Ab-initio studies of few-nucleon systems: Recent progress and applications

Laura E. Marcucci

University of Pisa

INFN-Pisa

Introduction: *ab-initio* studies

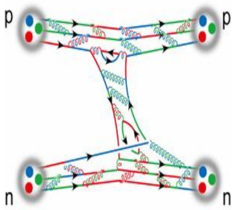
Nuclear observable X

1. Nucleus = system of A nucleons interacting among themselves and with external electro-weak probes
2. Realistic description of nuclear interactions and electro-weak currents
3. Exact* method to solve the quantum-mechanical problem (both bound and scattering states)
4. "True" predictions for the observable X
5. Ideal case: estimate the theoretical error

* Exact ≡ no uncontrolled approximations

- controlled approximations are allowed (expansion on a certain basis) → converged results = *ab-initio* results
- comparison of *ab-initio* results obtained with different *ab-initio* methods → benchmark calculations
- comparison of *ab-initio* results with data → test of input ingredients

The nuclear Hamiltonian: $H = T + V$



Nuclear interaction: $V = V_{NN} + V_{NNN}$

Until $\simeq$ 20–30 years ago: phenomenological potentials

- $V_{NN} + V_{NNN}$ semi-phenomenological
- V_{NN} with $\simeq$ 40 parameters fitted to $A = 2$ data $\rightarrow \chi^2$/datum $\simeq 1$
- V_{NNN} with 2-3 parameters fitted to $B(A = 3, 4)$

Very common models: AV18+UIX, AV18+IL7

Very successful, but

- many parameters
- no connection with QCD
- no estimate of theoretical uncertainty

$\Rightarrow$ **Chiral Effective Field Theory**

Chiral Effective Field Theory (χEFT): a very short summary

- QCD $\rightarrow$ quarks and gluons ("high-energy" d.o.f.)
- Nuclear physics $\rightarrow$ nucleons and pions ("low-energy" d.o.f.)
- EFT $\rightarrow$ processes with $E \simeq p \simeq m_\pi \ll \Lambda_{\rm QCD} \sim 1$ GeV
  - ⋆ keep the "l-e" d.o.f.: π and N
  - ⋆ Lagrangians describing the interactions of π and N are expanded in powers of $O(p/\Lambda_{\rm QCD})^\nu \rightarrow$ perturbative theory
  - ⋆ "h-e" d.o.f. integrated out $\rightarrow$ contact interactions with "l-e" d.o.f. and low-energy constants (LECs) obtained from experiment
- χEFT $\rightarrow$ EFT with spontaneous breaking of QCD's χ-symmetry
- Regularization of short-range terms with cutoff function $\rightarrow \Lambda \simeq 400 - 600$ MeV

Disadvantage: limited to processes with $E \leq [2 \div 3]\, m_\pi$

Advantages

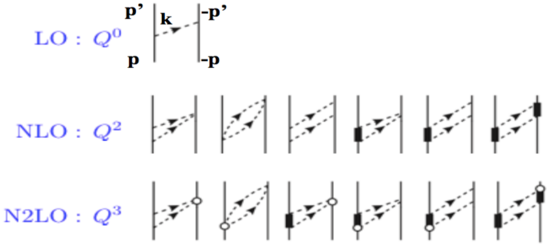
- nuclear force "hierarchy" $\rightarrow$ accurate $V_{NN} + V_{NNN}$
- consistent framework for interactions + currents (just add electro-weak field as d.o.f.)
- possibility to estimate the theoretical uncertainty (perturbative expansion)

Until very recently, everything developed in momentum-space
(for example: N3LO500/N2LO500)
$\rightarrow$ not user-friendly when charged particles

Local chiral V_{NN} with Δ's

M. Piarulli *et al.*, Phys. Rev. C **91**, 024003 (2015)

- $V_{NN} = v^{EM} + v^{LR} + v^{SR}$
- v^{EM}= electro-magnetic component including corrections up to α^2
- Chiral 1π and 2π exchange in v^{LR} with Δ's up to Q^3 (N2LO)

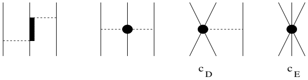


- v^{SR} contact terms up to Q^4 (N3LO) with 26 LECs
- v^{SR} regularized with $C_{R_S}(r) \propto \mathrm{e}^{-(r/R_S)^2}$ with $R_S = 0.8(0.7)$ fm [model *a* (*b*)]
- fit the 2013 Granada database
  - up to $E_{lab} = 125$ MeV ($\sim$ 2700 data) with χ^2/datum ≤ 1.1 (model I)
  - up to $E_{lab} = 200$ MeV ($\sim$ 3500 data) with χ^2/datum ≤ 1.4 (model II)

