Comments on Fermi's original papers on cosmic-ray acceleration(*)

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(ricevuto il 4 Novembre 2002; approvato il 4 Dicembre 2002)

Summary. — In his two 1949 papers on cosmic rays, Fermi introduced the concepts which form the basis of most theories of cosmic-ray acceleration today: magnetic trapping; repeated, small stochastic gains; energy derived fron the large-scale bulk kinetic energy of interstellar plasmas. I consider the historical context in which these concepts were proposed, and compare the questions which Fermi regarded as unresolved with those which we now regard as unresolved.

PACS 95.85.Ry – Neutrino, muon, pion and other elementary particles; cosmic rays. PACS 98.70.Sa – Cosmic rays (including sources, origin, acceleration, and interactions).

PACS 01.30.Cc - Conference proceedings.

1. - Introduction

In 1949 Fermi published two papers [1,2] which remain today the basis of most theories of the origin of cosmic rays. I should like to consider the influences which may have led him to take the approach he did to tackle the problem, and to reflect on some of his concerns which are still with us today.

Until recently theories of the origin of cosmic rays have created them by invoking a mechanism to accelerate particles to higher energy. Most which remain viable today are based on Fermi's original ideas. As the observational data has improved on ultrahighenergy cosmic rays with energy above 10^{19} eV, it is, however, questionable whether any acceleration theory can explain these particles. Alternative approaches invoke as yet unknown, decaying GUT particles, created with the GUT energy of about 10^{25} eV, which then cascade down in energy.

^(*) Paper presented at the IX ICRA Network Workshop "Fermi and Astrophysics" (Rome, Pescara, September 2001) held under the auspices of the Italian Committee for the Celebration of the Hundredth Anniversary of the birth of Enrico Fermi. Joint copyright SIF and World Scientific.

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2. – Historical perspectives

2.1. Cosmic rays in 1949. – Although a mysterious "radiation" with an extraterrestrial origin had been known for about 50 years, very little progress had been made determining either its nature or origin. Their cosmic origin was demonstrated in 1912 by Viktor Hess, using balloon flights to show that the penetrating, ubiquitous, ionizing radiation increases in intensity with altitude. It was not until the 1930's, with increased understanding of nuclear physics, that the "radiation" was recognized to be charged particles. Very little else was known about it, neither what the particles were, nor their energy, nor where they came from, nor how they acquired that energy. The particles were supposed to be subatomic, probably protons. The energy was supposed to be high, considerably suprathermal, since it was penetrating.

Although extensive air showers had been discovered in 1938 by Pierre Auger [3,4], progress in interpreting them had been hindered by both incomplete theory and inadequate technology. In order to derive the composition and energy of the particle which had caused the shower, a detailed calculation of the nuclear reactions must be made from the original interaction of the particle impinging on the top of the atmosphere, through all the cascades, down to the muons measured on the ground. One single particle at the top of the atmosphere produces millions of particles in the ground. The highest energy particles seen, in fact, produce over 10⁹ particles. Clearly not only was more detailed nuclear theory required, but also high-speed computers. Although cosmic rays are a piece of the cosmos which has arrived on our doorstep, and hence local measurements were possible, nevertheless deductions about their properties were inconclusive.

Considering the cosmic-ray's life in the cosmos rather than its local death, one would like to know their spatial extent. Were they confined to the Earth's vicinity, or to the solar system, or did they extend throughout the Galaxy? There was no evidence to favor either of these alternatives. An argument which was invoked to favor more local confinement was that the energy requirements would be enormous otherwise. If the cosmic rays filled interstellar space, a very large amount of energy would have to be channeled from some other source into the cosmic rays. One would not only have to explain how the energy was channeled efficiently, but also find such a large energy reservoir.

