IL NUOVO CIMENTO Vol. 121 B, N. 1 Gennaio 2006
DOI 10.1393/ncb/i2005-10196-x

Statistical difference at high temperature in particle creation

H. Majima and A. Suzuki

Department of Physics, Faculty of Science, Tokyo University of Science
1-3 Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan

(ricevuto il 2 Novembre 2005; approvato il 20 Gennaio 2006)

Summary. — The creation of particles is a natural consequence of quantum field theory in curved space-time. We study this phenomenon at finite temperature by using Thermo Field Dynamics (TFD). In the conventional TFD formulated in the thermal Schrödinger picture, temperature is included in the state vectors. We adopt another approach where the temperature is absorbed in operators, so that the double (time and thermal) Bogoliubov transformations of the operators in curved space-time can be unified consistently with TFD, giving a correct number of particles created from false vacuum at finite temperature. We found that the net number of particles created at time t due to thermal effects and curved space-time is given by $\Delta n_{\mathbf{k}}(t,\beta) = \{1 + \sigma 2 n_{\mathbf{k}}(\beta)\} n_{\mathbf{k}}(t)$, where $n_{\mathbf{k}}(t)$ is the number of particles at time t and $n_{\mathbf{k}}(\beta) = 1/(e^{\beta\omega_k} - \sigma)$ the initial distribution for bosons ($\sigma = 1$) and fermions ($\sigma = -1$) at temperature $T\,(= 1/k_{\mathrm{B}}\beta)$, respectively. Thermal state condition in THP is also given in a general form.

PACS `11.10.Wx` – Finite-temperature field theory.
PACS `04.62.+v` – Quantum field theory in curved spacetime.
PACS `98.80.Cq` – Particle-theory and field-theory models of the early Universe (including cosmic pancakes, cosmic strings, chaotic phenomena, inflationary universe, etc.).

1. – Introduction

Laciana [1] studied the problem of boson creation amplification in curved space [2] due to thermal effects. In his study, he formulated the problem by means of thermo field dynamics (TFD) along with the Bogoliubov transformation. TFD [3-5] is a field theory to describe elementary particles at finite temperature and has been recognized as a theory essential to treat a system with infinitely many quantum particles. We can expect that there are two possibilities in the creation of particles (boson, fermion) from the vacuum state: one is a *dynamical effect* due to a natural consequence of quantum field theory in curved space-time and the other is a *spontaneous phenomenon* due to thermal effects. In the latter case, those particles are created from false vacuum at temperature T.

In this paper, we shall consider the problem of creation of particles from false vacuum at finite temperature by using TFD for both *bosonic* and *fermionic* fields in a unified manner. We formulate the problem by introducing a *thermal Heisenberg operator* [6] analogous to a Heisenberg operator in quantum mechanics. In other words, we construct the theory of particle creation in terms of a thermal Heisenberg picture (THP) within the framework of TFD. When we take the statistical averaging of an observable A in

TFD, the averaging of a thermal Heisenberg operator $A(\beta) := U^\dagger(\beta)AU(\beta)$ should be taken so as to give a correct expression for $\langle A\rangle$ although $\langle A\rangle$ can be calculated in terms of a thermal Schrödinger picture (TSP), that is, $\langle A\rangle$ can be calculated from

$$\langle A\rangle \equiv \begin{cases} \langle 0,\widetilde{0}|A(\beta)|0,\widetilde{0}\rangle & \text{(THP)},\\ \langle 0(\beta)|A|0(\beta)\rangle & \text{(TSP)}, \end{cases} \tag{1}$$

