INGV Istituto Nazionale di Geofisica e Vulcanologia

Studying the rheology of geophysical flows with physical-mathematical models: An application of the GPUSPH particle engine

G. Bilotta[1], V. Zago[1,2], A. Herault[1,3], A. Cappello[1], G. Ganci[1], C. Del Negro[1]

[1]INGV-Italy [2]NW-USA [3]CNAM-France

L'Aquila, 2019-09-25

INGV Istituto Nazionale di Geofisica e Vulcanologia

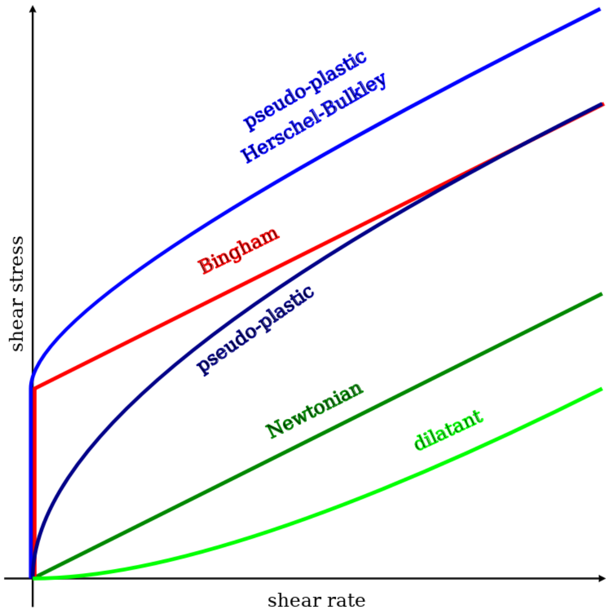
Geophysical flows

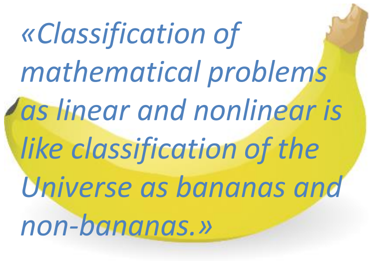
- Examples:
  - landslides
  - lahars
  - lava flows
- Complex flows with one or more of:
  - Multiple phases (fluid/solid)
  - Non-Newtonian rheology
  - Strong thermal effects
    - Temperature-dependent rheology
    - Phase transition

INGV Istituto Nazionale di Geofisica e Vulcanologia

Rheology

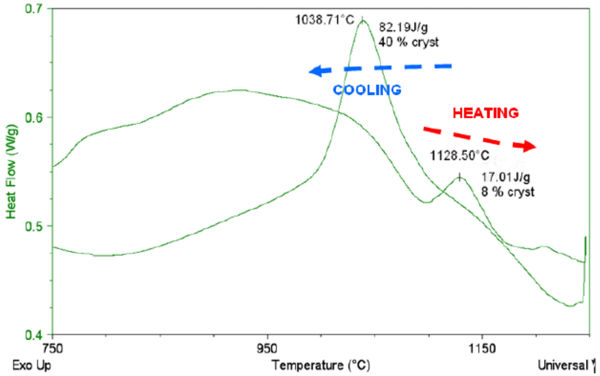
- Expresses the stress/strain(rate) relationship, determines how the fluid flows
- Newtonian rheology: classic linear relationship, parameter: μ (dynamic viscosity)
- Non-Newtonian: anything else. E.g.:
  - Bingham: Newtonian rheology with yielding (rigid-body behavior if stress less than yield strength). Parameters: μ_0, τ_0
  - Power-law rheology: stress is related to a power n of the strain rate;
    - Also polynomial, e.g. $\tau = \mu_1\dot{\gamma} + \mu_2\dot{\gamma}^2$
  - Herschel–Bulkley: power-law with yield strength
  - Exponential (De Kee & Turcotte)

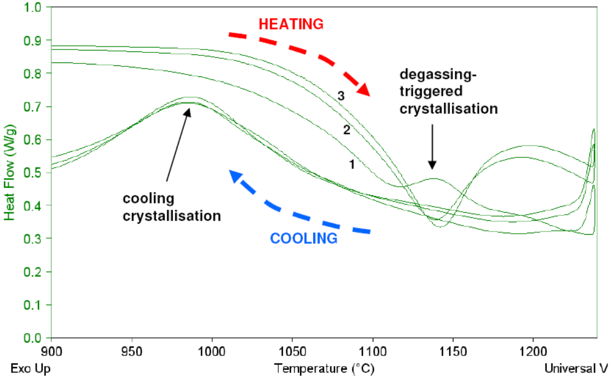




Rheology: the problem

Credits: Pinkerton *et al.* (2011) LAVA-V3 Final Report





- Potentially arbitrary laws: easy to fit to data, hard to determine parameter values;
- Complex dependency on fluid composition and physical properties (e.g. temperature, amount and types of gaseous or crystalline components, saturation, etc);
- Difficult to measure because of:
  - Change over time (e.g. settled landslide is different from the flowing landslide; re-melt lava is different from the originally effused);
  - Extreme conditions (e.g. high temperatures);
- Numerical methods for Computational Fluid Dynamics to the rescue!

