Semiclassical quantization of matter fields in gravity (*)

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Summary. — We study the problems of unitarity and quantum back reaction of matter fields in curved space-times from the point of view of the quantum field theory derived from the Wheeler-DeWitt equation in which both the quantum back reaction of matter fields to geometry and the quantum-gravitational corrections to matter fields are considered. Finally, we discuss the unitarity of 2D CGHS dilaton black holes.

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1. – Introduction

The problems of unitarity and quantum back reaction of quantum field theory in curved space-times have been an issue of extreme importance, long debated but still difficult to solve without any ambiguity. Since the quantum effects of matter fields cannot be safely neglected in the early Universe or black holes, one must select the right quantum field theory of matter fields. In particular, relativists mostly admit that the problems of particle creation, Hawking radiation, and information loss in black holes should be solved in the context of quantum gravity which can be written formally as $\widehat{G}_{\mu\nu} = 8\pi\kappa \widehat{T}_{\mu\nu}$, where κ is the gravitation constant, with both gravity and matter field quantized at the same time, but is beyond the present treatment. But they agree unanimously that at present there is no complete theory of quantum gravity free of all the conceptual and technical problems, so at best one can consider these problems in the context of semiclassical gravity which can also be written formally as $G_{\mu\nu} = 8\pi\kappa \langle \widehat{T}_{\mu\nu} \rangle$, in which one quantizes matter fields but treats the geometry as a fixed

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classical background space-time. The recent CGHS dilaton gravity model of black holes [1], in which one solves in part the classical back reaction problem, arouses the relativists' interest back to the problems of unitarity and information loss, and raises again the question of the final state of black-hole evaporation.

The main aim of this paper is to revisit the problems of unitarity and quantum back reaction of matter fields to geometry in curved space-times. For this purpose we study the quantum field theory derived from the Wheeler-DeWitt equation, which is self-consistent in that we solve not all but some part of the problems of unitarity and quantum back reaction as described above. We study the quantum Friedmann-Robertson-Walker cosmological model minimally coupled to a free massive scalar field to discuss the problem of time and unitarity and the problem of quantum back reaction. And, finally, we discuss the unitarity of quantum field theory in CGHS 2D dilaton gravity and Gowdy T^3 cosmological model.

We adopt the recently introduced semiclassical gravity based on the Wheeler-DeWitt equation [2]. By elaborating further the asymptotic expansion method for the Wheeler-DeWitt equation into semiclassical gravity [3], we have derived the quantum field theory for matter fields. The Wheeler-DeWitt equation separates into the gravitational field equation and the quantum field equation for the matter fields in the form of matrix equation with the introduction of a cosmological time through gravitational action. The gravitational field equation should lead to the Einstein-Hamilton-Jacobi equation equated with the expectation value of the matter Hamiltonian which, in turn, should lead to the classical Einstein equation coupled to matter fields. The full field equation for the matter fields is found to preserve unitarity asymptotically in the limit $\hbar/M_{\mathrm{P}} \to 0$ for an oscillatory gravitational wave function. In the new asymptotic expansion method we find the exact Fock space of matter fields on the basis of the generalized invariant of the matter field Hamiltonian.

En route to the problem of quantum back reaction we consider the quantum Friedmann-Robertson-Walker cosmological model and see how classical space-times can emerge from a limiting procedure from quantum gravity to semiclassical gravity obtained from the new asymptotic expansion of the Wheeler-DeWitt equation and finally down to classical gravity [4]. In the new asymptotic expansion method, the matter field obeys purely a Tomonaga-Schwinger equation equivalent to the time-dependent functional Schrödinger equation with higher-order gravitational quantum corrections, and semiclassical gravity is described by the Einstein-Hamilton-Jacobi equation equated with the quantum back reaction of matter fields [3].

