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Global properties of energy-truncated spheroidal stellar systems (*)

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Summary. — A family of self-consistent two-integral (energy and angular momentum along the symmetry axis) models for spheroidal stellar systems has been constructed, with differential rotation and anisotropy. The dependence on the angular momentum is the same as in the Wilson model and the energy dependence is the same as in the well-known King models. Our main interest is to study them as sequences of models showing many features observed both in galaxies and in globular clusters. In particular, we compute some global quantities (the potential energy, the cut-off energy, the adimensional mass) and examine the relationships among them. These can be useful for the understanding of their structure, with special emphasis on the connection between the dynamics and the flattening of the configurations. The structural stability of these models can be inferred comparing the relations among the global quantities with those obtained in the spherical models. Moreover, the study of the total energy in relation with the structural parameters, suitably constrained, gives information about their secular stability.

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1. – Introduction

Energy-truncated models represent an important step in the study of stellar systems, mainly as a fundamental improvement of the features exhibited by the classical isothermal sphere. Their most popular application is to fit the observational

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data of globular clusters (the well-known *King models* [1] and their multi-mass and anisotropic variants), but often they are also used to model elliptical galaxies, in particular when a simple spheroidal rotating model is needed (Prendergast and Tomer [2]; Wilson [3]). In this respect they were almost always used as tools to analyze the properties of single objects, whereas they were seldom studied as a *sequence* of models. As far as we know, one of the first papers in which this second perspective emerges is the one by Hunter [4], where he computes the spherical isotropic limit of the Prendergast-Tomer and Wilson models and some of their anisotropic spheroidal perturbations and performs an analysis of the relations among various global quantities. The aim of this paper is to present the global analysis of a family of models based on a distribution function that has a King-like dependence on energy and is a simple function of the angular momentum,

$$f(E, L_z) = \begin{cases} A(e^{-aE} - 1)\, e^{-bL_z - dL_z^2}, & E < 0\,, \\ 0\,, & E \geq 0\,, \end{cases} \tag{1.1}$$

so to construct rotating anisotropic spheroidal models.

The main point on which we focus our attention is the link among the global quantities (mass, cut-off energy, total energy and so forth) of the equilibrium configurations and the parameters appearing in the distribution function: the central potential Ψ_c and the parameters b and d, which are related, respectively, to the rotation and to the anisotropy in the velocity distribution (the spherical isotropic case is obviously parameterized only by Ψ_c). At a given fixed value of the pair b and d, the functional form of the relation between, *e.g.*, the true edge potential (the same as the negative of the cut-off energy) and Ψ_c provides a lot of information about the sequence of models: the *waviness* of this function was already remarked by Hunter [4] in the isotropic cases mentioned above and an analogous behavior is presented below for the standard King models (corresponding to $b \equiv d \equiv 0$). One of the main results of the present paper is that the relations among the total mass M, the equatorial tidal radius r_t and the equatorial edge potential $\Phi_t = \Phi(r_t) = -GM/r_t$, can be found also in the spheroidal anisotropic case and a systematic change of behavior is detected: in the oblate models (that is those with the equatorial radius greater than the polar, or $\hat{z}$, radius) the qualitative trend of, *e.g.*, the Φ_t *vs.* Ψ_c relation is preserved, with a general oscillatory form and several maxima and minima located approximately in the same locations of the corresponding King model. In the prolate models (that is those with the equatorial radius shorter than the polar, or $\hat{z}$, radius), after an initial agreement at low Ψ_c values, with the first maximum and first minimum locations almost unchanged, the first minimum gradually shifts towards higher values of Ψ_c. The relation tends to be almost monotone with no further oscillations, at least in the range of Ψ_c which is considered of interest for the description of realistic stellar systems until a point of inflection develops and, after that, a minimum which is again where the first minimum of the corresponding King model is. This transition occurs, more or less, at the transition between prolate and oblate configurations. It is well known that the nature of critical points in the relations between global quantities provides several pieces of information regarding the structural and the secular stability of the models. The change of behavior of the Φ_t *vs.* Ψ_c relation implies also a change in the nature (or, at least, in the location) of its critical points, suggesting important consequences on their stability properties. Stimulated by this observation, we have examined the possible application of the Poincarè theory of linear series (Lynden-Bell and Wood [5]; Katz [6]) to assess the secular stability of the anisotropic rotating systems. We found that a rigorous extension of the stability analysis performed in the spherical isotropic case (*e.g.*, in the case of King models, Katz [7]) is not possible in the framework of our treatment, for both theoretical and technical problems that will be illustrated below. However, a simplified analysis, based on the study of perturbations at fixed mass and total energy, can be used as a guess of the stability properties of these more general models. The results of this analysis are that, comparing models with b and d so to allow both prolate and oblate models, increasing the rotation *and/or* decreasing the anisotropy tend to gradually shift the stability limit (that, we recall, for the King model is $\Psi_c \simeq 7.4$ (Katz [7]) to higher values.

The plan of the paper is as follows: in sect. **2** we illustrate the integration of the self-consistent equations for the construction of the equilibrium models; in sect. **3** we describe the properties of the sequences of possible models; in sect. **4** we present a discussion of the results and in sect. **5** a brief exposition of their consequences.

2. – The integration algorithm

Introducing in a natural way the quantity with the dimension of length $r_a = \sqrt{a/d}$, we can define two dimensionless parameters by which we can characterize the properties of the models based on the function introduced in eq. (1.1). They are, respectively,

$$\Omega = \frac{r_c}{\sqrt{a}}\, b, \tag{2.1}$$

proportional to the rotation velocity at the core radius and

$$\gamma = \frac{1}{r_c}\sqrt{\frac{a}{d}} = \frac{r_a}{r_c}, \tag{2.2}$$

giving the dimensionless radius beyond which the radial anisotropy dominates, where r_c is defined as

$$r_c = 3/\sqrt{4\pi G a \varrho_c}$$

(with ϱ_c central density) that, in our models, is a natural measure of distance (King radius [1]).