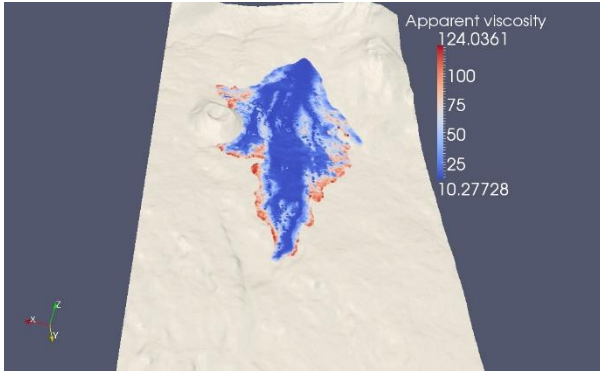
Rheology: the problem

Credits: Chevrel *et al.* (2017) *MEASURING THE VISCOSITY OF LAVA USING A FIELD VISCOMETER (KILAUEA NOV. 2016)*, MeMoVolc 2017, Catania.

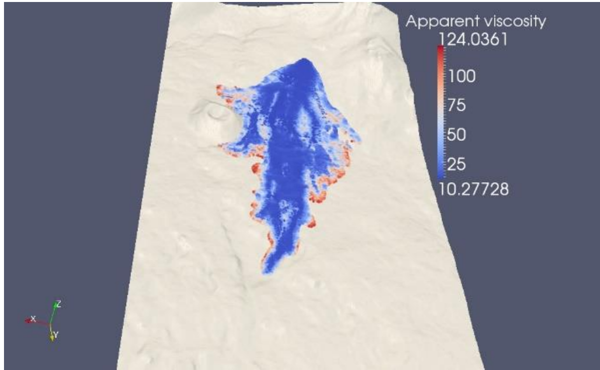
- Potentially arbitrary laws: easy to fit to data, hard to determine parameter values;
- Complex dependency on fluid composition and physical properties (e.g. temperature, amount and types of gaseous or crystalline components, saturation, etc);
- Difficult to measure because of:
  - Change over time (e.g. settled landslide is different from the flowing landslide; re-melt lava is different from the originally effused);
  - Extreme conditions (e.g. high temperatures);
- Numerical methods for Computational Fluid Dynamics to the rescue!

The numerical approach

1. Computational Fluid Dynamics:
   - convert analytical formulas and equations into discrete equations in space and time;
   - implement the discrete equations as computer programs.
2. Validation:
   - ensure code produces the correct result (numerically approximate, but converging to exact solution at higher resolution).
3. Study rheology with *validated* CFD code:
   - run multiple simulations varying rheology/dependency of rheology on fluid properties;
   - compare with field measurements/analog experiments.

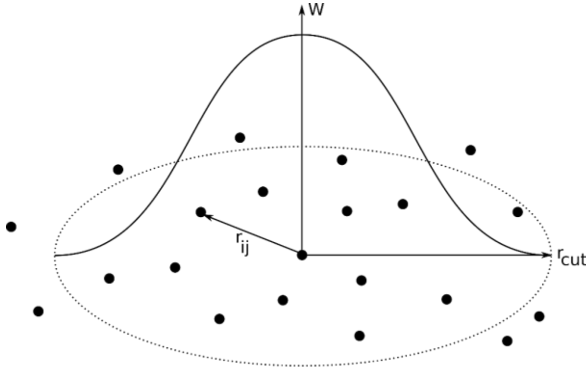


Comparison between GPUSPH simulations w/ Bingham (above) and DeKee&Turcotte ($t_1 = 0.01$) (below): emplacement and apparent kinematic viscosity.



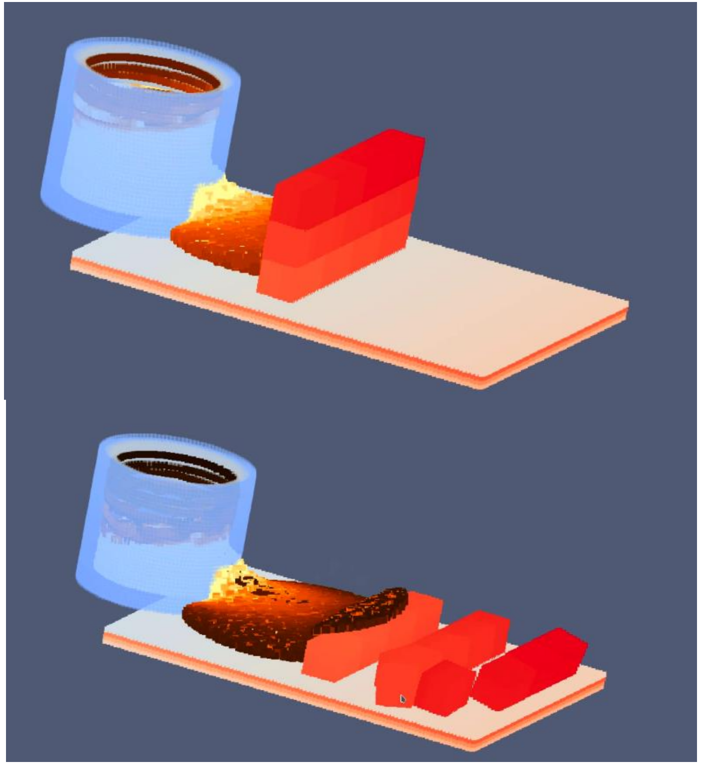
Smoothed Particle Hydrodynamics

- SPH: meshless/particle method (no grid or connectivity, as opposed to FVM, FEM):
    - automatic conservation of mass (each particle carries its own);
    - implicit tracking of free surface and other interfaces (e.g. solidification fronts);
    - no issue with large deformation;
- Weakly Compressible formulation: assume small (subsonic) density fluctuation, compute pressure from density;
    - no need to solve large implicit systems;
    - trivial to parallelize;
    - “natural” implementation on parallel HPC hardware such as GPUs.



