Monte Carlo simulations for a radio halo in a cluster of galaxies (*)

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Summary. — The nature of a radio halo in a cluster of galaxies is investigated taking account of the turbulent-wake theory with a series of Monte Carlo simulations. By allowing the injected electrons of $\gamma = 2.6$ to spread over the cluster core radius, the spectral index of the radio halo appears to be consistent with the numerical simulations when the parameters were adjusted to the wake filling factor 0.1 at the cluster center which follows the King profile and the energy gain via the first-order Fermi acceleration process was adjusted to $2 \cdot 10^{-16}$/s at 1 GeV. The diffusion coefficient and the strength of the intracluster magnetic field are assumed to be constants that are 10^{30} cm^2/s and 10^{-6} G over the entire cluster. More simulations covering larger parameter space would be fruitful for better understanding the origin of the radio halo.

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1. – Introduction

Continuum radio observations of the Coma cluster and other clusters [1, 2] revealed a type of nonthermal radio sources of very large angular size in the clusters, having steep spectra. Several explanations arise for the origin of the sources, radio halos: pseudohalo hypothesis, diffusion from cluster radio sources, relics of galaxies previously exhibiting head-tail behaviour, secondary-electron model [3] and turbulent galactic wakes [4, 5]. In this paper, we test the validity of the turbulent-wake theory with a series of Monte Carlo simulations. The analytic discussion focusing on this theory and parameters applied to the Coma cluster is found in ref. [6]. The purpose of

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this paper is to test, mainly, the agreement of spectral indices of the simulated halo with the observations. Therefore we will avoid discussions of detailed normalizations and other statistical techniques.

It is believed that galaxy motions in a cluster of galaxies are not highly supersonic. This implies that shocks associated with galaxy motions are likely to be collisionless. This means that energy dissipation should involve wave-wave or wave-particle interactions in a plasma. In this regard, the interaction of the intracluster magnetic field with galaxies becomes particularly important. The existence of such turbulent galactic wake provides particle's acceleration in intracluster medium (ICM) by the first-order Fermi process.

2. – Monte Carlo simulation

We performed the Monte Carlo simulation by following the history of each electron. The basic input values are as follows. The galaxy cluster is assumed to be 1 Mpc in radius and spherically symmetric. The electrons were assumed to be injected into the intracluster medium by cluster radio sources and the spatial distribution of the electrons is assumed to follow the galaxy distribution which is the King profile

$$f(r) = \frac{1}{1+(r/r_c)^2}\,, \tag{2.1}$$

for which the cluster core radius r_c is set to 350 kpc as for the Coma. The energy spectrum of the injected electrons is simply assumed as the power spectrum $P(E) \propto E^{-\gamma}$ with $\gamma = 2.6$ and 10 MeV $\leqslant E \leqslant$ 100 GeV, which is generally accepted by high-energy cosmic-ray observations. Therefore, the radio halo initially had a large size and the spectral index of the halo flux density at frequency ν ($F_\nu \propto \nu^{-\alpha}$) was everywhere $\alpha = (\gamma - 1)/2 = 0.8$. This is rather an unrealistic assumption for a halo but we shall see what changes would take place for the halo as the electrons diffuse away, encountering with the turbulent wakes.

As these electrons diffuse, they interact with the intracluster magnetic field which is random in orientation and whose strength is constant over the cluster on the order of microgauss. Such successive interactions with the magnetic field cause the diffusion more like a Brownian motion. We accept the diffusion coefficient $D = 10^{30}$ cm^2/s [6] which is equivalent to the mean free path length $l_c = 0.032$ kpc or the mean collision time $t_c = 105$ years. During this diffusion process electrons lose energies via synchrotron radiation ($-aE^2$). At the same time, they also gain energies in a turbulent wake via the first-order Fermi mechanism (bE). This gain is effective only when the particle is inside a wake,

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -aE^2 + \begin{cases} bE & \text{inside a wake}\,, \\ 0 & \text{outside a wake}\,. \end{cases} \tag{2.2}$$

The loss coefficient a is obtained from the standard synchrotron radiation theory [7] for $B = 10^{-6}$ G. However, the gain coefficient is not clear. As for the maximum value of b, we take $b_{\max} = 2\cdot 10^{-15}$/s $= 6.7\cdot 10^{-6}$ GeV/(collision time) at $E = 1$ GeV, as suggested in [6]. It turned out in the simulation that the best value is an order of magnitude lower $b_{\text{best}} \simeq 0.1\cdot b_{\max}$.