To find an equilibrium model based on a distribution function like the one of eq. (1.1), we have to solve the Poisson equation $\nabla^2 \Phi = 4\pi G \varrho$. Assuming $G = 1/4\pi$ and using dimensionless quantities $\mathcal{E} = -aE$, $\Psi = a(\Phi(r_t) - \Phi(r))$ and expressing the density ϱ as a function of energy and angular momentum (Binney and Tremaine [8]), we obtain

$$\nabla^2 \Psi = -\frac{4\pi}{R}\int_0^{\Psi} d\mathcal{E} \int_{-L_m}^{+L_m} f(\mathcal{E}, L_Z)\, dL_Z, \tag{2.3}$$

where

$$\mathcal{E} = -\frac{1}{2}\mathbf{v}^2 + \Psi(R, \varphi, Z), \qquad L_Z = Rv_\varphi$$

are the single-particle energy and angular momentum (in cylindrical coordinates), $L_m = R\sqrt{2(\Psi - \mathcal{E})}$ and the φ coordinate is integrated out by exploiting the axial symmetry.

For the purpose of the subsequent discussion we will limit ourselves to the assessment of the gross morphological features of the models (essentially the iso-density contours and the global quantities mentioned above), leaving aside for the moment the detailed study of their kinematics. The goal is therefore that of integrating the non-linear partial differential equation embodied by the Poisson equation (2.3). We follow the iterative scheme of Wilson [3], which is an improvement of that introduced by Prendergast and Tomer [2].

To initialize the iterative procedure, we choose as step-zero solution the potential $\Psi(r)$ of the spherical isotropic counterpart of the model, that is simply the King model [1], from which we compute the step-zero density

$$\varrho^{(0)}(r, \vartheta) = \frac{4\pi}{r \sin\vartheta} \int_0^\Psi \mathrm{d}\mathcal{E} \int_{-L_m}^{+L_m} f(\mathcal{E}, L_Z)\, \mathrm{d}L_Z, \tag{2.4}$$

where $R = r\sin\vartheta$.

Then we perform a Legendre-series expansion on the density and the potential

$$\Psi^{(0)}(r, \vartheta) = \sum_k \Psi_k^{(0)}(r)\, P_k(\cos\vartheta), \qquad \varrho^{(0)}(r, \vartheta) = \sum_k \varrho_k^{(0)}(r)\, P_k(\cos\vartheta),$$

having chosen the spherical coordinates r, ϑ, φ. Substituting these into the Poisson equation we obtain an "*improved*" solution $\Psi^{(1)}$, by which we generate a $\varrho^{(1)}$, that is used to construct a $\Psi^{(2)}$, by means of a step analogous to that of eq. (2.4), and so on.

There are some practical problems to solve in working out this algorithm to reach a good compromise between precision and CPU time consume. With regard to the choice of the coordinate grid, we found that for the spacing in ϑ an interval of 2 degrees is sufficient, whereas, for the radial coordinate the choice is dictated by the integration of the spherical *triggering* model. A fourth-order Runge-Kutta routine, with a step size varying as the inverse of the potential (normalized to one) was used. With regard to the choice of the order of the expansion in the Legendre series, since the coefficients decrease rapidly in amplitude, truncating the series at the eighth order is enough to obtain a convergence at the level of 10^{-3} in ~ 10 iteration (depending on the model).

3. – Sequences of models

The family of models studied here is characterized by three parameters: Ω, γ and the depth of the potential well $\Psi_c = a(\Phi_t - \Phi_c)$, which depends on the choice of Ψ_c of the triggering spherical King model. One can observe two main behavioral features: fixing Ω and γ and spanning the sequences along the central potential Ψ_c, one can see that for small Ψ_c values (*e.g.* $\Psi_c \leqslant 5$) the isodensity contours are practically spherical whereas, increasing Ψ_c ($6 \leqslant \Psi_c \leqslant 10$), one notices a richer morphology depending on Ω

and γ. Keeping Ω fixed, models range from prolate (small γ) to oblate (large γ) configurations with a smooth transition through the spherical symmetry, whereas, for large Ψ_c values ($\Psi_c \simeq 16$) all models exhibit an isophotal distortion, either in the shape of "lemons" (prolate) or "peanuts" (oblate). In the following we will refer to models with small γ as prolate and with large γ as oblate: although, as stated before, this does not hold true throughout the sequence, it is nevertheless true in the section of parameter space that is more relevant to physically interesting models.

In the present paper we construct a grid of the following sequences of models: the values of Ω chosen were the following: $\Omega = 0.033$, 0.05, 0.1. The $\Omega = 0.05$ sequence in particular represents quite well the characteristics described above: for γ ranging from 5 to ~ 40 we have prolate models with a roughly spherical transition at $\gamma = 46$ and oblate models for γ from 60 to 80. For each couple of the independent parameters, one has to choose the sequence of triggering models so to cover as uniformly as possible the range of Ψ_c between 1 and, more or less, 15. The global quantities relevant for the description of the models behaviour are the physical cut-off energy $\mathcal{E}_c = GaM/r_t$ and the total energy U. In terms of dimensionless quantities, using cylindrical coordinates (R, φ, Z), we compute the dimensionless mass and potential energy:

$$M_* = 4\pi \int_0^{z_t} dz \int_0^{x_t} \varrho_*(x, z)\, x\, dx\,, \qquad W_* = 2\pi \int_0^{z_t} dz \int_0^{x_t} \Psi(x, z)\, \varrho_*(x, z)\, x\, dx\,, \tag{3.1}$$

where ϱ_* is the normalized density $\varrho_* = \varrho / \varrho_c$, $x = R/r_c$ and $z = Z/r_c$ are the normalized coordinates so that we can introduce the dimensionless equatorial tidal radius $x_t = R_t / r_c$. An estimate of the (dimensionless) cut-off energy at the equatorial radius is defined as $\mathcal{E}_c = GaM/r_t$. Since the actual mass M is $M = \varrho_c r_c^3 M_*$, $\mathcal{E}_c$ can be computed by means of

$$\mathcal{E}_c = \frac{1}{4\pi} \frac{M_*}{x_t}\,. \tag{3.2}$$

The total energy U can be expressed in the form

$$U = -uGM^2/r_t\,,$$

where the factor

$$u = \frac{2\pi}{9} \frac{W_* x_t}{M_*^2} + \frac{1}{4} \tag{3.3}$$

can be computed from dimensionless quantities.

The results of the computation of the above quantities in the case of the spheroidal anisotropic variant of the King models can be interpreted from different points of view and can shed light on many aspects of the structure and also of the evolution of the stellar systems that can be described by these models. This is the aim of the rest of the paper.