3. – Fermi's approach

3.1. Influences from other disciplines. — At Chicago Fermi would have heard from the Yerkes Observatory astronomers of Adams' work on interstellar absorption lines. Although Adams had recently retired as director of Mt. Wilson Observatory, a post he had held for two decades, he had studied and worked at Yerkes in his youth, and would have maintained his ties there. Adams' high dispersion stellar spectra [5] allowed him to identify weak, narrow, absoption lines of molecules at different radial velocites than that of the star. These are due to cold interstellar clouds in the line of sight between us and the star. The concept of cold clouds between the stars, with molecules in them, moving with random velocites of the order of 15 km/s, was new. Although Adams' observations were of only nearby stars, since the stars had to be bright in order to obtain high dispersion spectra, if one were to suppose most of the Galaxy were similar, their kinetic energy represents a very significant energy reservoir.

In 1948 Alfvén visited Chicago. Many of his ideas of magnetic fields in plasmas, and throughout the Universe, were not received well by the physics community at large at that time, but Fermi found his arguments convincing. In particular Alfvén had proposed

a decade earlier that there was a large-scale magnetic field throughout the Galaxy, but since the general picture of the Galaxy at that time was of a vacuum between the stars, it was not seen how the electric currents could be supported to maintain such a field. But did not Adams' interstellar absorption lines indicate all was not a vacuum between the stars?

Fermi was familiar with what is sometimes called the Stoermer [6] problem, the motion of a charged particle in a magnetic dipole-field. Scandinavian physicists had been particularly interested in this problem as it applies to the aurora. In the appendix of his book which came out a year later, Fermi [7] considered the magnetic cut-off of charged particles in the Earth's magnetic field.

The ingredients were there: the theory of trapped charged particles moving between magnetic mirrors; Alfvén's conviction that not only was there a large-scale magnetic field in the Galaxy, but also inside interstellar clouds; and observational evidence for clouds moving with random velocities, occupying a significant volume of the Galaxy. One further ingredient for which he had no evidence but which he required, he had to postulate, that of a population of suprathermal seed partcles.

3'2. Order-of-magnitude estimate. – Fermi's approach to the problem clearly reflects the nuclear physicist who thinks in terms of particle decay and lifetimes, rather than the plasma physicist.

Consider a particle of mass m trapped between two moving mirrors. The energy gain per collision is

$$\delta w = B^2 w$$
.

where B = V/c, and V is the velocity of the mirrors. After N collisions,

$$w = mc^2 \exp[B^2 N].$$

For losses due to nuclear collisions with a mean free path Λ , a collision loss time T can be defined by

$$\Lambda = Tc$$
.

Let the time between collisions with the walls be τ . Then, if the only losses are nuclear collisions, the energy distribution is

$$\pi(w)dw = (\tau/B^2T)(mc^2)^{\tau/B^2T}dw/w^{1+\tau/B^2T}.$$

The success of this model is that it produces an inverse power law spectrum. Even then it appeared that was the spectrum which one should try to explain. Although at that time measurements were available only up to particle energies of about 10^{12} eV, the power law in fact extends up to above 10^{19} eV.

Figure 1 [8] shows a recent compilation of data over the range of 13 orders of magnitude in energy. The deviation at low energy is fairly well understood, is a local effect, and is due to the effect of solar modulation. The incoming galactic cosmic rays scatter off magnetic irregularities in the solar wind. The other two features marked have been dubbed anthropomorphically the knee and the ankle. The knee probably results from particles leaking out of the Galaxy. It occurs around the energy for which protons

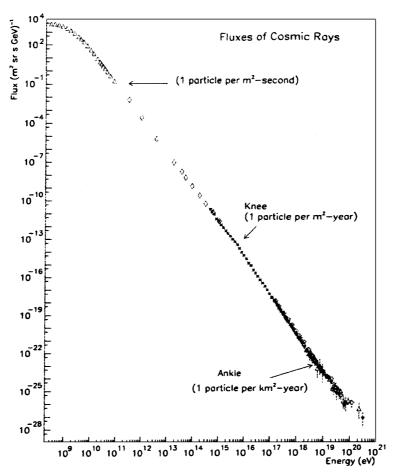


Fig. 1. – Differential energy spectrum of cosmic rays.

can no longer be confined by the magnetic field of the Galaxy. The ankle is less well understood, and may be where a new component, the ultrahigh-energy cosmic rays, is starting to appear, produced by an entirely different, as yet unknown, mechanism.

