High precision perturbative QCD predictions for Large Hadron Collider physics

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Milan University & INFN Milan

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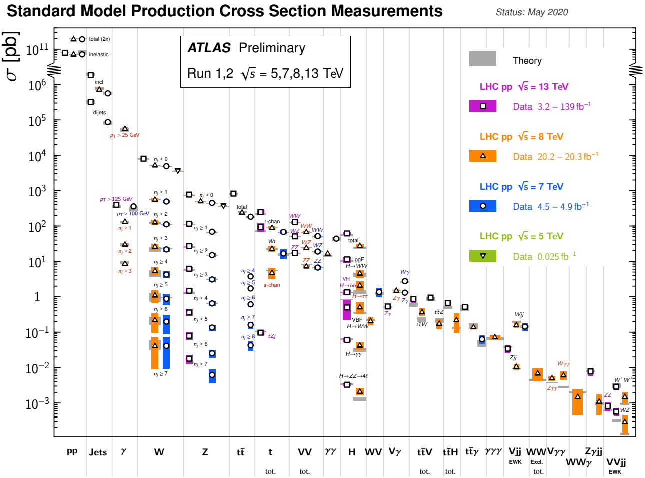
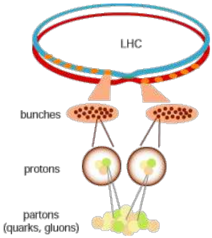
LHC key results: Higgs boson and much more



- Very good agreement between experimental results and SM theoretical predictions for hard processes (see M. Cobal talk).

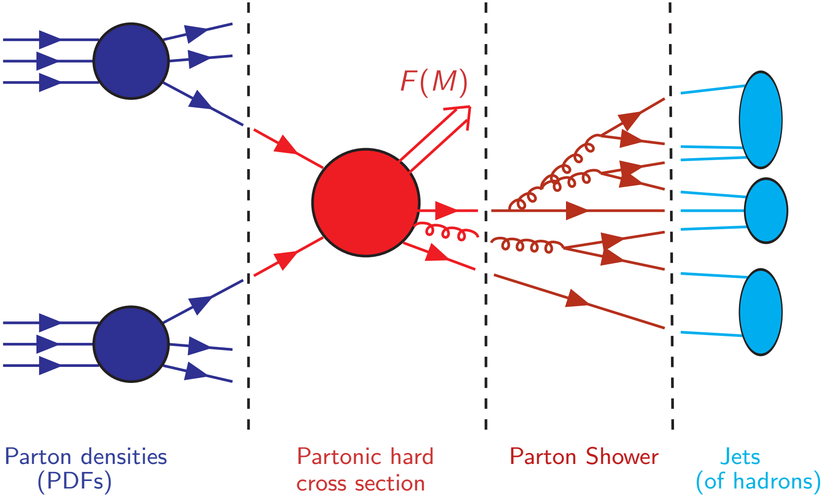
How to **increase the discovery power of the LHC?**

The LHC is a hadron collider machine: all the interesting reactions initiate by QCD hard scattering of partons: a good control of the QCD processes is necessary.

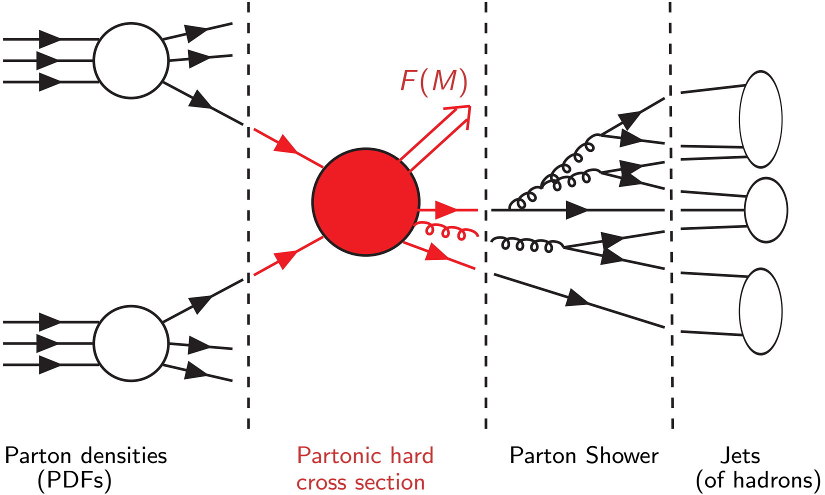


To fully exploit the information contained in the LHC experimental data, **precise theoretical predictions** of the Standard Model cross sections are needed.