GPUSPH

- GPUSPH is an implementation of Weakly Compressible SPH (WCSPH) running on GPUs.
- Fully three-dimensional.
- Aimed at the simulations of lava flows:
  - Temperature dependent viscosity
  - Thermal dissipation by radiation and surface air convection
  - Non-Newtonian rheology
  - Open boundaries
- Interaction with moving bodies
- Explicit or implicit integration of the viscous term



The model

- Radius 2 Wendland Smoothing Kernel
- Second order predictor corrector integrator
- Discretized momentum equation:

$$\frac{D\mathbf{u}_\beta}{Dt} = -\sum_\alpha \left(\frac{P_\alpha}{\rho_\alpha^2} + \frac{P_\beta}{\rho_\beta^2}\right) F_{\alpha\beta} m_\alpha \mathbf{x}_{\alpha\beta} + \sum_\alpha \frac{2\bar{\mu}_{\alpha\beta}}{\rho_\alpha \rho_\beta} F_{\alpha\beta} m_\alpha \mathbf{u}_{\alpha\beta} + \mathbf{G} \qquad P(\rho) = c_0^2 \frac{\rho_0}{\gamma}\left(\left(\frac{\rho}{\rho_0}\right)^\gamma - 1\right)$$

- Thermal model:
  - Thermal diffusion:

$$\frac{DT_\beta}{Dt} = \frac{1}{c_p} \sum_\alpha \frac{\kappa_{\alpha\beta} T_{\alpha\beta}}{\rho_\alpha \rho_\beta} F_{\alpha\beta}$$

  - Thermal radiation:

$$\frac{DT}{Dt} = \frac{K_B \kappa \epsilon}{m c_p}(T^4 - T_a^4)$$

  - Losses due to air convection:

$$\frac{DT}{Dt} = \frac{\eta}{m c_p}(T - T_a)S$$

The model

- Boundary model: Dummy boundary (Adami et al.)
  - Boundary particles velocity:
  - Boundary particles pressure:

$$\mathbf{u}_\beta = \mathbf{u}_w - \frac{\sum_{\alpha\in\mathcal{F}} \mathbf{u}_\alpha W_{\alpha\beta}}{\sum_{\alpha\in\mathcal{F}} W_{\alpha\beta}}$$

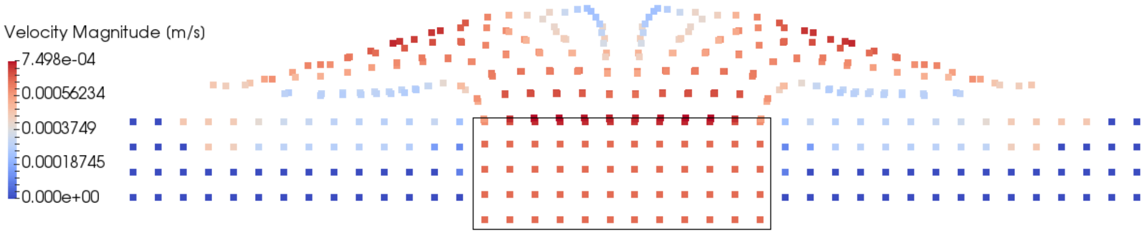
$$P_\beta = \frac{\sum_{\alpha\in\mathcal{F}} P_\alpha W_{\beta\alpha} + \mathbf{g}\sum_{\alpha\in\mathcal{F}} \rho_\alpha \mathbf{x}_{\beta\alpha} W_{\beta\alpha}}{\sum_{\alpha\in\mathcal{F}} W_{\beta\alpha}}$$

- Relative density: to reduce effects of numerical precision on the integration of the density we use a relative density written as: $\tilde{\rho} = (\rho/\rho_0 - 1)$

The continuity equation with relative density becomes: $\frac{D\tilde{\rho}}{Dt} = -\frac{\tilde{\rho}}{\rho_0}\nabla\cdot\mathbf{u}$

Inlet

- Open boundary with pre-determined mass flow rate (towards the domain).
- Modelled using multiple layers of particles, matching the boundary model, and including particle generation
- The velocity of the inlet particles is imposed, while the pressure is evaluated using the Riemann invariants, following M. Ferrand et al. (*Unsteady open boundaries for SPH using semi-analytical conditions and Riemann solver in 2D*, Computer Physic Communications, 2017)



Hydrostatic density correction has also been applied.