4. - Fermi's unfounded concerns

4.1. Seed particles. – Fermi's mechanism requires suprathermal seed particles. They must have energies above about 200 MeV in order to avoid that ionization losses, which are proportional to w^{-2} , are greater than the energy gains. Fermi considered this a major problem for his theory. He proposed a "chain reaction" by spallation, but clearly this can produce only light element cosmic rays, not heavies. If heavy elements were found in the cosmic rays, he felt that would pose a major problem for his model.

At the time solar flares and coronal mass ejections were not known. Figure 2 [9] shows energetic ions measured *in situ* in the solar wind. The sun clearly knows how to inject suprathermal particles into interstellar space, including heavies.

The mass spectrum of the injected particles extends all the way up to iron. Fur-

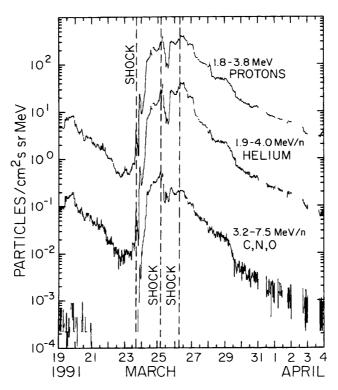


Fig. 2. – Energetic ions measured onboard the Ulysses spacecraft during the period of intense flare activity in March 1991.

thermore the characteristics of these suprathermal particles do not seem to change substantially between about 1 MeV and 100 MeV. Figure 3 [10] shows, for example, the ionic charge of $\approx~1~{\rm MeV}$ and $\approx~100~{\rm MeV}$ particles ejected from the Sun for the heavy elements up to iron.

Therefore, we now know that in principle the seed particles Fermi needed, and had to postulate in the absence of observational data, are indeed being injected into interstellar space. Their energy is probably derived from magnetic field energy in stellar-type objects. Detailed calculations by Decker and Vlahos [11] are able to produce ions in the range 10–100 MeV in solar flares from 100 keV particles, which can be obtained either from the tail of the distribution of particles directly heated in the flare, or from various direct "prompt" electromagnetic acceleration means in the flare. Providing Fermi with the 200 MeV seed ions he needs does not seem to be a problem in principle, either observationally, nor theoretically.

4.2. Heavy nuclei. – At that time very little was known about the composition of the cosmic rays. Fermi was aware that there was some indication that they were not all protons, and he viewed this as a potential problem. Not only would the injection of heavy seed particles pose a problem, but indeed their entire spectrum. Since loss rates for heavies would be different than for protons, the spectrum would be different. He expected the spectrum to be much steeper for heavies than for protons, so there should be very few at the higher energies.

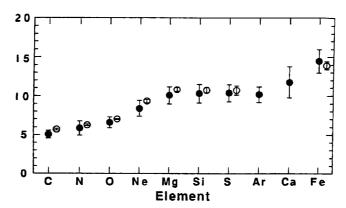


Fig. 3. – Mean ionic charge of solar energetic particles in the energy range 10–100 MeV/nuc (solid symbols) for the events Oct. 31-Nov. 7, 1992. The open symbols show the energy range 0.3–3 MeV/nuc.

Modern measurements show that the heavy elements are well represented. Figure 4 [12] shows that for a wide range of heavy elements the abundance in galactic cosmic rays, extrapolated back to the source, is within an order of magnitude of solar. The details of the deviations from solar abundance are interpreted as clues to the environment in which the particles were accelerated.

Deriving the mass of the primary particle from the air shower data still involves

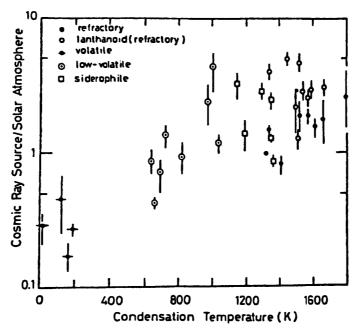


Fig. 4. – The chemical composition of galactic cosmic rays at their sources, as compared with that of the solar atmosphere, as a function of the condensation temperature of the elements.