where $|0(\beta)\rangle$ $(:= U(\beta)|0,\widetilde{0}\rangle)$ is a thermal vacuum state and $U(\beta)$ a unitary operator that generates $|0(\beta)\rangle$ upon operating the vacuum state $|0,\widetilde{0}\rangle$. In ref. [1], however, $\langle A\rangle$ is calculated from $\langle 0,\widetilde{0}|A_\beta|0,\widetilde{0}\rangle = \langle 0,\widetilde{0}|U(\beta)AU^\dagger(\beta)|0,\widetilde{0}\rangle$. Thus, the expectation value in ref. [1] is certainly *not* equal to the statistical averaging defined in TFD though his result for boson creation gives the correct result as will be derived in this paper. In order to avoid the inconsistency for the use of statistical averaging in TFD as seen in ref. [1] and some literatures, we shall present a theory in terms of THP within the framework of TFD and apply it to the problem of particle creation amplification for both bosonic and fermionic fields and study the effect of the statistical defference in particle creation, namely, the creation of bosons and fermions at finite temperature. We specifically consider the situation where we had a *curved* geometry at finite temperature initially and ask how many particles will be created by thermal effects at time t. We will show that, in the formulation, working with THP is advantageous over the conventional TFD formulated in TSP and unavoidable utility of the double Bogoliubov transformation to connect initial and final thermal states is proved to be justified under certain circumstances.

2. – Thermo field dynamics

2·1. *Preliminary.* – To begin with, we briefly summarize the most essential parts of TFD [3]. In TFD, statistical averaging of an observable A in the ordinary quantum-statistical mechanics is expressed by

$$\langle A\rangle = \mathrm{Tr}(\rho A) \equiv \langle 0(\beta)|A|0(\beta)\rangle, \tag{2}$$

where $\rho\,(= \exp[-\beta H]/\mathrm{Tr}[\exp[-\beta H]])$ is a density operator and $|0(\beta)\rangle$ is a thermal (inverse temperature $(\beta \equiv 1/k_\mathrm{B}T)$ dependent) vacuum state defined by

$$|0(\beta)\rangle := \rho^{1/2}\sum_n |n\rangle\otimes|\widetilde{n}\rangle =: \rho^{1/2}\sum_n |n,\widetilde{n}\rangle. \tag{3}$$

Here, $|n\rangle$ $(|\widetilde{n}\rangle)$ is the eigenstate of Hamiltonian H $(\widetilde{H})$ in the Hilbert (tilde conjugate Hilbert) space $\mathscr{H}$ $(\widetilde{\mathscr{H}})$ and each state satisfies the orthogonality conditions: $\langle m,\widetilde{m}|n,\widetilde{n}\rangle = \delta_{mn}\delta_{\widetilde{m}\widetilde{n}}$, where $|n\rangle \in \mathscr{H}$ and $|\widetilde{n}\rangle \in \widetilde{\mathscr{H}}$. By introducing the tilde conjugate Hilbert space $\widetilde{\mathscr{H}}$ corresponding to the ordinary Hilbert space $\mathscr{H}$, thermal states can be expressed in the space $\mathscr{H}\otimes\widetilde{\mathscr{H}}$. Any operators A, B on $\mathscr{H}$ and the corresponding tilde operators $\widetilde{A}$, $\widetilde{B}$ on $\widetilde{\mathscr{H}}$ must satisfy the following tilde conjugation rules:

- $(AB)^\sim = \widetilde{A}\widetilde{B}$,
- $(c_1A + c_2B)^\sim = c_1^*\widetilde{A} + c_2^*\widetilde{B} \quad (c_1, c_2 \in \mathbb{C})$,
- $(A^\dagger)^\sim = \widetilde{A}^\dagger$,

- $(\widetilde{A})^{\sim} = \sigma A \quad (\sigma = 1\ (-1)$ for boson (fermion)),
- $[A, \widetilde{B}]_\sigma = A\widetilde{B} - \sigma\widetilde{B}A = 0$,

where $\dagger$ indicates an Hermitian conjugate. It is now clear that the statistical average (expectation value) of an observable A on $\mathscr{H}$ can be in principle calculated from $\langle 0(\beta)|A|0(\beta)\rangle$ by use of the vacuum state (3), on which so-called the *thermal state condition* [3] in TSP, *i.e.*

$$A|0(\beta)\rangle = e^{\beta\hat{H}/2}\widetilde{A}^\dagger|0(\beta)\rangle \tag{4}$$

