Cavity optomechanics: manipulating mechanical resonators with light

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Congresso SIF 2011, L’Aquila, September 28 2011
Outline of the talk

1. Introduction to cavity optomechanics

2. The special case of a thin membrane within a Fabry-Perot cavity (membrane-in-the-middle setup): (theory and experimental results in Camerino)

3. Theory predictions on steady state continuous variable entanglement

4. Quantum information applications
Why entering the quantum regime for opto- and electro-mechanical systems?

1. **quantum-limited sensors**, i.e., working at the sensitivity limits imposed by Heisenberg uncertainty principle

Nano-scale: Single-spin MRFM
D. Rugar group, IBM Almaden

Macro-scale: gravitational wave interferometers (VIRGO, LIGO)
Toward single molecule nanomechanical sensors (tiny small mass $\Rightarrow$ detectable frequency shift)

M. Roukes group, Caltech
2. exploring the **boundary between the classical macroscopic world and the quantum microworld** (how far can we go in the demonstration of macroscopic quantum phenomena?)

Optical detection of Schrödinger cat states of a cantilever?
3. quantum information applications (optomechanical and electromechanical devices as light-matter interfaces and quantum memories), or transducers for quantum computing architectures
We focus on cavity optomechanics: many possible schemes

1. Fabry-Perot cavity with a moving micromirror
   - Micropillar mirror (LKB, Paris)
   - Monocrystalline Si cantilever, (Vienna)

2. Silica toroidal optical microcavities
   - Spoke-supported microresonator (Munich, Lausanne)
   - With electronic actuation, (Brisbane)
Evanescent coupling of a SiN nanowire to a toroidal microcavity (Munich, Lausanne)

“membrane in the middle” scheme: Fabry-Perot cavity with a thin SiN membrane inside (J. Harris group Yale, Kimble group Caltech, Camerino)

Photonic crystal “zipper” cavity (Caltech)

microdisk and a vibrating nanomechanical beam waveguide (Yale, H. Tang group)
**Example: the membrane-in-the-middle setup**

**Many cavity modes** (still Gaussian $\text{TEM}_{mn}$ for an aligned membrane close to the waist)

$$H_{cav} = \sum_k \hbar \omega_k(z_0) a_k^+ a_k$$

**Many vibrational modes** $u_{mn}(x,y)$ of the membrane

$$u_{mn}(x, y) = \sin \frac{n \pi x}{d} \sin \frac{m \pi y}{d}$$

$$\Omega_{nm} = \sqrt{\frac{\pi T}{\rho L_d d^2}} (m^2 + n^2)$$

Vibrational frequencies

T = surface tension
$\rho$ = SiN density,
$L_d$ = membrane thickness
d = membrane side length
$m,n = 1,2…$
Optomechanical interaction due to radiation pressure

\[ H_{\text{int}} = -\int dx dy \, P_{\text{rad}}(x, y) \, z(x, y) \]

(at first order in z)

Radiation pressure field

\[ P_{\text{rad}}(x, y) = \varepsilon_0 \left(n^2_M - 1\right) \int_{z_0-L_d/2}^{z_0+L_d/2} dz \left( \vec{E}(x, y, z) \times \vec{B}(x, y, z) \right)_z \]

Membrane axial deformation field

\[ z(x, y) = \sum_{n,m} \sqrt{\frac{\hbar}{M \Omega_{nm}}} q_{nm} u_{nm}(x, y) \]

Trilinear coupling describing photon scattering between cavity modes caused by the vibrating membrane

Let us now simplify the system: **single mechanical oscillator, nonlinearly coupled to a single optical oscillator**

Possible when:

- The external laser (with frequency \( \omega_L \approx \omega_c \)) **drives only a single cavity mode** \( a \) and scattering into the other cavity modes is negligible (no frequency close mode)

- A **bandpass filter** in the detection scheme can be used, isolating a single mechanical resonance

\[
\hat{H} = \frac{\hbar \omega_m}{2} (p^2 + q^2) + \hbar \omega(q) a^+ a + H_{\text{drive}}
\]

\[
\omega(q) = \omega_c - G_0 q
\]

