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$\Omega < 1$ polar inflation in a Kaluza-Klein cosmology (*)

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Summary. — We discuss a model 4-dimensional Friedmann cosmology which may have evolved from a model of $(4+D)$-dimensional Kaluza-Klein (KK) universe. After compactification of extra dimensions, the 4-dimensional BD parameter appearing in the dimensional reduction is negative. We find that if there had been an inflationary transition, the matter-dilaton coupling quickly becomes exponentially small. Then the universe undergoes a polar-type superluminal expansion, and the density parameter of the post-inflationary universe becomes a discrete quantity. In particular, for the universe compactified from 9 dimensions ($D = 5$), it is shown that $\Omega_{\text{total}} \approx 0.26$ even if the universe is geometrically flat ($k = 0$).

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1. – Introduction

For decades cosmologists have been wishing to know the average density ϱ of the universe. The reason is that if we set the critical density ϱ_c, which separates those models that recollapse in the future from those that expand forever, the density parameter $\Omega_{\text{total}} \equiv (\varrho/\varrho_c)_{\text{now}}$ determines the ultimate fate of the universe. The quantity $\Omega_{\text{total}} > 1$ if the Universe at some future time collapses into a second singularity, whereas if $\Omega_{\text{total}} < 1$ the universe will expand forever. For the critical density, $\Omega_{\text{total}} = 1$ and it corresponds to the ever-expanding Einstein-de Sitter universe.

Inflation theory favors the $\Omega_{\text{total}} = 1$ universe. Due to an enormous inflationary

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growth of the universe, any spatial curvature in the pre-inflationary universe is flattened close to the Euclidean one. Thus $\Omega_{\text{total}} = 1$ throughout the radiation- and matter-dominated epochs is one of the most robust prediction of inflation models [1]. This result, however, poses a serious problem to the standard big-bang model. This is the Ω-problem. The main dilemma is that most observed data indicate $\Omega_{\text{total}} < 1$. It appears that the directly observable luminous baryonic matter contributes only a tiny fraction of the critical density: $\Omega_{\text{baryonic}} \approx 0.005$. Rotation curves for spiral galaxies suggest that galaxies may contain more (baryonic) matter enough to increase Ω_{baryonic}, but at best, by a factor ten [2]. The problem is clear in that even the second estimate fails to yield the average density close to unity. (Also standard nucleosynthesis models which depend on many of the parameters such as neutron half-life and the number of light-neutrino types, yields an estimate for the present density $0.015 < \Omega_{\text{baryonic}} h^2 < 0.01$ [3]. Here $0 < h^2 < 1$ is the normalized Hubble parameter.) Thus the inflationary claim that $\Omega_{\text{total}} = 1$ does not seem compatible with observations.

Thus far, several prescriptions have been suggested. One can argue that $\Omega_{\text{total}} \neq \Omega_{\text{baryonic}}$. If the universe contains dark matter which so far evaded detection, then $\Omega_{\text{total}} = \Omega_{\text{baryonic}} + \Omega_{\text{dark matter}} = 1$. This explanation has an additional advantage in that if such "unseen" non-baryonic matter exists, it can improve the efficiency of mass clumping. However, a recent APM survey reports that the dark-matter prescription is incomplete. There are cosmic structures greater than ~ 30 Mpc where the even successful cold-dark-matter (CDM) model cannot accommodate [4]. One can also conjecture the possible existence of the cosmological constant, known as the Λ-term in the universe. In this case, $\Omega_{\text{total}} = \Omega_{\text{baryonic}} + \Omega_{\Lambda} = 1$. Unfortunately though, we hardly understand the nature of the cosmological constant. Further, if a non-zero cosmological constant Λ does exist in the universe, we also should be able to answer why the constant takes a value just right enough to explain the puzzle. Considering these difficulties, we reinvestigated whether the $\Omega_{\text{total}} < 1$ universe is indeed incompatible with inflation theory. The purpose of this paper is to show that the KK theory (or perhaps other higher-dimensional gravitation theories) can provide a solution to the Ω-problem.

2. – Kaluza-Klein cosmology

KK cosmology has been investigated by many authors in relation to inflation theory. However, those models consider the case where inflation takes place while the internal space is contracting [5]. We note that this work differs from previous works that are dealing with the inflation of the 4-dimensional space-time which takes place *after* compactification.