Instead of specifying the positions of wakes, we may use the following probability: at each step, the electron is being asked if it is located inside or outside a wake. This probability is the wake space filling factor f_W, which, of course, is proportional to the galaxy distribution, the King profile [6],

$$f_W(r) = \frac{1}{1+(r/r_c)^2} . \tag{2.3}$$

The total gain of energy in a wake becomes the product of the energy gain within a scattering and the number of scatterings in a wake. Therefore we have to specify the wake size, or, equivalently, the successive scattering number of the particle in a wake. We choose the simple exponential function for this successive scattering number distribution

$$P(N) = N_0 e^{-N/N_0}, \tag{2.4}$$

where the successive mean scattering number N_0 is taken as 10^6. This value $\sqrt{10^6}\, l_c =$ 32 kpc is based on the typical wake size 4–50 kpc [6].

An electron near the cluster center would experience $\sim 10^6$ collisions on the average before reaching the cluster boundary. Since we run 10^5 electrons for good statistics, the total number of scattering is obviously beyond the resonable range of computing time. Therefore we introduced the *bunch step technique* in the simulation. For example, during the 10^4 successive scatterings the travel distance is $l_b \approx 10^2 l_c$, while the elapsed time is $t_b \approx 10^4 t_c$. This technique greatly reduces the computing time by a factor of 10^4. Of course, the energy change during this bunch step is $\Delta E_b \approx 10^4 \Delta E_c$. In addition, for the numerical stability, we incorporated the predictor-corrector scheme to determine this energy change.

3. – Results and discussions

Given an energy distribution of relativistic electrons, it is straightforward to construct a radio spectrum

$$I(\nu)\, d\nu = \int_{E_{min}}^{E_{max}} P(\nu, E)\, N(E)\, d\nu . \tag{3.1}$$

The synchrotron radiation $P(\nu, E)$ at each electron energy E has broad spectrum and we used the interpolation following ref. [7] for the integration. The results are shown in fig. 1 and fig. 2.

The distribution of electrons and the radio spectrum shown in fig. 1 and 2 are obviously in good agreement with the observation of the Coma radio halo [1]. The overall spectral index of the halo is $\alpha = 1.3$ and the spectrum becomes steeper at frequencies over 1 GHz. However, the spatial variation of the spectral index seems not quite obvious in fig. 2 up to 900 kpc from the cluster. In other words, it does not show $\alpha \approx 0.8$ near the cluster center, as seen in the observations [2]. Further simulations might improve our understanding on this feature.

Here let us mention briefly the general properties of the turbulent wakes. Galaxies that move through the ICM produce the wakes, so the higher the galaxy density the larger the wake population, *i.e.* the larger the wake filling factor. Particles once entered into the wakes experience reacceleration while they continuously lose energy via synchrotron radiation. The size of wakes is about 30 kpc and an electron stays most of the time with its terminal energy in the wake. The possibility that an electron

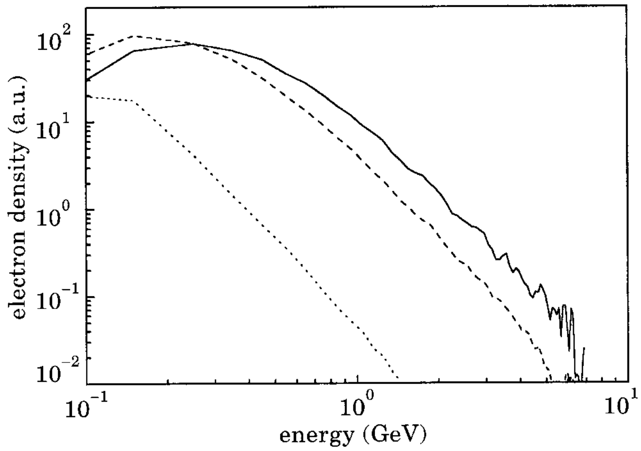

Fig. 1. – Plot of electron number density as a function of energy at several positions in a cluster. The data are not smoothed and the ripples should be understood as the poor statistics. —— 100 kpc, – – – 300 kpc, ····· 900 kpc.

encounter with a wake depends on the wake filling factor only. A too low wake filling factor thus results in an invisible radio halo, for the average electron energy becomes too low. Particles spread out over the ICM in a diffusive manner, since the magnetic field in the ICM is random in nature [1], and they experience gain and loss of energy as they encounter the wakes on their way of diffusion.