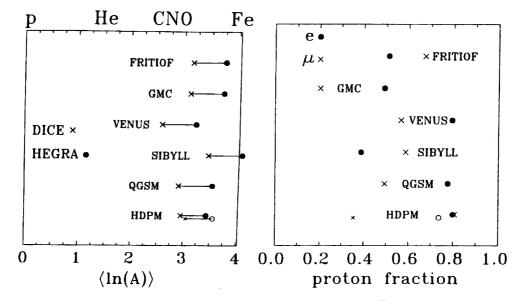


Fig. 5. – (a) Average logarithm of the primary particle mass at 10^{15} eV for different models of high-energy interactions. (b) Proportion of proton- and iron-induced showers in the event sample, obtained from different interaction models.

some uncertainty, both in the model used for the nuclear interactions, and in the details of the interactions themselves. Figure 5 [13] (a) shows the difference for two different experiments, DICE and HEGRA, using 6 different models of the nuclear interactions. The conclusion is that for primaries with energy of 10^{15} eV, a substantial fraction are heavier than CNO nuclei, whichever model or experimental data is used. Figure 5 [13] (b) demonstrates that all of the models agree that the primaries are neither all iron nor all protons.

4.3. Energy budget. — The extent of the confinement region of cosmic rays was not known, and there were no observational constraints. They could have been confined to the region around the Earth, or to the solar system, or to the Galaxy. Fermi's model requires that they fill a large portion, if not all, of the Galaxy. He was concerned that the objection could be raised that this would require the total energy channeled into cosmic rays to be very large. He does not directly counter this objection, except to point out that, therefore, the acceleration mechanism must be efficient.

The argument has often been invoked when new phenomena are discovered in astronomy, that the total energy required would be too large unless the phenomenon is restricted spatially, temporaly, directionally, etc. It has usually proved to be wrong, demonstrating that the Universe is more extreme than most astronomers would like to think it is. The argument that the energy required would be unreasonably high was used when the "nebulae" were proposed to be extragalactic, when quasars were proposed to be at cosmological distances, and, more recently, when gamma-ray bursts were proposed to be at cosmological distances.

In Fermi's model, the cosmic rays draw their energy from the random large-scale kinetic motion of interstellar clouds. The astronomical observations of interstellar absorption lines suggested that this might be a very large energy source. The details of

the magnetic mirror, however, depend on the existence of small-scale magnetic irregularities in the cloud, which presumably are indicative of turbulence. Energy dissipation in turbulence, as energy cascades from large eddies down to the smallest scale where it is dissipated in viscosity, is very efficient, probably more efficient than any mechanism to channel energy into cosmic rays could be, since the former is moving the state towards equilibrium and the latter is moving it away. So he postulated that magnetic fields could suppress small-scale turbulence. The energy in turbulence is stored in the largest eddies, but dissipated in the smallest. The problem of the energy balance is determined by the small-scale structure and physics, not the overall extent of the region. The total energy requirement was determined not by the extent of the confinement region per se, but by the physics of the dissipation on small scales.

5. - A nuclear physicist before a hydrodynamicist

Fermi's experience as a nuclear physicist influenced how he formulated the problem. Physicists and mathematicians try to reduce a new problem to one which they have solved before. The calculation of the cosmic-ray spectrum is treated as a problem in radioactive decay, and the creation of seed suprathermal particles is formulated as a nuclear chain reaction for which the question becomes whether or not it is self-sustaining. It was only later in collaboration with Chandrasekhar that he became a hydrodynamicist.