has to be imposed in order to satisfy the tilde conjugation rules listed above. It should be noted that $\hat{H} := H - \widetilde{H}$ is introduced so as to give the same expression for eq. (2) [5]. In this paper we shall consider a system of free particles (bosons, fermions). The Hamiltonian is given by $H = \sum_{\mathbf{k}} \hbar\omega_k a^\dagger_{\mathbf{k}} a_{\mathbf{k}}$. Here, $a_{\mathbf{k}}$, $a^\dagger_{\mathbf{k}}$ denote, respectively, the annihilation and creation operators for bosonic (fermionic) fields defined on $\mathscr{H}$ and satisfy the commutation (anti-commutation) relations: $[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}]_\mp = \delta_{\mathbf{k},\mathbf{k}'}$, $[a_{\mathbf{k}}, a_{\mathbf{k}'}]_\mp = 0$. Here, the upper $(-)$ sign corresponds to a commutation relation for bosons whereas the lower $(+)$ sign to an anti-commutation relation for fermions. $\mathbf{k}$ denotes the wave vector of a particle. In TFD, we also have to introduce the tilde operators $\widetilde{a}_{\mathbf{k}}$, $\widetilde{a}^\dagger_{\mathbf{k}}$, which are, respectively, the annihilation and creation operators for bosonic (fermionic) fields defined on $\widetilde{\mathscr{H}}$ and represent the quantum effect of particle distribution, namely, the thermal effect. These tilde operators obey the tilde conjugation rules [3] and also satisfy the usual commutation (anti-commutation) relations for bosonic (fermionic) fields.

In the following, we shall consider the problem of particle creation from false vacuum at finite temperature for a system consisting of free particles (bosons, fermions). Hereafter $\hbar$ and k_{B} will be taken as 1.

2·2. *Thermal Schrödinger picture*. – In conventional TFD, a thermal vacuum state (3) can be expressed by

$$|0(\beta)\rangle = U(\beta)|0, \widetilde{0}\rangle, \tag{5}$$

where the vacuum state $|0, \widetilde{0}\rangle$ belongs to the double Hilbert space $\mathscr{H} \otimes \widetilde{\mathscr{H}}$ and $U(\beta)$ is the unitary operator defined by [3]

$$U := \exp\left[-\sum_{\mathbf{k}} \theta_{\mathbf{k}}(\beta)(\widetilde{a}_{\mathbf{k}} a_{\mathbf{k}} - a^\dagger_{\mathbf{k}}\widetilde{a}^\dagger_{\mathbf{k}})\right], \tag{6}$$

where $\theta_{\mathbf{k}}(\beta)$ must be determined from $\sinh\theta_{\mathbf{k}}(\beta) = \frac{e^{-\beta\omega_k/2}}{\sqrt{1-e^{-\beta\omega_k}}}$ for bosons and $\sin\theta_{\mathbf{k}}(\beta) = \frac{e^{-\beta\omega_k/2}}{\sqrt{1+e^{-\beta\omega_k}}}$ for fermions, respectively. The unitary operator U provides the thermal Bogoliubov transformation, giving new operators $a_{\mathbf{k},\beta}$, $\widetilde{a}_{\mathbf{k},\beta}$ as

$$a_{\mathbf{k},\beta} := U a_{\mathbf{k}} U^\dagger = a_{\mathbf{k}} \cosh\theta_{\mathbf{k}}(\beta) - \widetilde{a}^\dagger_{\mathbf{k}} \sinh\theta_{\mathbf{k}}(\beta), \tag{7}$$

$$\widetilde{a}_{\mathbf{k},\beta} := U \widetilde{a}_{\mathbf{k}} U^\dagger = \widetilde{a}_{\mathbf{k}} \cosh\theta_{\mathbf{k}}(\beta) - a^\dagger_{\mathbf{k}} \sinh\theta_{\mathbf{k}}(\beta), \tag{8}$$

for bosons and

$$a_{\mathbf{k},\beta} := U a_{\mathbf{k}} U^{\dagger} = a_{\mathbf{k}} \cos\theta_{\mathbf{k}}(\beta) - \widetilde{a}_{\mathbf{k}}^{\dagger} \sin\theta_{\mathbf{k}}(\beta), \tag{9}$$

$$\widetilde{a}_{\mathbf{k},\beta} := U \widetilde{a}_{\mathbf{k}} U^{\dagger} = \widetilde{a}_{\mathbf{k}} \cos\theta_{\mathbf{k}}(\beta) + a_{\mathbf{k}}^{\dagger} \sin\theta_{\mathbf{k}}(\beta), \tag{10}$$