It is well known that the Brans-Dicke (BD) theory [6] can be derived from the Kaluza-Klein (KK) theory [7]. Consider a $(4 + D)$-dimensional KK theory. Under dimensional reduction, the $(4 + D)$-dimensional Einstein-Hilbert action takes the following form:

$$\mathscr{L}_{\text{KK}} = -\sqrt{g}\left[\Phi \mathscr{R} + \omega g^{\mu\nu} \frac{\partial_\mu \Phi \partial_\nu \Phi}{\Phi} + (\text{other terms})\right], \tag{1}$$

where $\omega = -(D-1)/D$, g is the determinant of the metric tensor, and $\mathscr{R}$ is the Ricci scalar. The quantity Φ is the KK dilaton. When extra dimensions $(D > 4)$ are compactified into the 4-dimension, the square root of the determinant of the metric in compactified manifold functions as the BD field. Thus the Kaluza-Klein theory does yield the BD Lagrangian, but the BD coupling parameter is negative [8]. In fact, the negative ω is a feature which naturally arises in compactified 4-dimensional gravitation theories such as superstring theory [9], or N-dimensional variants of the BD theory [10]. In this respect we stress that original BD theory in the strict sense does not follow from higher-dimensional gravitational theories. Conventional BD theory is characterized by the positive ω.

Now let us consider a compactified KK universe in which the cosmological constant (or terms proportional to it) is set identically zero. It will be shown that in this (4-dimensional) negative-ω KK universe, there is a polar-type inflation, and the density parameter of the post-inflationary universe becomes less than unity. We find that the deviation from unity critically depends on the number of compactified dimensions D. To show the details, let us write the full, 4-dimensional KK action:

$$\mathscr{A}(g_{\mu\nu}, \Phi) = \int \mathrm{d}^4 x \sqrt{-g}\left\{-\Phi \mathscr{R} - \omega \frac{\partial_\mu \Phi \partial^\mu \Phi}{\Phi} - W(\Phi) - \mathscr{L}_{\mathrm{matter}}(\sigma)\right\}. \tag{2}$$

Here, $g_{\mu\nu}$ is the metric tensor, and the Greek indices μ, ν run from 0 to 3. The quantity g is the determinant of $g_{\mu\nu}$; $M_{\mathrm{P}} = 10^{19}$ GeV is the Planck mass; and Φ is the KK dilaton (or the BD scalar field). The matter field σ does not couple directly to the dilaton field Φ. Instead, the Φ field has a source to the trace part of the energy-momentum tensor $T_{\mu\nu} = T_{\mu\nu}(\sigma)$. This retains the original structure of the Brans-Dicke theory.

When $W(\Phi) = 0$, which is the case for the original BD theory, the action appears to describe the conventional BD cosmology. However, there is a critical difference. The BD parameter is negative: $\omega = -|\omega|$. In the conventional BD Universe, the gravity $G_{\mathrm{N}} = G(\Phi(t))$ varies monotonically and the BD parameter is constrained by solar-system experiments: $\omega \geqslant 500$ [11]. In the KK universe where $\omega = -|\omega|$, there is no experimental constraint established. Thus ω is constrained only by the weak-energy condition: $-1 < \omega \equiv -(D-1)/D \leqslant 0$. In this respect, the inclusion of a non-trivial dilatonic potential $W(\Phi)$ to the action remains optional. In fact, it is stressed that the pure BD theory (where $W(\Phi) = 0$) does not seem natural from the particle-physics point of view. For scale-invariant theories or supersymmetric models, conformal invariance or supersymmetry can be spontaneously broken. In this way, the BD field gets its energy scale at low energies [12].

For completeness, in this paper, we will assume that

$$W(\Phi) \ll \varrho_{\mathrm{F}} \equiv -\langle 0 | \mathscr{L}_{\mathrm{matter}}(\sigma) | 0 \rangle. \tag{3}$$

In this way, the presence of the non-trivial $W(\Phi)$ works to suppress the time-variation of the gravitational "constant" $G(t) = 1/\Phi$, since as Φ reaches its local minima, Φ becomes a constant. When this happens, gravity gets its energy scale $M_{\mathrm{P}} = 10^{19}$ GeV, and the whole theory becomes indistinguishable to the big-bang cosmology in a background of the Einstein gravity.

