

Numerical modeling of magma mixing dynamics

Deepak Garg

Paolo Papale, Antonella Longo, Chiara Montagna and many others

Istituto Nazionale di Geofisica e Vulcanologia

Pisa, Italy

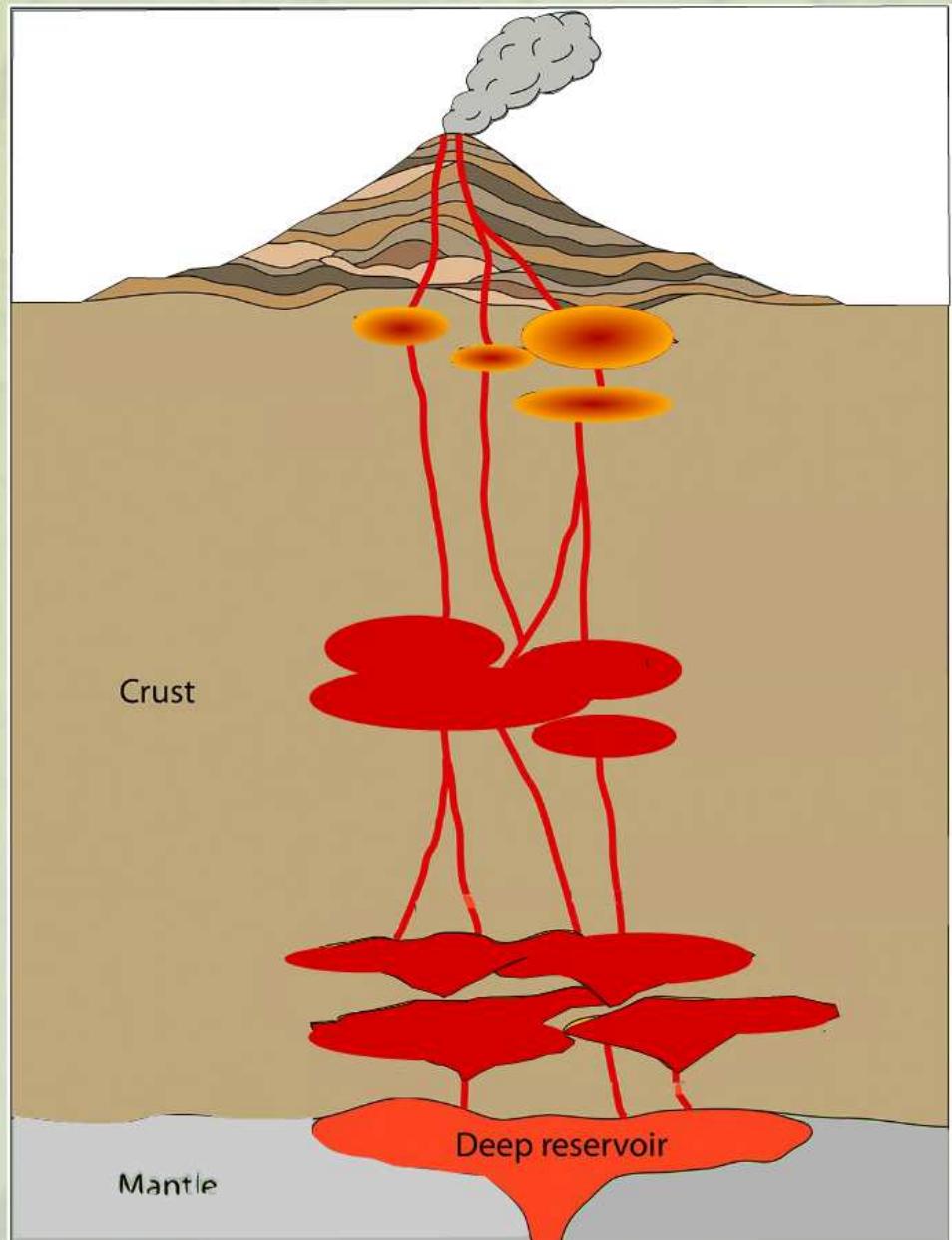
deepak.garg@ingv.it

Modeling magma and volcano dynamics: the challenges

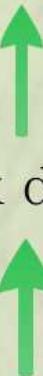
- Complex domain
- multi-component materials
- Non-linear dynamics
- Wide spatial scales
- Wide relevant temporal scales

⋮

- A substantial portion of a volcanic system is NOT accessible to direct observation

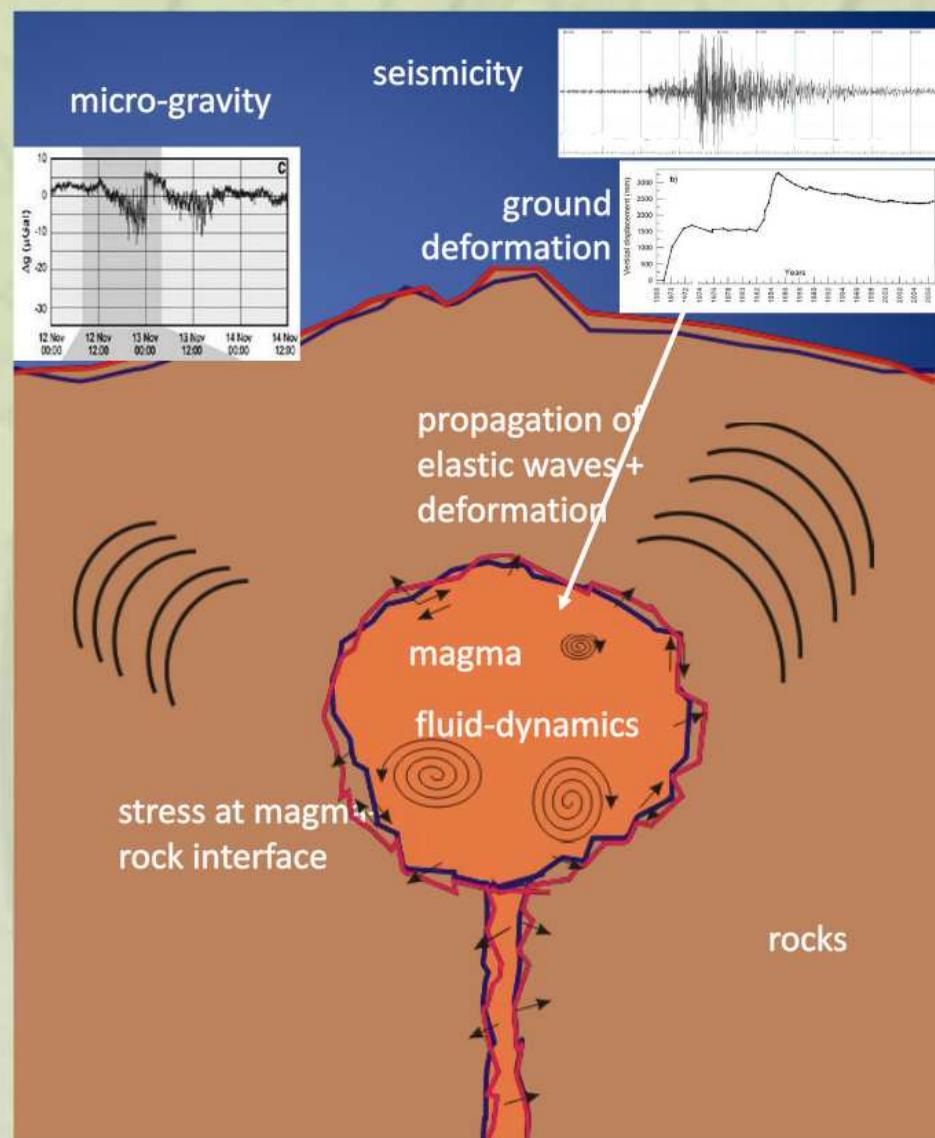


Signals



Rock deformation

Magma dynamics



- Computation of magma dynamics + rock dynamics + their mutual interaction
- Obtainment of synthetic signals and their comparison with monitoring data

We have been continuously developing a C++ finite element code GALES.

GALES can compute

- Compressible/incompressible flow for single/multi-component fluids
- Flow over deforming domains and with free surfaces
- Solid deformation for elastic, hyperelastic and visco-elastic materials
- Fluid-solid interaction

Fundamental transport equations for fluid flow

Eulerian formulation, best suited for fluid systems

Mass
$$\frac{\partial(\rho \mathbf{Y})}{\partial t} + \nabla \cdot (\rho \mathbf{Y} \otimes \mathbf{v}) = \nabla \cdot \mathbf{J},$$

mass change effect of mass diffusion

For each component

Momentum
$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g},$$

momentum change stress and body forces

For mixture

Energy
$$\frac{\partial(\rho E)}{\partial t} + \nabla \cdot (\rho E \mathbf{v}) = \nabla \cdot (\boldsymbol{\sigma} \mathbf{v} - \mathbf{q} - (\mathbf{h}' \mathbf{J})') + \rho(\mathbf{g} \cdot \mathbf{v}),$$

Energy change work against forces + heat exchange

- Constitutive equations for stress, heat flux and mass diffusion flux

Numerical method

- The equations are solved with **Galerkin LEast Squares** time-discontinuous space-time stabilized FEM.