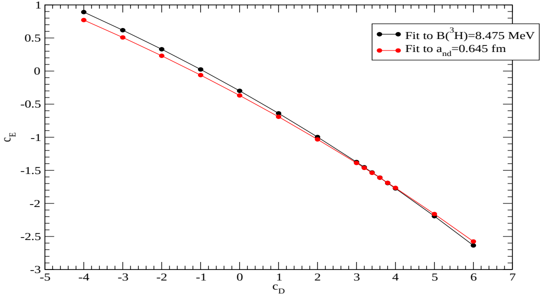
Local chiral V_{NNN} with Δ's (NV2+3/Ia & NV2+3/Ib)

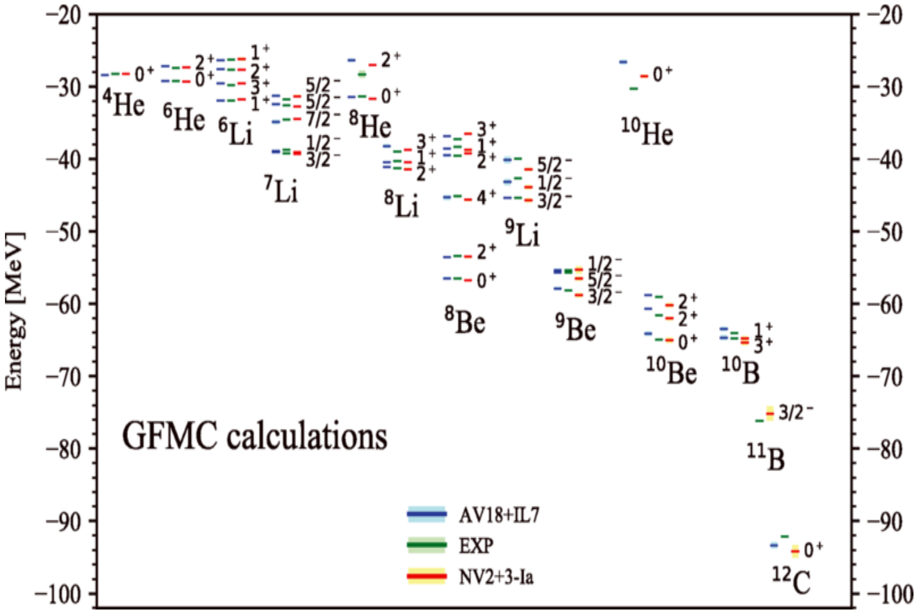
M. Piarulli *et al.*, Phys. Rev. Lett. **120**, 052503 (2018)

- Three-nucleon interaction V_{NNN} up to N2LO



- Type 1: fit c_D & c_E to $B(^3\mathrm{H})$ and $a_{nd}^{Exp} = (0.645 \pm 0.010)$ fm



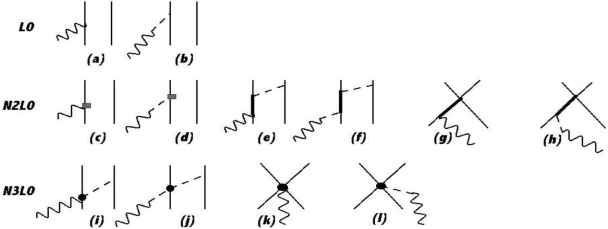


M. Piarulli *et al.*, Phys. Rev. Lett. **120**, 052503 (2018)

Local chiral V_{NNN} with Δ's (NV2+3/Ia* and NV2+3/Ib*)

A. Baroni *et al.*, Phys. Rev. C **98**, 044003 (2018)

To avoid correlation between $B(^3\mathrm{H})$ and a_{nd} $\rightarrow$ $B(^3\mathrm{H})$ and GT^{Exp} in ^{3}H β-decay $\Rightarrow$ $j_A(q)$



