The fifth force and neutron star structure revisited (*)

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Summary. — Assuming a hypothetical intermediate force which is composition dependent, we have studied the properties of a neutron star in terms of parameters, α and λ. We have found that the structure, size and mass of a neutron star are affected by the strength and range of this new force. In the experimental and theoretical limits of the new force, the effects of parameters on the size and mass of the star are not relevant. But, in a special case such as when the strength parameter α exceeds the experimental value, the structure of the star strongly changes and the size and mass increase exponentially. The effect of the range parameter λ on a neutron star is not relevant, but the size and mass have local maximum in the range from a few metres to several kilometres.

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1. – Introduction

Reanalyzing the Eötvös experimental data of 1922, Fischbach *et al.* [1] exclaimed that they found the existence of a new intermediate-range force which is sensitive to the composition of matter. From then it is called fifth force. The new fifth force is a repulsive interaction which acts between baryons or hypercharges [1]. The potential energy between masses m_i and m_j can be expressed by $V(r) = -G(r)\, m_i m_j / r$ [1, 2], where the coupling constant $G(r)$ which is defined by $G(r) = G_\infty (1 + \alpha e^{-r/\lambda})$ shows the dependence on the radius r due to the existence of the fifth force [3, 4] and G_∞ is the usual Newtonian constant. The parameter α determines the strength relative to the gravitation and the parameter λ is related to the range of the force. According to Fischbach *et al.*, $\alpha = (7.2 \pm 3.6)\cdot 10^{-3}$ and $\lambda = (200 \pm 50)$ m, respectively.

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Our problem is that, if we assume the existence of the fifth force, what are the effects on a self-graviting system composed by baryons such as neutrons? In 1987, Glass and Szamosi showed that, in the framework of the Lane-Emden equation, the effects of the fifth force on a self-graviting system with uniform mass density are comparable to the general relativistic effects. For a non-uniform mass distribution, Song and Lee [5] considered a Thomas-Fermi–type hydrostatic equilibrium equation, which is obtained by combining the gravitational Poisson equation and the Helmholtz equation for an intermediate-range force and applied this equation to a self-gravitating system of degenerate fermions like a neutron star. As a result, we have shown that, in the experimental limits, the size and mass of a neutron star are proportional to $\alpha/2$ and $3\alpha/2$, respectively. Furthermore, we have also shown that, in the ultrarelativistic regime, the mass converges to $M_{\mathrm{lim}} \propto (1-\alpha)^{-3/2} m_{\mathrm{pl}}^3/m^2$ for any α, and this should be a new mass limit for a self-gravitating fermion system. Here m is the mass of fermions and $m_{\mathrm{pl}} \equiv \sqrt{\hbar c/G_\infty}$ is the Planck mass.

As shown in the previous paragraphs, if there is a fifth force, the coupling constant $G(r)$ will be dependent on the radius, and will be determined by two parameters, α and λ. We have already considered the effects of the strength parameter α to estimate the size and mass of a self-gravitating system [6, 5], but we have not yet considered the effects of the range parameter λ, and this is the reason why we revisit this topic to complete our analysis.

2. – Hydrostatic equilibrium equation

In this article, we consider a neutron star as a self-gravitating fermion system because the size is roughly comparable with the result of Fischbach *et al.* and we could expect the largest effects of the fifth force. If we assume the existence of the fifth force, the hydrostatic equilibrium equation for the system, which is obtained by the combination of the gravitational Poisson equation and the Helmholtz equation for an intermediate-range force, becomes a modified Thomas-Fermi equation. If we include special relativistic regime for a state function of degenerate fermions and introduce a new adimensional scale parameter x which is defined by $r = a_x x$, then the governing, adimensional differential equation for a self-gravitating system of degenerate fermion gases is given by [5]

$$\nabla_x^2 \theta + (1-\alpha)\, \theta^{3/2} \left[1 + \left(\frac{N}{N_c} \right)^{4/3} \theta \right]^{3/2} = \alpha (a_x \lambda)\, F(N, x, x_b) \tag{1}$$