Compact form of eqns. $\mathbf{U}_{,t} + \mathbf{F}_{i,i}^a = \mathbf{F}_{i,i}^d + \mathcal{F}$

- Weak form

$$\int_{Q_n} \mathbf{W}^h \cdot \left(\mathbf{U}_{,t}(\mathbf{Y}^h) + \mathbf{F}_{i,i}^a(\mathbf{Y}^h) - \mathbf{F}_{i,i}^d(\mathbf{Y}^h) - \mathbf{S}\mathbf{Y}^h \right) dQ_n \\ + \int_{\Omega(t_n^+)} \mathbf{W}^h(t_n^+) \cdot \left(\mathbf{U}(\mathbf{Y}^h(t_n^+)) - \mathbf{U}(\mathbf{Y}^h(t_n^-)) \right) d\Omega + \text{Stabilization} = 0$$

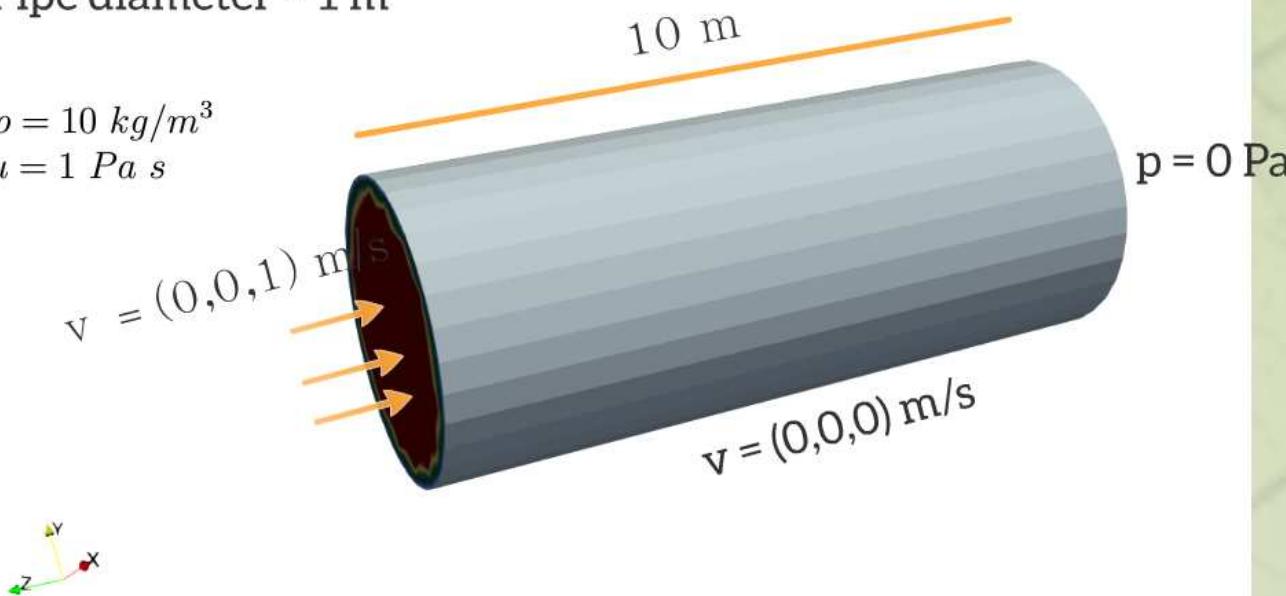
- Non-linear system of equations are solved by an implicit predictor-corrector method.
- The eqs. are solved for p, v, T, Y [Longo et al. 2012, Garg et al. 2018 (a,b)].
- The numerical approach is applicable to both compressible and incompressible flows.

Pipe flow

Pipe diameter = 1 m

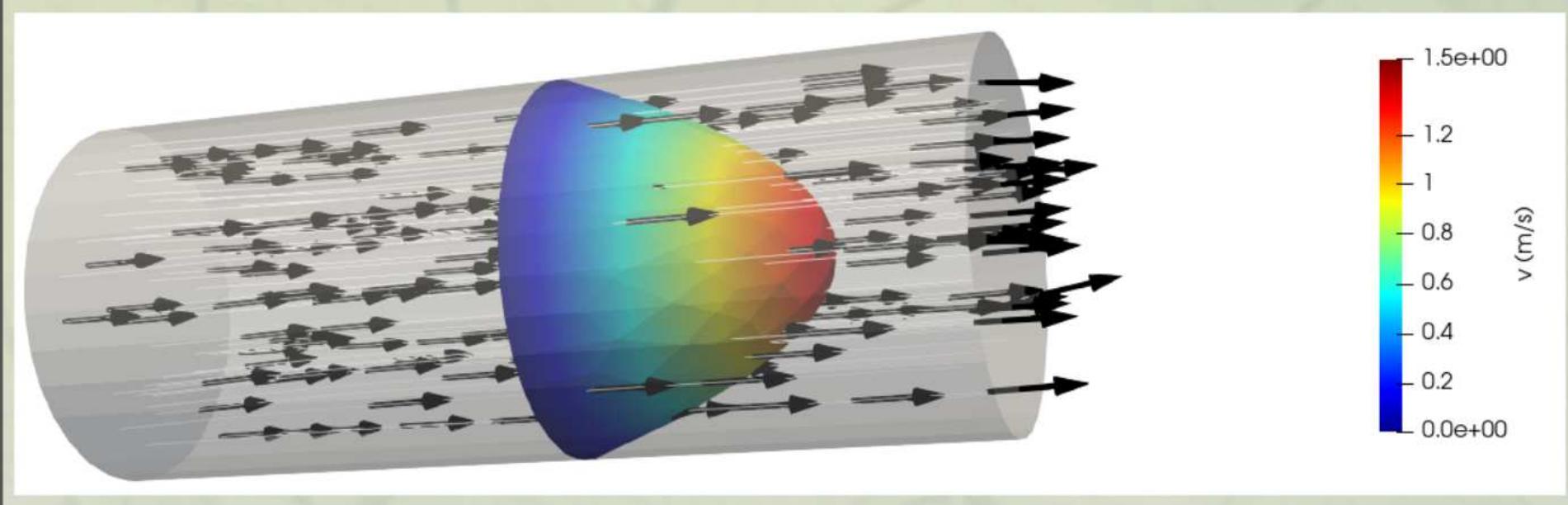
$$\rho = 10 \text{ kg/m}^3$$

$$\mu = 1 \text{ Pa s}$$



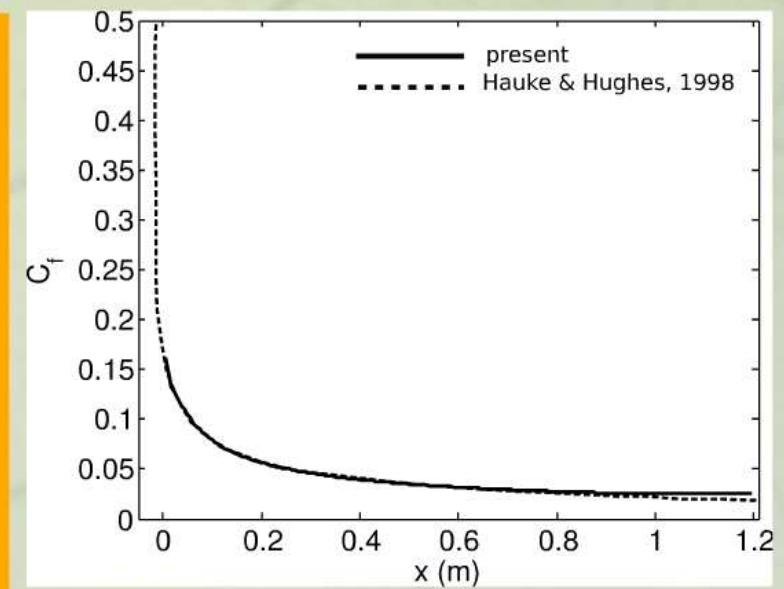
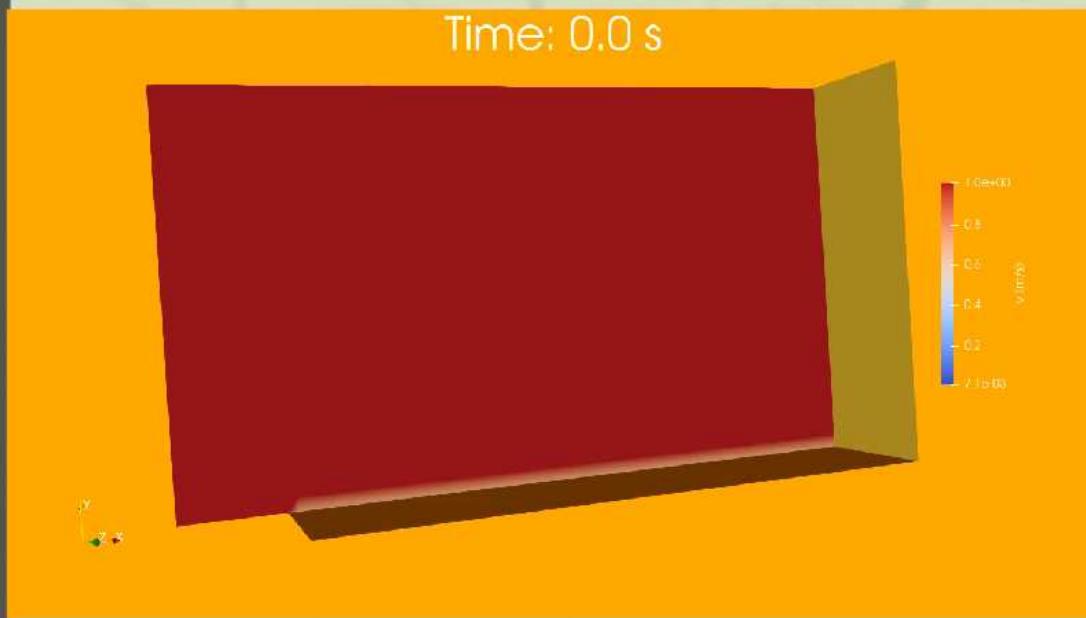
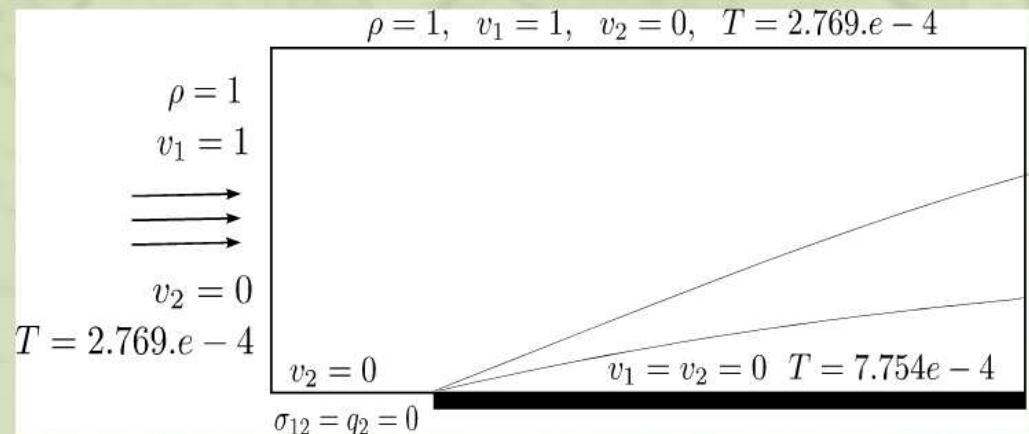
$$v(r) = \frac{-R^2}{4\mu} \frac{dp}{dz} \left(1 - \frac{r^2}{R^2}\right)$$