for fermions, respectively. These new operators $a_{\mathbf{k},\beta}$, $\widetilde{a}_{\mathbf{k},\beta}$ and their Hermitian conjugate operators $a_{\mathbf{k},\beta}^{\dagger}$, $\widetilde{a}_{\mathbf{k},\beta}^{\dagger}$ are called the *thermal quasi-particle operators* that operate on $|0(\beta)\rangle$ and satisfy the usual commutation (anti-commutation) rules for the bosonic (fermionic) fields [4, 5]. Solving eqs. (7), (8) (eqs. (9), (10)) for $a_{\mathbf{k}}$, $a_{\mathbf{k}}^{\dagger}$ and using the tilde conjugation rules and the appropriate commutation relations for these thermal quasi-particle operators of the bosonic (fermionic) field, we can obtain the number of particles (bosons, fermions) having momentum (wave vector) $\mathbf{k}$ at temperature $T(=1/\beta)$:

$$\begin{aligned} n_{\mathbf{k},\beta} &= \langle a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} \rangle \equiv \langle 0(\beta) | a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} | 0(\beta) \rangle \\ &= \begin{cases} \sinh^2\theta_{\mathbf{k}}(\beta) = (e^{\beta\omega_k} - 1)^{-1} & \text{(boson)}, \\ \sin^2\theta_{\mathbf{k}}(\beta) = (e^{\beta\omega_k} + 1)^{-1} & \text{(fermion)}. \end{cases} \end{aligned} \tag{11}$$

As expected, we obtained the Bose-Einstein and the Fermi-Dirac distributions for bosonic and fermionic fields, respectively. In the conventional TFD, the thermal effect (*viz.*, temperature) is generally taken into account in the state vectors as demonstrated in the above calculation for the particle number. Analogous to the Schrödinger picture in quantum mechanics, we call this picture "Thermal Schrödinger Picture" (TSP) in TFD.

2˙3. *Thermal Heisenberg picture.* – Analogous to the Heisenberg picture in quantum mechanics, where the time dependence is absorbed in an operator, we transfer the thermal effect into an operator, so that the operator depends on temperature. We call this picture "Thermal Heisenberg Picture" (THP) and those temperature-dependent operators in THP of TFD are called *thermal Heisenberg operator*, which may be defined through the thermal Bogoliubov transformation as $A(\beta) := U^{\dagger}(\beta) A U(\beta)$. Thermal state condition in THP corresponding to eq. (4) is then given by

$$A(\beta)|0,\widetilde{0}\rangle = e^{\beta \hat{H}(\beta)/2} \widetilde{A}^{\dagger}(\beta)|0,\widetilde{0}\rangle, \tag{12}$$

where $A(\beta)$, $\widetilde{A}(\beta)$ are thermal Heisenberg operators defined on $\mathscr{H} \otimes \widetilde{\mathscr{H}}$. It should be noted that the definition of a thermal Heisenberg operator $A(\beta)$ differs from the thermal quasi-particle operator A_{β} introduced in refs. [1,4,5], *viz.*, $U^{\dagger}(\beta) A U(\beta) =: A(\beta) \neq A_{\beta} := U(\beta) A U^{\dagger}(\beta)$.

Let us illustrate the THP within the framework of TFD. Since the thermal vacuum state introduced in TFD is given by eq. (5), the statistical averaging of an observable A (see eq. (1)) can be expressed in terms of the thermal Heisenberg operator $A(\beta)$:

$$\begin{aligned} \langle 0(\beta)|A|0(\beta)\rangle &= \langle 0,\widetilde{0}|U^{\dagger}(\beta) A U(\beta)|0,\widetilde{0}\rangle \\ &=: \langle 0,\widetilde{0}|A(\beta)|0,\widetilde{0}\rangle, \end{aligned} \tag{13}$$

where $U(\beta)$ is the unitary operator (generator) given by eq. (6). It is clear that eq. (13) is *not* equal to $\langle 0,\widetilde{0}|A_{\beta}|0,\widetilde{0}\rangle = \langle 0,\widetilde{0}|U(\beta) A U^{\dagger}(\beta)|0,\widetilde{0}\rangle$ defined in ref. [1] and some literatures. The statistical averaging of an observable A must be evaluated by taking the expectation value of the thermal Heisenberg operator $A(\beta)$ with the vacuum state $|0,\widetilde{0}\rangle$ as defined in eq. (13).