**Cavity optomechanics**

**Hamiltonian applicable in a wide variety of systems**

\[
\hat{H}_{\text{drive}} = i\hbar (E e^{-i\omega_L t} a^+ - E^* e^{i\omega_L t} a)
\]

\[
E = \sqrt{\frac{2 \kappa P_L}{\hbar \omega_L}}
\]

amplitude of the driving laser with input power \( P_L \)
The first order expansion based on the radiation pressure force is a poor approximation at nodes and antinodes; it is better to adopt the general dependence

\[ \omega(q) = \omega_0 - G_0 q \]

\[ \omega(q) = \omega_0 + (-1)^p \arcsin\left\{\sqrt{R} \cos[2k_0z_0(q)]\right\} \]

R = membrane reflectivity

Theory curves for a perfectly aligned membrane close to the waist
Misalignment and shift of the membrane from the waist couple via scattering the TEM$_{mn}$ cavity modes:

linear combinations of nearby TEM$_{mn}$ modes become the new cavity modes: $\alpha(q)$ is changed significantly: tunable optomechanical interaction

**experimental data in Camerino:** 50 nm membrane, misaligned by an angle of 0.77 mrad, and 1.2 mm shifted from waist
Results well fitted by a perturbative solution of the wave equation in the cavity with the membrane inside: the various $\omega(q)$ are well reproduced:

See also J. Sankey et al., Nat. Phys. 2010

Avoided crossings between TEM$_{00}$ and the TEM$_{20}$ triplet

$G_0 \sim 120$ Hz

$\frac{\omega'(q)}{2\pi} \approx 2 MHz/nm$

Linear coupling

Strong quadratic coupling

$\frac{\omega''(q)}{2\pi} = 4.46 MHz/nm^2$
At the membrane positions where there is an **avoided crossing** the optomechanical interaction becomes quadratic in the membrane position

\[
\frac{\omega''(q)}{2\pi} = 4.46\text{MHz/nm}^2
\]

\[H_{\text{int}} = \hbar \omega''(q)q^2 a^+ a\]

Dispersive interaction useful for the **nondestructive measurement** of mechanical energy: **possible detection of “phonon quantum jumps”**
Also damping and noise act on the system

• The mechanical element is in contact with its environment at temperature $T$

Fluctuation-dissipation theorem $\Rightarrow$ presence of a quantum Langevin force $\xi$ with correlation functions

$$
\langle \xi(t) \xi(t') \rangle = \frac{\gamma_m}{\omega_m} \int \frac{d\omega}{2\pi} e^{i\omega(t-t')} \omega \left[ \coth \left( \frac{\hbar \omega}{kT} \right) + 1 \right]
$$

Gaussian, generally non-Markovian (non-delta-correlated)

• The cavity mode is damped by photon leakage with decay rate $\kappa$ $\Rightarrow$ presence of a vacuum input Langevin noise $a_{in}(t)$ with correlation functions

$$
\langle a_{in}(t) a_{in}(t') \rangle = \langle a_{in}(t)^+ a_{in}(t') \rangle = 0 \quad \langle a_{in}(t) a_{in}(t')^+ \rangle = \delta(t-t')
$$

Gaussian, Markovian

Vacuum electromagnetic field outside the cavity $\Leftrightarrow$ reservoir at $T \approx 0$ at optical frequencies, because

$$
\frac{\hbar \omega_a}{kT} \gg 1 \quad \Rightarrow \quad \bar{N} = \left[ e^{\frac{\hbar \omega_a}{kT}} - 1 \right]^{-1} \approx 0
$$
Here we consider also an important noise of technical origin, which is usually neglected: phase noise of the driving laser.

It is practically unavoidable and it could be extremely destructive for continuous variable (CV) entanglement = quantum correlations between field and mechanical quadratures at a given phase.

Modification of the driving term: 

\[ E \rightarrow \varepsilon(t)e^{-i\phi(t)} \]

\( \varepsilon(t) \) laser amplitude fluctuations
\( \phi(t) \) (zero-mean) laser phase fluctuations

Noisy laser is also phase reference for any quadrature measurement \( \Rightarrow \) the dynamics must be described in the frame rotating at the fluctuating laser frequency:

\[ \omega_L + \dot{\phi}(t) \]

M. Abdi et al., PRA 84, 032325 (2011)
Heisenberg-Langevin equations in such a fluctuating frame

\[ \begin{align*}
\dot{q} &= \omega_m p, \\
\dot{p} &= -\omega_m q - \gamma_m p + G_0 a^\dagger a + \xi, \\
\dot{a} &= -\kappa a - i(\Delta_0 - \phi - G_0 q)a + \mathcal{E}(t) + \sqrt{2\kappa} \tilde{a}_{\text{in}}(t)
\end{align*} \]