Theoretical predictions at the LHC

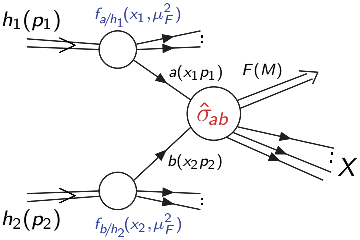


Partonic Hard Cross Section



QCD factorization

$h_1 + h_2 \;\to\; F(M) + X$



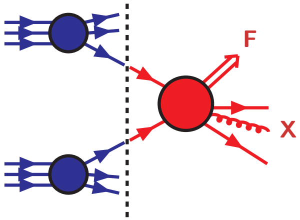
The framework: **QCD factorization formula**

$$\sigma^F_{h_1h_2}(p_1,p_2) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \hat{\sigma}^F_{ab}(x_1p_1, x_2p_2;\mu_F^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{M}\right)^p$$

- $f_{a/h}(x,\mu_F^2)$: Non perturbative universal parton densities (PDFs), $\mu_F \sim M$.
- $\hat{\sigma}_{ab}$: Hard scattering cross section. Process dependent, calculable with a perturbative expansion in the strong coupling $\alpha_S(M)$ ($M \gg \Lambda_{QCD} \sim 1\,\text{GeV}$).
- $\left(\frac{\Lambda_{QCD}}{M}\right)^p$ (with $p \geq 1$): Non perturbative power-corrections.

Precise predictions for σ depend on good knowledge of both $\hat{\sigma}_{ab}$ and $f_{a/h}(x,\mu_F^2)$

Higher-order calculations



- Factorization theorem

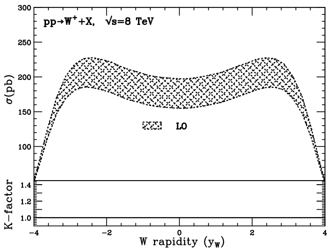
$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

- Perturbation theory at **leading order (LO)**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\, \hat{\sigma}^{(1)} + \alpha_S^2\, \hat{\sigma}^{(2)} + \mathcal{O}(\alpha_S^3)$$

- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO: precise prediction & **robust uncertainty**.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities**: impossible direct use of numerical techniques.



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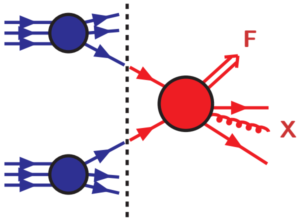

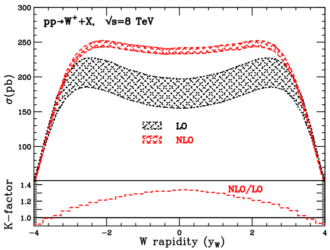

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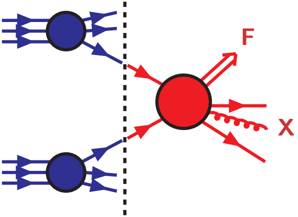
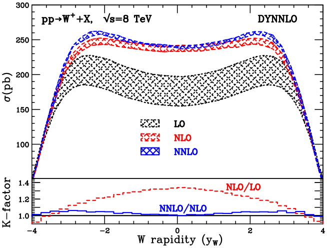


Parton densities (PDFs)

Partonic cross section $\hat{\sigma}$



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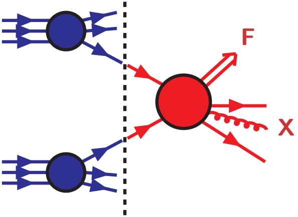

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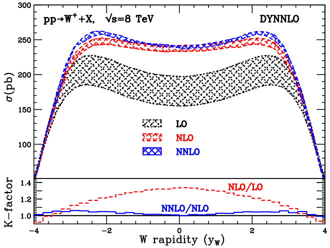

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Drell-Yan process

LO	**NLO**	**NNLO**
Drell,Yan (1974)	Altarelli,Ellis,Greco Martinelli,(1980-84)	Melnikov,Petriello; Catani,Cieri,deFlorian, G.F.,Grazzini (2007–09)