From the definition of a thermal Heisenberg operator $A(\beta) := U^\dagger(\beta)AU(\beta)$ along with eq. (5), those thermal Heisenberg annihilation operators, $a_{\mathbf{k}}(\beta)$ and $\widetilde{a}_{\mathbf{k}}(\beta)$, are explicitly given for boson by

$$a_{\mathbf{k}}(\beta) := U^\dagger a_{\mathbf{k}} U = a_{\mathbf{k}} \cosh\theta_{\mathbf{k}}(\beta) + \widetilde{a}^\dagger_{\mathbf{k}} \sinh\theta_{\mathbf{k}}(\beta), \tag{14}$$

$$\widetilde{a}_{\mathbf{k}}(\beta) := U^\dagger \widetilde{a}_{\mathbf{k}} U = \widetilde{a}_{\mathbf{k}} \cosh\theta_{\mathbf{k}}(\beta) + a^\dagger_{\mathbf{k}} \sinh\theta_{\mathbf{k}}(\beta) \tag{15}$$

and for fermion by

$$a_{\mathbf{k}}(\beta) := U^\dagger a_{\mathbf{k}} U = a_{\mathbf{k}} \cos\theta_{\mathbf{k}}(\beta) + \widetilde{a}^\dagger_{\mathbf{k}} \sin\theta_{\mathbf{k}}(\beta), \tag{16}$$

$$\widetilde{a}_{\mathbf{k}}(\beta) := U^\dagger \widetilde{a}_{\mathbf{k}} U = \widetilde{a}_{\mathbf{k}} \cos\theta_{\mathbf{k}}(\beta) - a^\dagger_{\mathbf{k}} \sin\theta_{\mathbf{k}}(\beta) \tag{17}$$

and their corresponding Hermitian conjugates are also obtained from these equations. It is noted that these new operators $a_{\mathbf{k}}(\beta), \widetilde{a}_{\mathbf{k}}(\beta)$ and their Hermitian conjugates $a^\dagger_{\mathbf{k}}(\beta)$, $\widetilde{a}^\dagger_{\mathbf{k}}(\beta)$ also satisfy the usual commutation (anti-commutation) relations for bosonic (fermionic) fields. Employing eqs. (14), (16) and their Hermitian conjugates in the calculation of particle (boson and fermion) number $\langle 0,\widetilde{0}|a^\dagger_{\mathbf{k}}(\beta)a_{\mathbf{k}}(\beta)|0,\widetilde{0}\rangle$ in THP, we can obtain the same expressions as in eq. (11) calculated in TSP. To show this, let us calculate the number of particles (bosons, fermions) having momentum $\mathbf{k}$ at finite temperature in THP. The transformations (14), (16) and the Hermitian conjugates of (15), (17) can be expressed for the bosonic and fermionic fields in a same simple matrix equation:

$$\begin{pmatrix} a_{\mathbf{k}}(\beta) \\ \widetilde{a}^\dagger_{\mathbf{k}}(\beta) \end{pmatrix} = \frac{1}{\sqrt{1-\sigma f_k}} \begin{pmatrix} 1 & f_k^{1/2} \\ \sigma f_k^{1/2} & 1 \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}} \\ \widetilde{a}^\dagger_{\mathbf{k}} \end{pmatrix}, \tag{18}$$

where we used $\sinh\theta_{\mathbf{k}}(\beta) = \frac{e^{-\beta\omega_k/2}}{\sqrt{1-e^{-\beta\omega_k}}}$ for bosons and $\sin\theta_{\mathbf{k}}(\beta) = \frac{e^{-\beta\omega_k/2}}{\sqrt{1+e^{-\beta\omega_k}}}$ for fermions, respectively. In eq. (18), f_k is defined by $f_k := e^{-\beta\omega_k}$ and $\sigma = 1\,(-1)$ corresponds to the case for boson (fermion). Since the vacuum state at 0 K in THP is given by $|\mathbf{0}\rangle := |0,\widetilde{0}\rangle$, the expectation value of the particle number at $T\ (= 1/\beta)$ can be calculated from $\langle\mathbf{0}|a^\dagger_{\mathbf{k}}(\beta)a_{\mathbf{k}}(\beta)|\mathbf{0}\rangle$ and is given by