and

$$F(N, x, x_b) = \frac{e^{-(a_x \lambda^{-1})\, x}}{x} G(N, x) + \frac{\sinh\left((a_x \lambda^{-1})\, x\right)}{x} G(N, x, x_b), \tag{2}$$

$$G(N, x) = \int_0^x \mathrm{d}x'\, x'\, \theta^{3/2}(x') \left[1 + \left(\frac{N}{N_c} \right)^{3/2} \theta(x') \right]^{3/2} \sinh\left((a_x \lambda^{-1})\, x'\right), \tag{3}$$

$$G(N, x, x_b) = \int_0^x \mathrm{d}x'\, x'\, \theta^{3/2}(x') \left[1 + \left(\frac{N}{N_c} \right)^{3/2} \theta(x') \right]^{3/2} e^{-(a_x \lambda^{-1})\, x'}. \tag{4}$$

The scale factor a_x in the above equations is expressed by $a_x = a_0 (N/N_c)^{-1/3}$ in terms of the number of neutrons N, and a_0 defined by $a_0 = 2^{-1}(3\pi/4)^{1/2}(\hbar/m)\, m_{pl}/m$ is a characteristic scale of a neutron star. N_c, which is defined by $N_c = (3\pi/4)^{1/2}(m_{pl}/m)^3$, is a characteristic number in which the neutron star could be contained. From this, we may obtain the characteristic mass of a neutron star as $M_c = (3\pi/4)^{1/2} m_{pl}^3/m^2$.

By x_b and $\theta(x)$ as solutions of eq. (1), we may define the radius, mass, and density of a neutron star, respectively,

$$R = a_x x_b, \tag{5}$$

$$M(R) = 4\pi \int_0^R \varrho(r)\, r^2\, dr\,, \tag{6}$$

$$\varrho(x) = \varrho_0 \theta^{3/2}(x) \frac{[1 + (N/N_c)^{4/3} \theta(x)]^{3/2}}{[1 + (N/N_c)^{4/3}]^{3/2}}\,. \tag{7}$$

We may notice that the differential equation given in eq. (1) is determined by three parameters, α, λ, and N. Therefore, we also notice that the size, mass, and structure of a self-gravitating system of neutron gases are dependent on these three parameters, and the detailed calculations are achieved by solving the inhomogeneous differential equation (1). As in the case of the Thomas-Fermi equation, taking the conditions that the density is finite and the derivative is zero at the center, and the density becomes zero at the boundary of a star, we put boundary conditions for eq. (1) as $\theta(0) = 1$, $\theta'(x)|_0 = 0$, and $\theta(x_b) = 0$. Using these boundary conditions and doing numerical analysis of eq. (1) to calculate $\theta(x)$ as a function of the scale parameter x, finally, in the following section, we investigate the dependences of size, mass of a neutron star on the strength parameter α, range parameter λ, and relativity parameter N/N_c. To show more distinctively the effects of the hypothetical fifth force, we consider the radius parameter x_b and mass parameter $m(x_b)$, instead of the radius R and mass $M(R)$, as functions of the three parameters.

3. – The radius and mass of a neutron star

3·1. *The radius.* – As in the previous section, the radius of a neutron star is in eq. (5) and, if we define the characteristic radius which is determined by the physical constants participating in the equilibrium processes of a self-gravitating system as $R_c = a_0$, then the radius of a neutron star can be rewritten by

$$R/R_c = (N/N_c)^{-1/3} x_b(\alpha, \lambda, N)\,. \tag{8}$$

Hence if we obtain x_b as a solution of eq. (1) with respect to three parameters, we can estimate the radius by eq. (8). In fig. 1*a*), *b*), *c*), we have shown the variations of x_b depending on the parameters, α, λ, and N/N_c.