2. – The problem of unitarity

To gain insight into the method that enables one to derive the Tomonaga-Schwinger equation from a relativistic theory, we first consider the well-known one-dimensional Klein-Gordon equation

$$\left[-\frac{\hbar^2}{c^2}\frac{\partial^2}{\partial t^2} + \hbar^2 \frac{\partial^2}{\partial x^2} - m^2 c^2 \right] \Psi(t, x) = 0 . \tag{2.1}$$

There are two asymptotic parameters, the speed of light c and the Planck constant $1/\hbar$. The Hamiltonian $\widehat{H} = \frac{\partial^2}{\partial x^2}$ is independent of the parameter time t, so it is a conserved operator in the Heisenberg picture. The eigenstates of the Hamiltonian are

$$|\Phi_k(\phi)\rangle = \exp[\pm ikx]. \tag{2.2}$$

There is no mixing of eigenstates. The expectation value $\langle\Phi_{k'}|\widehat{H}|\Phi_k\rangle$ gives the value $H_{k'k} = -\hbar^2 k^2 \delta_{k'k}$. By substituting (2.2) into (2.1) we separate the time-dependent equation

$$\left[\frac{\hbar^2}{c^2}\frac{\partial^2}{\partial t^2} + \hbar^2 k^2 + m^2 c^2\right]\psi(t) = 0, \tag{2.3}$$

whose solution is

$$\psi(t) = \exp[\pm i\omega t], \qquad \omega = \sqrt{k^2 c^2 + \frac{m^2 c^4}{\hbar^2}}. \tag{2.4}$$

It is to be noted that the method gives directly the exact result even at the asymptotic limit $c \to \infty$.

The strategem for the derivation of quantum field theory from the Wheeler-DeWitt equation is to separate the gravitational field equation and the Tomonaga-Schwinger equation of the matter fields just in analogy with the Klein-Gordon equation. In the Wheeler-DeWitt equation

$$\left[-\frac{\hbar^2}{2M_\mathrm{P}}\nabla^2 - M_\mathrm{P}U(h_a) + \widehat{H}\left(\frac{i}{\hbar}\frac{\delta}{\delta\phi}, \phi, h_a\right)\right]\Psi(h_a, \phi) = 0, \tag{2.5}$$

for a quantum cosmological model, in which $\nabla^2 = G_{ab}(\delta^2)/(\delta h_a \delta h_b)$ is the Laplace-Beltrami operator on the superspace with the DeWitt metric G_{ab}, $U(h_a)$ denotes the superpotential of three-curvature, and $\widehat{H}$ represents the matter field Hamiltonian, there are again two asymptotic parameters, the Planck mass squared M_P and the Planck constant $1/\hbar$. It is known [5] that there are a spectrum of wave functions of the general form

$$\Psi(h_a, \phi) = \psi(h_a)\,\Phi(\phi, h_a), \tag{2.6}$$

where $\Phi(\phi, h_a)$ is a gravitational field-dependent quantum state of the matter field which is to be determined later on. Assuming that the quantum states belong to a Hilbert space, one can expand the quantum state by some orthonormal basis

$$\Phi(\phi, h_a) = \sum_k c_k(h_a)\,|\Phi_k(\phi, h_a)\rangle, \langle\Phi_k|\Phi_n\rangle = \delta_{kn}. \tag{2.7}$$

Substituting eqs. (2.6) and (2.7) into eq. (2.5) and acting $\langle\Phi_n|$ on both sides, the Wheeler-DeWitt equation equals to the following matrix equation:

$$\begin{aligned} c_n(h_a)\left(-\frac{\hbar^2}{2M_\mathrm{P}}\nabla^2 - M_\mathrm{P}U(h_a) + H_{nn}(h_a)\right)\psi(h_a) - \frac{\hbar^2}{M_\mathrm{P}}\nabla\psi(h_a)\cdot\nabla c_n(h_a) + \\ + i\frac{\hbar^2}{M_\mathrm{P}}\nabla\psi(h_a)\cdot\sum_k \mathbf{A}_{nk}(h_a)\,c_k(h_a) + \psi(h_a)\sum_{k\neq n} H_{nk}(h_a)\,c_k(h_a) - \\ - \frac{\hbar^2}{2M_\mathrm{P}}\psi(h_a)\sum_k \Omega_{nk}(h_a)c_k(h_a) = 0, \end{aligned} \tag{2.8}$$