- Ignore pion-pole terms [(b), (d), (f), (h), (j), (l)]
- diagrams (g) and (h) vanish; diagram (e) $\rightarrow c_3^\Delta$; c_4^Δ (similar to c_3; c_4 of diagram (i))
- CTs in (i) and (k)

$$\mathbf{j}_{5,a}^{N3LO}(\mathbf{q};CT) = z_0\,\mathrm{e}^{i\mathbf{q}\cdot R_{ij}}\,\frac{\mathrm{e}^{-(r_{ij}/R_S)^2}}{\pi^{3/2}}(\tau_i\times\tau_j)_a(\sigma_i\times\sigma_j)$$

$$z_0 = \frac{g_A}{2}\frac{m_\pi^2}{f_\pi^2}\frac{1}{(m_\pi R_S)^3}\left[-\frac{m_\pi}{4g_A\Lambda_\chi}c_D+\frac{m_\pi}{3}(c_3+2c_4+c_3^\Delta+2c_4^\Delta)+\frac{m_\pi}{6m}\right]$$

$$\text{but}\quad c_3^\Delta+2c_4^\Delta=-\frac{h_A^2}{9m_{\Delta N}}+2\frac{h_A^2}{18m_{\Delta N}}=0\quad\text{with } h_A\equiv g_A^*$$

	NV2+3/Ia	NV2+3/Ib
c_D	3.666	–2.061
c_E	–1.638	–0.982
GT	0.9885	0.9730

	NV2+3/Ia*	NV2+3/Ib*
c_D	–0.635	–4.71
c_E	–0.09	0.55
a_{nd} [fm]	0.638	0.650

$\mathrm{GT}^{Exp} = 0.9511 \pm 0.0013$

$a_{nd}^{Exp} = (0.645 \pm 0.010)$ fm

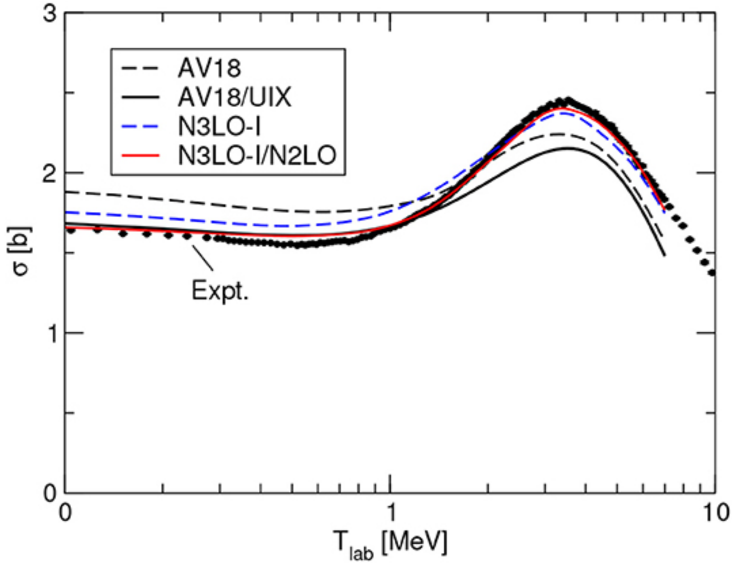
$A = 3, 4$ binding energies and scattering lengths with NV potentials

Model	B(^{3}H)	B(^{3}He)	B(^{4}He)	a^2_{nd}	a^4_{nd}	a^2_{np}	a^4_{pd}
NVIa	8.718	7.090	25.15	1.119	6.326	0.959	13.596
NVIb	7.599	6.885	23.96	1.307	6.327	1.294	13.597
NV2+3/Ia*	<u>8.477</u>	<u>7.727</u>	28.30	0.638	6.326	0.070	13.596
NV2+3/Ib*	<u>8.469</u>	<u>7.724</u>	28.21	0.650	6.327	0.070	13.597
Exp.	8.475	7.725	28.30	0.645(10)	6.35(2)	–0.13(4)	14.7(2.3)

L.E. Marcucci *et al.*, Front. Phys. **8**, 69 (2020)