$$v_{max} = \frac{-R^2}{4\mu} \frac{dp}{dz}$$



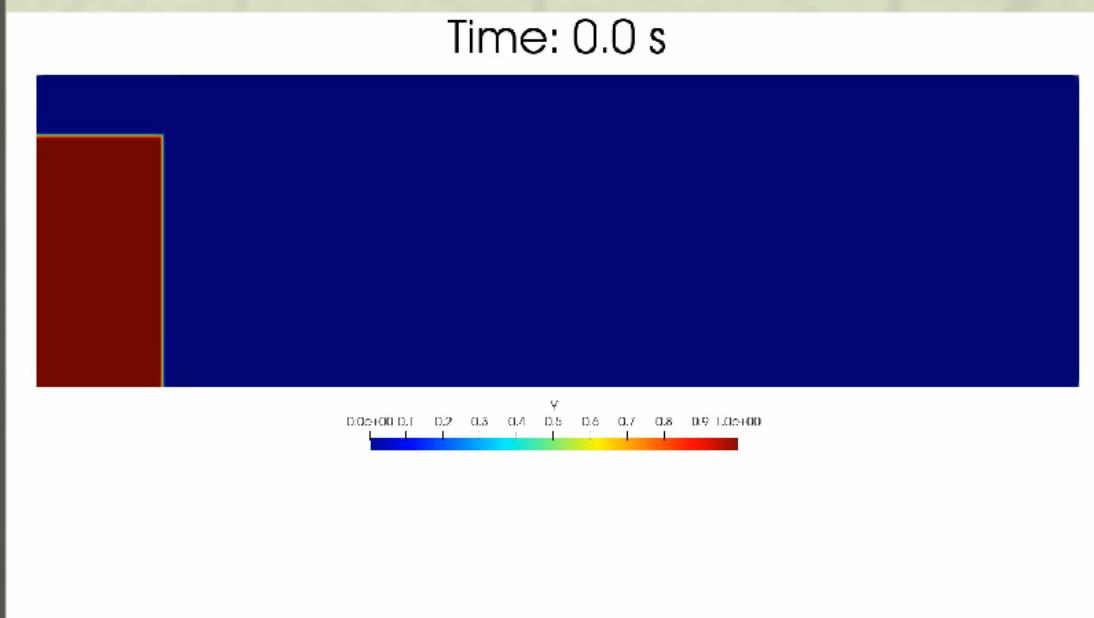
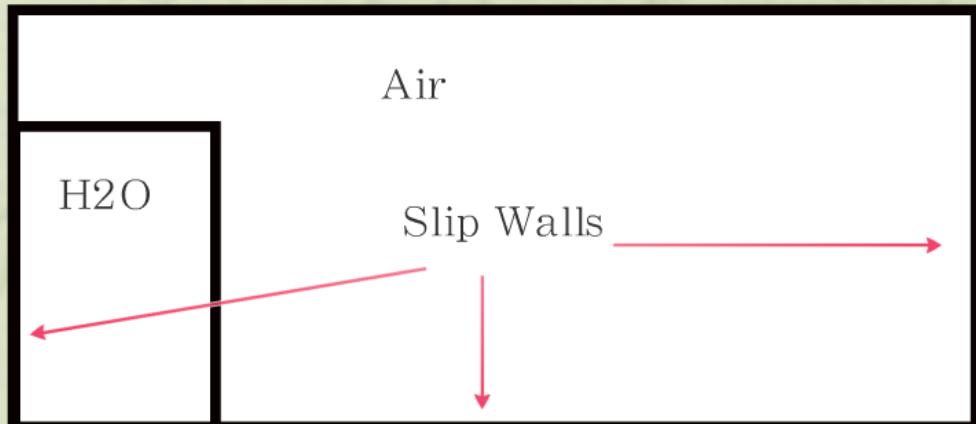
Flow over a flat plate

- Mach 3 flow
- Shock
- boundary layer



Broken dam

$p = 1 \text{ atm}$



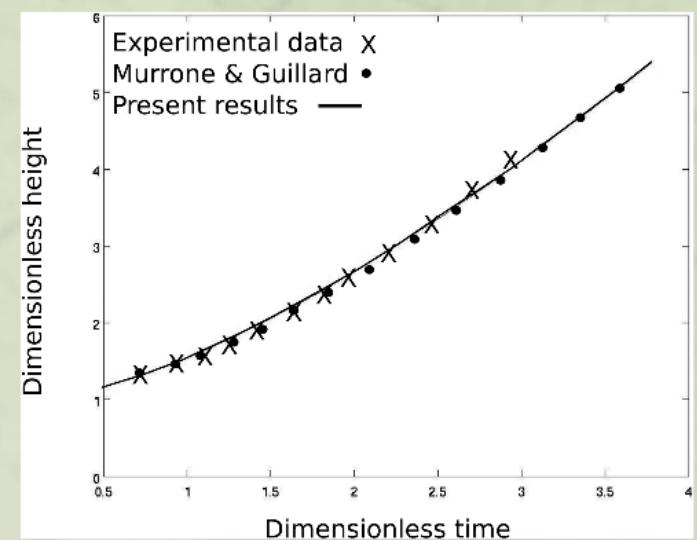
Air

$$\rho = 1.0 \text{ kg/m}^3$$
$$\mu = 10^{-5} \text{ Pa s}$$

H₂O

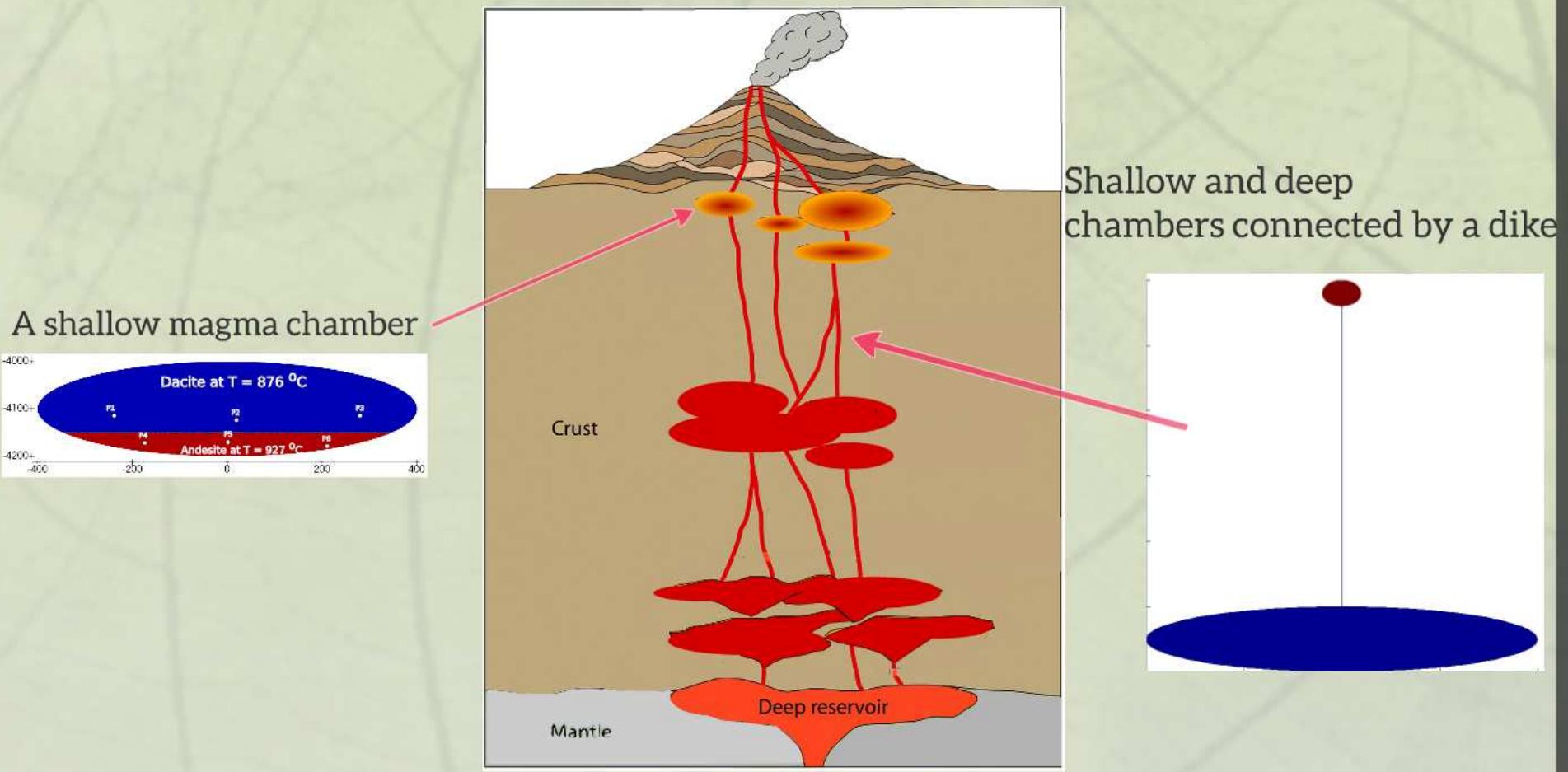
$$\rho = 1000 \text{ kg/m}^3$$
$$\mu = 0.001 \text{ Pa s}$$

