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Channeling radiation on quartz stimulated by acoustic waves

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Summary. — The stimulation of channeling radiation by acoustic waves excited in the single crystal has been predicted in early works of the 1980's. Based on quantum calculations, the described experiment aimed at verification of theoretical considerations. Making use of the reverse piezoelectric effect, hypersonic waves of frequency 12 GHz, which corresponds to a dedicated transition between bound states of planar channelled relativistic electrons, were excited in a single-crystal quartz plate. The spectrum of channeling radiation measured under the influence of acoustic waves reveals enhanced radiation intensity. The obtained results are discussed and may be phenomenologically understood assuming electron diffraction on an acoustic superlattice.

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1. – Introduction

Channeling radiation (CR) is a well-described source of X- or γ-rays (see, *e.g.*, [1-3]). At medium energies ($\leq 100\,\mathrm{MeV}$), CR emission has to be treated as a quantum process [4, 5]. The CR intensity depends linearly on the beam current over many orders of magnitude [6].

Considerations of radiation-induced amplification of CR led to very high current densities ($10^6 \div 10^{12}\,\mathrm{A/cm^2}$) [3, 7-9]. Another principle to stimulate CR emission, suggested in the 1980's [10-13], is based on manipulating the crystal potential along the path of channeled particles by excitation of longitudinal hypersonic (GHz) waves [14-22]. According to [12, 13], the wavelength of hypersound has to be resonant with the wavelength of the propagating oscillatory motion of channeled particles. At quantized motion, this

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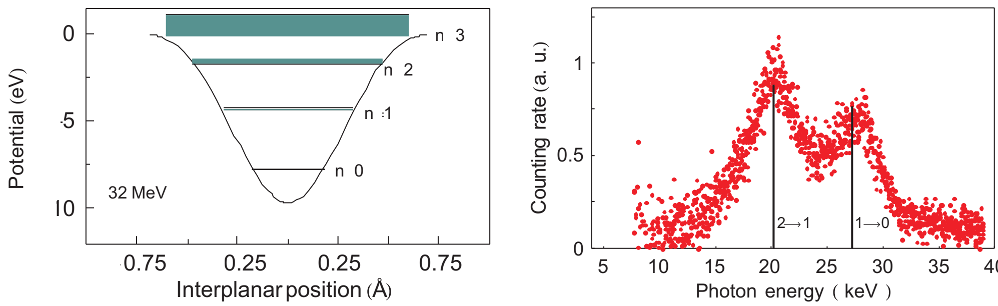

Fig. 1. – Left: continuous potential of the $(0\bar{2}23)$ plane of quartz with channeling states. Right: spectrum of channeling radiation measured at the electron energy of 32 MeV on a 200 μm thick quartz crystal. Vertical lines—calculated photon energies.

requires the knowledge of the transition energies between bound states in the continuous planar potential, which may be calculated by means of the many-beam formalism [23,24]. Since α-quartz (SiO_2) is a piezoelectric material, acoustic waves of defined frequency may easily be excited in a sample by applying the reverse piezoelectric effect. The excitation of hypersonic waves (HW) in a thin crystal plate, however, requires its positioning inside an RF-cavity which is operated in the TM_{010} mode [22]. The predicted influence of HW on CR has for the first time been verified in [25]. The present work deals with a continuative experiment carried out at ELBE [26].

2. – Planar CR measured on quartz

Results of our systematic CR measurements on quartz at electron energies of 17–32 MeV are published in [24,27,28]. The following example illustrates the motivation for this work. From the bound states drawn in the continuous potential of the rather exotic $(0\bar{2}23)$ plane of quartz (fig. 1, left) it is obvious that CR emitted by electrons of energy 32 MeV channeled along this plane may result from only two possible transitions. The measured CR spectrum (fig. 1, right) shows two pronounced peaks at photon energies which are in excellent agreement with calculated values [24]. Stimulation of CR emission in this plane by appropriate HW should, therefore, be clearly observable.