Laser phase noise

Laser amplitude noise

\[ \Delta_0 = \omega_c - \omega_0 \]

\[ G_0 = -\left(\frac{d\omega_c}{dx}\right) \sqrt{\frac{\hbar}{m\omega_m}} \] coupling

CV entanglement = strong quantum correlations between quadratures better achievable with strong driving \( \Rightarrow \) steady-state with an intense intracavity field (amplitude \( \alpha_s \)) and deformed/displaced resonator. Linearized dynamics of the quantum fluctuations around this steady state \( \Rightarrow \) exact for \( |\alpha_s| >> 1 \)

\[ a \rightarrow \alpha_s + \delta a \quad q \rightarrow q_s + \delta q \]
Linearized Quantum Langevin equations for the quantum fluctuations

\[ \delta q = \omega_m \delta p, \]
\[ \delta p = -\omega_m \delta q - \gamma_m \delta p + G_0 (\alpha_s \delta a^\dagger + \alpha_s^* \delta a) + \xi, \]
\[ \delta \dot{a} = -[\kappa + i\Delta] \delta a + iG_0 \alpha_s \delta q + i\phi \alpha_s \]
\[ + \varepsilon + \sqrt{2\kappa} \tilde{a}_{in}. \]

Large phase noise (\(|\alpha_s| \gg 1\))

amplitude noise negligible

Nonlinear terms \( \frac{\delta a^\dagger \delta a}{i[G_0 \delta q + \phi] \delta a} \) have been neglected

\[ \Delta = \Delta_0 - G_0 q_s = \Delta_0 - \frac{G_0^2 |\alpha_s|^2}{\omega_m} \]

Effective cavity detuning
Phase noise statistics and spectrum

\( \phi(t) = \text{zero-mean Gaussian noise} \), related to laser spectrum \( S_L(\omega) \)

\[
S_L(\omega) = \int d\tau e^{i\omega \tau} C(\tau) = \int d\tau e^{i\omega \tau} \langle \exp\{i\phi(t+\tau) - i\phi(t)\} \rangle
\]

\[C(\tau) = \left\langle \exp\left\{ i \int_0^\tau ds \dot{\phi}(s) \right\} \right\rangle
\]

\[= \exp\left\{ -\frac{1}{2} \int_0^\tau ds \int_0^\tau ds' \langle \dot{\phi}(s) \dot{\phi}(s') \rangle \right\}
\]

\[
S_\phi(\omega) = \int d\tau e^{i\omega \tau} \langle \dot{\phi}(t+\tau) \dot{\phi}(t) \rangle
\]

frequency noise spectrum;

\[
S_\phi(\omega) = 2\Gamma_l \frac{\Omega^4}{(\Omega^2 - \omega^2)^2 + \omega^2 \gamma^2}
\]

we take a bandpass noise spectrum

\( \Gamma_l = \text{laser linewidth} \)

\( \Omega = \text{center of the frequency noise band} \)

\( \gamma = \text{frequency noise bandwidth} \)
When the system is stable, it reaches for $t \to \infty$ a **Gaussian steady state** $\rho_S$, due to:

1. Linearized dynamics
2. Gaussian quantum noises

$\rho_S$ Gaussian $\iff$ Gaussian characteristic function

$$
\Phi(\vec{\lambda}) = \text{Tr}[\rho e^{-i\vec{\lambda}^T \vec{\xi}}] = \exp\left[-\frac{\vec{\lambda}^T V \vec{\lambda}}{2} + i\vec{d}^T \vec{\lambda}\right] \vec{\xi}^T = (\delta q, \delta p, \delta X, \delta Y)
$$

**correlation matrix (CM)**

fully characterizing the steady state and its entanglement properties (we use log-negativity $E_N$)

$$
V_{ij} = \frac{\langle \xi_i \xi_j + \xi_j \xi_i \rangle}{2} - \langle \xi_i \rangle \langle \xi_j \rangle
$$

EFFECT OF PHASE NOISE ON log-neg $E_N$

(no phase noise)