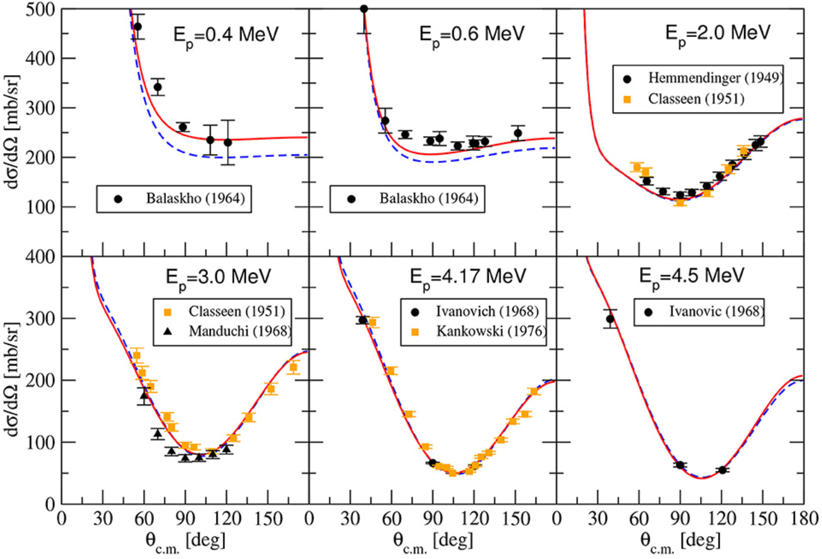
Ab-initio method: the Hyperspherical Harmonics (HH) method

$A = 4$ scattering observables: σ_{tot} for $n + {}^3$H



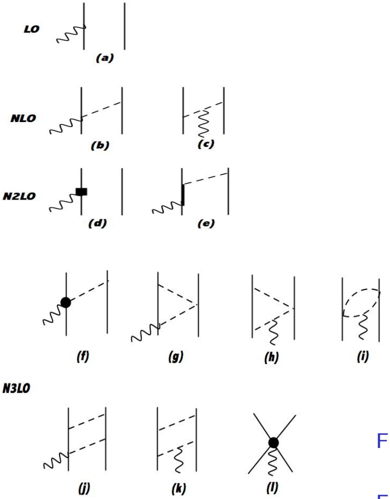
L.E. Marcucci *et al.*, Front. Phys. **8**, 69 (2020)

$A = 4$ scattering observables: $d\sigma/d\Omega$ for $p + {}^3$H



L.E. Marcucci *et al.*, Front. Phys. **8**, 69 (2020)

Electromagnetic current in χEFT



- $\mathbf{j}_{\Delta}^{\rm N2LO}(\mathbf{q})$ in panel (e) absent in $\not\Delta$-EFT
- not included the Δ intermediate states at N3LO
- $\mathbf{j}_{\rm OPE}^{\rm N3LO}(\mathbf{q}) \to d_2^S, d_2^V \,;\, d_3^V$
- $\mathbf{j}_{\rm MIN}^{\rm N3LO}(\mathbf{q}) \to$ from πN scattering
- $\mathbf{j}_{\rm NM}^{\rm N3LO}(\mathbf{q}) \to d_1^S \,;\, d_1^V$

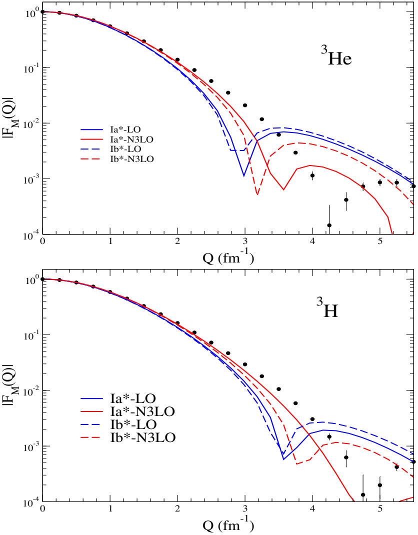
To be noticed:

$$\mathbf{j}_{\rm OPE}^{\rm N3LO}(\mathbf{q}) \propto \frac{\sigma_j \cdot \mathbf{k}_j}{(m_\pi^2 + \mathbf{k}_j^2)}\, \mathbf{q} \times [(d_2^S \tau_i \cdot \tau_j + d_2^V \tau_j^z)\mathbf{k}_j + d_3^V (\tau_i \times \tau_j)^z \sigma_i \times \mathbf{k}_j]$$

FIT 1 $d_2^V \,;\, d_3^V \to$ saturated with Δ-current of panel (e); d_1^S, d_2^S, d_1^V fitted to $A = 2, 3$ magnetic moments