4. – Discussion

Following Hunter [4] we can plot the relation between $\mathcal{E}_c$ and Ψ_c, and make a comparison among the curve $\mathcal{E}_c$ *vs.* Ψ_c for the spherical King model (fig. 1) with the analogous curves for the spheroidal models (fig. 2-4).

Fixing Ω we see that the trend for the models with low values of γ is similar to the King model, but with a value of $\mathcal{E}_c$ at the first minimum which is much lower in the spheroidal models. There is a tendency to shift the minimum to higher values of Ψ_c increasing the value of γ, until gradually a point of inflection appears and, after that, the first minimum manifests itself again at, approximately, the same value of Ψ_c of the spherical model and at the same level of $\mathcal{E}_c$. Therefore an interesting peculiarity is that there is a certain similarity in the properties of extreme prolate models with the

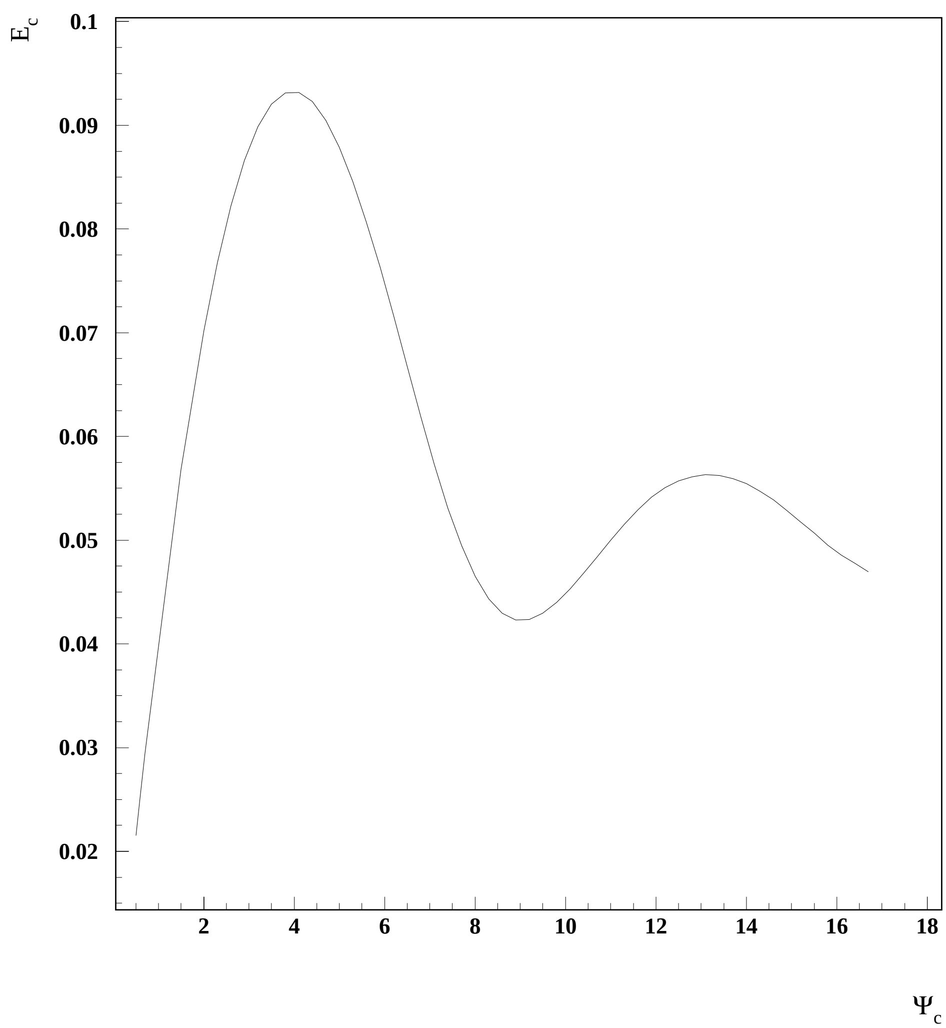

Fig. 1. $-\mathcal{E}_c$ *vs.* Ψ_c curve for a spherical King model.

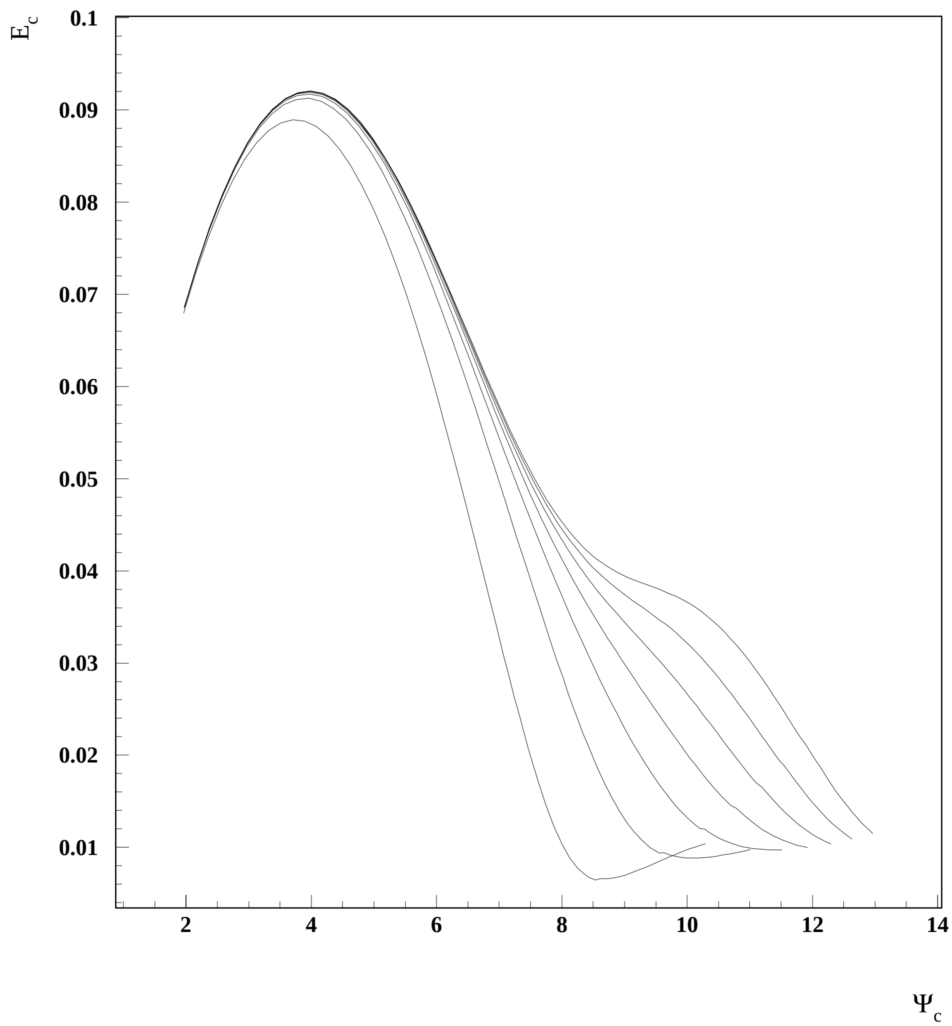

Fig. 2. – $\mathcal{E}_c$ *vs.* Ψ_c curve for a spheroidal model with $\Omega = 0.033$ and $\gamma = 5$, 10, 15, 20, 25, 30, 35.