In 1949 the existence of interstellar galactic magnetic fields was still controversal. An interstellar magnetic field was necessary for Fermi's cosmic-ray acceleration model to work. The large-scale Galactic magnetic field proposed by Alfvén could also be used to confine the cosmic rays within the Galaxy, if the geometry proved suitable. But was there one? It was just then that some actual observational data became available which confirmed that there indeed was a large-scale galactic field, as Fermi was quick to recognize. In that year Hiltner [14] published measurements of the optical polarization of stars across the sky, and found large-scale patterns. This is attributed to scattering by non-spherical dust grains alligned in a magnetic field in the line of sight to the star. With the first concrete evidence that Alfvén (and he himself as well) was correct, he pursued other consequences of the large-scale interstellar field. Together with Chandrasekhar in 1953 [15] he devised two methods to estimate the field strength using hydrodynamics. One method attributes observed small-scale transverse motions to Alfvén's new waves, and the other estimates the magnetic pressure needed to satisfy hydrostatic equilibrium. The Chandrasekhar-Fermi method is still used today as one of the few ways to estimate interstellar large-scale magnetic-field strength. For a recent review of the Chandrasekhar-Fermi method applied to current-day observations, see Zweibel [16].

6. – Persistent concerns

6.1. Composition. – As for Fermi and his model, the composition is also a crucial test for the current theories of ultrahigh energy (UHE) cosmic rays. Much of the current excitement in cosmic-ray studies focuses on the UHE cosmic rays above 10^{19} eV. Fermi's mechanism cannot explain them, and it is doubtful any modification of the theory would be able to explain them. In fact, they should not exist. Aside from the difficulty of finding a means of accelerating them to that energy, another problem enters for energies above about 5×10^{19} called the Greisen-Zatsepin-Kuz'min cut-off [17, 18]. At those energies they interact with the cosmic microwave background photons, and are destroyed by pion photodissociation. A particle with a relativistic γ of 10^{11} sees a very hard gamma-ray,

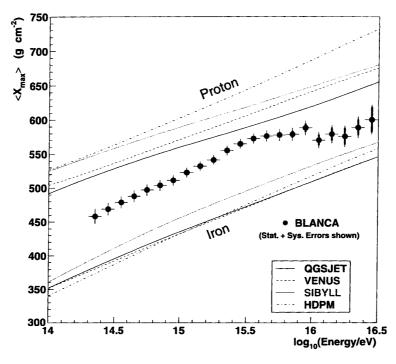


Fig. 6. – The mean depth of shower maximum X_{max} as a function of energy. The curves represent the values for pure proton and pure iron as predicted by different hadronic interaction models.

not a microwave photon. The source must be closer than about 100 Mpc, or it would not have survived. Since particles of such high energy are not deflected by the galactic magnetic field, they must point back to their source. There is, however, nothing in their direction. If it were only 100 Mpc from us and capable of producing 10^{20} eV particles, we should presumably see the photons it also produces.

The so-called "top-down" class of models, which produce particles with energy of the GUT energy of about 10^{25} eV from the decay of exotic supersymmetric particles or topological defects, could not conceivably produce anything heavier than a proton. If heavies are found, top-down exotic models would appear to be ruled out, and the UHE cosmic rays must have been accelerated from thermal matter.

Figure 6 [19] shows the comparison of the data from one air shower array with several different model calculations for the middle energies around the knee. As the energy increases to the highest energies considered by Fermi, the proton fraction increases slightly, as he predicted, although perhaps by considerably less than he would have expected. This is due to the higher losses for heavy nuclei caused by nuclear interactions during propagation through the interstellar medium. By 10^{16} eV, however, the fraction of iron has increased significantly. This is due to light particles escaping from the Galaxy, since the gyroradius in the Galactic magnetic field of a proton is much larger than that of an Fe nuclei. At the highest energies for which there are composition measurements, which correspond to the energies above which even Fe nuclei are not confined in a 3 μ G field, there is a tantalizing indication that the proton fraction begins to increase again. Figure 7 [20] shows a similar plot for a different set of model calculations compared with

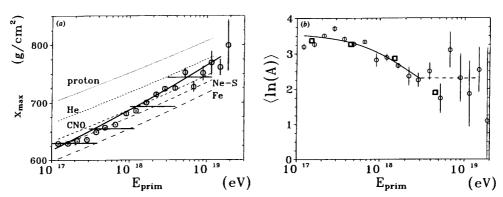


Fig. 7. – (a) Depth of X_{max} vs. the primary energy E_{prim} . (b) $\ln A$ vs. E_{prim} derived by two different methods. See original reference for details.

data from the Fly's Eye experiment at the highest energies. It may be that the proton fraction is, indeed, increasing with energy.