$$g = 9.81 \text{ m/s}^2$$



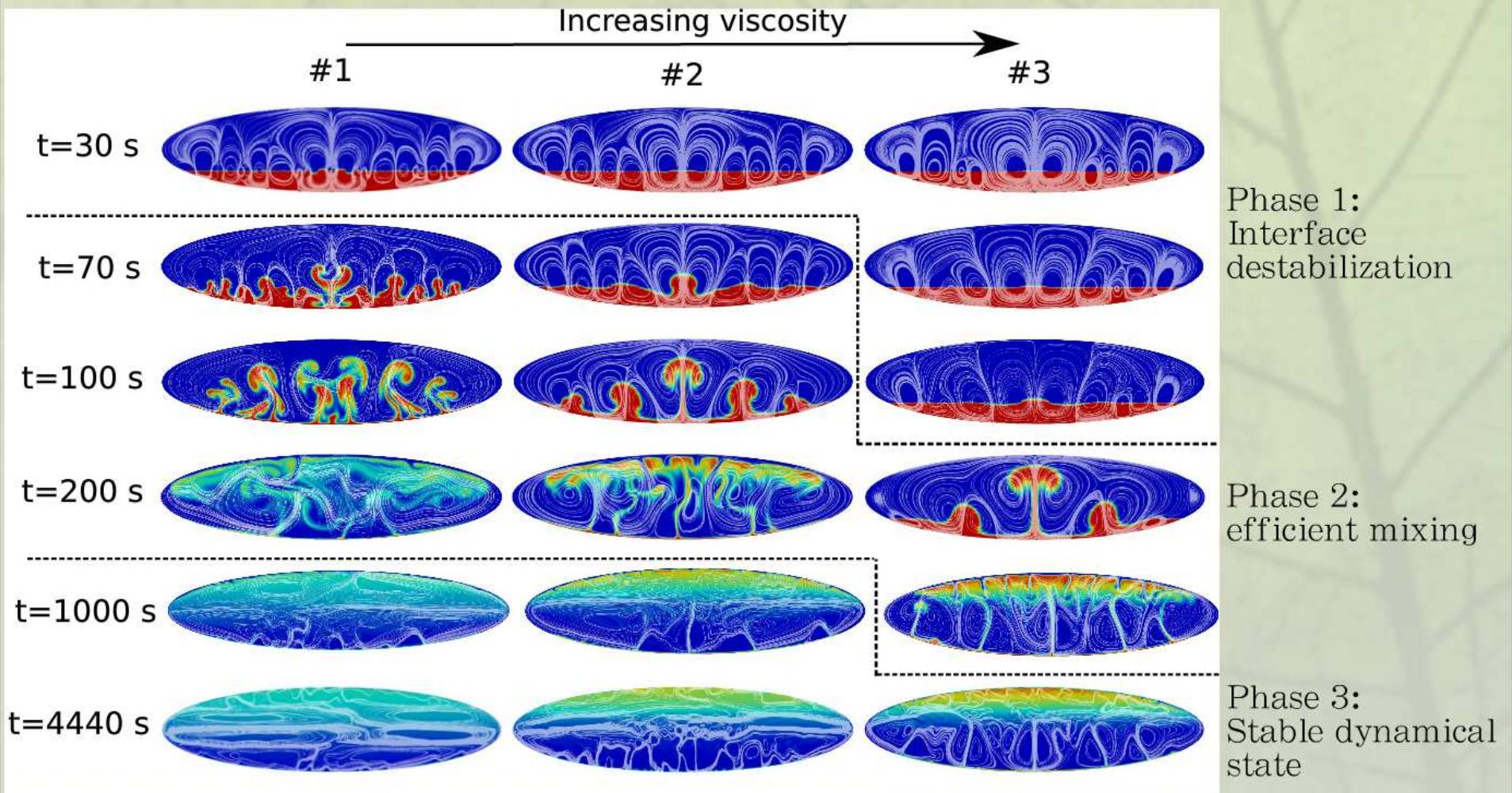
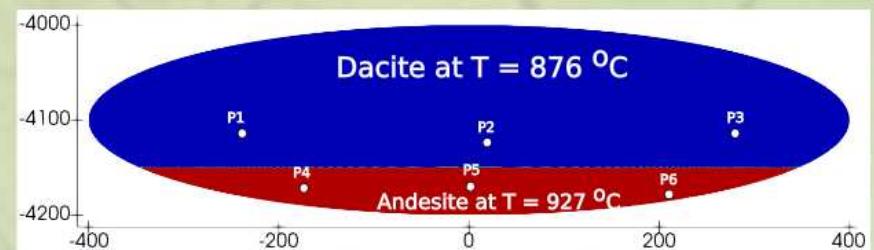
Volcano dynamics

- Melt phase with 10 major oxides, and a given amount of volatiles ($H_2O + CO_2$)
- P-T-X dependent melt density, volatile distribution and rheology
- Crystals + gas bubbles accounted for (1 velocity field)
- All properties and thermodynamics are locally (space-time) computed

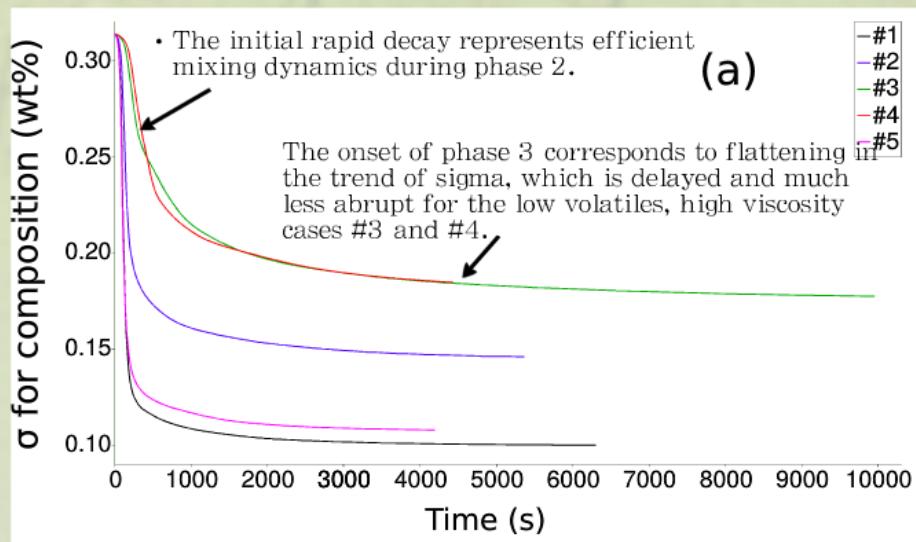


Magma mixing in shallow chamber

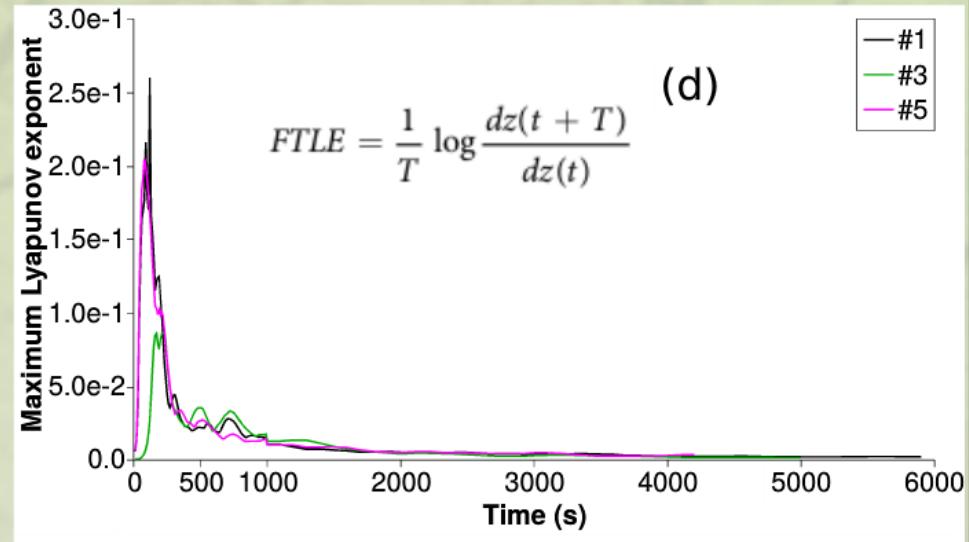
- Varying viscosity and volatile contents
- H₂O = 2-6 wt%, CO₂ = 0.1-3 wt%
- Viscosities (up to 10-fold increase)
- Natural convection



- In phase 3 no significant compositional evolution is further observed (a stable dynamic equilibrium state)
- Local stretching is quickly hindered everywhere



(a)

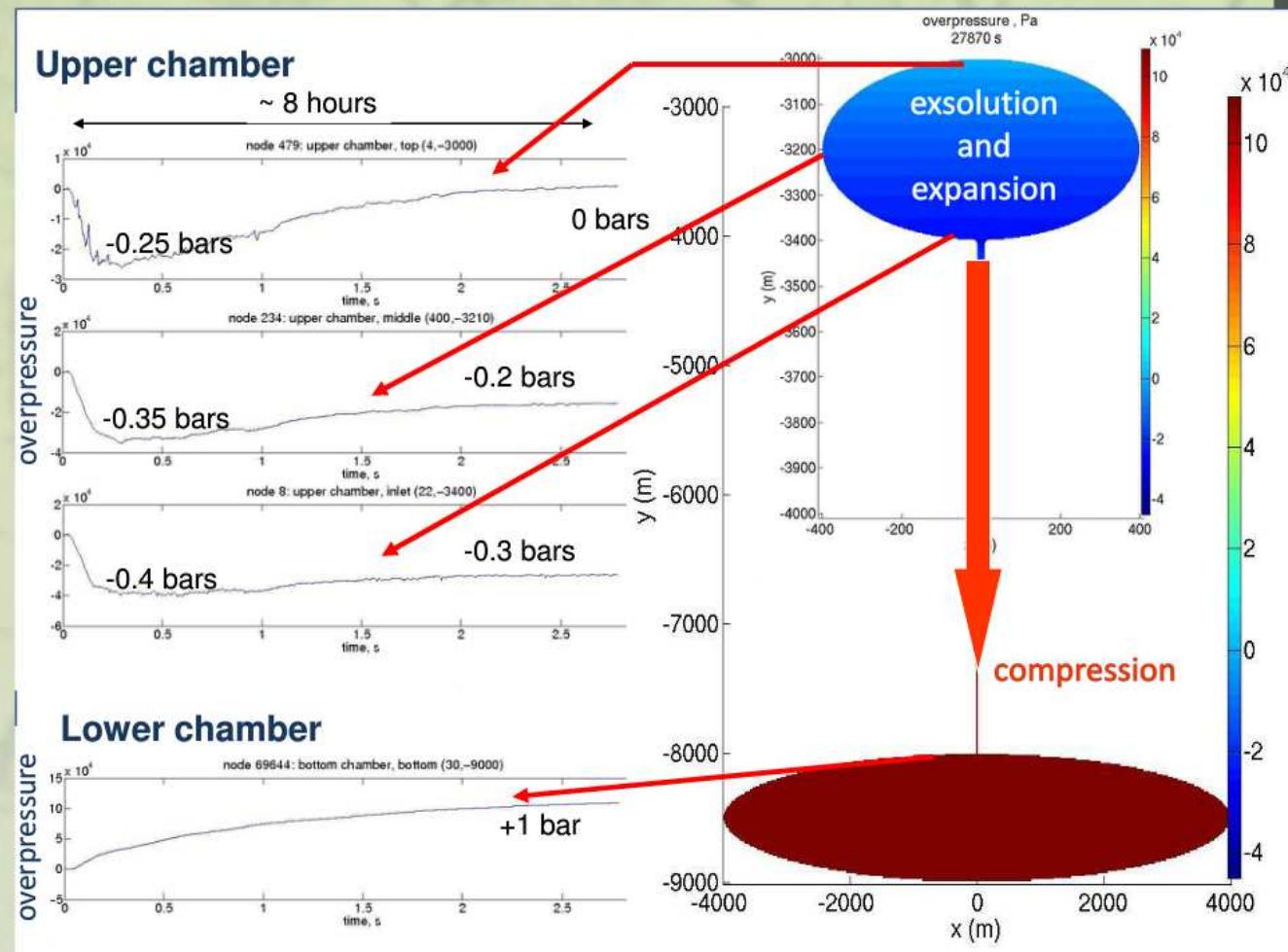
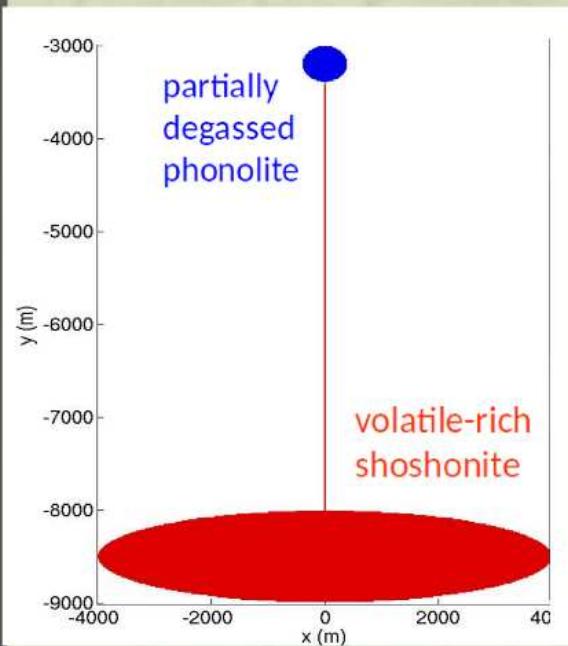


