.

The results of simulations are quite stable: the fault grows by **increasing the dip** angle in the **shallow and less rigid layer**.

An example of a r > 1 configuration is when shallow layers of recently formed **sedimentary rocks,** with **low rigidity**, are superimposed to stiffer layers whose rigidity increases with depth. This condition is **very common** as in the shallow and brittle crust, the rigidity generally increases with depth, as confirmed by depth-increasing S-wave speed, $V_s = \sqrt{\mu/\rho}$

Examples





Tung & Masterlak, 2018

Discussion and conclusions

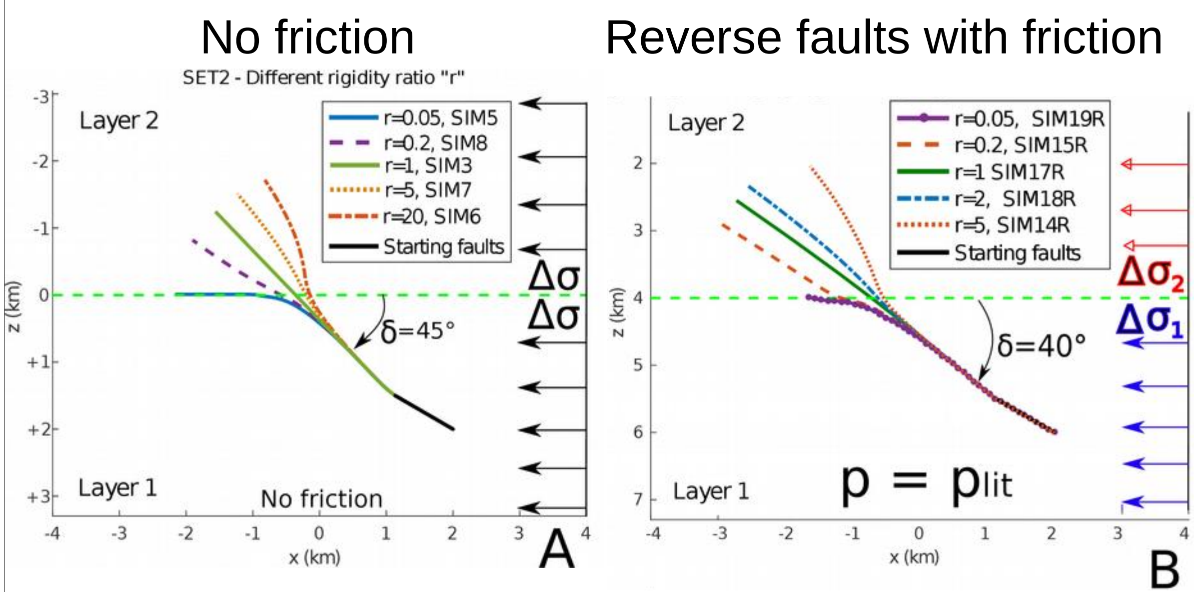
- **Detachment faults**

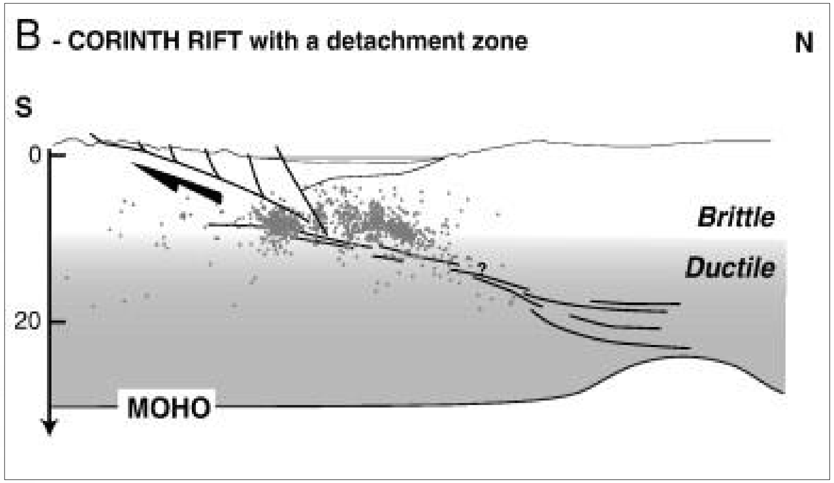
According to our results, detachment faults can be realized only if $\boldsymbol{r < 1}$. Beside this condition, a **low dynamic friction** coefficient in the deeper layer or a **high pore pressure** is necessary.

Only if $\boldsymbol{r \ll 0.2}$ (Fig. 6a) the fault can propagate along the elastic discontinuity producing the **horizontal detachment** of the deeper layer with respect to the shallower one (i.e. a **decollement** fault), without penetrating the upper stiffer layer.

The interface above the much softer deeper layer might be identified with the **brittle-ductile transition** within the crust.