6². Trapping. – The essential elements Fermi's model supplies are an energy source and particle trapping. The energy in cosmic rays is channeled from the kinetic energy of the random motion of magnetic irregularities, identified with cold interstellar clouds. Particle trapping is the new ingredient, prompted by his work on the Stoermer problem. One step, direct "prompt" acceleration processes, such as in the initial acceleration in a solar flare, are faster and the energy gain per step is much greater, but without a means of trapping the particle within the "accelerator" the total energy gains are modest, since the particle escapes the accelerator. The maximum energy attainable would be considerably below even the highest energies known at the time. Constructing a model which as a byproduct also trapped the particles, allowed him to reach the highest energies he wished. Since he considered only ionization losses, which become negligible for energies above about 200 MeV, he was not worried about the high-energy end of the distribution. He considered the only limitation to the maximum energy attainable was the energy equipartition value, although the process was slow, and it would take a very long time to reach. For the energies known at the time, this was probably so. Now that we know the energy spectrum continues many orders of magnitude above 10^{15} eV (fig. 1), trapping again becomes a problem, and particle leakage becomes the dominant loss process. Theoreticians who try to extend Fermi acceleration to high-energy cosmic rays are forced to resort to postulating intricate, and usually implausible, magnetic field configurations in order to trap the particles. A detailed analysis by Lagage and Cesarsky [21] of the non-linear effects on the magnetic field within the shock, and the resulting changes in the diffusion coefficient, lead to the conclusion that many estimates of the maximum energy attainable are one to two orders of magnitude too large. Even using the most favorable conditions and configurations, 10^{16} eV seems to be a robust upper bound to the maximum energy attainable.

6^{*}3. The injection problem. – The injection of seed particles is a problem which is still with us in the details. The relative abundances measured in cosmic rays do not correspond exactly, either in element nor isotope, with the thermal plasma of any known environment. Presumably this is due to selective acceleration during the injection pro-

cess. Attempts to find simple correlations, such as with the first ionization potential, always result in several exceptions. Neither have we addressed the question of whether there are enough of them. That requires detailed study of the different galactic stellar populations, and how much each type would contribute.

7. - Self-regulating mechanisms and equipartition

The cosmic-ray energy density is of the same order of magnitude as several other energy densities in the Galaxy. The interstellar medium and the Galaxy as a whole are, however, many orders of magnitude removed from equilibrium. The photon energy density is too weak by many orders of magnitude for its spectral distribution or "black-body temperature". Therefore, if two energy densities are found to be equal, one must find a mechanism which makes, and keeps, them equal. Fermi's model provides a natural relation between the random motions in the Galaxy, the dynamical temperature of the disk, and the cosmic-ray energy density. Equipartition is common in astrophysical systems, even if equilibrium is not.

7¹. Astrophysical coincidences. – Fermi was intrigued by puzzles and conundrums, and astrophysics offers many. Fermi's model of cosmic-ray acceleration seeks to make the link

$$\epsilon_{\rm cr} \approx \epsilon_T$$

where $\epsilon_{\rm cr}$ is the energy density present in cosmic rays, and ϵ_T is the energy density present in large-scale random kinetic motions of gas in the disk. What about

$$\epsilon_{\rm cr} \approx \epsilon_T \approx \epsilon_*,$$

where ϵ_* is the energy density present in starlight? To understand

$$\epsilon_{\rm cr} \approx \epsilon_*$$

one needs to understand magnetohydrodynamics. The energy density in cosmic rays can influence the formation of interstellar clouds via the Parker instability [22], and the formation of clouds is the first step towards contraction to a protostar. The energy density in cosmic rays enhances the effect of the anisotropic pressure of the magnetic field until the field ballons up out of the disk, and cosmic rays can escape, as, at the same time, the thermal gas falls back and collects at the bottom of the curved field lines to form clouds.