where

$$(2.9)\qquad \begin{cases} H_{nk}(h_a) = \langle \Phi_n(h_a) \,|\, \widehat{H} \,|\, \Phi_k(h_a) \rangle, \\ \mathbf{A}_{nk}(h_a) = i \langle \Phi_n(h_a) \,|\, \nabla \,|\, \Phi_k(h_a) \rangle, \\ \Omega_{nk}(h_a) = \nabla^2 \delta_{nk} - 2i \mathbf{A}_{nk} \cdot \nabla + \Omega^{(2)}_{nk}, \end{cases}$$

where

$$(2.10)\qquad \Omega^{(2)}_{nk}(h_a) = \langle \Phi_n(h_a) \,|\, \nabla^2 \,|\, \Phi_k(h_a) \rangle .$$

$\mathbf{A}$ is physically interpreted as the induced gauge potential in analogy with the molecular system. $i\nabla$ acts on $c_k(h_a)$ as a Hermitian operator, so do H, $\mathbf{A}$, $\Omega^{(2)}$, and thereby Ω.

The key point of the method is the observation that the Wheeler-DeWitt equation separates into the gravitational field equation

$$(2.11)\qquad \left(-\frac{\hbar^2}{2M_{\mathrm{P}}} \nabla^2 - M_{\mathrm{P}} U(h_a) + H_{nn}(h_a) \right) \psi(h_a) = 0 ,$$

and into the matter field equation

$$(2.12)\qquad -\frac{\hbar^2}{M_{\mathrm{P}}} \nabla\psi(h_a) \cdot \nabla c_n(h_a) + i \frac{\hbar^2}{M_{\mathrm{P}}} \nabla\psi(h_a) \cdot \sum_k \mathbf{A}_{nk}(h_a)\, c_k(h_a) + \\ + \psi(h_a) \sum_{k \neq n} H_{nk}(h_a) c_k(h_a) - \frac{\hbar^2}{2M_{\mathrm{P}}} \psi(h_a) \sum_k \Omega_{nk}(h_a) c_k(h_a) = 0 .$$

Now, the effective potential of the gravitational wave function is $M_{\mathrm{P}} U - H_{nn}$. We shall confine the gravitational wave functions to a region of the Lorentzian universe, in which they oscillate and are peaked around the classical trajectories. In this region the gravitational wave function has an oscillatory form,

$$(2.13)\qquad \psi(h_a) = f(h_a) \exp\left[\frac{i}{\hbar} S_n(h_a) \right].$$

In the asymptotic (semiclassical) limit $1/\hbar \to \infty$ the gravitational field equation (2.11) leads to the Einstein-Hamilton-Jacobi equation for the gravitational action with the quantum back reaction of matter fields

$$(2.14)\qquad \frac{1}{2M_{\mathrm{P}}} (\nabla S_n(h_a))^2 - M_{\mathrm{P}} U(h_a) + H_{nn}(h_a) = 0 .$$

It is known that the Einstein-Hamilton-Jacobi equation equated with an energy-momentum tensor is equivalent to the classical Einstein equation [6]. An infinite number of wave functions for the Wheeler-DeWitt equation are known to exist, so we find an infinite number of gravitational wave functions which depend on the quantum states of matter fields. Each quantum state of matter fields gives rise to a classical background geometry

$$(2.15)\qquad G_{\mu\nu} = 8\pi\kappa \langle \Phi_n \,|\, \widehat{H} \,|\, \Phi_n \rangle .$$

The peak of each gravitational wave function prescribes a classical background geometry with the quantum back reaction of matter fields and describes a history of evolution of the universe. Along each gravitational wave function one can introduce the cosmological time

$$\frac{\partial}{\partial \tau^{(n)}} := \frac{1}{M_{\mathrm{P}}} \nabla S_n(h_a) \cdot \nabla . \tag{2.16}$$