3. – Frequencies of resonant hypersound

The quantum-mechanical description of the impact of HW on CR is developed in [18, 19] and discussed in [25]. Let us only briefly reconsider essential facts. Since bound states of planar channeled electrons are in general not equidistant, resonant HW may influence only CR transitions between dedicated states. The relation between CR energy and resonance frequency of HW is approximately given by eq. (1):

$$f_s = \frac{v_s \omega_0}{\pi c} \approx \frac{v_s}{2\pi c\hbar} \frac{E_{if}^{\mathrm{CR}}}{\gamma^2}, \tag{1}$$

where v_s denotes the velocity of sound in the crystal ($\approx$ units $\times\, 10^3$ m/s), $2\gamma^2\hbar\omega_0 = E_{if}^{\mathrm{CR}}$ is the maximum photon energy at zero observation angle ($\theta = 0$) and γ is the Lorentz factor. According to [13], the resonance condition for longitudinal HW, $\lambda_s = \lambda_0/2$, leads

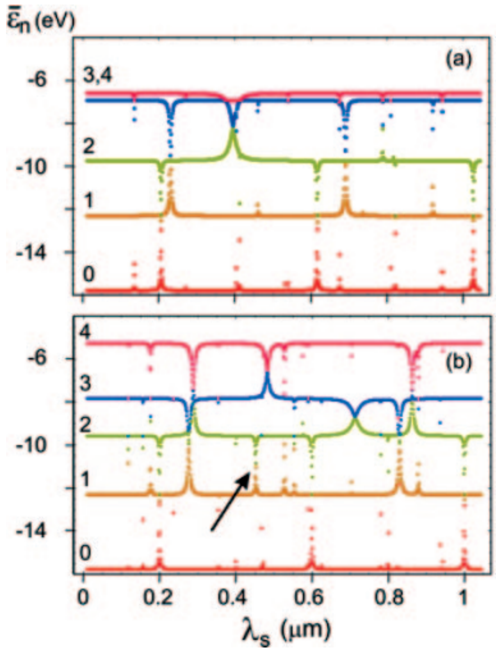

Fig. 2. – Calculated effective eigenvalues for electrons of energy 32 MeV planar channeled in an acoustic superlattice. (a) $k_x = 0$; (b) $k_x = 0.6\, g/2$.

to $\beta_s = v_s/c$ (c—velocity of light) what considerably lowers f_s compared to the frequency of emitted CR. Nevertheless, as resulting from [22, 24], the frequencies of resonant HW reach values near or even higher than 10 GHz. Calculations of effective eigenvalues for electrons of energy 32 MeV channeled in an acoustic superlattice along the $(0\bar{2}23)$ plane are illustrated in fig. 2 (cf. [25]). Due to the alternating parity of bound channeling states at zero Bloch momentum, $k_x = 0$, resonant mixing of states at dedicated wavelengths of HW is observed only for $\Delta n = 2$ (fig. 2a). As recently discovered in [25], the zone structure of bound states related to the variation $-g/2 < k_x < g/2$ (g—reciprocal lattice vector) also allows dispersive mixing of states with $\Delta n = 1$. This is illustrated in fig 2b where 1↔2 resonance exemplarily occurs at $\lambda_s \approx 0.45\,\mu$m for $k_x = 0.6\, g/2$. Variation of k_x (k_x is connected with the electron entrance angle into the channeling plane) shifts the calculated eigenvalues through the corresponding zones. Therefore, the wavelength of resonant HW is also shifted, and this circumstance came out to be useful for practical realization of the resonance condition.

The measurement setup is described in [25]. The oscillating electric field in a pillbox cavity excites HW of about 12 GHz near the surface of the thin crystal plate. Since propagation of HW through the crystal is damped, the relativistic electrons rather interact with a frozen acoustic superlattice of traveling HW than with standing ones.

4. – Measurements

The radiation emitted at zero degree with respect to the beam direction into a solid angle $\leq 10^{-7}$ sr was registered by a CdTe detector. The average electron current of ≈ 1 nA has been monitored by a transmission beam-monitoring system [29].