(d)

Some insights

- Compositionally different magmas can coexist for long time in a shallow chamber
- Magma mixing over short time scales can be a consequence, and not necessarily a trigger, of a volcanic eruption

Magma mixing in extended system



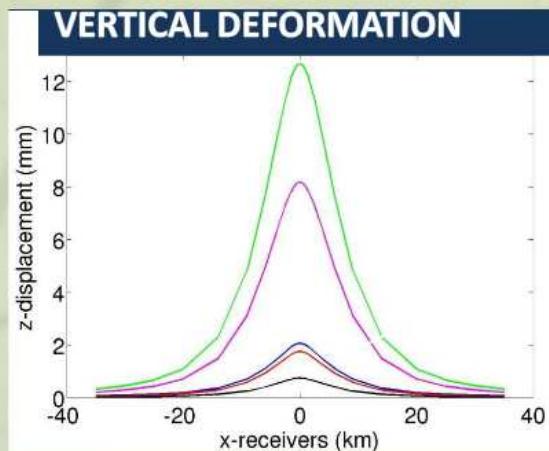
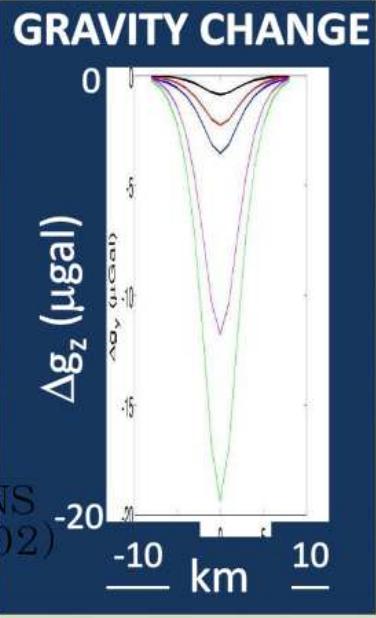
Interplay between gas expansion and mass loss

Gas expansion \uparrow \longrightarrow Pressure \uparrow

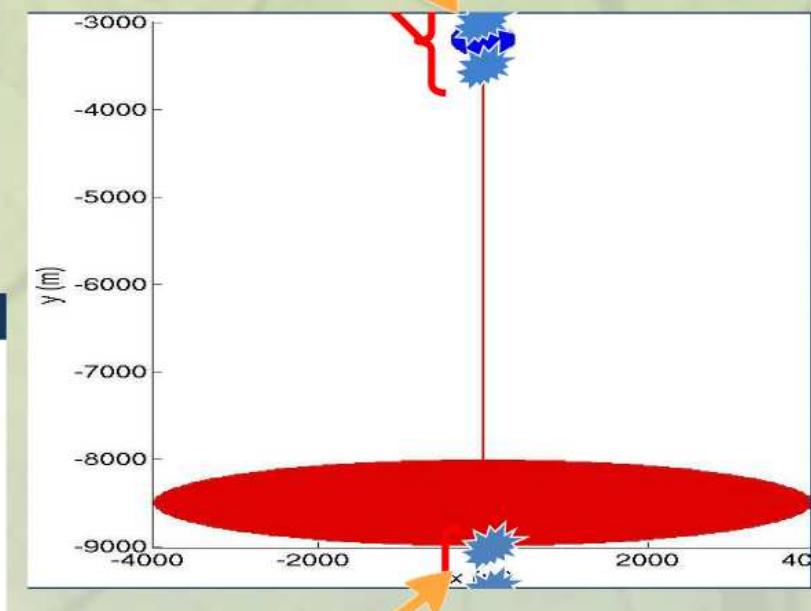
Mass \downarrow \longrightarrow Pressure \downarrow

GALES

GREEN' S FUNCTIONS
(Aki and Richards, 2002)



Source from inversion
of gravity change

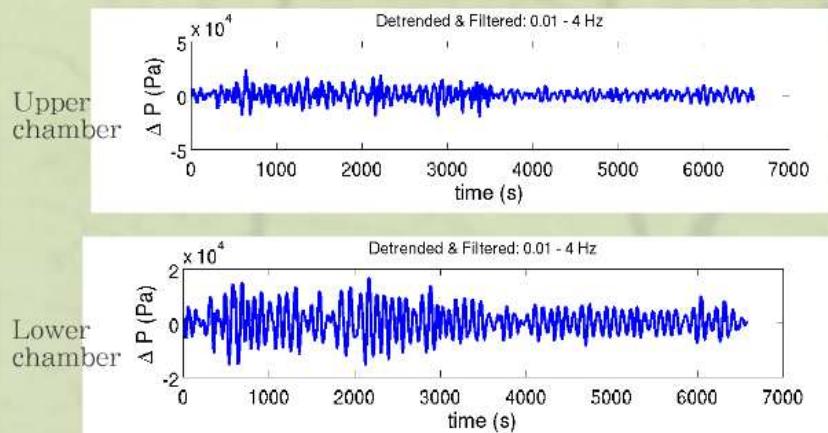


(Mogi model)
Source from inversion
of ground deformation

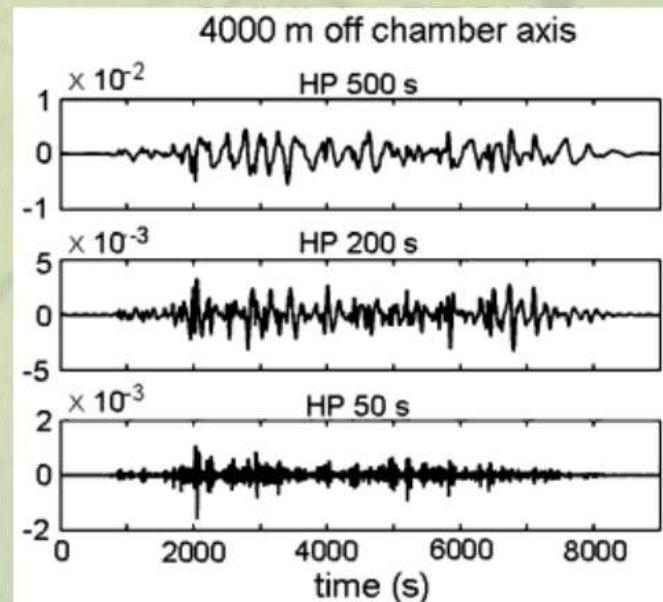
Insights

- Inversion of ground displacement and micro-gravity changes can provide a more complete picture of the magmatic system complexities

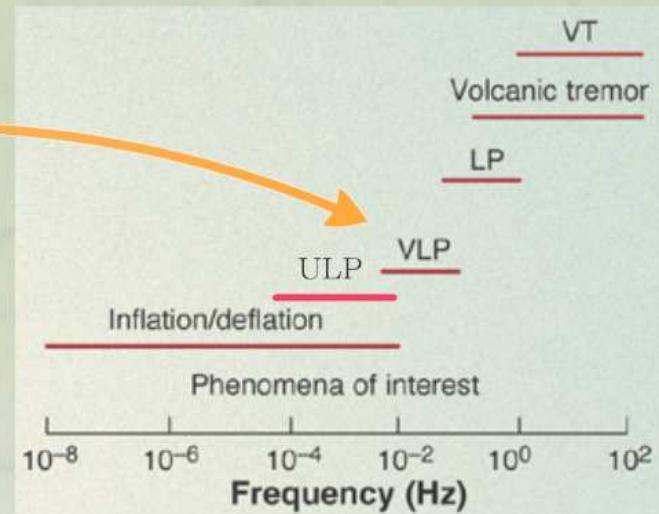
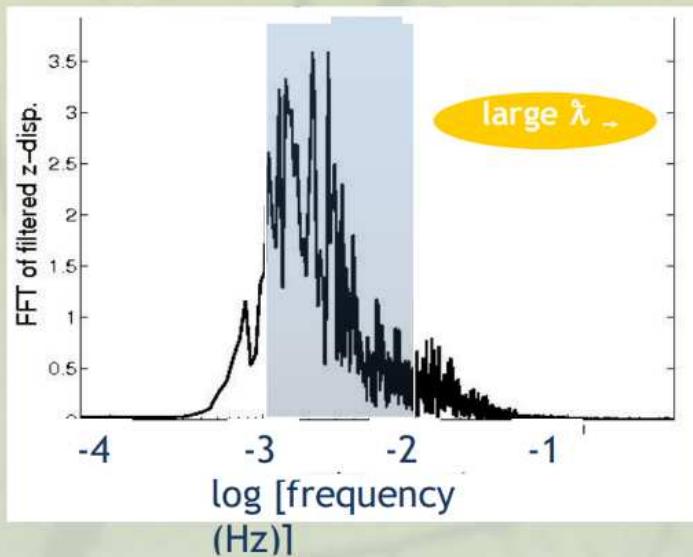
Pressure signals