Then the matter field (2.12) becomes an evolution equation,

$$i\hbar \frac{\partial}{\partial \tau^{(n)}} c_n + \Omega_{nn}^{(1)} c_n + \sum_{k \neq n} (\Omega_{nk}^{(1)} - H_{nk}) c_k + \frac{\hbar^2}{2 M_{\mathrm{P}}} \sum_k \Omega_{nk}^{(3)} c_k = 0 , \tag{2.17}$$

where

$$\Omega_{nk}^{(1)} = i\hbar \langle \Phi_n | \frac{\partial}{\partial \tau^{(n)}} | \Phi_k \rangle = \frac{\hbar}{M_{\mathrm{P}}} \nabla S_n \cdot \mathbf{A}_{nk}, \tag{2.18}$$

and

$$\Omega_{nk}^{(3)} = \Omega_{nk} + 2 \frac{1}{f} \nabla f \cdot \nabla \delta_{nk} - 2i \frac{1}{f} \nabla f \cdot \mathbf{A}_{nk}. \tag{2.19}$$

The full field equation (2.17) for the matter field is not unitary due to the terms $2(1/f)\nabla f \cdot \nabla \delta_{nk} - 2i(1/f)\nabla f \cdot \mathbf{A}_{nk}$, which do not obviously act as unitary operators.

The second step of the strategem is to use the generalized invariant

$$\begin{cases} \dfrac{\partial}{\partial \tau^{(n)}} \hat{I} - \dfrac{i}{\hbar} [\hat{I}, \widehat{H}] = 0 , \\ \hat{I} | \Phi_k \rangle = \lambda_k | \Phi_k \rangle, \end{cases} \tag{2.20}$$

whose eigenstates decouple the field equation in the asymptotic limit $\hbar/M_{\mathrm{P}} \to \infty$ canceling $H_{nk} = \Omega_{nk}^{(1)}$, $n \neq k$, the off-diagonal gauge potential and the quantum back reaction [7]. On the basis of the generalized invariant the full matter field equation takes the simpler form

$$i\hbar \frac{\partial}{\partial \tau^{(n)}} c_n + \Omega_{nn}^{(1)} c_n + \frac{\hbar^2}{2 M_{\mathrm{P}}} \sum_k \Omega_{nk}^{(3)} c_k = 0 . \tag{2.21}$$

The asymptotic ($\hbar/M_{\mathrm{P}} \to \infty$) quantum state of matter field can be easily integrated out:

$$\Phi(\phi, h_a) = c_n(\tau_0^{(n)}) \exp \left[\frac{i}{\hbar} \int \Omega_{nn}^{(1)}(h_a)\, \mathrm{d}\tau^{(n)} \right] | \Phi_n(\phi, h_a) \rangle . \tag{2.22}$$

The time-dependent Tomonaga-Schwinger equation

$$i\hbar \frac{\partial}{\partial \tau^{(n)}} \Phi(\phi, h_a) = \widehat{H} \left(\frac{i}{\hbar} \frac{\delta}{\delta \phi}, \phi, h_a \right) \Phi(\phi, h_a), \tag{2.23}$$

has the solution (2.22) up to an additional phase factor $\exp[-(i/\hbar) \int H_{nn} \,\mathrm{d}\tau^{(n)}]$. The exact quantum state (2.7) can be obtained by solving perturbatively (2.21), which is a linear superposition of eigenstates of the generalized invariant with gravitational field-dependent coefficient functions. The norm of the quantum states need not be preserved due to unitarity-violating terms. But in the asymptotic limit $\hbar/M_{\rm P} \to 0$ these unitarity-violating terms are suppressed and the norm of vector, $|\mathbf{c}|$, is preserved. This implies a unitary operator

$$\mathbf{c}(\tau^{(n)}) = \mathbf{U}_c(\tau^{(n)}, \tau_0^{(n)})\, \mathbf{c}(\tau_0^{(n)})\,. \tag{2.24}$$

Recollecting that the basis of the generalized invariant has been chosen as an orthonormal basis, one also finds a unitary operator

$$|\Phi(\tau^{(n)})\rangle = \mathbf{U}_\Phi(\tau^{(n)}, \tau_0^{(n)})\, |\Phi(\tau_0^{(n)})\rangle \tag{2.25}$$

for the evolution of the column vector of $|\Phi_k\rangle$. The physical implication is that the quantum field theory of the matter field is asymptotically unitary in this sense.