From fig. 1*a*) in which is drawn the change of x_b with respect to N/N_c for given values of α and λ, we have that the x_b, in a nonrelativistic case ($N/N_c \ll 1$) with $\alpha = 0.0$, is the same as $x_b = 3.65375$ of the Lane-Emden equation of index $n = 3/2$. On increasing the relativity parameter N/N_c, x_b is being decreased and reaches

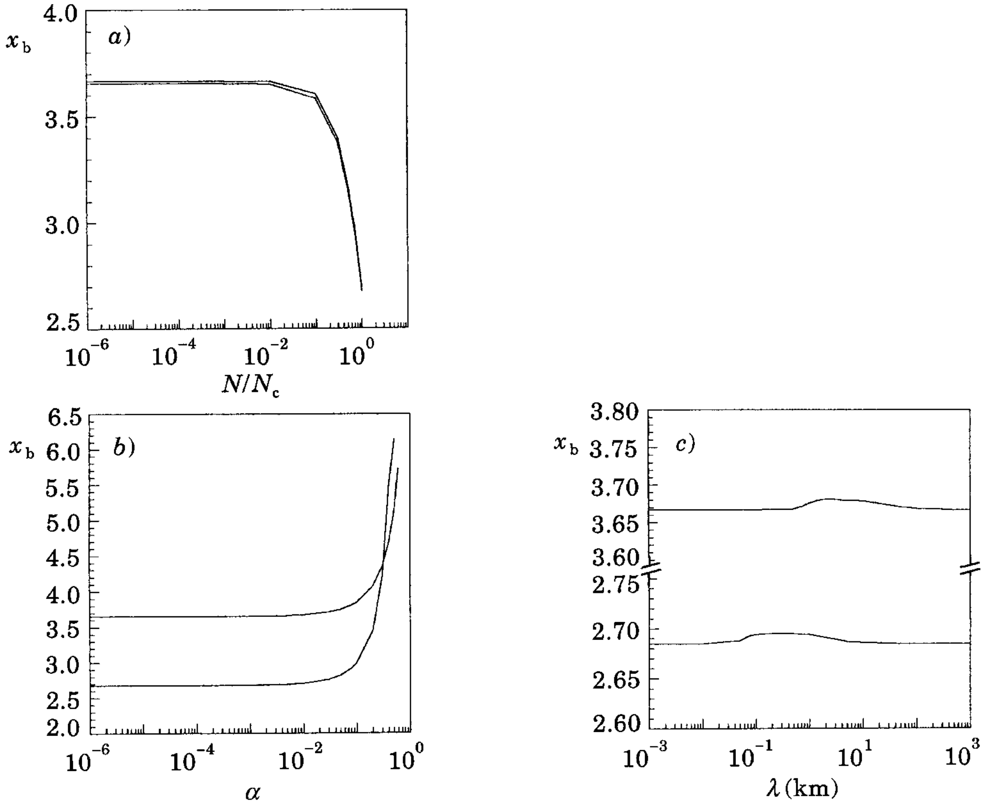

Fig. 1. – *a*) For fixed values of $\alpha = 0.0$ and $\lambda = 0.2$ km (upper curve), $\alpha = 7.2 \cdot 10^{-3}$ and $\lambda = 0.2$ km (lower curve), the dependence of the radius parameter x_b on the relativity parameter N/N_c is plotted. *b*) For fixed sets of $N/N_c = 10^{-3}$ and $\lambda = 0.2$ km (upper curve), $N/N_c = 1.0$ and $\lambda =$ 0.2 km (lower curve), the dependence of x_b on α is plotted. *c*) For fixed sets of $N/N_c = 10^{-3}$ and $\alpha = 7.2 \cdot 10^{-3}$ (upper curve), $N/N_c = 1.0$ and $\alpha = 7.2 \cdot 10^{-3}$ (lower curve), the dependence of x_b on λ is plotted.

$x_b = 2.6760$ at $N/N_c = 1.0$, which is in conflict with the classical $x_b = 6.89685$ of the Lane-Emden equation of index $n = 3.0$. This is the natural result of the Thomas-Fermi equation with considering special relativistic effects.