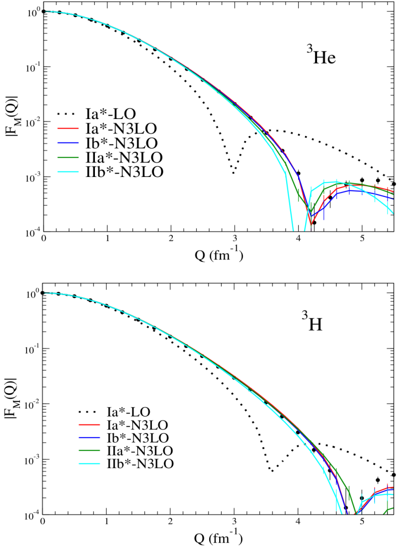
FIT 2 all 5 LECs fitted to $A = 2, 3$ magnetic moments and $d(e, e')pn$ at threshold

R. Schiavilla *et al.*, Phys. Rev. C **99**, 034005 (2019) with FIT 1



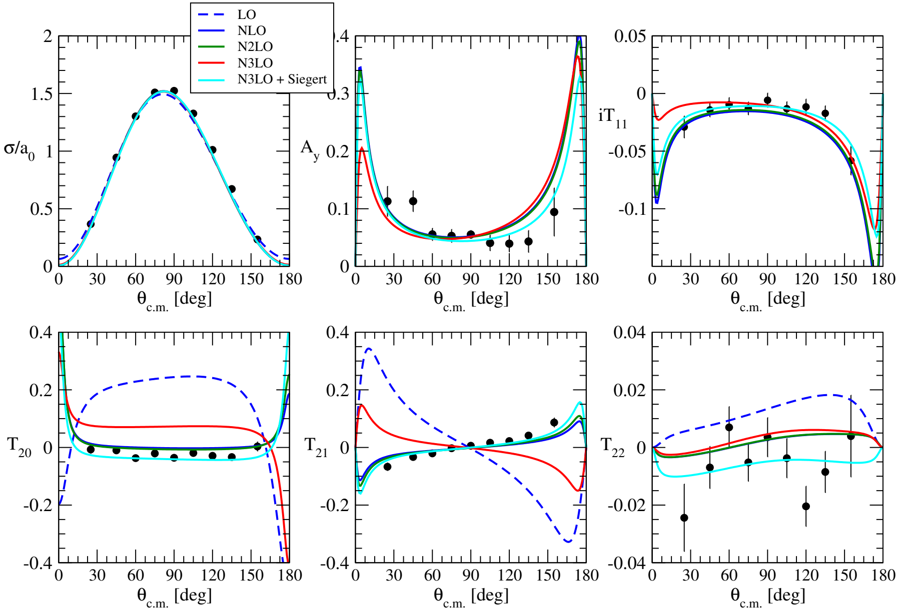
R. Schiavilla *et al.*, Phys. Rev. C **99**, 034005 (2019)

with FIT 2

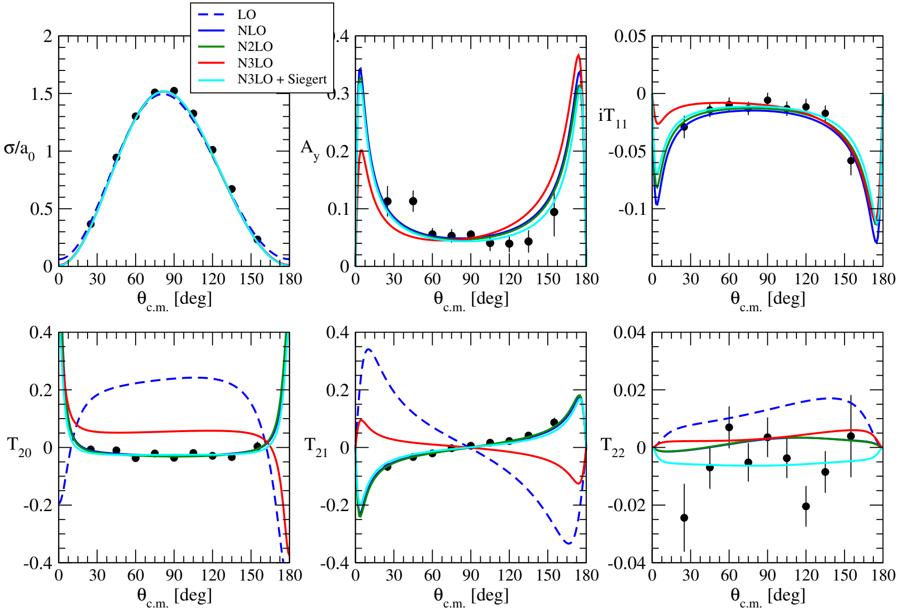


R. Schiavilla, private communication

The $p + d \to {}^3\mathrm{He} + \gamma$ reaction at $E_{cm} = 2$ MeV: PRELIMINARY (NV2+3/Ia*)