$$\begin{aligned} n_{\mathbf{k}}(\beta) &= \langle 0,\widetilde{0}|a^\dagger_{\mathbf{k}}(\beta)a_{\mathbf{k}}(\beta)|0,\widetilde{0}\rangle \\ &= \frac{1}{1-\sigma f_k}\langle\mathbf{0}|f_k^{1/2}\widetilde{a}_{\mathbf{k}} f_k^{1/2}\widetilde{a}^\dagger_{\mathbf{k}}|\mathbf{0}\rangle = \frac{1}{e^{\beta\omega_k}-\sigma}. \end{aligned} \tag{19}$$

This is the Bose-Einstein (Fermi-Dirac) distribution for free bosons (fermions). From eqs. (11) and (19), we see that $n_{\mathbf{k},\beta} = n_{\mathbf{k}}(\beta)$, proving the equivalence between TSP and THP in TFD. The definition of thermal Heisenberg operators, $A(\beta) := U^\dagger(\beta)AU(\beta)$, introduced in this theory is therefore justified.

Now we consider another approach to treat the problem of particle creation amplification for bosonic and fermionic fields from false vacuum by thermal effects, namely, to treat it in terms of THP within the framework of TFD.

3. – Particle creation from false vacuum at finite temperature

Let us consider the situation where we had a curved geometry at finite temperature initially and ask how many particles will be created by thermal effects at time t. For this purpose, let us first consider another Bogoliubov transformation that relates the annihilation and creation operators at an initial time, say $t = 0$ and time t in curved space-time at zero temperature [2]:

$$a_{\mathbf{k}}(t) = U_k a_{\mathbf{k}} + V_k a^{\dagger}_{-\mathbf{k}}, \quad a^{\dagger}_{\mathbf{k}}(t) = U^*_k a^{\dagger}_{\mathbf{k}} + V^*_k a_{-\mathbf{k}}, \tag{20}$$

where U_k, V_k represent some physical (c-number) parameters characterizing the system (at time t) and $a_{\mathbf{k}}$, $a^{\dagger}_{\mathbf{k}}$ are, respectively, the annihilation and the creation operators for a boson (fermion) with momentum (wave vector) $\mathbf{k}$ at an initial time: $a_{\mathbf{k}}(t = 0) \equiv a_{\mathbf{k}}$, $a^{\dagger}_{\mathbf{k}}(t = 0) \equiv a^{\dagger}_{\mathbf{k}}$. The annihilation and the creation operators, $a_{\mathbf{k}}(t)$, $a^{\dagger}_{\mathbf{k}}(t)$ defined at time t satisfy

$$a_{\mathbf{k}}(t)|0; t\rangle = 0, \quad a^{\dagger}_{\mathbf{k}}(t)|0; t\rangle = |1_{\mathbf{k}}; t\rangle, \tag{21}$$

where $|0; t\rangle$, $|1_{\mathbf{k}}; t\rangle$ denote the vacuum and the one-particle states at time t, respectively. It is noted that $a_{\mathbf{k}}(t)$, $a^{\dagger}_{\mathbf{k}}(t)$ also satisfy the usual commutation (anti-commutation) relations for bosons (fermions), so that U_k and V_k satisfy the relations: $|U_k|^2 - \sigma|V_k|^2 = 1$. The number of particles at time t is then given by

$$n_{\mathbf{k}}(t) = \langle 0|a^{\dagger}_{\mathbf{k}}(t)a_{\mathbf{k}}(t)|0\rangle = |V_k|^2 \tag{22}$$

for *both* bosonic and fermionic fields. It is interesting to note that the number of particles created at time t due to the change in curved space-time gives the *same* result for bosonic and fermionic fields although these fields obey different statistics.