At first sight, the action of eq. (2) takes the same form as original extended inflation model [13], except 1) the sign of the BD parameter has been changed, $\omega < 0$; and 2) the physical meaning of the BD parameter has also changed. The quantity $\omega = -(D-1)/D$, so that the negative BD parameter is fixed by the number of compactified extra dimensions. In the following, we will show that those seemingly minor changes yield dramatic results. First, as the universe enters the inflationary epoch, there is a polar inflation; and the magnitude of the negative ω, as constrained by the weak-energy condition, fixes the density parameter of the 4-dimensional post-inflationary universe. In this way, we will conclude that the "Ω-problem" could be an indication that we may live in a KK universe which has evolved from 8–9-dimensions.

Now we proceed to the main issue. In the Friedmann-Robertson-Walker (FRW) universe, where the line element is $ds^2 = -(dt)^2 - R(t)^2((dr)^2/(1-kr^2) + r^2 d\Sigma^2)$ and the matter energy-momentum tensor represented in a perfect-fluid form is $T_{\mu\mu} = p_\sigma g_{\mu\nu} + (\varrho_\sigma + p_\sigma) U_\mu U_\nu$, the equations of motion for the cosmic scale factor R and the KK dilaton (or the BD field) Φ are

$$H^2 = \frac{8\pi\varrho_\sigma}{3\Phi} - \frac{k}{R^2} + \frac{\omega}{6}\left(\frac{\dot\Phi}{\Phi}\right)^2 - H\left(\frac{\dot\Phi}{\Phi}\right), \tag{4}$$

$$\dot\varrho_\sigma = -3H(\varrho_\sigma + p_\sigma), \tag{5}$$

$$\ddot\Phi + 3H\dot\Phi = \frac{8\pi[\varrho_\sigma(t) - 3p_\sigma(t)]}{3+2\omega}. \tag{6}$$

Here, the dot represents the derivative with respect to cosmic time; $H = \dot R/R$ is the Hubble parameter, and $k \pm 1, 0$ represents the 3-space curvature $r^2 d\Sigma$. The quantities $\varrho_\sigma(t)$, $p_\sigma(t)$ are the energy density and the pressure of cosmic matter, respectively.

For $\omega > 0$, as the universe supercools in the false phase, the energy density approaches a constant value $\varrho_\sigma \to \varrho_F$. This acts as an effective cosmological constant. Then $-p_\sigma = +\varrho_F$ and for $k = 0$,

$$\Phi = m_P^2\left(1 + \frac{H_B t}{\alpha}\right)^2, \tag{7}$$

$$R(t) = \left(1 + \frac{H_B t}{\alpha}\right)^{\omega + 1/2}. \tag{8}$$

Here, H_B is the Hubble parameter at the beginning of inflation, and

$$\alpha^2 \equiv (3+2\omega)(5+6\omega)/12\,; \qquad \alpha = \pm\sqrt{(3+2\omega)(5+6\omega)/12}. \tag{9}$$

In order that inflation occurs, it is obvious that the quantities $\omega + 1/2 > 1$, and $\alpha > 0$. The quantity m_P is an arbitrary constant corresponding to the effective Planck mass at the beginning of inflation, $t = 0$. The (Planck mass)^2 today is the inverse of the Newtonian gravitational constant ($1/G_N$). We are familiar with this solution [13]. This solution, which is a basis of extended inflation models, describes the inflationary expansion ($\ddot R > 0$) of the universe. The power law expansion, in particular, enables the universe to exit "gracefully" from the false phase via bubble nucleation process.

For the density parameter of the post-inflationary universe [14],

$$\Omega_{total} \equiv \frac{8\pi\varrho_\sigma}{3\Phi H^2} = 1 + \frac{4(4\omega+3)}{3(2\omega+1)^2} = 1 + \text{Order}\left(\frac{1}{\omega}\right). \tag{10}$$

Thus for $\omega > 500$ as constrained by solar-system experiments, or $1.5 < \omega \leqslant 20$ as allowed in the EI model, the quantity Ω_{total} deviates from unity insignificantly. This is a disappointing result as far as the Ω-problem is concerned. Of course, we are well aware why this happens. The main culprit is a rather weak coupling of the BD field (or $\omega \gg 1$) with matter fields in the universe.