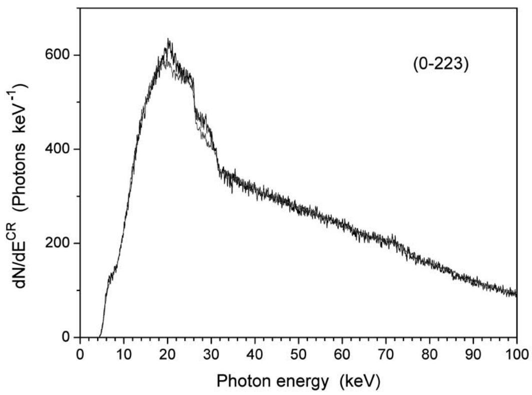

Fig. 3. – Radiation spectra as measured at aligned channeling plane for electrons of energy 34MeV without (grey) and with excitation of hypersound (black) in a 500 μm thick quartz crystal.

The x-cut of a quartz single crystal is characterized by a comparatively large number of crystallographic planes [22,24]. Application of conventional scanning procedures provides the orientation of planes with intense CR such as $(01\bar{1}1)$, $(0\bar{1}11)$ and $(01\bar{1}0)$ [24, 27, 28]. The precise alignment of the searched $(0\bar{2}23)$ plane has been proved by reproduction of the characteristic double-humped CR spectrum (see fig. 1). Spectra of CR measured with and without excitation of HW and normalized to each other at the high-energy tail are shown in fig. 3. Due to the crystal thickness of 500 μm, one observes a huge background of bremsstrahlung. At photon energies between 16 and 30 keV, deviations between the two spectra are clearly seen. Subtraction of these spectra from each other eliminates the background and separates the effect of HW on CR emission. The data processing, however, consisted in individual subtraction of adequately normalized bremsstrahlung spectra separately measured at misaligned crystal orientation (for more clearness not shown in fig. 3). Because of rather poor resulting statistics, moderate smoothing had to be applied for better visualisation of the effect. The spectral contributions of CR to the spectra of fig. 3 are presented in fig. 4.

The effective resonance frequency of the RF-cavity, checked at fully equipped setup by means of a network analyzer, amounted to 11.782 GHz. Since it is fixed, resonance had been achieved by tuning of the electron energy according to eq. (1). For narrow channeling zones, *i.e.* for transitions between low-lying states, this procedure may be troublesome because the resonance curve of a GHz-cavity is usually also quite narrow [25].

5. – Discussion of experimental results

As shown in fig. 4, CR emission may be stimulated by resonant HW. Other than in our previous work, where at given electron energy there were only two bound states in the $(01\bar{1}5)$ plane [25], the frequency of HW here corresponded to a CR transition between states with quantum numbers $n = 2$ and $n = 1$ (fig. 1, left). It seems plausible that enhanced CR emission resulting from higher transitions is accompanied with successive enhancement of CR intensities from lower transitions. In the present case, radiant $2 \to 1$ transitions feed the state $n = 1$ which represents the initial state for radiant $1 \to 0$ transitions to the ground state $n = 0$.

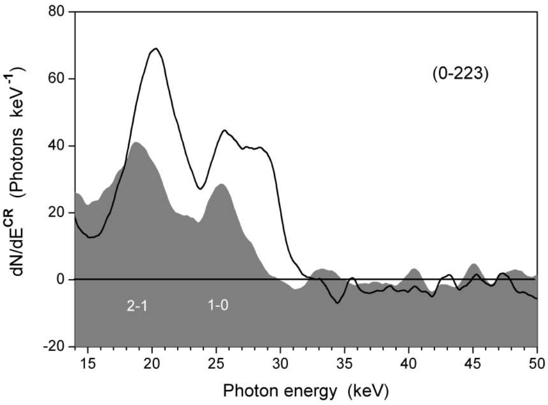

Fig. 4. – Bremsstrahlung-background subtracted spectra of channeling radiation. The lower spectrum (filled) results from the measurement without, the upper one (line) from the measurement with excitation of hypersound in a 500 μm thick quartz crystal.