Validation of GPUSPH

Proposed “benchmark” test-cases for validation of lava-flow simulation (Cordonnier *et al.* 2015):

1. BM1: viscous bam break: mechanical model, no thermal effects
2. BM2: inclined viscous isothermal spreading
3. BM3: axisymmetric cooling and spreading
4. BM4: analog experiment with real lava flow

B. Cordonnier et al., Benchmarking lava-flow models, Geological Society, London, 426, special publications, 2015.

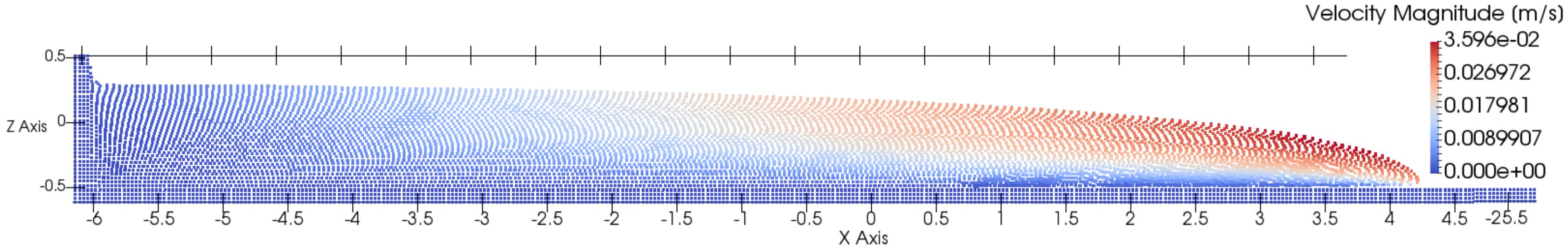
BM1: Viscous Dam Break

Dam break of a viscous fluid spreading on a horizontal plane.

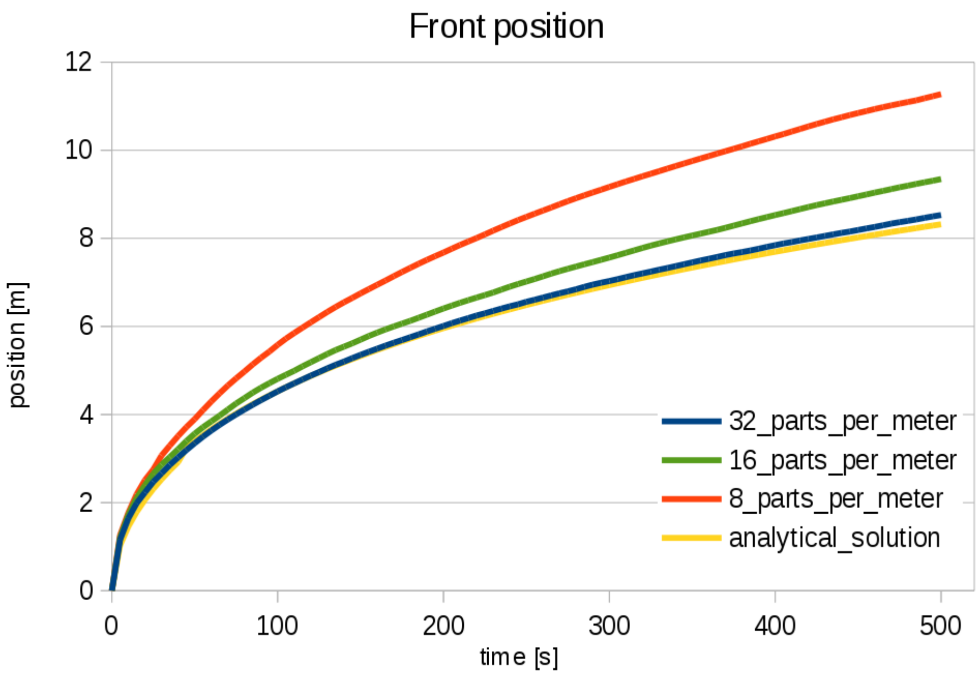
Validation according to the front position over time.

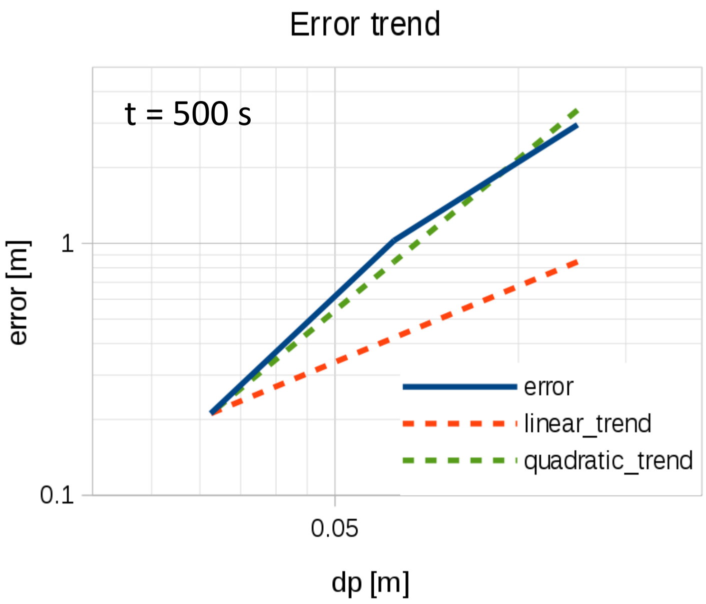
- Fluid initial size: H=1m, L=6.6m, W=1m;
- Density $\rho = 2700 \frac{kg}{m^3}$
- Dyn visc $\mu = 104\ Pa\ s$

- Convergence test: we use three levels of discretization: 8, 16 and 32 particles per meter. Speed of sound is set to $c = 443$ m/s



BM1: results





INGV Istituto Nazionale di Geofisica e Vulcanologia

BM2: Inclined viscous isothermal spreading

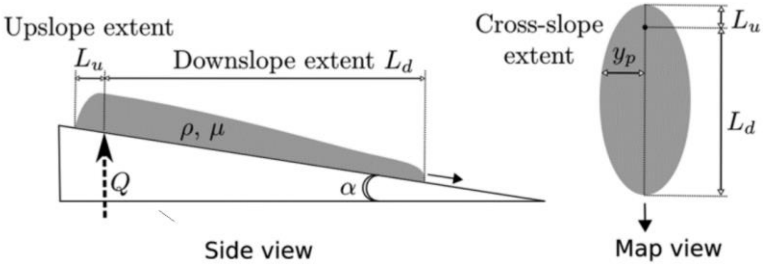
A viscous fluid from a point source spreading onto an inclined plane.