Higher orders: NLO and NNLO

- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- Beyond LO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques.
- Subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission. Fully formalized at NLO [Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]. It allows (relatively) straightforward calculations (once the QCD amplitudes are available).

$$\sigma^{\rm NLO} = \int_{m+1} d\sigma^{\rm R}(\epsilon) \; + \; \int_{m} d\sigma^{\rm V}(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^{\rm R}(\epsilon) - d\sigma^{\rm A}(\epsilon) \right]_{\epsilon=0} \; + \; \int_{m} \left[d\sigma^{\rm V}(\epsilon) + \int_{1} d\sigma^{\rm A}(\epsilon) \right]_{\epsilon=0}$$

where $\epsilon = (D-4)/2$ is the (dimensional) infrared regularazion parameter.
A fully general formalism to perform NNLO calculations is still lacking.

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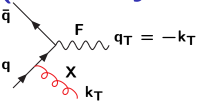
The q_T-subtraction method at NNLO (and beyond)



$$h_1 + h_2 \;\rightarrow\; F(M, q_T) + X$$

- **Key point I:** at lowest-order the q_T of the **F** is exactly zero [Catani,Grazzini('07)].

$$d\sigma^{F}_{N^nLO}|_{q_T\neq 0} = d\sigma^{F+jets}_{N^{n-1}LO}$$

At $q_T \neq 0$ the N^nLO IR divergences cancelled with a $N^{n-1}LO$ calculation.

- **Key point II**: At $q_T = 0$ the remaining singularities treated by a subtraction term $d\sigma^{CT}$ exploiting the universal all-order q_T resummation formalism [Bozzi,Catani,de Florian,G.F.,Grazzini('00,'06,'10)].
- **Key point III**: At $q_T = 0$ the *process dependent* IR finite part of the corresponding QCD amplitude included through a universal all-order formula in the function $\mathcal{H}^F$ [Catani,Cieri,de Florian,G.F.,Grazzini('13)].
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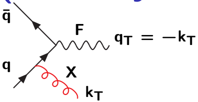

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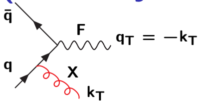

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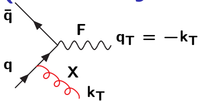

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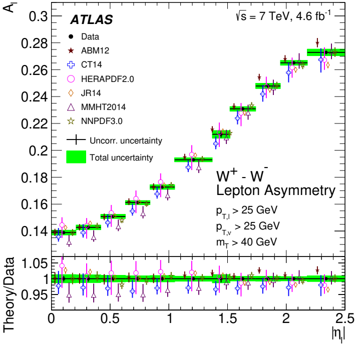
q_T subtraction at NNLO: numerical implementation

We have performed various fully exclusive NNLO QCD calculations implemented in publicly available Monte Carlo programs which includes spin correlations, interference and finite-width effects, and compute distributions in form of bin histograms:

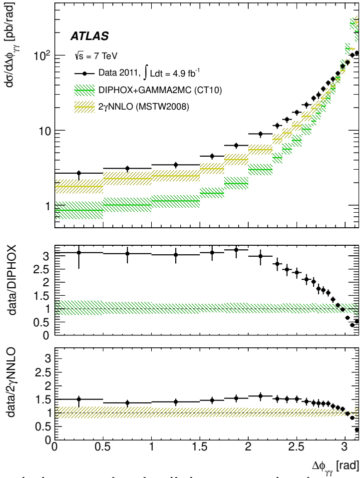
- DYNNLO vector boson ($Z/\gamma^*, W^\pm$) (Drell-Yan) production [Catani,Cieri,de Florian,G.F.,Grazzini ('09)],
- 2γNNLO diphoton production [Catani,Cieri,de Florian,G.F.,Grazzini ('12),('18)],
- HVNNLO associated Higgs and vector boson production [G.F.,Grazzini,Tramontano,Somogyi ('11),('14),('15),('18)].

http://pcteserver.mi.infn.it/~ferrera/research.html.