Next we consider the situation where we had a curved geometry at finite temperature initially and ask how many particles will be created by thermal effects at time t. Noticing the similarity between the two Bogoliubov transformations (time and thermal) (see eqs. (20) and (18)), we can conveniently work out this problem by introducing the quadrivectorial operators [1]: $\mathbf{A}_{\mathbf{k}}$, $\mathbf{A}_{\mathbf{k}}(\beta)$ and $\mathbf{A}_{\mathbf{k}}(t)$ defined by

$$\mathbf{A}_{\mathbf{k}}(l) := [\, a_{\mathbf{k}}(l), a^{\dagger}_{\mathbf{k}}(l), \widetilde{a}_{\mathbf{k}}(l), \widetilde{a}^{\dagger}_{\mathbf{k}}(l)\,]^{\mathrm{T}}, \tag{23}$$

where l is a generic variable for t and β. We also introduce the 4×4 matrix $\mathbf{\Lambda}(t, \beta)$, so that the two transformations (time and thermal) can be expressed by the single matrix form

$$\mathbf{A}_{\mathbf{k}}(t, \beta) =: \mathbf{\Lambda}(t, \beta)\mathbf{A}_{\mathbf{k}}, \tag{24}$$

where $\mathbf{\Lambda}(t, \beta)$ is expressed by

$$\mathbf{\Lambda}(t, \beta) = \frac{1}{\sqrt{1 - \sigma f_k}} \begin{pmatrix} \mathbf{M} & f_k^{1/2}\mathbf{R} \\ \sigma f_k^{1/2}\mathbf{R}^* & \widetilde{\mathbf{M}} \end{pmatrix}, \tag{25}$$

and the $\mathbf{R}$ and the $\mathbf{M}$ are given in terms of V_k and U_k as

$$\mathbf{R} = \begin{pmatrix} V_k\mathbf{P} & U_k \\ U_k^* & V_k^*\mathbf{P} \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} U_k & V_k\mathbf{P} \\ V_k^*\mathbf{P} & U_k^* \end{pmatrix}. \tag{26}$$

Here $\mathbf{P}$ denotes the parity operator: $\mathbf{P}a_{\mathbf{k}} = a_{-\mathbf{k}}$.

The total number of created particles (bosons, fermions) comes from two sources: a generic one due to the *curvature of space-time* and the other one due to *thermal effects*. Using the thermal Heisenberg operators $a_{\mathbf{k}}(t,\beta)$, $a_{\mathbf{k}}^{\dagger}(t,\beta)$ and applying the Bogoliubov transformation (24), the number of particles $n_{\mathbf{k}}(t,\beta)$ created from the initial vacuum due to the change of curved space-time and also to the distribution of particles is thus given by

$$\begin{aligned} n_{\mathbf{k}}(t,\beta) &:= \langle \mathbf{0}| a_{\mathbf{k}}^{\dagger}(t,\beta) a_{\mathbf{k}}(t,\beta) |\mathbf{0}\rangle \\ &= n_{\mathbf{k}}(\beta) + n_{\mathbf{k}}(t) + \sigma 2 n_{\mathbf{k}}(\beta) n_{\mathbf{k}}(t). \end{aligned} \tag{27}$$

From this, the net number of created particles (bosons, fermions), $\Delta n_{\mathbf{k}}(t,\beta) = n_{\mathbf{k}}(t,\beta) - n_{\mathbf{k}}(\beta)$, at time t, temperature T, is given by

$$\Delta n_{\mathbf{k}}(t,\beta) = \{1 + \sigma 2 n_{\mathbf{k}}(\beta)\} n_{\mathbf{k}}(t), \tag{28}$$

where $n_{\mathbf{k}}(t)$ is given by eq. (22) and $n_{\mathbf{k}}(\beta)$ by eq. (19) for bosons ($\sigma = 1$) and fermions ($\sigma = -1$), respectively. It is clearly seen from this that the number of particles created in curved space-time, $n_{\mathbf{k}}(t)$, is further modified by the factor $\{1 + \sigma 2 n_{\mathbf{k}}(\beta)\}$ due to the change in particle distribution (*i.e.* temperature). If at the beginning we had a curved geometry, there will be $n_{\mathbf{k}}(t,\beta)$ particles created at time t. Considering the boson (fermion) distribution at time t, we see that the number of bosons (fermions) are further increased (decreased) by the factor $\{1 + \sigma 2 n_{\mathbf{k}}(\beta)\}$ due to thermal effects. This quantum effect can be seen distinctively at low temperature due to the fact that bosons obey the Bose-Einstein statistics while fermions obey the Fermi-Dirac statistics. As temperature is raised, the energy of each particle becomes high, so that $n_{\mathbf{k}}(\beta)$ in eq. (28) can be approximated by the Boltzmann distribution ($\sim \exp[-\beta\omega_k]$), indicating that the statistical nature of particles (bosons and fermions) does *not* appear although the quantum effect due to the bosonic and fermionic nature of particles still remains even at high temperature.