$\Gamma_l/2\pi = 0$

$\Gamma_l/2\pi = 0.1$ kHz

$\Gamma_l/2\pi = 1$ kHz

State-of-art experimental parameters: $m = 10$ ng, $\omega_m/2\pi = 10$ MHz, $\gamma_m/2\pi = 5$ Hz, $G_0 = 1$ kHz, $L = 1$ mm, $T = 400$ mK, $\Omega/2\pi = 50$ kHz, $\gamma = \Omega/2$
EFFECT OF PHASE NOISE SPECTRUM ON log-neg $E_N$

\[ \Omega/2\pi = 30 \text{ kHz} \quad \Omega/2\pi = 80 \text{ kHz} \quad \Omega/2\pi = 140 \text{ kHz} \]

$2\kappa = \omega_m$

$\Gamma_l/2\pi = 0.1 \text{ kHz}$

$\Delta_l = \omega_m$

(the bandwidth parameter $\tilde{\gamma}$ is correspondingly adjusted so that $\tilde{\gamma} = \Omega/2$)

M. Abdi et al., arXiv:1106.0029v1 [quant-ph], in press on PRA
Approximate Analytical Results

Relevant quantity: \( S_{\phi}(\omega_{m}^{\text{eff}}) \)  
Frequency noise spectrum at the effective mechanical resonance \( \omega_{m}^{\text{eff}} \)

\[
\omega_{m}^{\text{eff}} \approx \sqrt{\omega_{m}^{2} - \frac{G^{2} \Delta \omega_{m} \left( \kappa^{2} - \omega_{m}^{2} + \Delta^{2} \right)}{\kappa^{2} + (\omega_{m} + \Delta)^{2} \left( \kappa^{2} + (\omega_{m} - \Delta)^{2} \right)}}
\]

Frequency modified by the optomechanical coupling

Without phase noise: \( E_{N} \) is maximum at the bistability threshold: close to it one has

\[
E_{N} = \max(0, -\ln 2\eta^{-})
\]

\[
\eta^{-} \approx \frac{1}{\sqrt{2}} \sqrt{\frac{a + bS_{\phi}(\omega_{\text{eff}}) + cS_{\phi}(\omega_{\text{eff}})^{2} + dS_{\phi}(\omega_{\text{eff}})^{3}}{f + gS_{\phi}(\omega_{\text{eff}})}}
\]

\[
\eta^{-} \approx \sqrt{\frac{a}{2f}} = \sqrt{\frac{4\Delta^{4} + 4\Delta^{2}(\kappa^{2} + \omega_{m}^{2}) + \omega_{m}^{4}}{16\Delta^{2}(\Delta^{2} + \kappa^{2} + 5\omega_{m}^{2})}}
\]

Maximizing over \( \Delta \) \( E_{N} = -\ln \left[ \frac{1}{5} \sqrt{9 + \frac{128\kappa^{2}}{8\kappa^{2} + 45\omega_{m}^{2}}} \right] \) \( \leq \ln(5/3) \approx 0.51 \) (maximum achievable \( E_{N} \))

With phase noise, one easily has \( \eta > 0.5 \Rightarrow E_{N} = 0 \) close to bistability, and \( E_{N} \) becomes maximum FAR FROM threshold
The steady state CM, $V$, contains also the info about the stationary energy of the membrane mode, $U$

$$V_{11} = \langle \delta q^2 \rangle \quad V_{22} = \langle \delta p^2 \rangle$$

$$U = \frac{\hbar \omega_m}{2} \left[ \langle \delta q^2 \rangle + \langle \delta p^2 \rangle \right] \equiv \hbar \omega_m \left( n_{eff} + \frac{1}{2} \right)$$

Cavity laser-cooling, realized by the radiation pressure of the cavity mode, which behaves as an effective additional zero-temperature reservoir for the oscillator

Phase noise is much less detrimental on laser cooling to ground state of the mechanical resonator (recently achieved at JILA and in Caltech)
State-of-art experimental parameters: \( m = 10 \) ng, \( \omega_m / 2\pi = 10 \) MHz, \( \gamma_m / 2\pi = 5 \) Hz, \( G_0 = 1 \) kHz, \( L = 1 \) mm, \( T = 400 \) mK, \( \Omega / 2\pi = 50 \) kHz, \( \gamma = \Omega / 2 \)
HOW TO EXPLOIT OPTOMECHANICAL ENTANGLEMENT?