Ground deformation



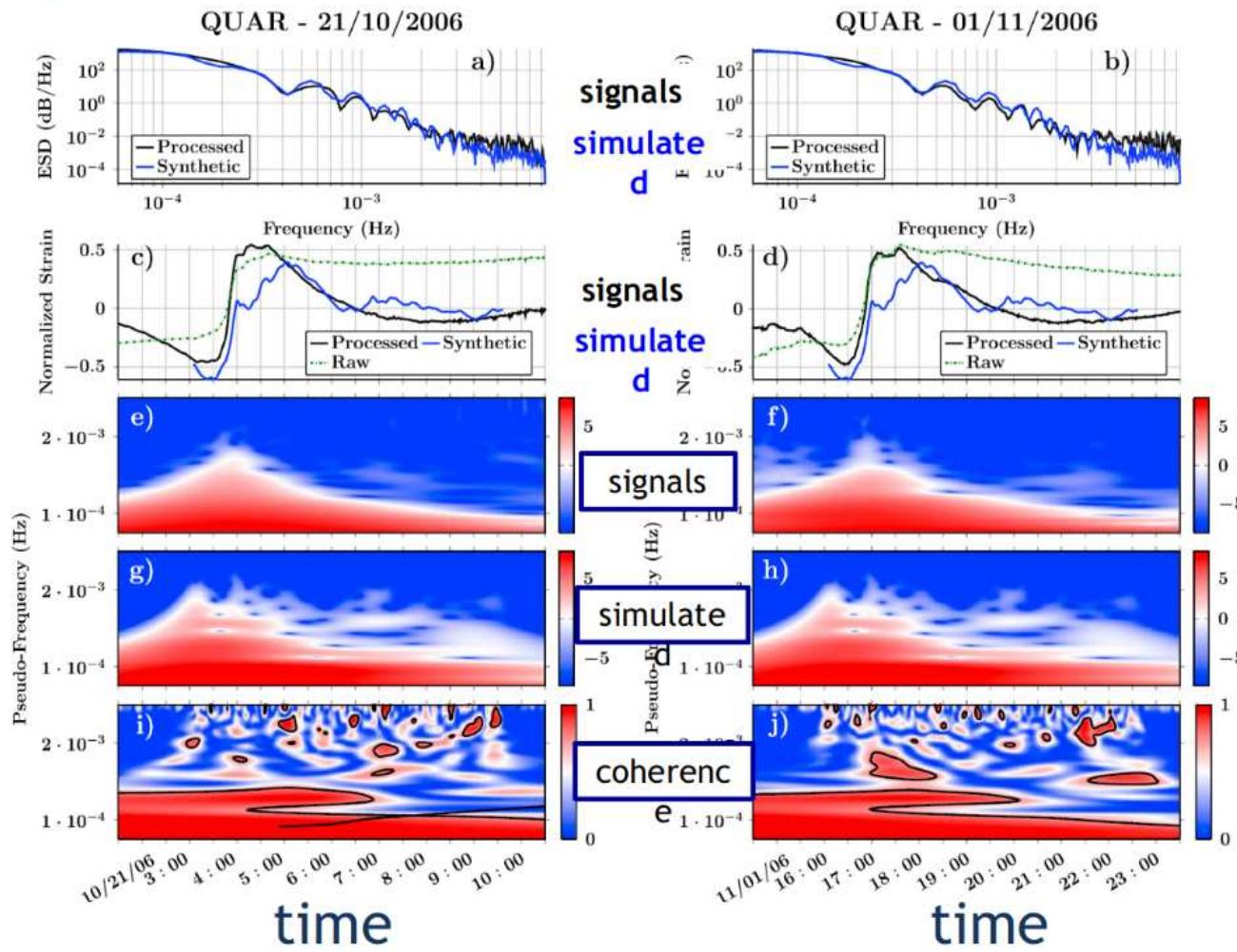
- Ground shaking energy concentrates in periods of 100 to 1000 s : ULP ground oscillations



Fourier spectra

Time series of strain

Continuous wavelet transform spectrogra m



- ULP signals like those predicted from modeling are revealed in strainmeter records at Campi Flegrei

Fluid flow with deforming domains and free surface

- Eulerian  Arbitrary Lagrangian Eulerian (ALE)
- Automatic mesh deformation method (elastostatic equations)

$$\nabla \cdot \boldsymbol{\sigma} = 0 \quad \text{on } \Omega$$

$$\boldsymbol{\sigma} = \lambda \operatorname{tr}(\boldsymbol{\epsilon}) \mathbf{I} + 2\mu \boldsymbol{\epsilon}$$

$$\boldsymbol{\epsilon} = (\nabla \mathbf{u} + \nabla \mathbf{u}^T)/2$$

Weak formulation for FEM

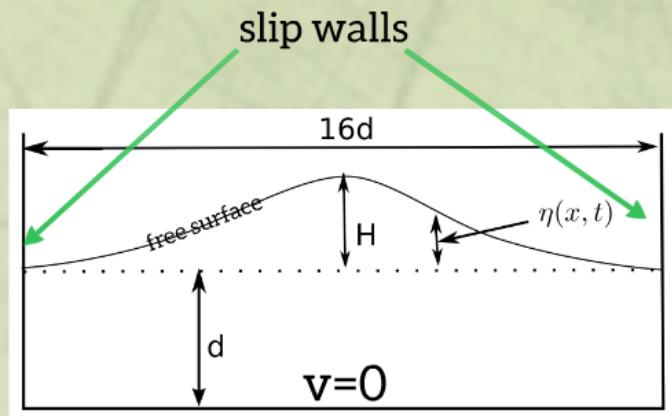
$$\int_{\Omega} \boldsymbol{\epsilon}(w) : \boldsymbol{\sigma}(u) \, d\Omega = \int_{\Gamma} w \cdot \mathbf{h} \, d\Gamma$$

\mathbf{u} is mesh deformation

Algorithm:



Solitary wave propagation



$$\rho = 1 \text{ kg/m}^3$$

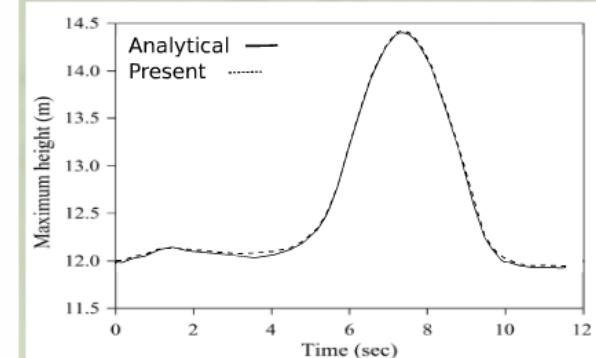
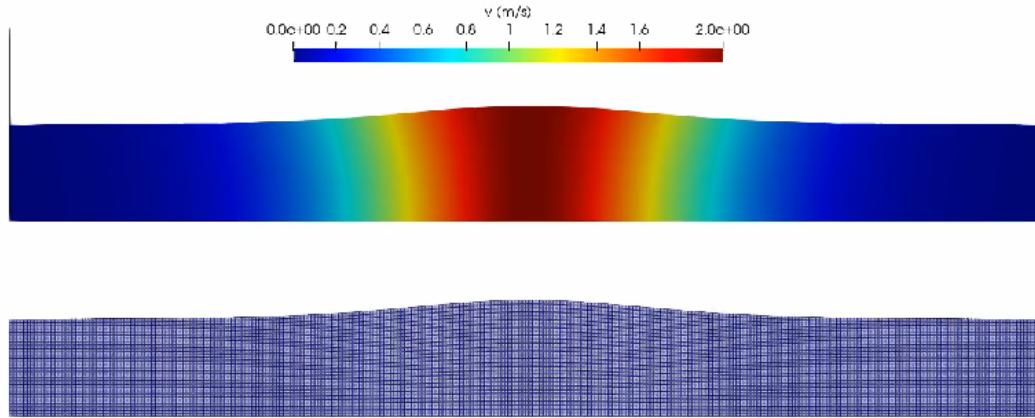
$$\mu = 1 \text{ kg/ms}$$

$$g = 9.81 \text{ m/s}^2$$

$$\eta = d + H \operatorname{sech}^2 \left(\sqrt{\frac{3H}{4d^3}} x \right)$$

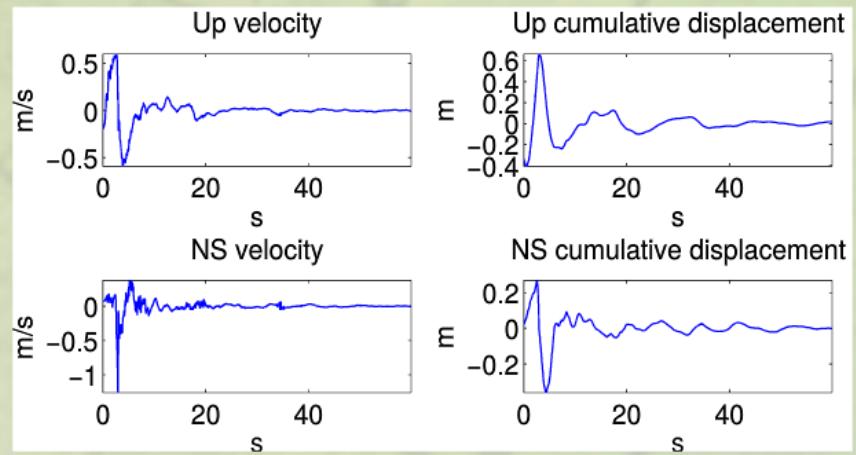
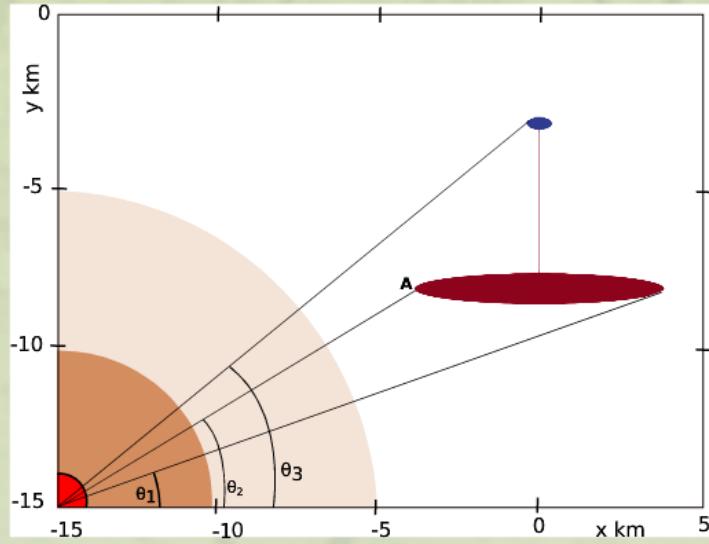
$$d = 10 \text{ m}, \quad H = 2 \text{ m}$$