The dispersive behavior of eigenvalues at resonant wavelengths of HW (see fig. 2) means that the wave functions of channeled electrons near resonance turn out to be superpositions of two components with different quantum numbers. Since the total electron energy, E, is an integral of motion, the dispersion relation reads $E \cong cp_z + \epsilon(n, k_x)$ (p denotes the electron momentum). The eigenvalues $\epsilon(n, k_x)$ represent the known zone structure of channeling states in the transversely periodic planar continuum potential. The energies of spontaneously emitted CR relate to differences such as $\Delta\epsilon = \epsilon_{n+1} - \epsilon_n$, $n = 0, 1, 2\ldots$ Since the Bloch momentum k_x may vary within $-g/2 < k_x < g/2$, one actually has $\epsilon(n, k_x) \to \widehat{\epsilon}(n, k_x)$ and $\Delta\epsilon \to \Delta\widehat{\epsilon}$, where averaging over the beam divergence implies $k_x \to \widehat{k}_x$ with p_z being an integral of motion. Then $\epsilon(n, k_x) \to \widehat{\epsilon}_n$ represents the eigenvalue of a multiply degenerated state.

Recently it was shown for channeling in an acoustic superlattice that p_z and ϵ_n are no longer integrals of motion [30]. The crystal potential periodically perturbed in channeling direction ($z = ct$) changes the electron wave functions and eigenvalues essentially because spatial averaging of the planar potential along z has broken. Similar to transverse direction x, the periodicity of the potential along z (or time dependence) suggests that the electron wave functions are also Bloch functions depending on some k_z.

Let us make a little simplified consideration. As well known, squared wave functions represent electron probability distributions. The periodic acoustic field may be characterized by a reciprocal lattice vector $G = 2\pi/\lambda_s$ or quasi-momentum $\hbar G$. Imagine the electron beam as a wave packet which is diffracted from an acoustic lattice. The Bragg condition would then read $\Delta k_z = G = 2\pi f_s/v_s = \Delta p_z/\hbar$ from which one obtains

$$f_s = \frac{v_s}{2\pi\hbar}\Delta p_z = \frac{v_s}{2\pi c\hbar}\Delta(cp_z). \tag{2}$$

Comparison of eq. (2) with eq. (1) at conservation of the total electron energy gives

$$\pm\Delta(cp_z) = \mp\Delta\epsilon. \tag{3}$$

This means that at resonance with HW neither p_z nor ϵ are integrals of motion, but the dispersion relation holds for an arbitrary number of combinations of kinetic and potential electron energy,

$$E \cong cp_z + \epsilon(n, k_x, k_z) \cong cp_0 + \epsilon_0 \cong cp_1 + \epsilon_1 \cong cp_2 + \epsilon_2 \cong \ldots cp_i + \epsilon_i. \tag{4}$$

If the frequency of HW relates to realistic differences of eigenvalues $\Delta\epsilon = \epsilon_{i+1} - \epsilon_i$, it follows from eq. (3) that $\Delta(cp_z) = c\hbar\Delta k_z = -\Delta\epsilon(\Delta k_z) = -(\epsilon_{i+1} - \epsilon_i) = \epsilon_i - \epsilon_{i+1}$. In other words, the initial and final states are indistinguishable. At resonance they are merged. One observes a quantum uncertainty as demonstrated in fig. 2, and that even at small magnitudes of hypersound [30].

This situation may be interpreted twice, namely, that the electron on its path through the crystal is resonantly scattered between two states due to diffraction on an acoustic lattice, or that the matrix element of perturbative overlap of electron wave functions due to the impact of resonant HW is different from zero. At different parity of states, this overlap will be zero ($k_x = 0$, fig. 2a). The zone structure of channeling states, especially pronounced near the top of the potential, *i.e.* for larger n, involves $k_x \neq 0 \rightarrow \delta\epsilon(n, k_x, k_z)$. Realizing δk_x, *i.e.* $\delta\Theta_i \neq 0$ for the electron entrance angle Θ_i (no plane electron wave), one may find some $\delta\lambda_s$ (or δf_s, respectively) where a finite overlap is realized. In the present experiment we made use of this circumstance (fig. 2b).

Since incoming electron bunches and HW, excited in the crystal, are not synchronized to each other, the initial population of states is an average over all possible phase differences. Intense thermal (multiple) scattering in quartz rapidly equalizes the occupation of states and finally leads to dechanneling [22]. The radiationless $1 \leftrightarrow 2$ scattering of channeled electrons between states due to resonant HW has consequences on the occupation probability of states and, therefore, on CR emission. If the CR intensity from the transition $2 \rightarrow 1$ is enhanced (fig. 4), the corresponding initial state $n = 2$ was populated additionally. Obviously, this may be caused by resonant $1 \rightarrow 2$ scattering during channeling. The small shift of the $2 \rightarrow 1$ CR peak, observed in fig. 4, indicates this process which is connected with $k_x \neq 0$. Comparison of fig. 2a with fig. 2b also reveals that $\widehat{\epsilon}\,(n, k_x)$ is shifted with respect to $\epsilon(n, k_x = 0)$.