As one can see in fig. 1*b*) and *c*), the change of x_b is small for α smaller than about 10^{-2}. But for α larger than 10^{-2} the changes become remarkable and, for the extreme case, they become exponential. On the other hand, the dependence of x_b on the range parameter is not relevant for a certain λ. But, from fig. 1*c*), we may notice that x_b has slightly enhanced and has local maximum in the neighborhood of several kilometres for the nonrelativistic regime and in the neighborhood of several hundred metres for the ultrarelativistic regime. Outside of these ranges the effects of the parameter λ have disappeared. For example, the radius parameter $x_b = 2.6950$ for the experimental values of Fischbach *et al.* [1] and for $N/N_c = 1.0$ is slightly larger than for $\alpha = 0.0$ and the fractional increasing is $\delta x_b / x_b \simeq 7.1 \cdot 10^{-3}$. Therefore it should be very interesting to factor out this effect by real observations.

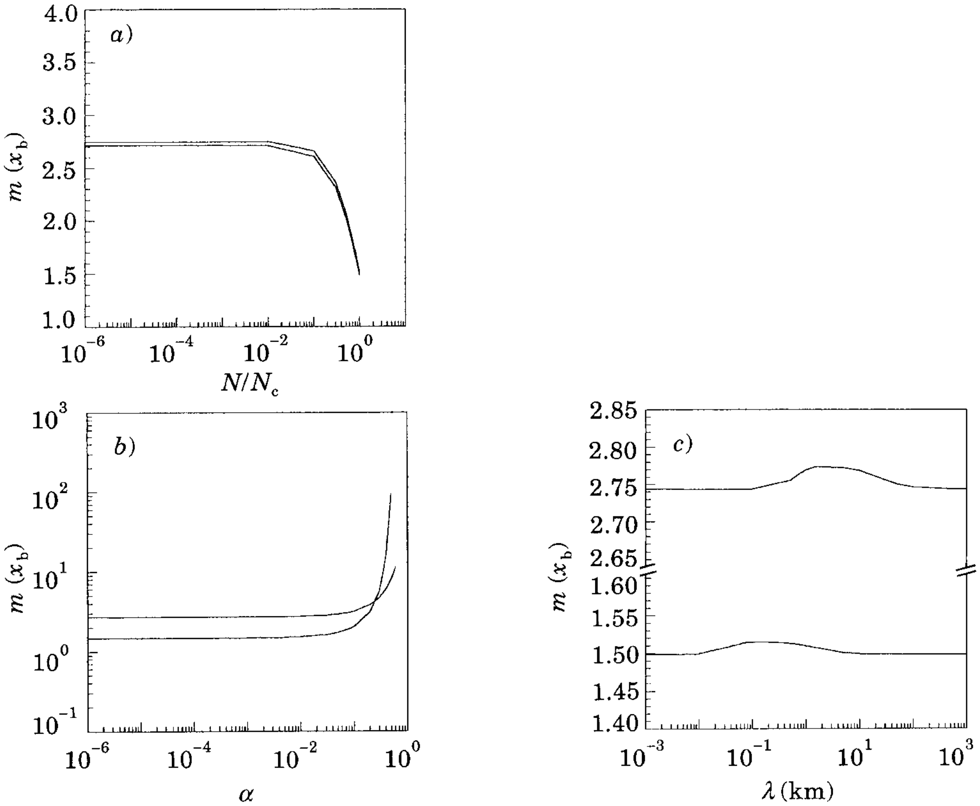

Fig. 2. – *a*) For fixed values of $\alpha = 0.0$ and $\lambda = 0.2$ km (upper curve), $\alpha = 7.2 \cdot 10^{-3}$ and $\lambda = 0.2$ km (lower curve), the dependence of the mass parameter $m(x_b)$ on the relativity parameter N/N_c is plotted. *b*) For fixed sets of $N/N_c = 10^{-3}$ and $\lambda = 0.2$ km (upper curve), $N/N_c = 1.0$ and $\lambda = 0.2$ km (lower curve), the dependence of $m(x_b)$ on α is plotted. *c*) For fixed sets of $N/N_c = 10^{-3}$ and $\alpha = 7.2 \cdot 10^{-3}$ (upper curve), $N/N_c = 1.0$ and $\alpha = 7.2 \cdot 10^{-3}$ (lower curve), the dependence of $m(x_b)$ on λ is plotted.