NNLO QCD predictions compared with LHC data

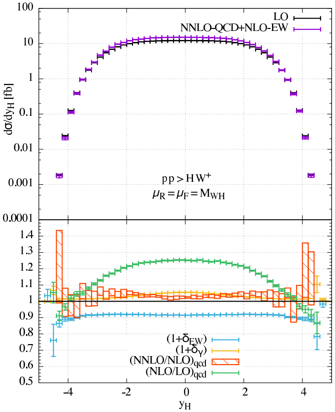


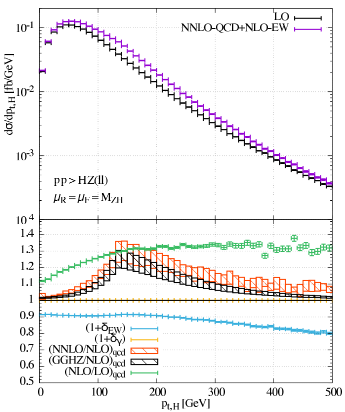
Lepton charge asymmetry from $W \to l\nu_l$ decay. Comparison between experimental data and NNLO predictions (DYNNLO [Catani,Cieri,de Florian, G.F.,Grazzini ('09),('10)]) using various PDFs (from [ATLAS Coll.('12)]).



Azimuthal separation in diphoton production. Comparison between exp. data, NLO and NNLO predictions (2γNNLO[Catani,Cieri,de Florian,G.F., Grazzini ('12,'18)]) (from [ATLAS Coll.('13)]).

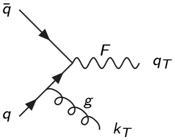
NNLO QCD predictions for H production at LHC





Left panel: rapidity of the Higgs boson in W^+H production. Right panel: p_T of the Higgs boson in ZH production. NNLO QCD prediction (HVNNLO [G.F.,Grazzini,Tramontano('11),('14),('15); G.F.,Tramontano,Somogyi('18)]) for LHC ($\sqrt{s} = 13$ TeV). From LHC Higgs Cross Section Working Group[CERN report(2016)].

All order Sudakov resummation



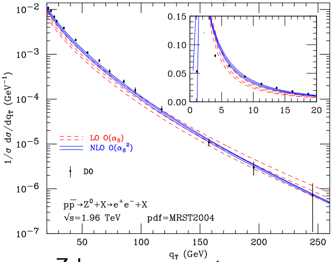
High-mass state production at small q_T ($q_T \ll M$).
The standard fixed-order QCD perturbative expansions gives:

$$\int_0^{Q_T^2} dq_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \sim 1 + \alpha_S \left[c_{12} \log^2(M^2/Q_T^2) + c_{11} \log(M^2/Q_T^2) + c_{10}(Q_T) \right]$$
$$+ \alpha_S^2 \left[c_{24} \log^4(M^2/Q_T^2) + \cdots + c_{21} \log(M^2/Q_T^2) + c_{20}(Q_T) \right] + \mathcal{O}(\alpha_S^3)$$

The logs are the residue of the cancellation of the real-virtual infrared singularities due to soft/collinear gluon emissions.

Fixed order calculation reliable only for $q_T \sim M$

For $q_T \to 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$: need for resummation of logarithmic corrections.



Z boson q_T spectrum.

Idea of large logs (Sudakov) resummation: reorganize the perturbative expansion by all-order summation ($L = \log(M^2/q_T^2)$).

$\alpha_S L^2$	$\alpha_S L$	$\cdots$	$\mathcal{O}(\alpha_S)$
$\alpha_S^2 L^4$	$\alpha_S^2 L^3$	$\cdots$	$\mathcal{O}(\alpha_S^2)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\alpha_S^n L^{2n}$	$\alpha_S^n L^{2n-1}$	$\cdots$	$\mathcal{O}(\alpha_S^n)$
dominant logs	$\cdots$	$\cdots$	$\cdots$

- Ratio of two successive rows $\mathcal{O}(\alpha_S L^2)$: fixed order expansion valid when $\alpha_S L^2 \ll 1$.
- Ratio of two successive columns $\mathcal{O}(1/L)$: resummed expansion valid when $1/L \ll 1$.

Sudakov resummation feasible when:
dynamics AND kinematics factorize $\Rightarrow$ exponentiation.