Each quantum state of matter fields gives rise to a classical background geometry determined by eq. (2.15). The perturbative solution to (2.21)

$$|\Phi^{(1)}\rangle = |\Phi^{(0)}\rangle + |\delta\Phi^{(0)}\rangle, \tag{2.26}$$

where $|\delta\Phi^{(0)}\rangle$ is of the order of $\hbar/M_{\rm P}$, yields a classical background geometry $g_{\mu\nu}^{(1)}$ shifted from $g_{\mu\nu}^{(0)}$ owing to the transition among eigenstates.

3. – Problem of back reaction

In order to see how the Einstein-Hamilton-Jacobi equation (2.14) equated with the quantum back reaction is related with the classical Einstein equation and includes the quantum effect of matter field, we shall consider here the quantum Friedmann-Robertson-Walker cosmological model minimally coupled to a free massive scalar field. The spatially flat Friedmann-Robertson-Walker space-time manifold has the topology $R \times R^3$ with a homogeneous and isotropic metric

$$ds^2 = -N^2(t)\, dt^2 + R^2(t)\, d\Omega_3^2, \tag{3.1}$$

where N is the lapse function and $R(t)$ is the scale factor depending only on t. The time will be scaled in units of $c = 1$ and the Planck mass squared will thus be equal to $M_{\rm P} = 1/8\pi\kappa$. The action is

$$S = \int dt \left[-M_{\rm P} R^3 \left(\frac{1}{2N} \left(\frac{\dot R}{R} \right)^2 + N \left(-\frac{k}{2R^2} + \frac{\Lambda}{6} \right) \right) + R^3 \left(\frac{1}{2N} \dot\phi^2 - \frac{Nm^2}{2} \phi^2 \right) \right]. \tag{3.2}$$

The classical equations of motion are obtained by varying δN, δR, and $\delta\phi$, respectively:

$$-M_{\rm P} \left(\frac{R\dot R^2}{2} + \frac{kR}{2} - \frac{\Lambda R^3}{6} \right) + R^3 \left(\frac{\dot\phi^2}{2} + \frac{m^2}{2} \phi^2 \right) = 0, \tag{3.3}$$

$$M_{\rm P} \left(\frac{d}{dt} (R\dot R) - \frac{\dot R^2}{2} + \frac{k}{2} - \frac{\Lambda R^2}{2} R^2 \right) + 3R^2 \left(\frac{\dot\phi^2}{2} - \frac{m^2}{2} \phi^2 \right) = 0\,, \tag{3.4}$$

$$\frac{d}{dt} \left(R^3 \dot\phi \right) + m^2 R^3 \phi = 0\,. \tag{3.5}$$

The classical equation (3.3) can be rewritten in the form

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} - \frac{\Lambda}{3} = 16\pi\kappa \frac{1}{R^3} T_{00}, \tag{3.6}$$

where T_{00} is the Hamiltonian, that is the time-time component of the energy-momentum stress tensor

$$T_{00} = \frac{1}{2} R^3 \dot{\phi}^2 + \frac{1}{2} m^2 R^3 \phi^2. \tag{3.7}$$