From eq. (7), we can express the central number density ϱ_0 in terms of the characteristic number of neutrons N_c as $\varrho_0/\varrho_c = (N/N_c)^2[1 + (N/N_c)^{4/3}]^{3/2}$, then, by eq. (8), we can rewrite the radius R/R_c as a usual function of ϱ_0/ϱ_c for given values of α and λ [5].

3˙2. *The mass.* – The total mass, $M(R)$, of a neutron star in the radius $R = a_x x_b$ is given in eq. (6). And then, in terms of radius parameter x_b and the density in eq. (7), the mass of a neutron star can be rewritten by an adimensional form,

$$M(x_b)/M_c = (N/N_c)\, m(x_b)\,, \tag{9}$$

and

$$m(x_b) = \int_0^{x_b} \mathrm{d}y\, y^2 \theta^{3/2}(y)[1 + (N/N_c)^{4/3}\theta(y)]^{3/2}, \tag{10}$$

where $m(x_b)$ is a mass parameter which is determined by the three parameters α, λ, and N/N_c the same as the radius parameter x_b. The results are drawn in fig. 2*a*), *b*) *c*), respectively.

In fig. 2*a*) for the dependences of the mass parameter on the relativity parameter N/N_c, we may see that, for $\alpha = 0.0$ and nonrelativistic regime, $m(x_b) = 2.71406$ which is in excellent agreement with the value of the classical Lane-Emden equation of index $n = 3/2$. But, for the extremely relativistic case, we have $m(x_b) = 1.482723$ which is in conflict with the solution of the Lane-Emden equation of index $n = 3.0$. Although $m(x_b)$ does not agree with the solution of the classical Lane-Emden equation in the ultrarelativistic regime, $\widetilde{m}(x_b)$ defined by $\widetilde{m}(x_b) = (N/N_c)\, m(x_b)$ converges to a constant for a given α [5].

As in the previous subsection for x_b, the mass parameter also changes with respect to α and λ. As one can notice in fig. 2*b*) and *c*), $m(x_b)$ increases with α, but very slowly up to $\alpha \simeq 10^{-2}$. The mass parameter grows exponentially for the range of $\alpha \geqslant 0.1$ and changes more steeply for the relativistic case than for the nonrelativistic case. The dependences of $m(x_b)$ on λ has somewhat interesting results. Even though the changes on mass parameter are not relevant, throughout the values of the range parameter, to a neutron star mass, $m(x_b)$ has slightly enhanced values and maximum in certain ranges of λ, which depend on the relativity parameters. For the nonrelativistic case, $m(x_b)$ has a maximum in the range of λ from about several kilometres to about several tens of kilometres. On the other hand, for the relativistic case, $m(x_b)$ has a maximum in the range of λ from about several hundred metres to about several kilometres.

For the experimental data given by Fischbach *et al.*, we have $m(x_b) = 1.514862$ for the relativity parameter $N/N_c = 1.0$ (see fig. 2*a*)) and we can estimate the fractional change in $m(x_b)$ with respect to the absence of the fifth force as $\delta m(x_b)/m(x_b) \simeq 2.17 \cdot 10^{-2}$. This implies that mass of a neutron star is significantly increased by a hypothetical intermediate-range force.