Time: 0.0 s

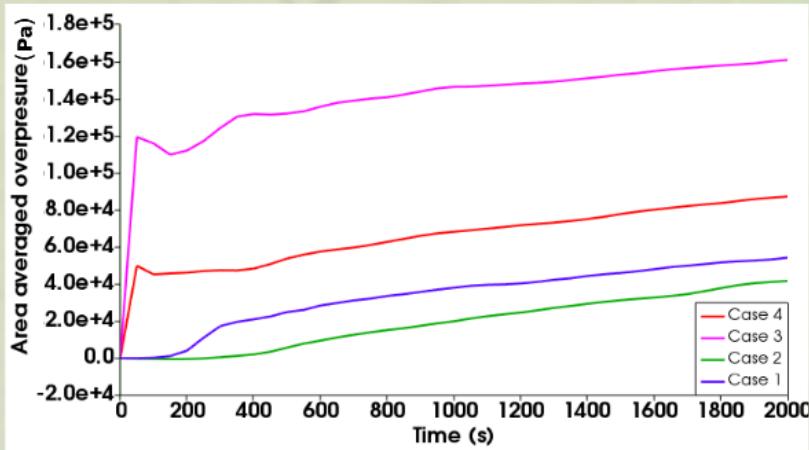


An application with moving meshes: Earthquake - Magma interaction

Problem statement



Results



A positive overpressure in chamber
Earthquake Trigger? → Volcanic eruption

Solid deformation

Elastodynamic equations in Lagrangian frame

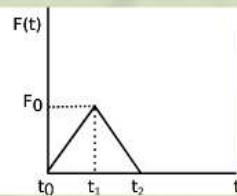
$$\rho_0 \mathbf{a} = \rho_0 \mathbf{f} + \nabla \cdot \mathbf{P}$$

\mathbf{a} – acceleration
 ρ_0 – density
 \mathbf{P} – stress tensor

FEM formulation:

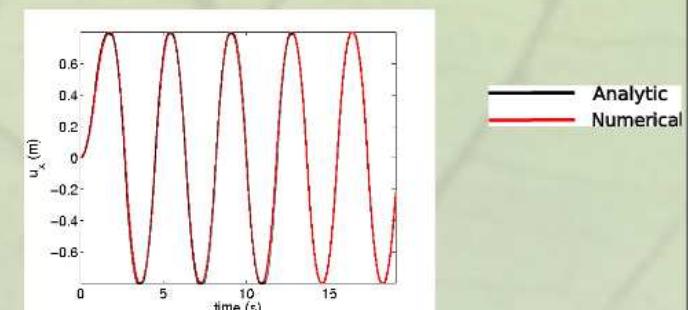
$$\int_{\Omega_0^s} \mathbf{w}^h \rho_0 \frac{\partial^2 \mathbf{d}^h}{\partial t^2} d\Omega + \int_{\Omega_0^s} \nabla \mathbf{w}^h : \mathbf{P}^h d\Omega - \int_{\Omega_0^s} \mathbf{w}^h \mathbf{f}^h d\Omega - \int_{\Gamma_0^s} \mathbf{w}^h \mathbf{P} \mathbf{N} d\Gamma = 0$$

Uniaxial bar

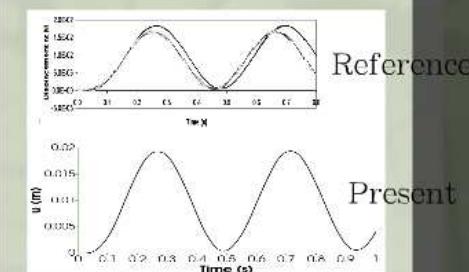
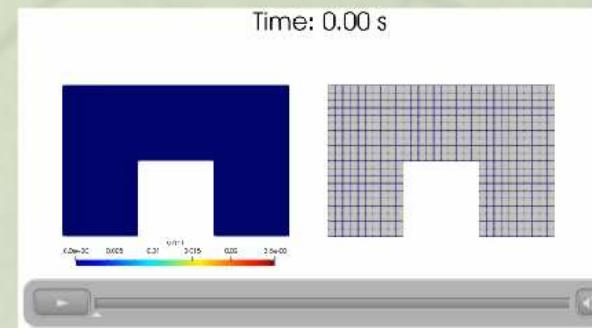
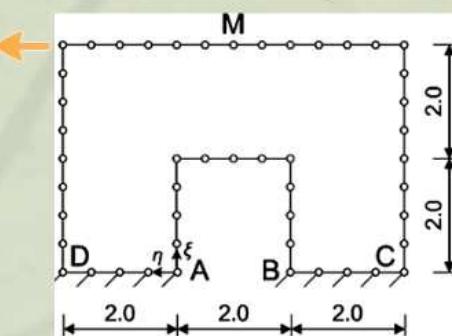
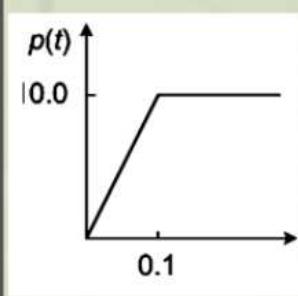


Parameters:
Young's modulus $E = 1.0$
Density $\rho = 1.0$

Area $A = 1.0$
Length $L = 1.0$
Peak load $F_0 = 1.0$
Triangular Pulse:
 $t_0 = 0, t_1 = 0.875, t_2 = 1.75$

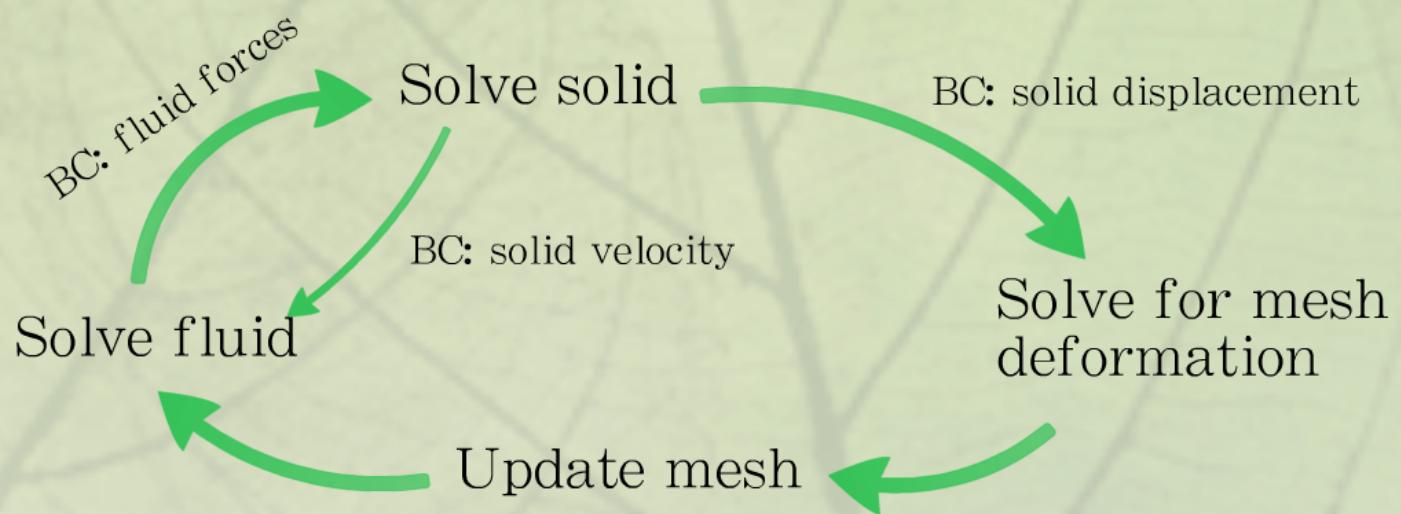


Frame like structure



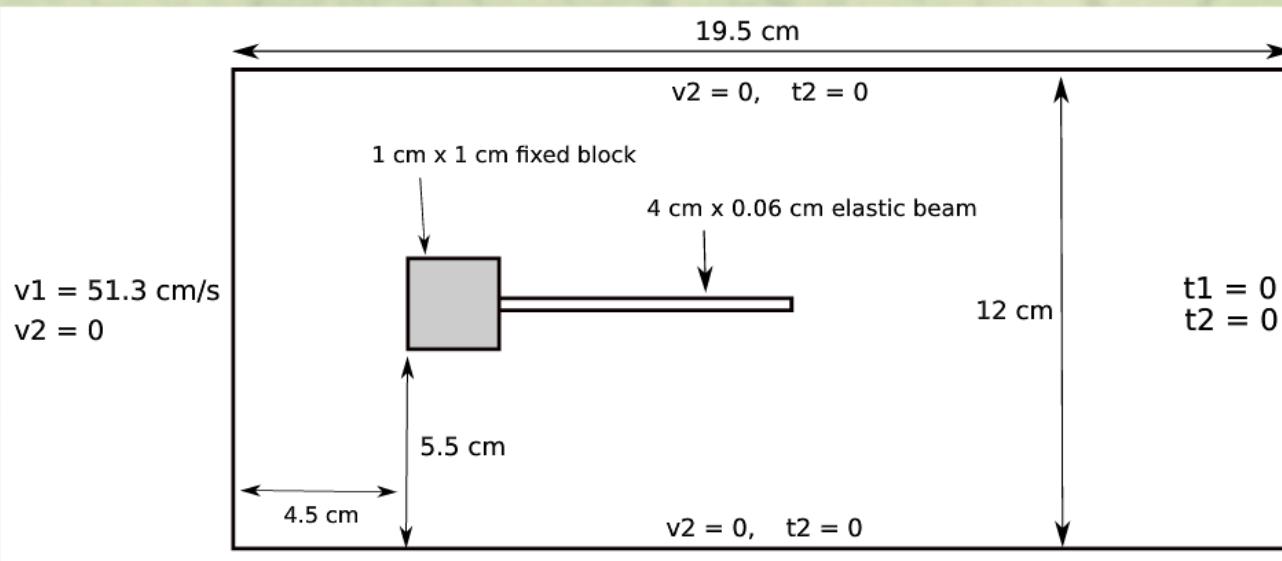
FSI - Fluid-solid (magma-rock) interaction