Examples





Jolivet et al., 2010

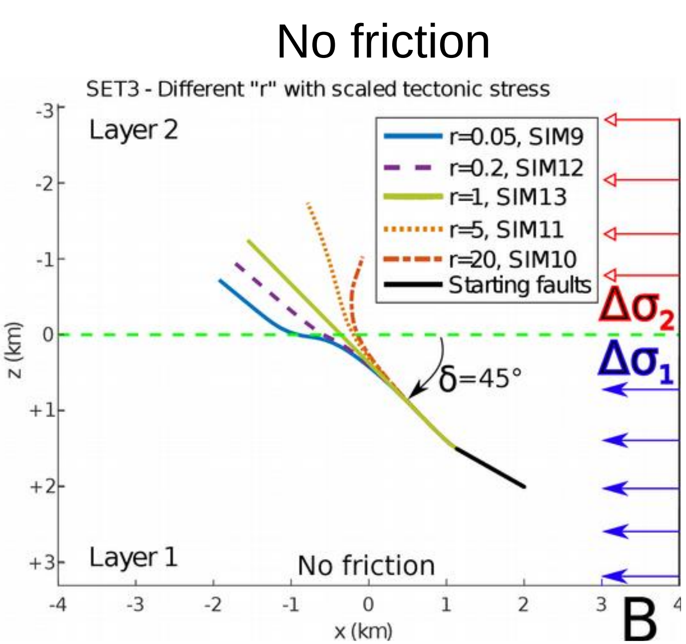
Discussion and conclusions

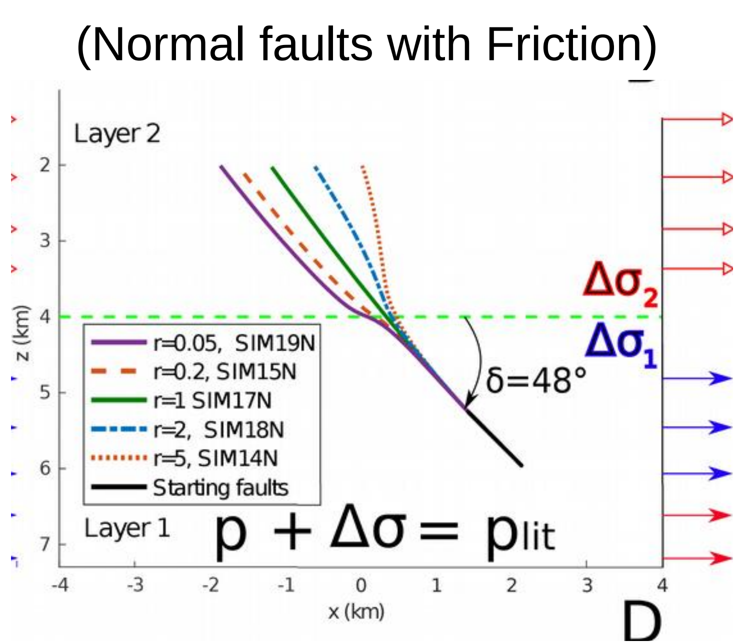
- **Ramp-flat-ramp faults**

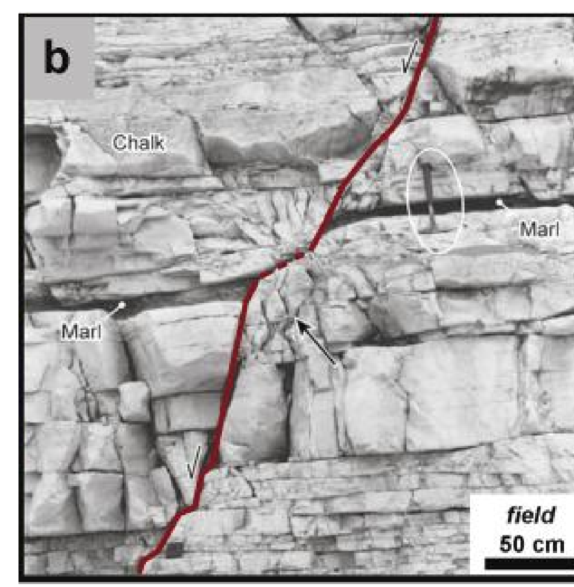
According to our results, as for detachment faults, ramp-flat-ramp faults can be realized if ***r* < 1.**

A ramp-flat-ramp fault growing almost horizontally in correspondence of the elastic discontinuity, and then allowed to rise in the stiffer layer after some iterations can be obtained for **strong rigidity contrasts**, **low friction** or **high pore pressure,** if a **vertically uniform strain** is assumed in elastic medium.

Examples







Vasquez et al. 2018

Thanks for your attention !

Geophysical Journal International

Geophys. J. Int. (2020) **220,** 1095–1111
doi: 10.1093/gji/ggz505
Advance Access publication 2019 November 05
GJI General Geophysical Methods

Fault dip variations related to elastic layering

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Accepted 2019 November 2. Received 2019 October 31; in original form 2019 May 3