For quantum communication applications one manipulates \textit{traveling rather than intracavity photons} \Rightarrow it is more important to analyze the \textit{entanglement of the mechanical mode with the optical cavity output}.

By considering the output, one can manipulate a \textit{multipartite system}: in fact, by means of spectral filters, one can select \textit{many different traveling output modes originating from a single intracavity mode}.
PROPER DEFINITION OF OUTPUT MODES

input-output relation

\[
a_{k}^{\text{out}}(t) = \sqrt{2\kappa}\delta a(t) - a^{\text{in}}(t)
\]

From the continuous cavity output \( a^{\text{out}}(t) \), one can extract \( N \) independent output modes by selecting appropriate time (or frequency) intervals.

\[
a_{k}^{\text{out}}(t) = \int_{-\infty}^{t} ds g_{k}(t - s) a^{\text{out}}(s), \quad k = 1, \ldots N
\]

\( g_{k}(t) \) causal filter function

Independent modes

\[
\int_{0}^{\infty} dt g_{j}^{\ast}(t)g_{k}(t) = \int_{-\infty}^{\infty} d\omega \tilde{g}_{j}^{\ast}(\omega)\tilde{g}_{k}(\omega) = \delta_{jk}
\]

C. Genes et al., PRA 78, 032316 (2008)
SELECTING ONE OUTPUT MODE

By appropriately choosing the center and the bandwidth of the detected output mode, the entanglement with the mechanical resonator can be distilled and increased.

\[ \Omega = \text{output center frequency} \]
\[ \varepsilon = \omega_m \tau, \tau = \text{inverse bandwidth} \]

Entanglement with the mechanical mode is optimally carried by the output mode centered at the Stokes sideband of the laser; optimal width \( \approx \Gamma = \text{effective width of the mechanical resonance} \).
One can use this optomechanical entanglement with the cavity output to teleport the state of an optical mode (Victor) onto the mechanical mode.

**Braunstein-Kimble teleportation protocol**

1. Homodyne measurements by Alice
2. Communication of the results to Bob
3. Conditional actuation (phase-space displacement) by Bob
• The Braunstein-Kimble protocol is designed for optical two-mode squeezed states (high correlation between amplitude and phase quadrature of the two modes).

• The present optomechanical entanglement implies field-mechanical correlations between generic quadratures, which are not easy to find

PROBLEMS

1. One has to make local transformations (local quadrature rotations, squeezing,…) in order to optimize teleportation. Which ones?

2. For a given optomechanical entanglement, what is the maximum achievable fidelity after the local optimization?

We consider a **generic bipartite Gaussian state with CM** $V$ and entanglement $E_N$ as shared resource, and **local trace-preserving Gaussian CP (TGCP) maps** (unitary Gaussian operations, eventually with additional ancillas in Gaussian states which are then traced out) as operations for improving the teleportation fidelity.

Under a TGCP map

\[
V \rightarrow V' = SVS^T + G
\]

\[
G + i\Omega - iS\Omega S^T \geq 0
\]

we proved that…

\[\Omega\] symplectic matrix
1. The optimizing local transformation is always a local Gaussian UNITARY at Alice and Bob site, supplemented at most by an attenuation either on Bob or Alice mode (i.e., mixing with vacuum at a beam splitter)

2. This optimal local TGCP map leads to a final CM, which can always be written in the following normal form

\[
\begin{pmatrix}
    n + \lambda & 0 & -d - \lambda & 0 \\
    0 & n & 0 & d \\
    -d - \lambda & 0 & m + \lambda & 0 \\
    0 & d & 0 & m
\end{pmatrix}
\]

3. For a given log negativity \( E_N \), the locally optimized fidelity \( F \) (for teleporting coherent states) always satisfies

\[
\frac{1 + e^{-E_N}}{1 + 3e^{-E_N}} \leq F \leq \frac{1}{1 + e^{-E_N}}
\]

The upper bound is achieved if and only if \( V \) is symmetric
CONCLUSIONS

1. One can create stationary (virtually infinite lifetime) optomechanical entanglement between the cavity mode and a nanomechanical resonator

2. Laser phase noise can seriously affect optomechanical entanglement (more than ground state cooling), but can be still achieved by appropriately filtering phase noise (it depends upon $\mathcal{S}_\phi(\omega_{\text{eff}}^m)$)