spherical isotropic ones. In particular, in the plots of $\mathcal{E}_c$ *vs.* Ψ_c for $\Omega = 0.05$ (fig. 3) and $\gamma = 5$ it can be seen that the oscillatory behavior of the relation is preserved, with the location of the maxima and the minima approximately unchanged. This situation gradually changes by increasing γ *i.e.* going towards the oblate models, with a point of inflection at $\gamma \simeq 32$ and a transition through a spherical model which happens at $\gamma \simeq 46$. We see that models with $\gamma > 46$ (oblate models) provide again a curve very similar to that of the spherical isotropic models. The situation is qualitatively the same in the other cases, with the minimum that, increasing Ω, gradually shifts towards higher values of Ψ_c, at corresponding values of γ.

A straightforward indication emerging from these plots is what we may call a *structural stability* of this class of energy-truncated models and which can be interpreted as the possibility of constructing sequences of rotationally and anisotropi-

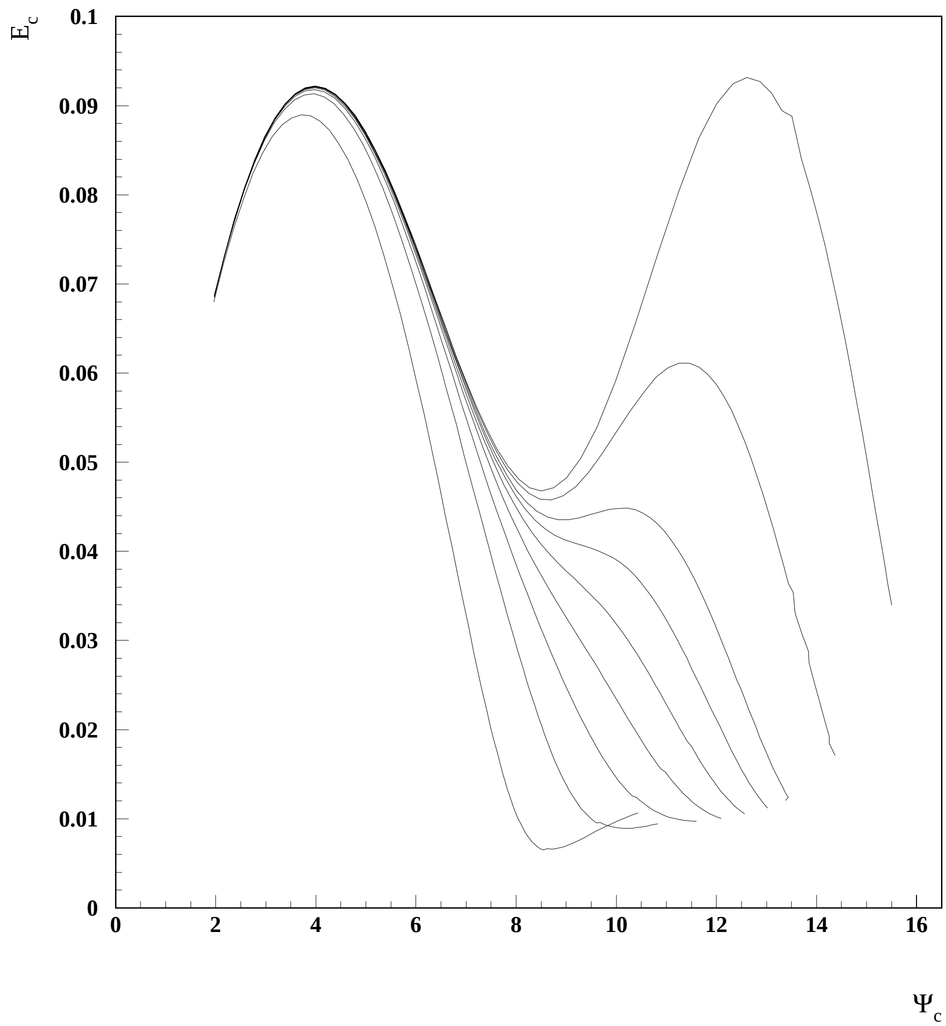

Fig. 3. $-\mathcal{E}_c$ *vs.* Ψ_c curve for a spheroidal model with $\Omega = 0.05$ and $\gamma = 2, 5, 10, 15, 20, 25, 30, 35, 46, 60$.

cally perturbed King-like models which preserve their original global properties. More specifically, the introduction of a moderate rotation and of a not too strong radial anisotropy does not substantially change the relations between the mass, the radius and the binding energy of the structures. The same kind of structural stability can be observed by displaying plots of U *vs.* $\mathcal{E}_c$.

But the plots of U *vs.* $\mathcal{E}_c$ have also the purpose of giving information about a fundamental question related to another notion of stability: the thermodynamical or, with a more appropriate stellar dynamical definition, the *secular* stability. Plots of the form U *vs.* $\mathcal{E}_c$ are in fact essential in the treatment of the stability of isothermal sphere (Lynden-Bell and Wood [5]; see also, Binney and Tremaine [8]) and of other isotropic spherical stellar models (Katz [6]) in the context of the Poincaré theory of linear series.