3. – Kaluza-Klein polar inflation

Now then let us ask what happens for $\omega = -|\omega|$. Could there be another inflationary solution? If so, then we may open a more direct avenue which links inflationary cosmology to particle physics. Fortunately, there exists a solution. From eqs. (7) and (8): simply take $\omega + 1/2 < 0$, and take the negative α of eq. (9). Then

$$R(t) = \left(1 - \frac{H_B t}{\alpha}\right)^{-|\omega| + 1/2}, \tag{11}$$

$$\Phi(t) = \beta_p^2 \left(1 - \frac{H_B t}{\alpha}\right)^2. \tag{12}$$

Here, $\alpha = +\sqrt{((3 + 2\omega)(5 + 6\omega))/12}$. In this way, the original contracting pair of extended inflation solution has been transformed into the polar inflation solution. Note that the polar inflation solution also has a consistent Einstein limit. For $|\omega| \to \infty$, $(1 - H_B t/\alpha)^{-|\omega| + 1/2} \to e^{+H_B t}$. This limit corresponds to the familiar exponentially expanding inflationary solution characteristic of the Einstein gravity.

In the KK cosmology, we restrict $-1 < \omega \leqslant 0$ due to the weak-energy condition. Due to the condition $\omega + 1/2 < 0$, the negative BD parameter is further constrained as $-1 < \omega < -1/2$. Also the quantity α should remain real: thus, ω is further restricted as $\omega > -5/6$, and $\omega < -3/2$. Therefore, for a successful KK polar inflation of this kind to take place, the BD parameter (or the number of compactified extra dimensions) should satisfy the following non-trivial condition:

$$-\frac{5}{6} < \omega < -\frac{1}{2}. \tag{13}$$

Since $\omega \equiv -(D-1)/D$ this implies that the number of compactified extra dimensions should fall in the range

$$2 < D < 6. \tag{14}$$

It is apparent that during inflation, β_p^2 decreases until $\beta_p^2 \to M_P^2$. Thus $\Phi(0) = \beta_p^2 \gg M_P^2$, and inflation terminates when $\Phi \to M_P^2$. For the KK polar inflation, we are reminded that $\chi t \equiv (H_B/\alpha)\, t = 0$, $\chi t \to 1$ refers to the moment when the Universe enters into and exits from the inflationary epoch, respectively.

The polar inflation is much faster than conventional exponential or power law expansion. One can check this via the Hubble parameter H. During the polar inflation H increases as $\propto (1 - \chi t)^{-|\omega| - 1/2}$, whereas for exponential inflation, H is constant, and it is time-decreasing for the power law extended inflation. The faster expansion implies that the KK polar inflation is not likely to be terminated via bubble nucleation and percolation processes. The bubble nucleation parameter $\varepsilon \propto H^{-4}$ which determines whether the universe can percolate or not, will be time-decreasing, and makes it harder for the universe to percolate [15]. Therefore, at the moment, the KK universe is the more likely to exit from inflationary epoch via a second-order phase transition.

For epochs that follow, the evolution of the universe becomes similar to the conventional big-bang model. As the universe reheats, the Φ-field is: 1) fixed to a constant (*i.e.* to the Newtonian value $\Phi \rightarrow \Phi_{max} = 1/G_N$) as the Φ-field reaches local minima of $W(\Phi)$, or 2) varies monotonically. This occurs when $W(\Phi) = 0$.

For the former case, the Φ-field can further oscillate with respect to its minima which may produce several astrophysically interesting effects [16]. Ultimately, as the Φ-field settles to one of its local minima, $\Phi \rightarrow \Phi_{max} = \text{const}$, and the KK gravity becomes indistinguishable from the Einstein gravity. (This implies that, effectively, $\omega = -|\omega|$ is transformed into $\omega \rightarrow \infty$.)

For the latter case, $W(\Phi) = 0$, the Φ-field will grow like conventional BD gravity, but the rate of time-variation of the gravitational "constant" will be different from that of the BD gravity. (At the moment, an experimentally acceptable range of the negative ω is not known to us.)

4. – The Ω-problem in the KK universe

The most interesting feature which emerges from the KK polar inflation is that the density parameter Ω_{total} of the post-inflationary universe can be much less than unity even if the universe is geometrically flat ($k = 0$). Further, Ω_{total} at the present epoch is quantized. The pair of the polar inflation solution eqs. (11) and (12) is originated from that of original extended inflation solution of eqs. (7) and (8). Thus the density of matter in the 4-dimensional Friedmann universe takes the same form as eq. (10):

$$\Omega_{total} = 1 + \frac{4(4\omega + 3)}{3(2\omega + 1)^2} = 1 - \frac{4}{3}\,\frac{D(D-4)}{(D-2)^2}\,, \tag{15}$$