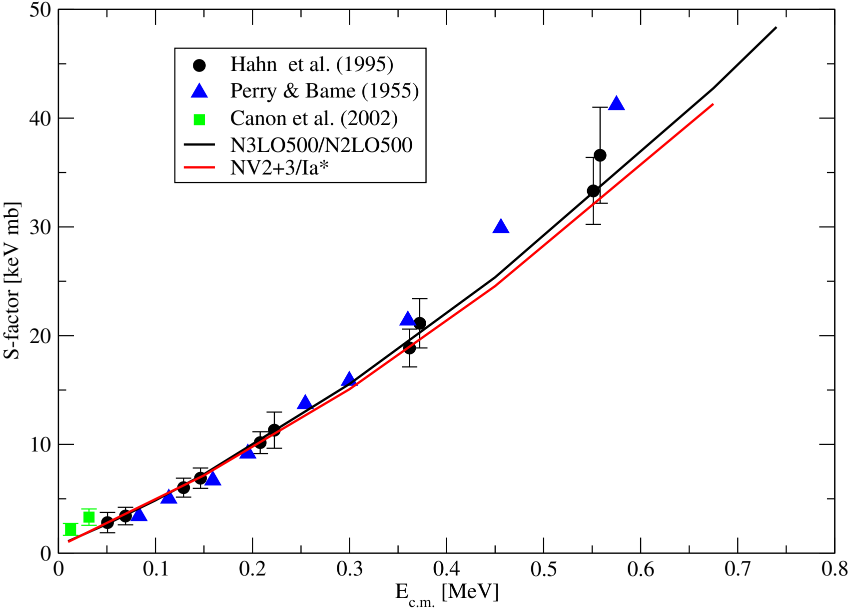
The $p + d \to {}^3\mathrm{He} + \gamma$ reaction at $E_{cm} = 2$ MeV: PRELIMINARY (NV2+3/Ib*)



Contribution	NV2+3/Ia*	NV2+3/Ib*	AV18/UIX
LO/IA	22.6(1)	22.3(1)	22.01(2)
NLO	24.4(1)	24.3(1)	–
N2LO	24.8(1)	24.9(1)	–
N3LO/FULL	25.5(1)	25.6(1)	25.86(5)
N3LO-Siegert	27.2(2)	26.2(2)	–

- Uncertainties on NV2+3/Ia/b* results → numerics
- Uncertainties on AV18/UIX results → scattering w.fs.
- Nice **agreement** between NV2+3/Ia/b* vs. AV18/UIX
- For the NV2+3/Ia/b* cases, we need to:
  - **increase numerics** (MC integration, here only 50k configs.)
  - study **convergence on bound/scattering w.fs.**
  - study **convergence on chiral order** ⇒ **theoretical uncertainty**
  - explore other c.m. energy ranges (**of interest for BBN**)

The $p + {}^3\mathrm{H} \rightarrow {}^4\mathrm{He} + \gamma$ reaction: PRELIMINARY



Conclusions and outlook

- We have all the ingredients for a systematic study in χEFT of
  - $A \leq 4$ nuclear reactions of astrophysical interest
  - More processes like
    - muon captures
    - photo/electro-disintegration
    - reactions involved in the dark-photon search
      [$\mathrm{d}(p, e^+e^-)^3\mathrm{He}$, $^3\mathrm{H}(p, e^+e^-)^4\mathrm{He}$ or $^3\mathrm{He}(n, e^+e^-)^4\mathrm{He}$]
    - $A = 3, 4$ momentum distributions
- $A > 4$ nuclei $\rightarrow$ HH method for $A = 6$
  (A. Gnech *et al.*, Phys. Rev. C **102**, 014001 (2020))

In collaboration with:

- A. Kievsky and M. Viviani (INFN Pisa - Italy)
- L. Girlanda (INFN Lecce and Univ. of Salento - Italy)
- A. Gnech (GSSI & INFN Pisa - Italy $\rightarrow$ JLab - USA)
- M. Piarulli (WUSTL - USA)
- R. Schiavilla (ODU & JLab - USA)

THANK YOU!