3. Optomechanical entanglement can be used in quantum communication protocols, as for example, to teleport the state of an optical field onto a nanomechanical resonator
Classical-like description of cavity cooling

\[
\gamma_m^{\text{eff}}(\omega) = \gamma_m + \frac{G \Omega_m \left\{ 2G \Delta \kappa_T(q_s) - \Gamma \left[ \kappa_T^2(q_s) + \omega^2 - \Delta^2 \right] \right\}}{[\kappa_T^2(q_s) + (\omega - \Delta)^2] [\kappa_T^2(q_s) + (\omega + \Delta)^2]}
\]

**frequency-dependent effective damping >> \gamma_m \Rightarrow** suppression of susceptibility at resonance \(\Rightarrow\) negligible sensitivity to thermal noise \(\Leftrightarrow\) cooling

Laser-cooling description of cavity cooling (I. Wilson-Rae et al., 2007)

Rates at which photons are scattered by the moving oscillator, simultaneously with the absorption (Stokes, \(A_+\)) or emission (anti-Stokes, \(A_-\)) of vibrational phonons

\[
A_{\pm} = \frac{G^2 \kappa_T + G \Gamma (\Delta \pm \Omega_m)}{\kappa_T^2(q_s) + (\Delta \pm \Omega_m)^2}
\]

\[
\Gamma_{\text{opt}} = A_- - A_+
\]

net laser cooling rate

\[
S(\omega) \quad \left(\text{Hz}^{-1}\right)
\]

\[
\omega / \omega_m
\]
$$\gamma_m^{\text{eff}}(\omega_m) = \gamma_m + \Gamma_{\text{opt}} = \gamma_m + A_- - A_+$$

In the limits

\[
\omega_m \gg \bar{n}\gamma_m, G \\
\kappa \gg \gamma_m, G
\]

$$n_{\text{eff}} = \frac{\gamma_m n_0 + \frac{\Gamma^2}{8\kappa_1(q_s)} + A_+}{\gamma_m + A_- - A_+}$$

**Ground state cooling** achieved when

$$A_- \gg A_+, \gamma_m$$

$$n_0, \frac{\Gamma^2}{8\kappa_1(q_s)}$$ not too large

If these conditions are satisfied, absorption by the membrane does not hinder reaching the quantum regime.
For parameters similar to those of our current experiment: $M = 35$ ng, 
$\omega_m/2\pi = 250$ KHz, $Q_m = 10^6$, $P_L = 650$ $\mu$W, $L = 9$ cm, $F_0 = 20000$, $T = 4$ K, $L_d = 50$ nm, $\Delta \sim \omega_m$, $n_M = 2.0 + i \times 10^{-4}$

Blue: $n_{\text{eff}} = \text{ground state occupancy}$
Red: $E_N$, Log-negativity

Quantum regime achievable!

C. Biancofiore et al., arXiv:1102.2210v1 [quant-ph]
Cavity resonant with the laser blue sideband

(e) Finesse, $\bar{F}_0 = 22000$

t (50 nm) membrane thickness
Example: Equivalence with capacitively-coupled nanoresonator-superconducting microwave cavity system

The Hamiltonian in the frame rotating at the driving frequency is equivalent to the Fabry-Perot case:

\[ H = \frac{p_x^2}{2m} + \frac{m\omega_m^2 x^2}{2} + \frac{\Phi^2}{2L} + \frac{Q^2}{2[C + C_0(x)]} - e(t)Q \]

\[ \frac{Q^2}{2[C + C_0(x)]} = \frac{Q^2}{2C_\Sigma} + \frac{\beta}{2dC_\Sigma} x(t)Q^2 \]

\[ a = \sqrt{\frac{\omega_cL}{2\hbar}} \hat{Q} + \frac{i}{\sqrt{2\hbar\omega_cL}} \hat{\Phi} \]

\[ H_I = \hbar \Delta_0 a^+ a + \frac{\hbar \omega_m}{2} (\hat{q}^2 + \hat{p}^2) + \hbar G_0 \hat{q} a^+ a - i\hbar E(a - a^+) \]

\[ G_0 = \beta \omega_c \left( \frac{1}{2d} \sqrt{\frac{\hbar}{m\omega_m}} \right) \]

Hamiltonian in the frame rotating at the driving frequency is equivalent to the Fabry-Perot case.