In summary, we have considered the simple problem of particle creation from false vacuum at finite temperature by applying the theory of TFD formulated in THP by introducing a thermal Heisenberg operator defined by $A(\beta) := U^{\dagger}(\beta) A U(\beta)$. We showed that if at the beginning we had a curved geometry at zero temperature there would be $n_{\mathbf{k}}(t,\beta)$ particles created at time t (see eq. (27)), and obtained the net particles created at time t and temperature T due to the change of the particle distribution as $\{(1 + \sigma 2 n_{\mathbf{k}}(\beta)\} n_{\mathbf{k}}(t)$ for bosonic ($\sigma = 1$) and fermionic ($\sigma = -1$) fields (see eq. (28)).

4. – Conclusions

Due to the difference in the statistics of quantum particles, our result (eq. (28)) shows that the number of bosons *increases* due to thermal effects (*i.e.* the change in the distribution of particles) by the factor $\{1 + 2 n_{\mathbf{k}}(\beta)\}$ whereas the number of fermions *decreases* by the factor $\{1 - 2 n_{\mathbf{k}}(\beta)\}$ in contrast to the bosonic field, indicating the

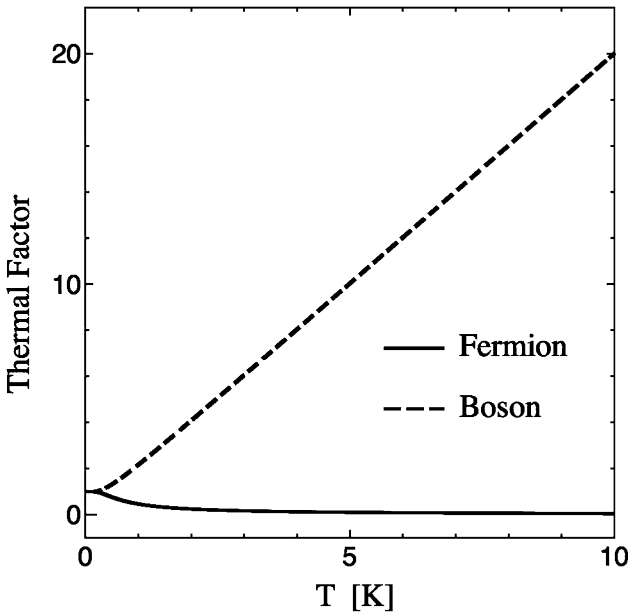

Fig. 1. – Thermal factor $(1+\sigma 2n_{\mathbf{k}}(\beta))$ *vs.* temperature (T). Here, $\beta = 1/T$ and $\omega = 1$. ($\sigma = +1$ for bosons, $\sigma = -1$ for fermions.)

quantum effect. As temperature is raised, energy of each particle becomes higher, so that the particle (boson, fermion) distribution $n_{\mathbf{k}}(\beta)$ becomes the Boltzmann distribution ($\sim \exp[-\beta\omega_k]$ for $\beta\omega_k \gg 1$), indicating that the statistical nature of quantum particles does *not* appear at high temperature although the quantum effect still can be seen even at such high temperatures. Another extreme, when temperature approaches zero (*i.e.* $\beta \to \infty$), $n_{\mathbf{k}}(\beta) \to 0$, and our result (eq. (28)) recovers the zero temperature result (eq. (22)) accordingly, where *no* distinction can be seen in the statistical nature of bosonic and fermionic fields. These distinctive features can be clearly seen in fig. 1.

Finally, we should note that our results are independent of the particular model. Therefore, the present theory could be applied to the system of photons, phonons, and the Cooper pairs in the condensed phase for bosonic fields or to the system of He^3 for the fermionic field, depending on the system considered.

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We wish to thank Prof. H. Matsumoto for valuable discussion on the foundation of thermo field dynamics.

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