Algorithm:



- Fluid forces cause solid deformation.
- Solid deformation cause fluid domain deformation + change in flow field
- Two-way coupling

Flow past a flexible beam



Cantilever

$$\rho_s = 100 \text{ kg/m}^3$$

$$\nu_s = 0.35$$

$$E = 2.5 \times 10^5 \text{ Pa}$$

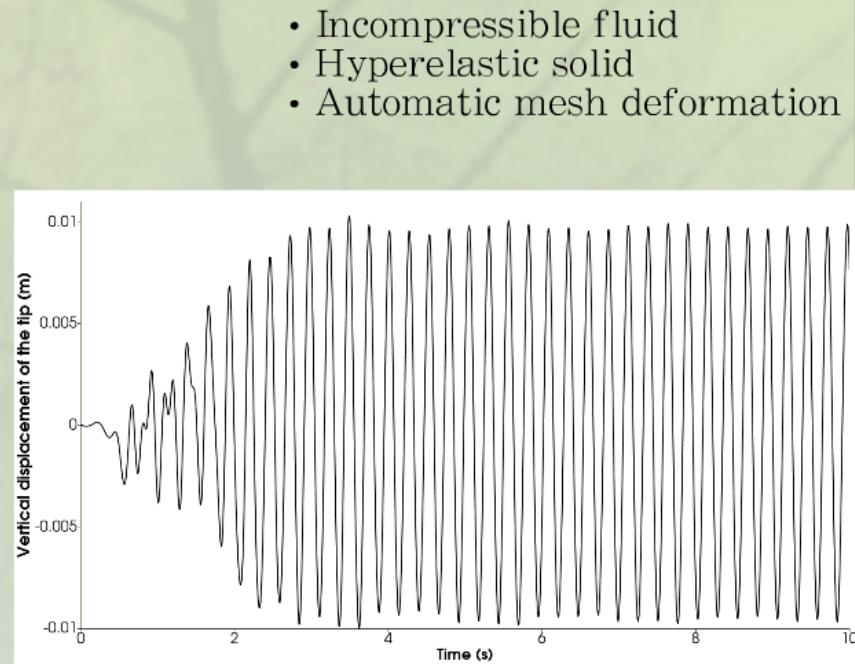
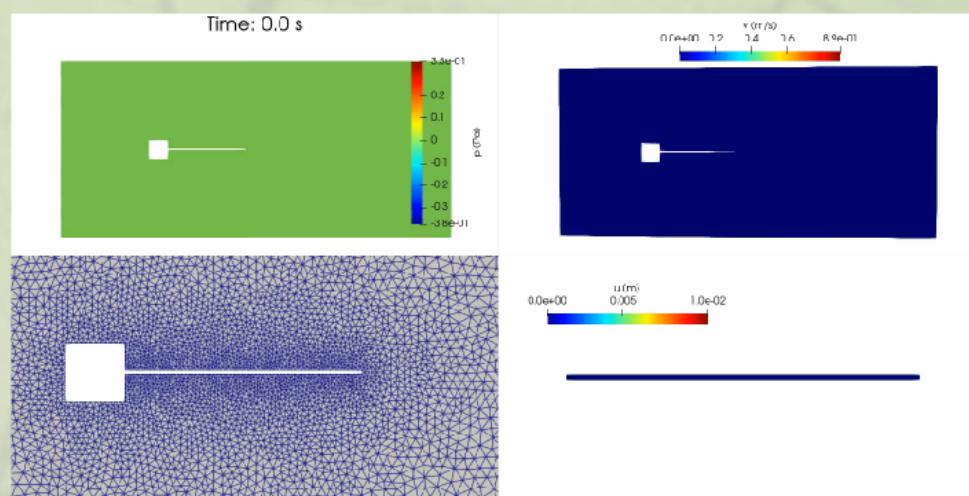
Fluid & Flow

$$\rho_f = 1.18 \text{ kg/m}^3$$

$$\nu_f = 1.54 \times 10^{-5} \text{ m}^2/\text{s}$$

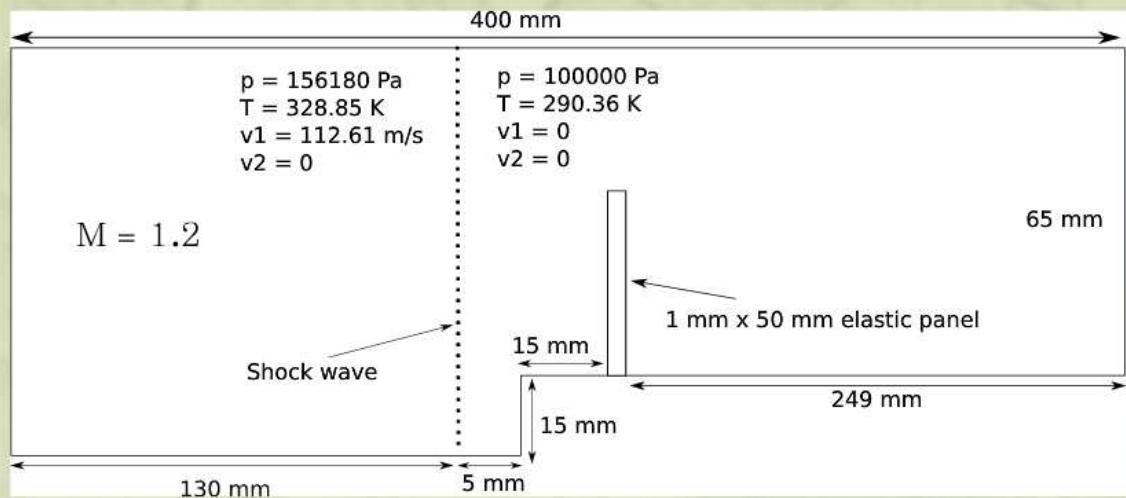
$$v_f = 0.513 \text{ m/s}$$

$$\text{Re} = 333$$



- Incompressible fluid
- Hyperelastic solid
- Automatic mesh deformation

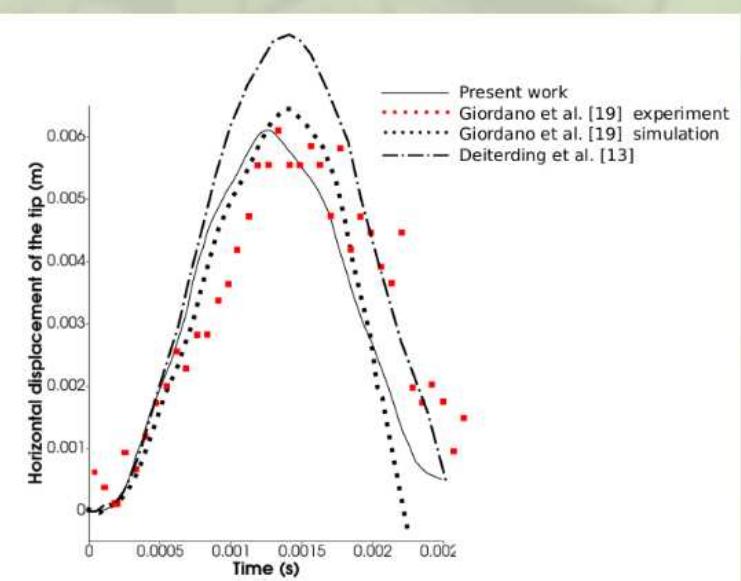
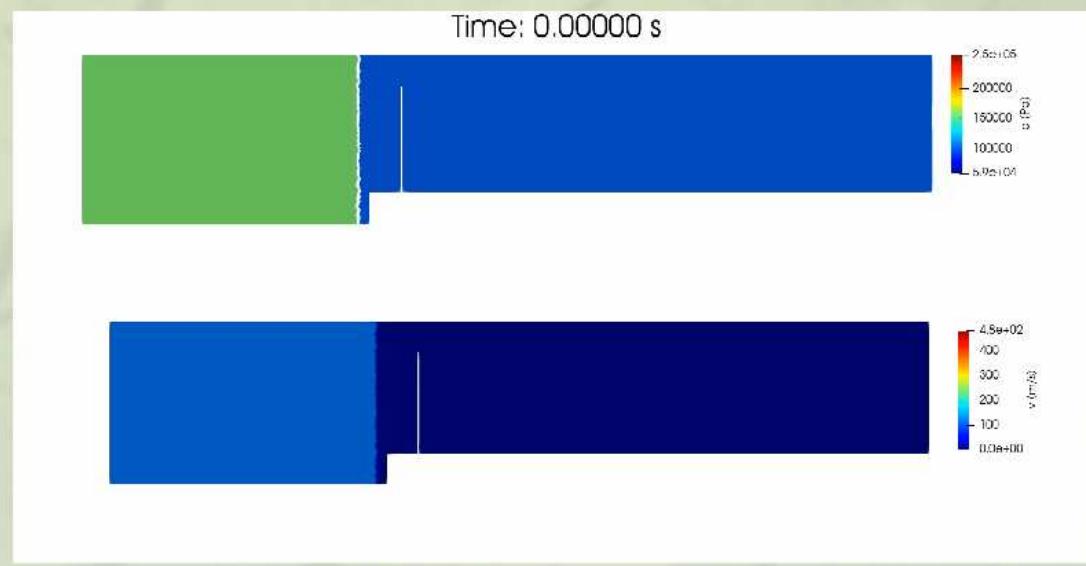
Blast wave interacting with flexible beam



Material: air
EOS: ideal gas

Elastic panel:
 $\rho = 7600 \text{ kg/m}^3$
 $E = 220 \text{ GPa}$
 $\nu = 0.3$

- Compressible air + shock
- Hyperelastic solid
- Automatic mesh deformation



Ongoing and future developments

- Two-way coupled magma-rock interaction
 - Comparison of synthetic signals with monitoring data
 - Early warning system for volcanic eruptions
-
- Multiphase (separated flow)
 - Variational multi-scale modeling
 - Crack propagation
-
- Code optimization
 - GALEs open source ([github](#))

Thank you !!!