In the impact parameter (Fouier conjugated) space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \equiv L \gg 1$

$$\frac{d\hat{\sigma}}{dq_T^2} \stackrel{q_T \ll M}{=} \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}} \mathcal{H}(\alpha_S) \otimes \exp\left\{\mathcal{G}(\alpha_S, L)\right\}$$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim \alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

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$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, ($\hat{\sigma}^{(0)}$); NLL ($\sim \alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g^{(3)}$, $\mathcal{H}^{(2)}$;

Idea of large logs (Sudakov) resummation: reorganize the perturbative expansion by all-order summation ($L = \log(M^2/q_T^2)$).

$\alpha_S L^2$	$\alpha_S L$	$\cdots$	$\mathcal{O}(\alpha_S)$
$\alpha_S^2 L^4$	$\alpha_S^2 L^3$	$\cdots$	$\mathcal{O}(\alpha_S^2)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\alpha_S^n L^{2n}$	$\alpha_S^n L^{2n-1}$	$\cdots$	$\mathcal{O}(\alpha_S^n)$
dominant logs	$\cdots$	$\cdots$	$\cdots$

- Ratio of two successive rows $\mathcal{O}(\alpha_S L^2)$: fixed order expansion valid when $\alpha_S L^2 \ll 1$.
- Ratio of two successive columns $\mathcal{O}(1/L)$: resummed expansion valid when $1/L \ll 1$.

Sudakov resummation feasible when:
dynamics AND kinematics factorize $\Rightarrow$ exponentiation.

In the impact parameter (Fouier conjugated) space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \equiv L \gg 1$

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$\mathbf{q_T}$ resummation: numerical implementations

- We have implemented the q_T resummed calculation in various publicly available codes:

HqT/DYqT: computes Higgs and vector boson $(Z/\gamma^*, W^\pm)$ resummed q_T spectrum, inclusive over other kinematical variables

[Bozzi,Catani,de Florian,G.F.,Grazzini('06,'09,'11,'12)]

HRes/DYRes: computes Higgs and vector boson $(Z/\gamma^*, W^\pm)$ resummed q_T spectrum and related distributions retaining the full kinematics of the boson and of its decay products (possible to apply arbitrary cuts on these variables, and to plot the corresponding distributions)

[Catani,de Florian,G.F.,Grazzini,Tommasini('11,'15)]

http://pcteserver.mi.infn.it/~ferrera/research.html.

DYTurbo: Optimised version of DYqT, DYRes (and DYNNLO) with significant enhancement in time performance. Main application W mass measurement at the LHC.
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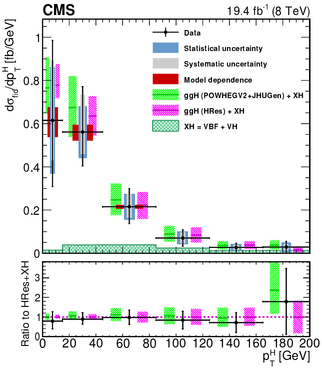
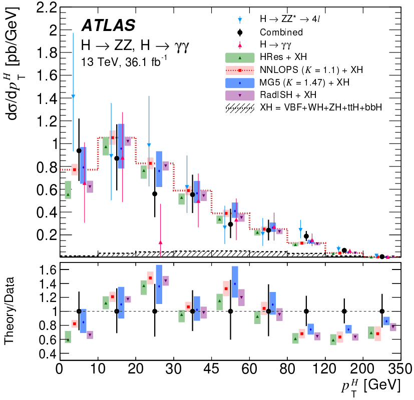
Higgs results: q_T-resummation with H boson decay

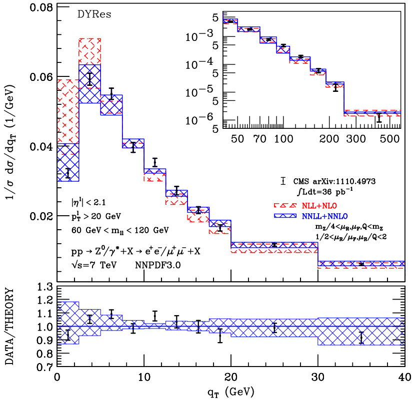


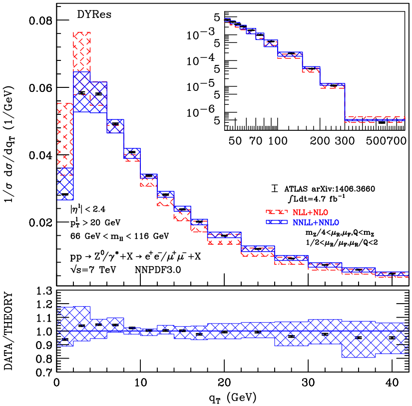
H q_T spectrum ($H \to WW$): theory predictions (HRes [de Florian, G.F., Grazzini, Tommasini ('12)]) compared with CMS data (from [CMS Coll. ('16)]).
Lower panel: ratio to theory (HRes).