In order for stars to form, the gas must cool. The cosmic rays influence the cooling of the clouds through their non-adiabatic effects within the cloud. The electrons released when a cosmic ray ionizes an ambient atom, then thermalizes and heats the gas. When the atom then recombines it releases a photon which cools the gas. This is a highly non-linear process and depends on the detailed conditions within each cloud, as well as its environment, but there are enough interdependences that the rough equipartition could plausibly be accounted for. The energy density in cosmic rays can influence not only the rate of star formation, but probably at least as important, the Initial Mass Function of the stars formed. The large-scale kinetic energy needed in Fermi's model, the random motion of the clouds, is probably contributed mainly by the higher mass stars, not only

through supernovae explosions, and mass ejection at the late stages of stellar evolution, but also by the expansion of an ionization front into the surrounding gas clouds during its main sequence life. The rate of injection of the seed particles needed by Fermi's acceleration mechanism from stellar flares and magnetic activity also depends on the star formation rate and the initial mass function.

There is another "coincidence" which is much more puzzling, however.

$$\epsilon_{\rm cr} \approx \epsilon_{\rm CMB}$$
,

where ϵ_{CMB} is the energy density in the cosmic microwave background. In most models of cosmic-ray propagation, there is a halo of cosmic rays around the Galaxy. Radio continuum observations of the synchrotron emission from energetic electrons show that some external galaxies do have a halo of energetic particles, at least of electrons. Diffuse gamma-ray emission in our Galaxy produced by cosmic rays colliding with ambient cold interstellar matter may also indicate a cosmic-ray halo. Such a halo is invoked by theoreticians as a storage ring, where the cosmic rays can propagate with low losses to explain certain isotopic ratios measured in secondary particles produced by spallation in the interstellar medium. Could the extent of the cosmic ray halo be dependent on cosmological epoch?

REFERENCES

- [1] FERMI E., Phys. Rev., **75** (1949)1169.
- [2] FERMI E., Nuovo Cimento Suppl., Vol. VI, no. 3, International Congress on Cosmic Ray Physics, Como, 11-16 Settembre 1949, p. 317.
- [3] Auger P., Maze R. and Grivet-Meyer T., C. R. Acad. Sci., 206 (1938) 1721.
- [4] AUGER P. and MAZE R., C. R. Acad. Sci., 207 (1938) 228.
- [5] Adams W. S., Astrophys. J., **97** (1943) 105.
- [6] Stoermer C., Arch. Sci. Phys. Nat. Ser. 4 (1911) 32, 117.
- [7] FERMI E., Nuclear Physics (The University of Chicago Press, Chicago) 1950.
- [8] ZAVRTANIK D., J. Phys. G, 27 (2001) 1597.
- [9] SANDERSON T. R., MARSDEN R. G., HERAS A. M., WENZEL K.-P., Proc. 22nd ICRC Dublin, 3 (1991) 173.
- [10] OETLIKER M. et al., Proc. 24th ICRC Roma, 4 (1995) 470.
- [11] DECKER R. B. and VLAHOS L., Astrophys. J., 306 (1986) 710.
- [12] SAKURAI K., Proc. 22nd ICRC Dublin, 2 (1991) 416.
- [13] Wibig T., J. Phys. G, 27 (2001) 1633.
- [14] HILTNER W. A., Astrophys. J., 109 (1949) 471.
- [15] CHANDRASEKHAR S. and E. FERMI, Astrophys. J., 118 (1953) 113.
- [16] ZWEIBEL E. G., Polarimetry of the Interstellar Medium, edited by W. G. ROBERGE and D. C. B. WHITTET, ASP Conf. Ser., 97 (1996) 486.
- [17] GREISEN K., Phys. Rev. Lett., 16 (1966) 748.
- [18] ZATSEPIN G. T. and KUZ'MIN V. A., JETP Lett., 4 (1966) 78.
- [19] Kampert K-H., J. Phys. G, **27** (2001) 1663.
- [20] WOLFENDALE A. W. and WIBIG T., J. Phys. G, 27 1625 (2001).
- [21] LAGAGE P. O. and CESARSKY C., J. Astron. Astrophys., 125 (1983) 249.
- [22] Parker E. N., Astrophys. J., 145 (1966) 811.