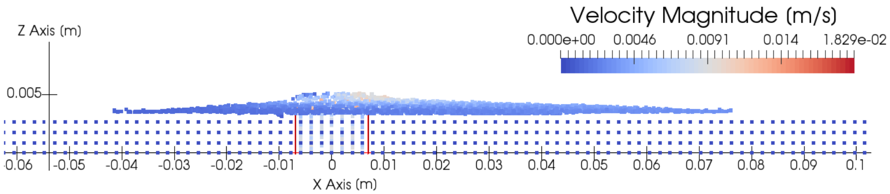
Validation according to cross-slope and downslope extent (long–time):

$$L_d \approx \left[\frac{(\rho g)^3 Q^4}{(3\mu)^3}\frac{\sin^5\alpha}{\cos^2\alpha}\right]^{1/9} t^{7/9} \qquad y_p \approx \left(\frac{Q\cos\alpha}{\sin\alpha}\right)^{1/3} t^{1/3}$$

- Fluid initial size: $\mathrm{H} = 1\mathrm{m}$, $\mathrm{L} = 1\ \mathrm{m}$, $\mathrm{W} = 6.6\ \mathrm{m}$.
- Density $\rho = 2700\,\frac{kg}{m^3}$
- Kin visc $\nu = 11.3\ 10^{-4}\ m^2/s$
- Plane angle $\alpha = 2°$

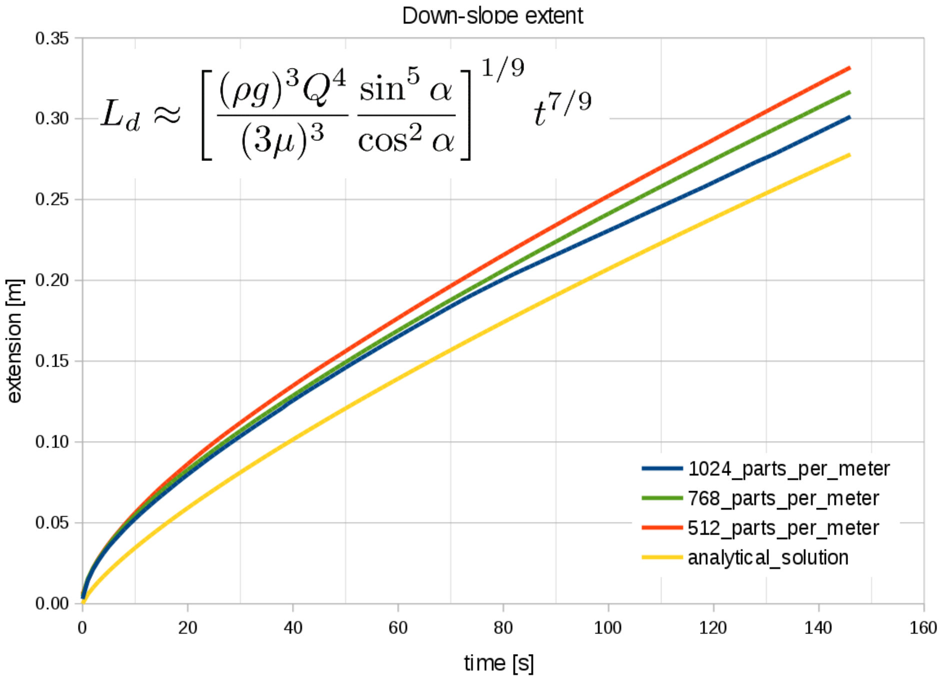
- Convergence test: we use three levels of discretization: 512, 768 and 1024 particles per meter. Speed of sound is set to $c = 19$ m/s. Vent size is 6 particles across.