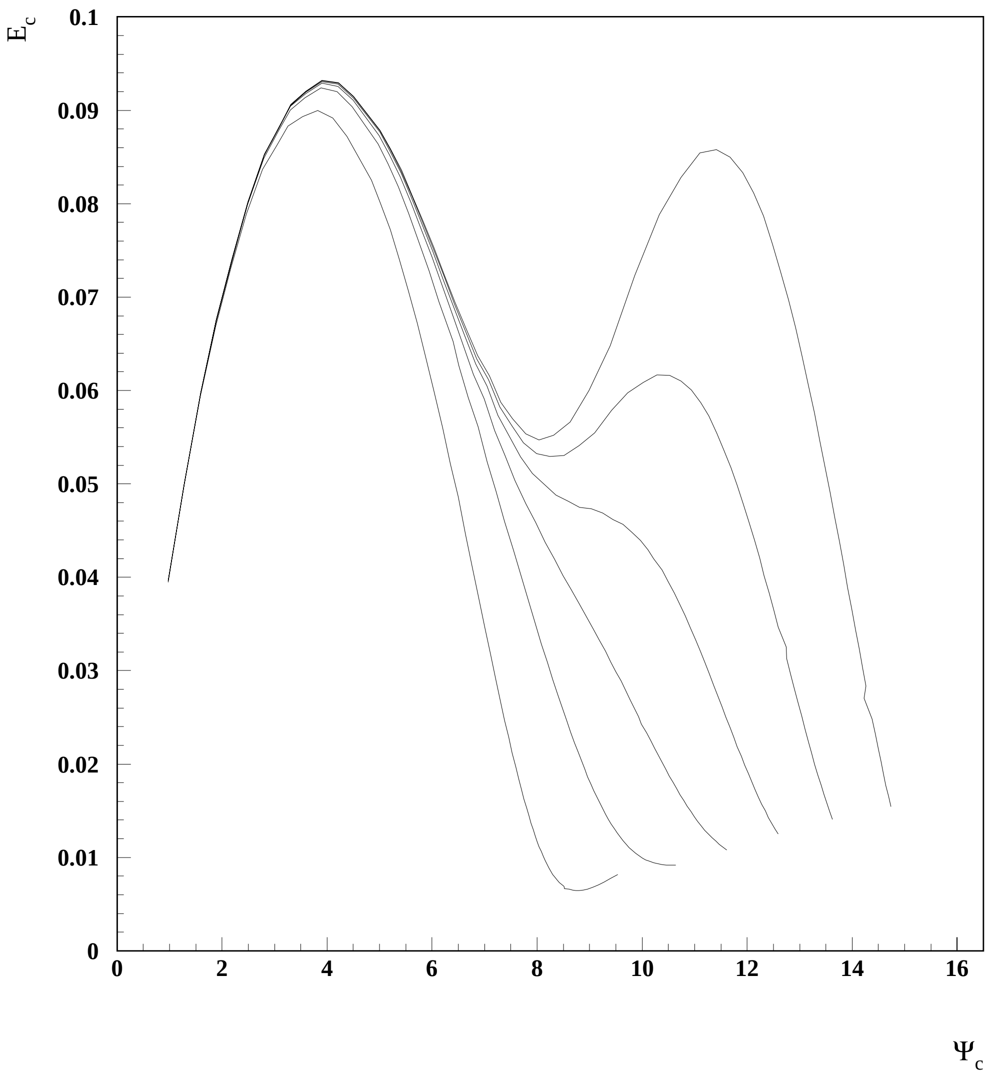

Fig. 4. $-\mathcal{E}_c$ *vs.* Ψ_c curve for a spheroidal model with $\Omega = 0.1$ and $\gamma = 5, 10, 15, 20, 25, 30$.

On the basis of this observation, it is tempting to try to perform the same kind of stability analysis for the rotating anisotropic models. In this respect a fundamental question to solve is that of clarifying the nature of the perturbation under which the stability of the model is tested. In the case of secular stability, we are interested in the fate of the structure if the exchange of a small quantity of energy among adiacent shells of the configuration is allowed. It is natural to assume, therefore, perturbations leaving constant the *total mass*, *total energy* and, following the arguments of Katz [7], the parameter A in the distribution function, which is related to the border chemical potential. These constraints to be imposed in the construction of the equilibrium series, parametrized in a natural way by the parameter Ψ_c, are inherited by the theory of spherical isotropic systems. In the attempt to apply the same theory to more general