The parameters that produce the results given in the figures are as follows (for more details, see [6]). The wake filling factor is 0.1, and the injected electron spectrum is that of -2.6 in power index. The energy gain is due to the first-order Fermi mechanism which is bE, in which case $b_{\mathrm{best}} = 0.1 b_{\mathrm{max}}$. And the energy loss is constant

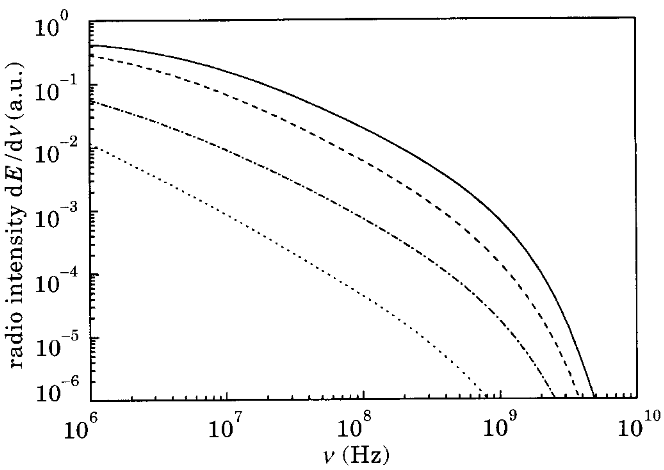

Fig. 2. – Plot of radio wave density as a function of frequency at several positions in a cluster. The electron's terminal energy in equilibrium of energy loss and gain in a wake is ≈ 7 GeV. The spectral index is $\alpha \approx 1.3$. —— 100 kpc, – – – 300 kpc, ····· 900 kpc, –·–·– average.

over the space, for the magnetic-field strength is assumed to be 10^{-6} G over the entire ICM. The terminal energy in a wake that an electron can reach is determined as $E_t = b/a \approx 0.11$ erg $\simeq 7$ GeV, in about 10^6 years. If b_{best} is chosen to be 0.01, the electron's terminal energy becomes too low, so the whole radio spectrum shifts toward the lower-frequency region. When b_{best} is chosen to be on the order of, say, 0.3, there appear too many high-energy electrons, hence the resulting radio spectrum inadequately shifts towards high frequencies. The best choice of b is therefore around 0.1 meaning that $E_t \approx 7$ GeV to match the observations of the radio halos, especially in the Coma cluster.

Noteworthy is the point that in the simulation, the choice of a proper electron source distribution was of some crucial importance. The distribution we chose in the foregoing results was that the distribution of the electrons injected into the ICM follows the galaxy distribution. But this is an arbitrary choice. When the particle injection is limited to the cluster center, the results are not satisfactory. Since the relativistic electrons are hardly influenced by the cluster potential, their distribution seems to be related to the particle suppliers. *i.e.* cluster radio sources. We have not simulated the case where the source distribution is offset from the cluster center. It might be intriguing to see if this distribution develops a radio halo on the cluster centroid.

In sum, it seems that the turbulent wake theory fits the observational properties of a radio halo without serious problems, as long as the magnetic field B in an order of 10^{-6} G is assumed. However, in order to make the turbulent wake theory complete, it should also provide us with B. The existence of B in Abell clusters seems to be more than possible as reported in [8], but the nuisance is the possibility that the turbulent wakes are not likely to amplify a week seed field $B \approx 10^{-8}$ G to the wanted strength $\approx 10^{-6}$ G, as demonstrated in [9, 10]. If the wakes are not able to produce the B, reacceleration of particles also is not expected to be effective as we assumed in the simulations. Further study on the nature of the ICM turbulence is required for better understanding on this subject.

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