Neglecting some part of the operator ordering ambiguity, we quantize the Hamiltonian *à la* Dirac quantization to obtain the Wheeler-DeWitt equation

$$\left[\frac{\hbar^2}{2M_P R}\frac{\partial^2}{\partial R^2} + M_P^2\left(-\frac{kR}{2} + \frac{\Lambda R^3}{6}\right) - \frac{\hbar^2}{2R^3}\frac{\partial^2}{\partial \phi^2} + \frac{m^2 R^3}{2}\phi^2\right]\Psi(R, \phi) = 0 . \tag{3.8}$$

Following sect. **2**, we set the wave function of the form

$$\Psi(R, \phi) = \psi(R)\, \Phi(\phi, R). \tag{3.9}$$

Here ψ and Φ are still unknown quantum states of gravity and the scalar field, respectively. The classical meaning of space-time will follow later after prescribing the interpretation of the wave function for the gravity equated with the quantum back reaction of scalar field. Any quantum state of the scalar field can be expanded by the basis of some Hermitian operator relevant to the matter field Hamiltonian:

$$\Phi(\phi, R) = \sum_k c_k(R)\,|\Phi_k(\phi, R)\rangle. \tag{3.10}$$

The gravitational field equation is given by

$$\left[\frac{\hbar^2}{2M_P R}\frac{\partial^2}{\partial R^2} + M_P\left(-\frac{kR}{2} + \frac{\Lambda R^3}{6}\right) + H_{nn}(R)\right]\psi(R) = 0 , \tag{3.11}$$

and in the asymptotic limit $\hbar/M_P \to 0$ the matter field equation is

$$i\hbar\frac{\partial}{\partial t} c_n + \Omega_{nn}^{(1)} c_n + \sum_{k \neq n} (\Omega_{nk}^{(1)} - H_{nk})\, c_k = 0 . \tag{3.12}$$

When the gravitational wave function has the WKB form

$$\psi(R) = f(R) \exp\left[i\frac{M_P}{\hbar} S(R)\right], \tag{3.13}$$

the cosmological time becomes

$$\frac{\partial}{\partial t} = \frac{1}{R}\frac{\partial S(R)}{\partial R}\frac{\partial}{\partial R} . \tag{3.14}$$

and the gauge potential

$$\begin{cases} \Omega_{nk}^{(1)} = \dfrac{i\hbar}{R}\dfrac{\partial S}{\partial R}\langle \Phi_n(\phi, R) | \dfrac{\partial}{\partial R} | \Phi_k(\phi, R)\rangle, \\ H_{nk} = \langle \Phi_n(\phi, R) | \widehat{H} | \Phi_k(\phi, R)\rangle. \end{cases} \tag{3.15}$$

From the gravitational wave equation we obtain the Einstein-Hamilton-Jacobi equation

$$\frac{1}{2R}\left(\frac{\partial S}{\partial R}\right)^2 + \frac{kR}{2} - \frac{\Lambda R^3}{6} = 8\pi\kappa H_{nn}(R), \tag{3.16}$$

which can be rewritten as

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} - \frac{\Lambda}{3} = 8\pi\kappa\frac{1}{R^3}H_{nn}(R). \tag{3.17}$$

The matter field Hamiltonian is a time-dependent harmonic oscillator of the form

$$H = T_{00} = \frac{1}{2R^3}\pi_\phi^2 + \frac{1}{2}m^2R^3\phi^2. \tag{3.18}$$

Then the asymptotic matter field equation becomes a diagonal equation whose solution is given by

$$c_n(t) = c_n(t_0)\exp\left[\frac{i}{\hbar}\int \Omega_{nn}^{(1)}\,\mathrm{d}t\right]. \tag{3.19}$$