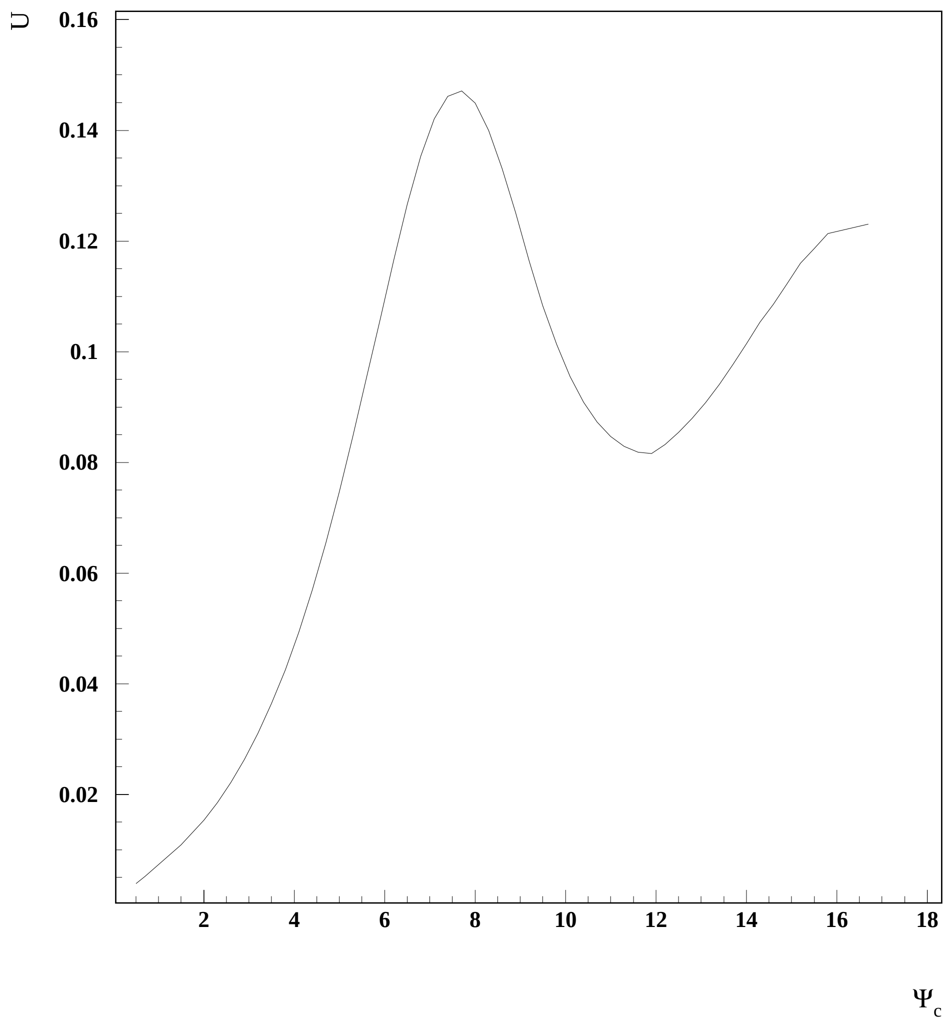

Fig. 5. – Total energy U *vs.* Ψ_c (at fixed true mass and A) for the sequence of spherical King models.

systems, with a phase space occupancy determined by more parameters, it is evident that other constraints might be imposed on the equilibrium sequences. However, in this respect, we are faced with several difficulties of various nature. Specifically, in the case of our models, we are concerned with the constraints related to the parameters Ω and γ and we will see soon that they are connected with two different kinds of problems. As regards γ, it is quite natural to think of some global quantity determining the amount of anisotropy: the problem is that, since an adequate modeling of the detailed process going on in the evolution is lacking, it is difficult to guess which specific quantity to evaluate and try to keep constant. The question is different regarding Ω, since, in this case, one would think natural to impose the constraint of the total angular momentum kept fixed during the perturbation. But, if we leave aside the problem that also in this

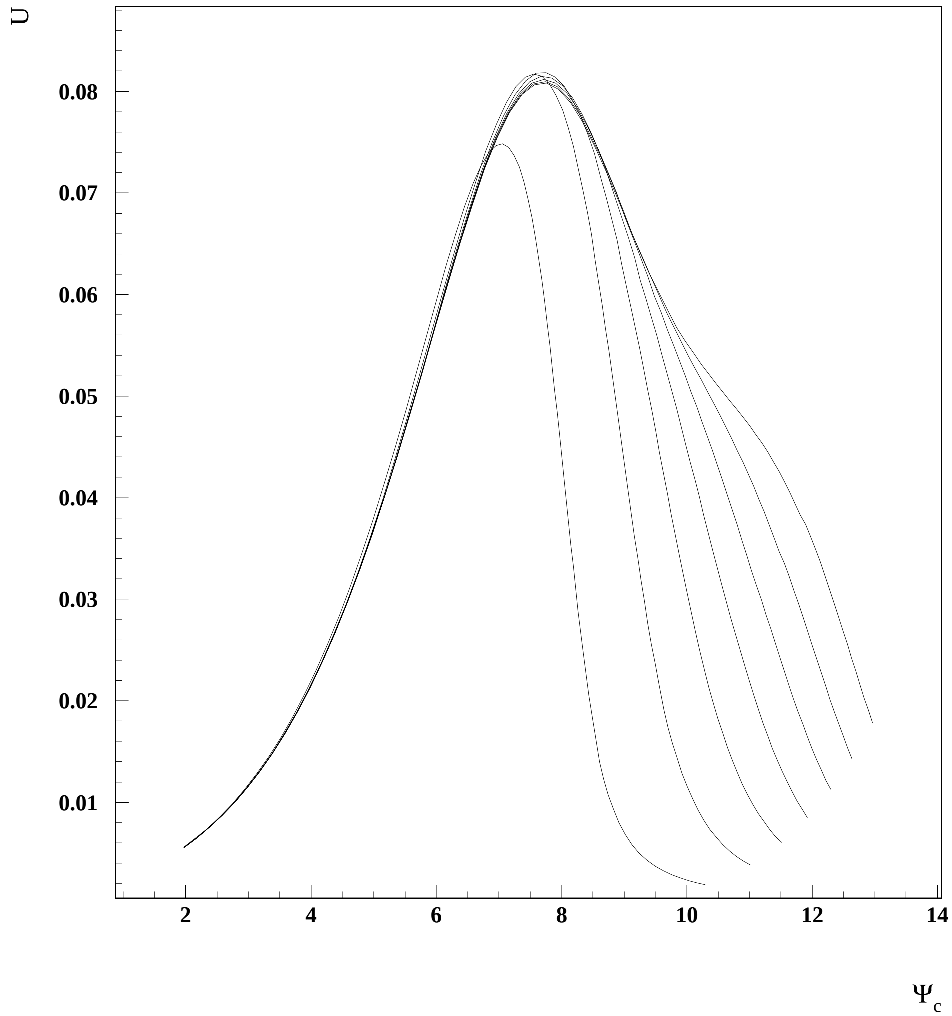

Fig. 6. – Total energy U *vs.* Ψ_c (at fixed true mass and A) for the sequence of spheroidal models with $\Omega = 0.033$ and $\gamma = 5, 10, 15, 20, 25, 30, 35$.

case we lack a rigorous argument to enforce this constraint, its implementation faces technical difficulties that will be examined below.

Let us recall the procedure for the analysis of the stability at constant A, mass and energy in the case of the King models. Recalling that with $G = 1/4\pi$ we have

$$r_c^2 = \frac{9}{a\varrho_c},$$

the central density is determined by the parameters of the model in the following