By the equation of $M(x_b)/M_c$ expressed as a function of central density ϱ_0/ϱ_c, we should notice that, for a given α, $M(x_b)/M_c$ converges to a constant in the ultrarelativistic regime [5], and this implies that we may define a new mass limit of a neutron star such as

$$M_{\lim} = (1-\alpha)^{-3/2}\, m N_c\, \widetilde{m}(x_b) = \left(\frac{3\pi}{4}\right)^{1/2} (1-\alpha)^{-3/2}\, m \left(\frac{m_{\mathrm{pl}}}{m}\right)^3 \widetilde{m}(x_b)\,. \tag{11}$$

This results should be confirmed by the ultrarelativistic approximation equation obtained from eq. (1). Since $\widetilde{m}(x_b)$ becomes constant for ultrarelativistic approximation and depends only on the strength parameter (see [5]), the classical Chandrasekhar mass [7]) may be replaced by a new mass definition in eq. (9). This implies that, in addition to a classical mass limit of a compact star of a self-gravitating system of degenerate fermions, which is determined by the mass of a fermion, light speed, Planck constant, and the gravitational constant, the strength parameter α of the fifth force also takes part in the determination of a self-gravitating system. By this result we may say that the size and mass of a star are not produced by chance but are determined by the physical interactions which take part in the process of stellar equilibrium.

4. – Summary

Assuming an intermediate-range force, we have solved a Thomas-Fermi–type equation which is obtained by the combination of the gravitational Poisson equation and the Helmholtz equation for an intermediate-range force and we have shown the results as in the following.

For sufficiently larges values α, the radius parameter and the mass parameter of a neutron star increase very rapidly. But, for the range of values smaller than 10^{-2}, the changes are very slow. The tendency of increasing with α is a natural consequence of the repulsive properties of the fifth force. At the range of values which are estimated experimentally [1] and theoretically [8], the estimated effect are not relevant to the size and mass of a neutron star. But, for a relativity parameter $N/N_c = 1.0$, the fractional changes in x_b and $m(x_b)$ between $\alpha = 0.0$ and $\alpha = 7.2 \cdot 10^{-3}$ are $\delta x_b / x_b \simeq 7.1 \cdot 10^{-3}$ and $\delta m(x_b)/m(x_b) \simeq 2.17 \cdot 10^{-2}$, respectively.

The dependences of the mass and size parameters on the range parameter are not relevant on a neutron star, but one may notice that, from fig. 1*c*) and 2*c*), x_b and $m(x_b)$ have enhanced values and maxima in certain ranges of λ. The interesting point is that, for a given relativity parameter $N/N_c = 1.0$ and a strength parameter $\alpha = 7.2 \cdot 10^{-3}$, x_b and $m(x_b)$ have maxima at λ around several hundred metres. These results are interestingly consistent with the estimates of Fischbach *et al.* [1].

Finally, if there is a new intermediate-range force which couples the baryons, then the mass limit of a self-gravitating system of degenerate fermions may be replaced by $M_{\rm lim} = (3\pi/4)^{1/2}(1-\alpha)^{-3/2} m(m_{\rm pl}/m)^3 \widetilde{m}(x_b)$.

REFERENCES

[1] FISCHBACH E., SUDARSKY D., SZAFER A., TALMADGE C. and ARONSON S. H., *Phys. Rev. Lett.*, **56** (1986) 3.
[2] FISCHBACH E. and TALMADGE C., in *5th Force, Neutrino Physics*, edited by O. FACKLER and J. TRAN THANH VAN (Editions Frontières) 1988.
[3] CHO Y. M., *Phys. Rev. Lett.*, **68** (1992) 3133.
[4] CHO Y. M. and PARK D. H., *Gen. Relativ. Gravit.*, **23** (1991) 7441.
[5] SONG D. J. and LEE H. S., *J. Astron. Space Sci.*, **11** (1994) 115.
[6] GLASS E. N. and SZAMOSI G., *Phys. Rev. D*, **35** (1987) 1205.
[7] CHANDRASEKHAR S., *An Introduction to the Study of Stellar Structure* (University of Chicago, Dover) 1967.
[8] CHO Y. M., *Proceedings of the International Workshop on Gravitation and Fifth Force, December 20-21, 1993*, edited by S.-W. KIM (Kyung Moon Sa, Seoul) 1993.