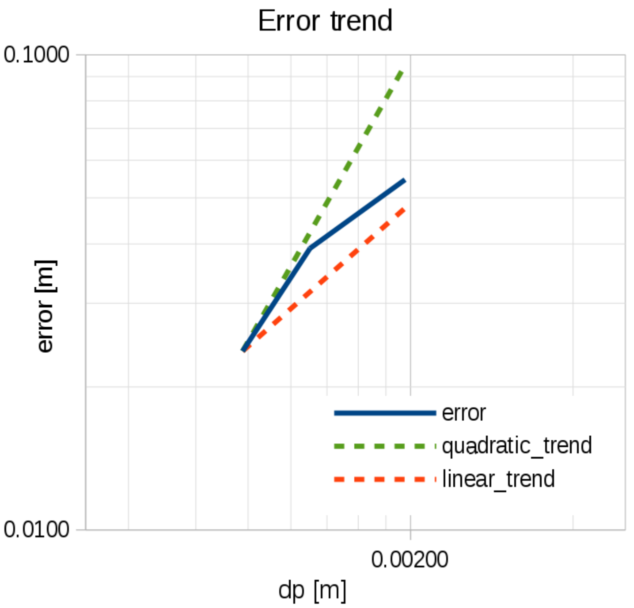




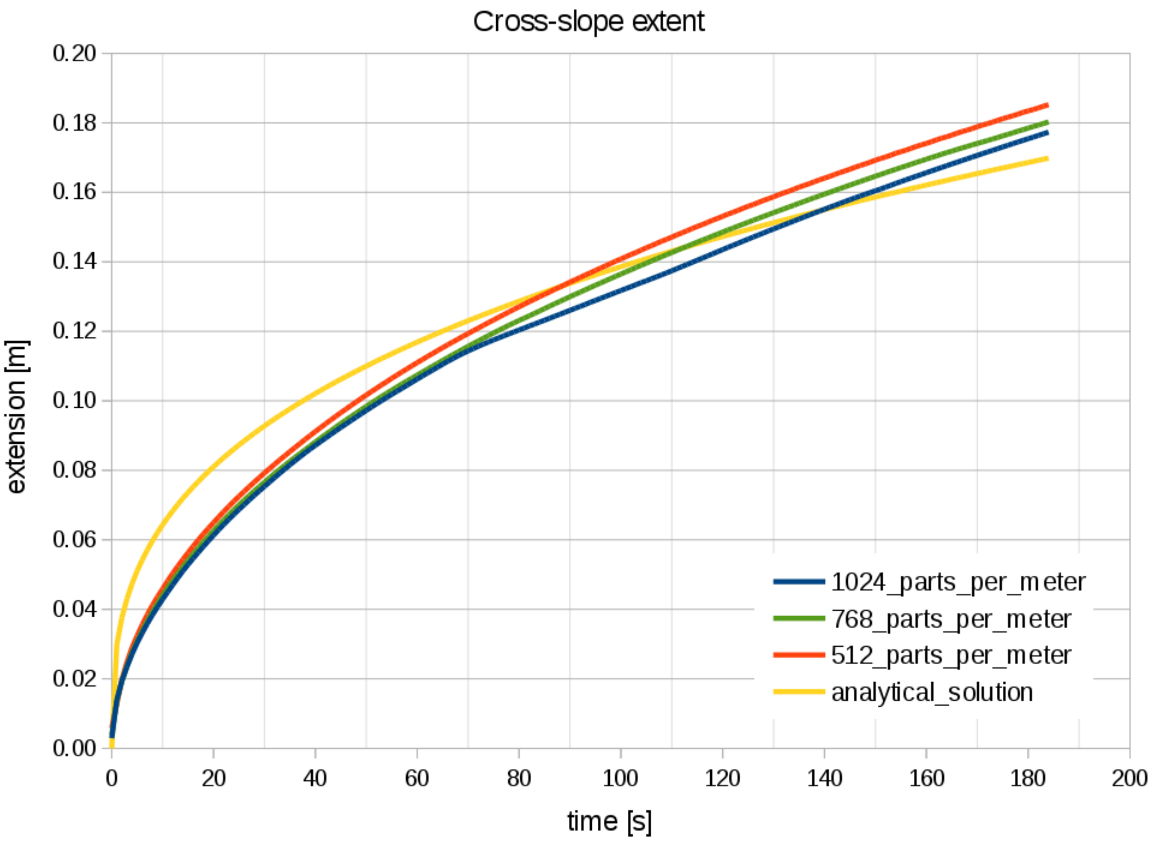
INGV Istituto Nazionale di Geofisica e Vulcanologia

BM2: Convergence of the Down-slope extent





BM2: Convergence of the Cross-slope extent



$$y_p \approx \left(\frac{Q \cos \alpha}{\sin \alpha}\right)^{1/3} t^{1/3}$$

The analytical solution for the cross slope extent is intended for very long time.

Further investigation is needed to understand the level of accuracy of the given solution and the reason for the discrepancy.

BM3: axisymmetric cooling and spreading

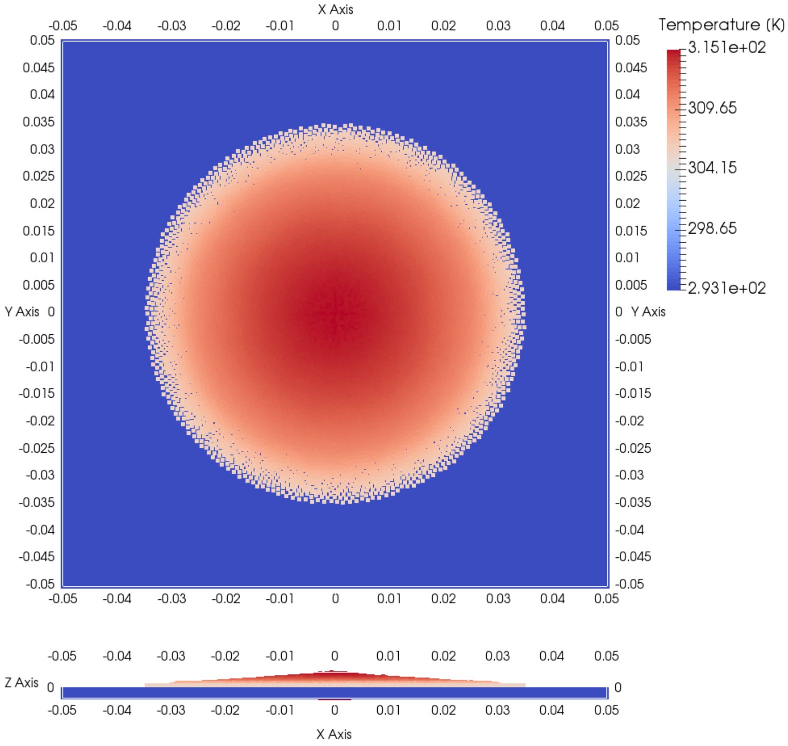
Non isothermal spreading from a point source on a horizontal plane. Temperature is **a passive tracer** (does not affect rheology).