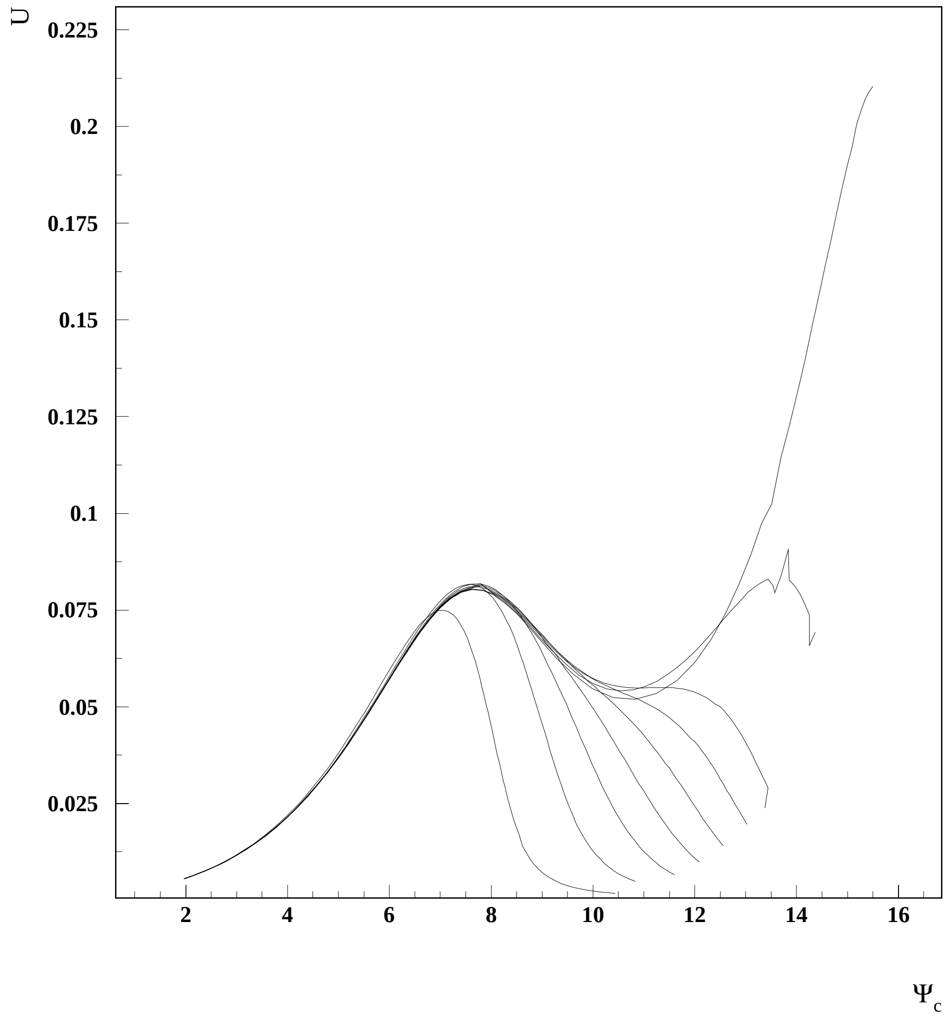

Fig. 7. – Total energy U *vs.* Ψ_c (at fixed true mass and A) for the sequence of spheroidal models with $\Omega = 0.05$ and $\gamma = 2, 5, 10, 15, 20, 25, 30, 35, 46, 60$.

implicit way:

$$\varrho_c = \frac{4\pi A}{a^{3/2}} D(\Psi_c),$$

where $D(\Psi_c) = \int_0^{\Psi} d\mathcal{E}(e^{\mathcal{E}} - 1)\sqrt{2(\Psi - \mathcal{E})}$. Obviously, a mass constraint is to be imposed on the *true mass*, so that the true mass is fixed at an arbitrary value (*e.g.*, $M = 1$), the relation between the true and dimensionless mass provides the value of a to be used consistently in all the other relations:

$$a = \frac{81\, M_*^{4/3}}{(4\pi A D(\Psi_c))^{2/3}} \,.$$

If, for example, we want to construct the equilibrium sequences using as main function the (true) total energy (and use Ψ_c as the independent parameter along the sequence), we have to substitute the value obtained for a in the definition of U, so to have the desired expression of the energy in terms of Ψ_c at fixed mass and A,

$$U = -\frac{uD(\Psi_c)^{2/3}}{9x_t(4\pi M_*)^{1/3}} \,.$$

According to the Poincaré theory, maxima and minima of U along the curve $U(\Psi_c)$ are the only places where a change in the stability may occur: in the case of the King model, there is a general agreement in locating the maximum at $\Psi_c \simeq 7.4$ [7] which, by examining the "*direction of rotation*" of the corresponding diagram u *vs.* $\mathscr{E}_c$, can be identified as a point of transition from stability to instability. As we said above, in the case of the spheroidal models, it would appear natural to impose, in addition to A and M, the constraint of a fixed amount of the total angular momentum J, along the sequence of equilibrium models. The attempt to apply a procedure analogous to that followed for the mass constraint fails, however, in finding a similar direct semi-analytical solution. In fact, since J can be written in the form

$$J = \frac{r_c^4 \varrho_c}{\sqrt{a}} J_* \,,$$

where J_* is the dimensionless total angular momentum, the *specific angular momentum* is

$$\frac{J}{M} = \frac{r_c}{\sqrt{a}} \frac{J_*}{M_*} \,.$$

Fixing the ratio J/M equal to an arbitrary constant, one would try to find an expression of Ω such that

$$\frac{r_c}{\sqrt{a}} \frac{J_*(\Psi_c, \Omega, \gamma)}{M_*(\Psi_c, \Omega, \gamma)} = \text{const} \,,$$

provided that a still satisfies the condition established above for the mass constraint. Unfortunately, there is no linear relationship between J and Ω, so that the only possibility to extract Ω in such a way to satisfy the constraint $J/M = \text{const}$ would be that of constructing a fine grid (in Ψ_c and Ω) of models and, after that, tracing a sequence at fixed J, but this is a very cumbersome approach to be attempted in due course.

In the light of the above discussion, we think that, if one wishes to carry on a stability analysis for the rotating isotropic models, this can be preliminarily done by imposing the constraints on the mass, energy and A (which can be made without