First we find the particular second-order generalized invariant of the form [4]

$$\widehat{I}(t) = \frac{1}{2}(\widehat{I}_+(t)\,\widehat{I}_-(t) + \widehat{I}_-(t)\widehat{I}_+(t)), \tag{3.20}$$

where

$$\begin{cases} \widehat{I}_+(t) = \phi^*(t)\,\widehat{\pi}_\phi - R^3(t)\,\dot{\phi}^*(t)\,\widehat{\phi}, \\ \widehat{I}_-(t) = \phi(t)\,\widehat{\pi}_\phi - R^3(t)\,\dot{\phi}(t)\,\widehat{\phi}, \end{cases} \tag{3.21}$$

in terms of one of the classical solutions of eq. (3.5) such that

$$\begin{cases} R^3(t)(\phi^*(t)\,\dot{\phi}(t) - \phi(t)\,\dot{\phi}^*(t)) = i, \\ \mathrm{Im}\left(\dfrac{\dot{\phi}(t)}{\phi(t)}\right) < 0. \end{cases} \tag{3.22}$$

Then $\widehat{I}_+(t)$ acts as the creation operator $\widehat{A}^\dagger(t)$ and $\widehat{I}_-(t)$ as the annihilation operator

$\widehat{A}(t)$. The ground-state quantum state is given by

$$\langle \phi | \Phi_0(\phi, R) \rangle = \frac{1}{(2\pi\hbar |\phi(t)|^2)^{1/4}} \exp\left[i \frac{R^3 \dot{\phi}(t)}{2\hbar\phi(t)} \phi^2 \right], \tag{3.23}$$

and the n-th quantum state by

$$\langle \phi | \Phi_n(\phi, R) \rangle = \tag{3.24}$$

$$= \frac{1}{(2\pi\hbar |\phi(t)|^2)^{1/4}} \frac{1}{\sqrt{2^n n!}} \left(i \frac{\phi^*(t)}{|\phi(t)|} \right)^n H_n\left(\frac{\phi}{\sqrt{2\hbar} |\phi(t)|} \right) \exp\left[i \frac{R^3 \dot{\phi}(t)}{2\hbar\phi(t)} \phi^2 \right],$$

where H_n is the n-th Hermite polynomial. From the quantum back reaction of the scalar field

$$H_{nn}(t) = R^3(t)(\dot{\phi}(t)\, \dot{\phi}^*(t) + m^2 \phi(t)\, \phi^*(t))\hbar \left(n + \frac{1}{2} \right) \tag{3.25}$$

we obtain the Einstein-Hamilton-Jacobi equation with the quantum back reaction in the nonadiabatic basis

$$\left(\frac{\dot{R}}{R} \right)^2 + \frac{k}{R^2} - \frac{\Lambda}{3} = 8\pi\kappa(\dot{\phi}(t)\, \dot{\phi}^*(t) + m^2 \phi(t)\, \phi^*(t))\hbar \left(n + \frac{1}{2} \right). \tag{3.26}$$

Even the ground state ($n = 0$) of the scalar field does not lead to the Einstein vacuum equation $G_{\mu\nu} = 0$. Quantum vacuum energy is left over from the quantum fluctuation and uncertainty of the field in its ground state. Furthermore, one can show that the semiclassical gravity reduces to the classical gravity by identifying the amplitude of classical field $\phi_c = \phi_0 \phi_q$, where $\phi_0 = \sqrt{\hbar(n + 1/2)}$ and ϕ_q satisfies the condition (3.22). In particular, the classical field energy is proportional to the field squared and the quantum energy to $n\hbar$, and therefore for a large quantum number n one may expect the correspondence $\phi_0 = \sqrt{n\hbar}$.

As a partial solution to the quantum back reaction problem in curved space-times we showed how classical space-times obeying the Einstein-Hamilton-Jacobi equation or, equivalently, the classical Einstein equation with the quantum back reaction of matter field can emerge through the investigation of the quantum Friedmann-Robertson-Walker cosmological model minimally coupled to a free massive scalar field. Compared with the conventional approach in which one just quantizes the matter fields and keeps the background space-time fixed, the quantum field theory using the new asymptotic expansion of the Wheeler-DeWitt equation takes into account the quantum gravitational corrections and the quantum back reaction of matter fields to the space-time geometry.