H q_T spectrum ($H \to \gamma\gamma$): various theory predictions (HRes [de Florian, G.F., Grazzini, Tommasini ('12)]) compared with ATLAS data (from [ATLAS Coll. ('18)]).
Lower panel: ratio to theory (HRes).

Drell–Yan production: q_T spectrum of Z boson at the LHC



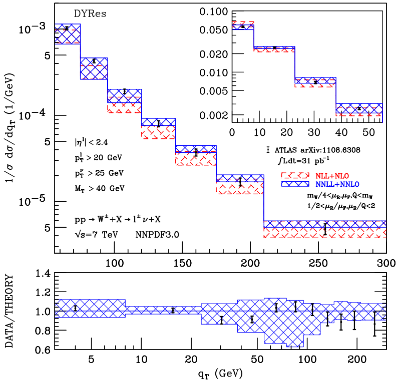


NNLL+NNLO q_T resummation for the Drell–Yan process implemented in public numerical codes DYqT/DYRes [Bozzi,Catani,G.F,de Florian,Grazzini('08,'10)],

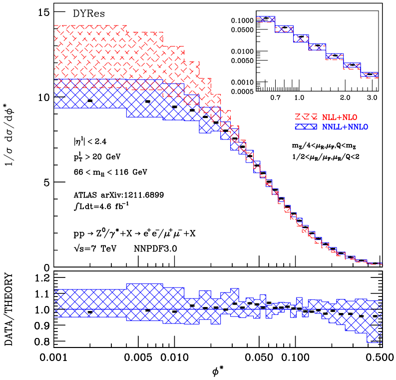
NLL+NLO and NNLL+NNLO bands for Z/γ^* q_T spectrum compared with CMS (left) and ATLAS (right) data.

Lower panel: ratio with respect to the NNLL+NNLO central value.

DY results: q_T spectrum of W and ϕ^* spectrum of Z boson at the LHC



NLL+NLO and NNLL+NNLO bands for $W^\pm$ q_T spectrum compared with ATLAS data.
Lower panel: ratio with respect to the NNLL+NNLO central value.

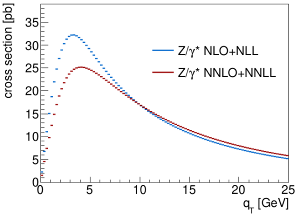


NLL+NLO and NNLL+NNLO bands for Z/γ^* ϕ^* spectrum compared with ATLAS data.
Lower panel: ratio with respect to the NNLL+NNLO central value.

Fast predictions for Drell-Yan processes: DYTurbo

[Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott ('19)]

Example calculation



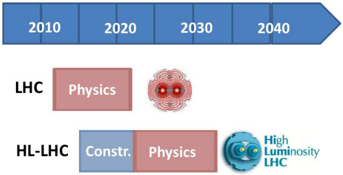
- Example calculation for Z p_T spectrum at 13 TeV
  - No cuts on the leptons
  - Full rapidity range
  - 100 p_T bins
  - 20 parallel threads

Time required	RES	CT	V+jet
NLO+NLL	6 s	0.2 s	4 min
NNLO+NNLL	10 s	0.7 s	3.4 h

- The most demanding calculation is V+jet

  → can use APPLgrid/FASTnlo for this term

Conclusions



- Very precise experimental data have been collected by the LHC and much more data are forthcoming with the High-Luminosity (HL-LHC) upgrade.

- To fully exploit the information contained in the experimental data precise theoretical predictions of the SM cross sections are necessary ⇒ computation of **higher-order and all-order pQCD corrections**.
- Discussed the formalisms necessary to perform NNLO QCD calculation and all order resummation which have been applied to some important processes at the LHC.