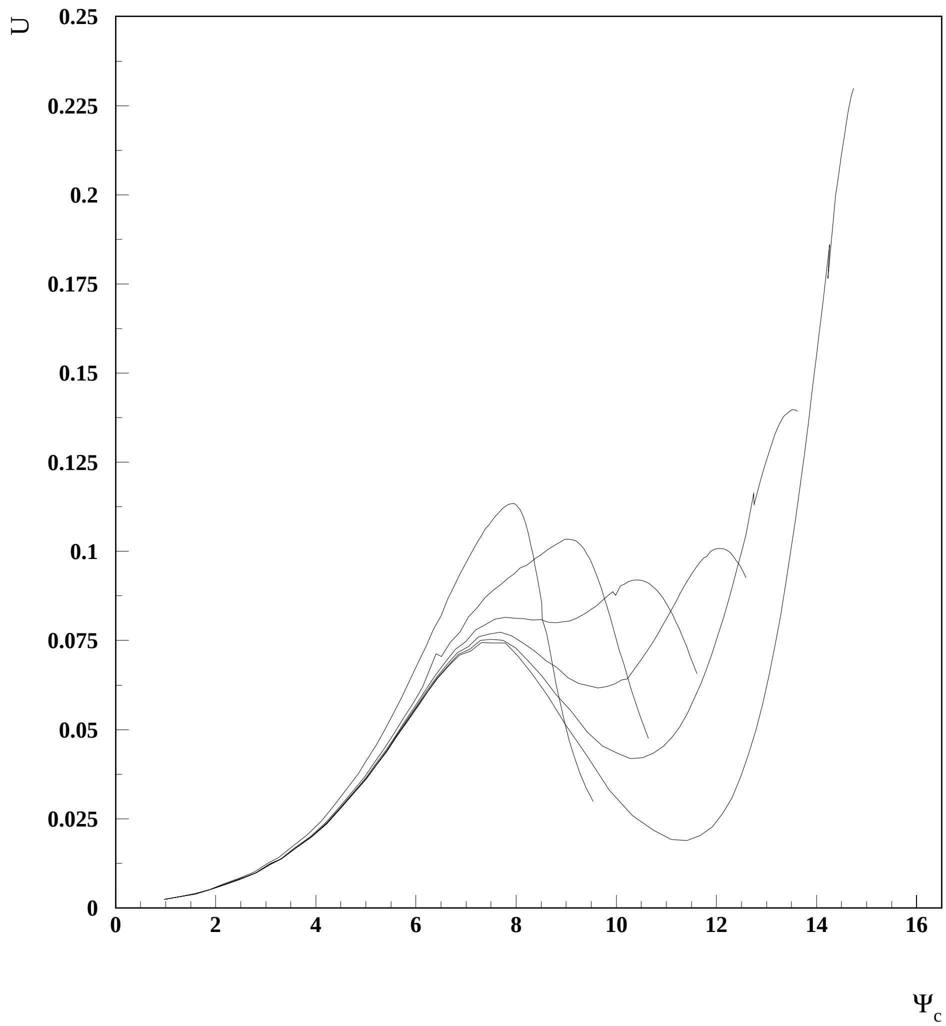

Fig. 8. – Total energy U *vs.* Ψ_c (at fixed true mass and A) for the sequence of spheroidal models with $\Omega = 0.1$ and $\gamma = 5, 10, 15, 20, 25, 30$.

difficulties following the same procedure of the isotropic case, with the only price of a much bigger computation time) and assuming that the simple condition of keeping constant the parameters Ω and γ is a not too bad approximation of the processes going on in the evolution along the sequence. This approach must be considered as a guess to be checked in a more refined elaboration of the theory.

In the figures from 5 to 8 we have the plots of the curve $U(\Psi_c)$ in the spherical King models (fig. 5) and in the set of spheroidal models with $\Omega = 0.033$ and γ ranging from 5 to 35 (fig. 6), $\Omega = 0.05$ and γ ranging from 2 to 60 (fig. 7), $\Omega = 0.1$ and γ ranging from 5 to 30 (fig. 8); we chose the γ interval with a regular spacing for all Ω values from 5 to 35 with step 5 and, as an exception, we selected $\gamma = 46$ and 60 as examples of oblate models in the $\Omega = 0.05$ case and $\gamma = 2$ as an example of extreme prolateness. Moreover, we were forced to exclude the $\gamma = 35$ value from the $\Omega = 0.1$ sequences as it would lead to extremely distorted configurations that could not give a reliable estimate of the total energy.

To summarize the results obtained, calling Ψ_m the value of the parameter for the transition to instability (so that $\Psi_m = 7.7$ in the spherical case), we have $\Psi_m < 7.7$ in the cases with small γ (see, for example, the case $\Omega = 0.05$ and $\gamma = 2$ of fig. 7), where we find the maximum to be $\Psi_m \simeq 6.5$, whereas increasing γ the maxima shift towards higher Ψ_c values. The critical value $\Psi_m \simeq 7.7$ is crossed at a value of γ approximately equal to 5. In the range from 5 up to $\gamma \simeq 20$ (this limit depends on the value of Ω) the maxima are located at gradually increasing values of Ψ_c (*e.g.* $\Psi_m \simeq 9.5$ for $\Omega = 0.05$ and $\gamma \simeq 15$, fig. 7) until when the maxima change their nature: points of inflection appear and the maxima are abruptly relocated at $\Psi_m = 7.7$. We see therefore that strongly anisotropic models $\gamma < 5$ are more "unstable" than the isotropic models and there is a range of the parameter space in which the stability threshold is raised to higher Ψ_c values and, finally, that for a small value of the anisotropy, we come back to the isotropic value.

5. – Conclusions

Energy-truncated models are still a very useful "*laboratory*" to explore some of the properties exhibited by stellar systems when they are in an environment that can heavily influence their characteristics. This is the case of globular clusters, but can be of relevance even in the case of elliptical galaxies in rich clusters. The purpose of the present paper was that of a general reinvestigation of this kind of models, mainly to assess their global properties as sequences of configurations. The first important result is that there is a general agreement between the spherical isotropic and the spheroidal anisotropic King models, so that the introduction of rotation and anisotropic velocity dispersion does not change their global properties.

Other results that, while awaiting to be checked by an analysis based on a more rigorous treatment of the constraints, are of notable importance, are those related to the assessment of the secular-stability limits along the sequences of equilibrium. Stimulated by a correlation with the model parameters favoured by the fits of observed profiles of globular clusters and elliptical galaxies, it is tempting to attach a physical meaning to the above behavior as a selection criterion for the possible equilibrium of collisionless self-gravitating systems. A preliminary analysis of the results obtained suggests that, referring to the onset value for the gravothermal catastrophe, the introduction of a moderate amount of rotation (that, in the present case, consists in an almost solid-body rotation of the core and a rapid decrease in the outher regions) and of a certain degree of anisotropy tends to hinder the process, shifting the value of the critical concentration towards higher values. Gradually removing the anisotropy, at fixed rotation parameter, the stability limit is increased, until, abruptly, it comes back to spherical-isotropic-models value. At the same time, the introduction of a quite high radial anisotropy tends to favor the process, shifting the value of the critical concentration towards lower values. These conclusions must be verified in the framework of a more rigorous analysis and, if confirmed, could well have consequences in the theory of the secular evolution of globular clusters and compact ellipticals.

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