On the other hand, concerning resonant $1 \leftrightarrow 2$ electron scattering, there should be no reason that $1 \rightarrow 2$ scattering occurs with another probability than $2 \rightarrow 1$ one. So the state $n = 1$ is fed by radiant and radiationless transitions as well. Consequently, radiant $1 \rightarrow 0$ transitions will also be resonantly enhanced, as observed in fig. 4. However, when $\Delta\widehat{\epsilon}_{21}(k_x) = \widehat{\epsilon}(2, k_x) - \widehat{\epsilon}(1, k_x)$ increases at $k_x \neq 0$, the difference $\Delta\widehat{\epsilon}_{10}(k_x) = \widehat{\epsilon}(1, k_x) - \epsilon_0$ decreases, so one should expect that the CR energy from a $1 \rightarrow 0$ transition would be $\Delta\widehat{\epsilon}_{10}(k_x \neq 0) \leq \epsilon(1, k_x = 0) - \epsilon_0$. The quantum uncertainty of eq. (4) actually declares that resonance leads to a large dispersion $\delta\epsilon$. Assuming for some CR emission accompanied with $2 \rightarrow 1$ scattering $\Delta\widehat{\epsilon}_{10}(k_x \neq 0) \in [(\widehat{\epsilon}_1 + \delta\epsilon) - \epsilon_0; \widehat{\epsilon}_1 - \epsilon_0]$ where $\delta\epsilon(k_x) \leq (\epsilon_2 - \epsilon_1)/2$ [25], then $\Delta\widehat{\epsilon}_{10}(k_x \neq 0) \geq \widehat{\epsilon}_1(k_x \neq 0) - \epsilon_0$ would explain the rather broad distribution of CR energies observed in fig. 4 for transitions $1 \rightarrow 0$. Summing up one may state that resonant HW provoke electron scattering where $1 \rightarrow 2$ scattering additionally populates the state $n = 2$. Due to the dispersion $\delta\epsilon$, the transition energies may be shifted.

The reduced level scheme shown in the left panel of fig. 1 implicates only bound states including the first zone of quasi-channeling. The top of the potential extends in fig. 2a to a value of about $-8\,\mathrm{eV}$. At $k_x = 0$ the eigenvalues of the quasi-free states $n = 3; 4$ represent maximum and minimum values, respectively. For $k_x \neq 0$ (fig. 2b) the corresponding values decrease and increase accordingly, smearing out over the entire zones of quasi-channeling. In the present measurement, the frequency of HW was chosen to cover at some $k_x \neq 0$ the energy gap $\Delta\epsilon_{12}$. It is imaginable that a value of $\Delta\epsilon = \Delta\epsilon_{12}$ is also valid for states in the quasi-channeling zones $n = 2; 3$ and $n = 3; 4$, etc. Therefore, at the given f_s, resonant electron scattering should also occur between these zones and might contribute to the radiation production. One example is the 3↔4 resonance found in fig. 2b at $\lambda_s \approx 4.8\,\mu$m. Comparison with fig. 2a reveals that for $k_x = 0$ this resonance occurs at a smaller value and for $k_x = 0.6\ g/2$ at a larger value than $\lambda_s \approx 4.5\,\mu$m found for the 1↔2 resonance. Due to δk_x both resonances may overlap.

6. – Conclusions

The effect of CR intensity amplification at channeling in a hypersonic superlattice was for the first time verified in [25] and has been approved in this work for another crystallographic plane of a single quartz crystal. The influence of HW on CR was predicted in [12,13] and theoretically investigated in [14-22]. Since the presentation of exact calculations would go beyond the scope of this work, experimental results were discussed in a slightly simplified manner. Nevertheless, main features of the CR spectrum observed under the influence of HW may be explained.

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