4. – Quantum 2D black holes

The one-loop effective action for the CGHS dilaton gravity has the form [8]

$$S = \frac{1}{\pi} \int d^2 x \left[-\frac{1}{\kappa} \partial_+ \chi \partial_- \chi + \frac{1}{\kappa} \partial_+ \Omega \partial_- \varphi + \lambda^2 e^{2(\chi - \Omega)/\kappa} + \frac{1}{2} \sum_{j=1}^{N} \partial_+ f_j \partial_- f_j \right], \tag{4.1}$$

where

$$\chi = \kappa \varrho - \frac{\kappa}{2} \phi + e^{-2\phi} \tag{4.2}$$

represents a Liouville-type field,

$$\Omega = \frac{\kappa}{2} \phi + e^{-2\phi} \tag{4.3}$$

is a rescaled version of the dilaton field ϕ, f_j are conformal scalar fields, and $1/\kappa = 12/(N-24)$ plays the role of an asymptotic parameter like the Planck mass squared in quantum cosmological models. The corresponding Wheeler-DeWitt equation takes the form

$$\left[\frac{\hbar^2 \kappa}{4} \frac{\partial^2}{\partial \chi_0^2} - \frac{\hbar^2 \kappa}{4} \frac{\partial^2}{\partial \Omega_0^2} - \sum_{j=1}^{N} \hbar^2 \frac{\partial^2}{\partial f_{j0}^2} - 4\lambda^2 e^{2(\chi - \Omega)/\kappa} - \kappa - 2 \right] \Psi = 0 , \tag{4.4}$$

where the subscript zero represents the zero mode of field expansion. Use the light-cone coordinates $\sigma_0 = \chi_0 + \Omega_0$ and $\tau_0 = \chi_0 - \Omega_0$, then the Wheeler-DeWitt equation becomes

$$\left[\hbar^2 \kappa \frac{\partial^2}{\partial \sigma_0 \partial \tau_0} - \sum_{j=1}^{N} \hbar^2 \frac{\partial^2}{\partial f_{j0}^2} - 4\lambda^2 e^{2\tau_0/\kappa} - \kappa - 2 \right] \Psi(\tau_0, \sigma_0, f_{j0}) = 0 . \tag{4.5}$$

There is a similarity between the Wheeler-DeWitt equations for the CGHS dilaton gravity and for the Gowdy T^3 inhomogeneous cosmological model [9]. In both Wheeler-DeWitt equations there is a unique selection of the cosmological time. The gravitational action S in the previous sections depends only on σ_0, and so the gravitational wave function here takes a trivial form:

$$\psi(\sigma_0) = f(p_{\sigma_0}) \exp\left(\frac{i}{\hbar} S(\sigma_0) \right). \tag{4.6}$$

The cosmological time is

$$\frac{\partial}{\partial \tau} = \kappa \frac{\partial S}{\partial \sigma_0} \frac{\partial}{\partial \tau_0} . \tag{4.7}$$

Finally, one obtains the Tomonaga-Schwinger equation

$$i\hbar \frac{\partial}{\partial \tau} \Phi(\tau, f_{j0}) = \left[\sum_{j=1}^{N} \hbar^2 \frac{\partial^2}{\partial f_{j0}^2} + 4\lambda^2 e^{2\tau/\kappa} + \kappa + 2 \right] \Phi(\tau, f_{j0}) = 0 . \tag{4.8}$$

It should be noted that the Tomonaga-Schwinger equation for the conformal fields in the CGHS dilaton gravity and for the inhomogeneous modes in the Gowdy T^3

cosmological model is completely unitary and there are no unitarity-violating terms as in generic quantum-cosmological models. The unitarity of field equation is a feature peculiar to the CGHS dilaton gravity and Gowdy T^3 cosmology. The unitarity of field equation may be relevant to the information loss problem of black holes if the CGHS dilaton gravity is to provide a realistic model for quantum black holes.

Note added in proofs

We come to notice that the ambiguity of separation of the Wheeler-De Witt equation into a gravitational field equation and a Tomonaga-Swinger equation for matter fields allows also a unitary semiclassical gravity in the oscillating regime [10].

* * *

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