Validation according to the spatial extent and radial temperature profile.

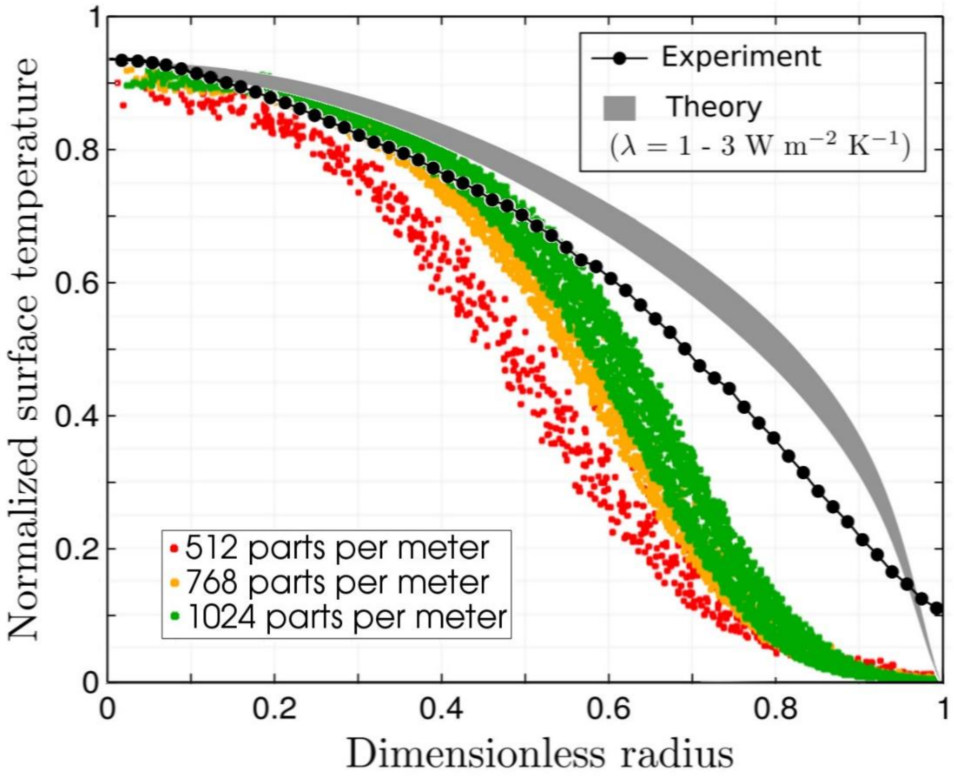
Parameter	Value
Density	886 kg m^{-3}
Viscosity	3.4 Pa s
Specific heat	1500 Jkg^{-1}K^{-1}
Bed slope	0°
Effusion rate	$2.2 \cdot 10^{-8}$ m^{3}s^{-1}
Convective heat transfer	2 Wm^{-2}K^{-1}
Emissivity	0.96
Eruption temperature (T_0)	42 °C
Ambient temperature (T_a)	20 °C
Thermal Conductivity	0.15 Wm^{-1}K^{-1}

Three levels of discretization: 1024, 1536 and 2048 particles per meter.

The speed of sound is set to c =28 m/s



BM3: Temperature profile



- free-surface detection (for radiation and cooling) overestimates surface area on outer edge

BM4: real lava flow

Real lava flow obtained experimentally with natural basalt heated over the solidus temperature.

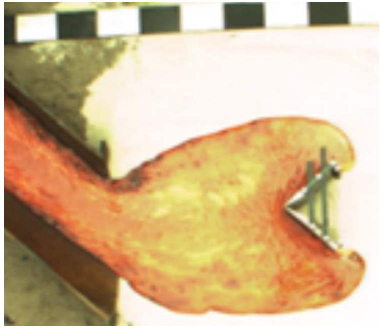
A triangular steel obstacle is 45 cm far from the source.

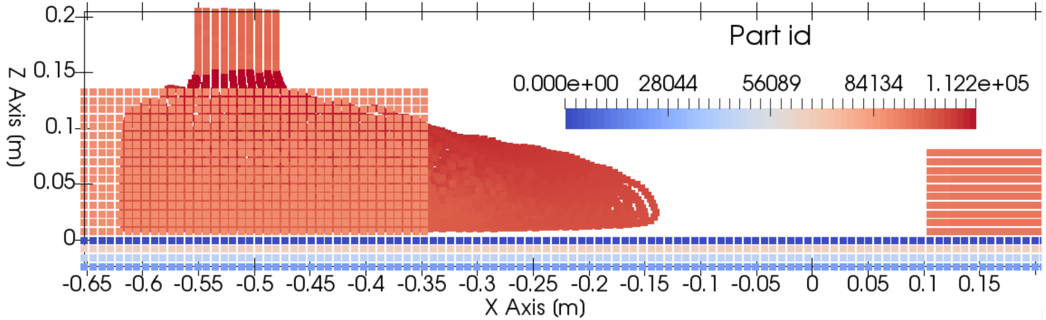
Newtonian rheology with temperature dependent viscosity:

$$\log_{10}\mu = -4.55 + \frac{5550}{T - 610}$$

We run three simulations with 64, 128 and 256 particles per meter. The speed of sound is set to $c = 43$ m/s.