except that here the quantity ω now depends on the number of compactified dimension D. Therefore, for $-(5/6) < \omega < -(1/2)$, one finds $0 < \Omega_{total} < \infty$. Now it is obvious that the average density of matter in the 4-dimensional Friedmann universe becomes $0 < \Omega_{total} \leqslant 1$ when the number of compactified dimension is constrained as

$$4 \leqslant D < 6\,, \tag{16}$$

or $-(5/6) < \omega \leqslant -3/4$. For $D = 4, 5$, $\Omega_{total} = 1, 0.26$, respectively. Thus within the context of the KK cosmology, the Ω-problem indicates that the number of compactified dimensions D is not greater than six ($D = 6$)-dimension.

It is indeed an interesting structure uncovered that the number of extra dimensions compactified at an earlier epoch is constrained by the astronomical observations of the low-energy universe. Further, the 4-dimensional Ω_{total} appears as a discrete quantity. In fact why this happens is simple to understand. Normally, when a higher-dimensional universe is compactified into that of the 4-dimension, the compactified extra dimensions appear as the diagonal term of 4-dimensional energy-momentum tensor. Therefore, the average density of the 4-dimensional universe, hence Ω_{total}, should be affected accordingly.

Before closing, let us list several comments. First, in the simple KK universe, the polar inflation as well as the $\Omega_{total} < 1$ feature appear to occur only in the rather narrow range of the negative ω. This restriction, however, is not an unnatural feature, since the natural range of the (negative) ω in the KK cosmology is intrinsically narrow. (Otherwise, the theory breaks down due to the presence of tachyons).

Second, when $W(\Phi) \neq 0$, an almost explosive increase of the gravitational coupling $G_N = 1/\Phi$ at the end of the inflationary epoch ($H_B t/\alpha \approx 1$) produces a strong fine-tuning problem. For $W(\Phi) \sim \lambda_\Phi \beta_p^4$, the condition of eq. (3) yields $\lambda_\Phi \sim (\varrho_F/\beta_p^4) \ll 1$. (Note that as the quantity $\Phi = (1 - H_B t/\alpha) \to 0$, $\beta_p^2 \gg M_P^2$. This implies that the Planck mass before inflation should be very large [17].) Of course, this tuning problem remains optional at the moment, since the problem vanishes if $W(\Phi) = 0$. This corresponds to the case when our negative BD parameter model is indeed free from the solar-system limits $\omega \geqslant 500$.

Third, we stress that this model works strictly in the Jordan-Brans-Dicke frame in which the BD coupling is negative, $\omega < 0$. One can perform conformal transformation of action (2) by rescaling the metric $g_{\mu\nu} \to g'_{\mu\nu}$. In this way one can produce a new frame in which the gravitational coupling strength becomes a constant, like the Einstein gravity, but inertia of cosmic matter becomes time-varying [6, 14]. In the latter frame, however, it is easy to show that the polar inflationary feature vanishes for any $-1.5 < \omega \leqslant 0$. This contrasts with original extended-inflation scenario which is based on the solution equations (7) and (8). In this case, the power law inflation occurs both before and after the metric scaling. (This feature is, in fact, a manifestation that the action of eq. (2) is not conformally invariant.)

Finally, we have shown that for $D = 4, 5$, $\Omega_{\text{total}} = 1, 7/27 \approx 0.26$, respectively. It is somewhat disappointing that this model fails to yield satisfactory Ω_{total}'s for $D = 6$ or 7 (10- or 11-dimensions). We note, however, that this result is sensitive to the functional relation between ω and the number of compactified dimension D, $\omega = \omega(D)$. Thus it remains to be seen what other particle-physics models of gravity can yield the more general relation $\omega = \omega(D)$: and hence, the more realistic relation $\Omega_{\text{total}} = \Omega_{\text{total}}(D) \leqslant 1$.

To summarize, we considered a 4-dimensional KK cosmology characterized by negative BD parameter. The main conclusion is that as the universe enters the inflationary epoch, there is a polar-type inflation, and the number of compactified extra dimensions fixes the 4-dimensional density parameter Ω_{total}. Thus Ω_{total} in the post-inflationary universe is quantized. We stress that this will be a generic feature for other polar inflation models based on (compactified) higher-dimensional gravitation theories. In particular, if we live in the 9-dimensional KK universe, $\Omega_{\text{total}} = 0.26$ even if the universe is geometrically flat ($k = 0$).

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