Parameter	Value
Density	$2350\ \mathrm{kg\ m^{-3}}$
Specific heat	$1500\ \mathrm{Jkg^{-1}K^{-1}}$
Bed slope	$14°$
Effusion rate	$0.77 \cdot 10^{-3}\ \mathrm{m^3s^{-1}}$
Convective heat transfer	$2\ \mathrm{Wm^{-2}K^{-1}}$
Emissivity	0.95
Eruption temperature	1100 °C
Ambient temperature	25 °C
Thermal Conductivity of the bed	$0.2\ \mathrm{Wm^{-1}K^{-1}}$

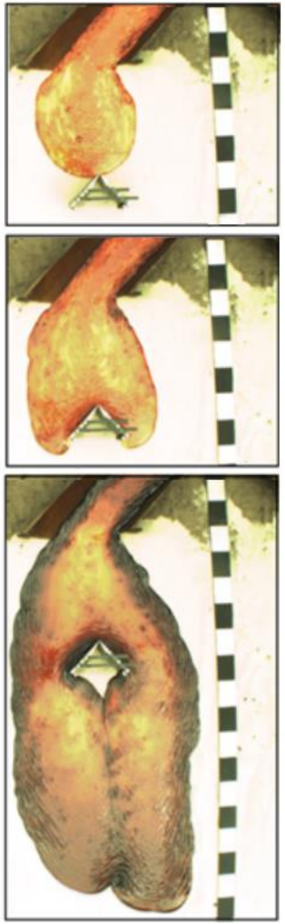


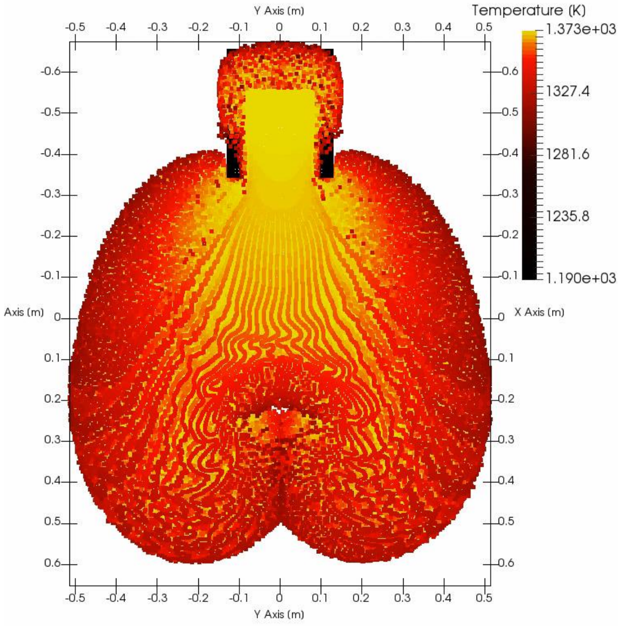


BM4: Uncertainty in the viscosity model

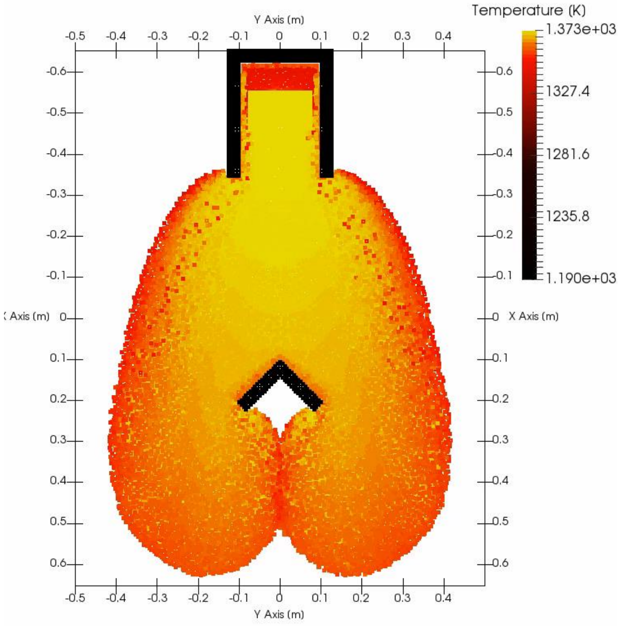
4 cm/s

2 cm/s





$$\log_{10}\mu = -4.55 + \frac{5550}{T - 610}$$



$$\log_{10}\mu = -5.94 + \frac{5500}{T - 610}$$

H. Dietterich, USGS, private communication

Conclusions

- Geophysical flows have complex rheology which is often difficult to study on the field;
- Validated numerical models can provide a tool to explore the effect of rheology on flow emplacement;
- Numerical results can complement field data and analog experiments to provide insights on the flow rheology.

Thank you for your attention