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Primordial nucleosynthesis with varying gravitational constant and neutrino degeneracy (*)

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Summary. — We discuss the effects of the possible variation of G from the present value G_0 and nonzero neutrino degeneracies on the primordial nucleosynthesis. We obtain the permitted ranges of parameters which are consistent with the inferred primordial abundances of light elements. It is found that a wider range of η_{10} is possible than in the standard BBN. We also calculate the primordial abundances of ^{9}Be and ^{11}B within the permitted ranges. While the abundance of ^{9}Be is found to be indistinguishable from that of the standard BBN, the primordial ^{11}B abundance is found to be sensitive enough to constrain the variation in gravitational constant and the neutrino degeneracy.

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The standard big-bang nucleosynthesis (BBN) [1-3] provides valuable limits on the number of cosmological and particle physics parameters through a comparison between the predicted and the inferred primordial abundances of the light nuclei up to ^{7}Li. However, given the uncertainties of the inferred abundances, the possibility of variations from the standard BBN has been studied in various contexts [4]. The evolution of the Universe is described by the expansion rate H,

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G\varrho_{\rm rad}, \tag{1}$$

where R is the scale factor and G is the gravitational constant. $\varrho_{\rm rad}$ is the energy

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density of the light particles which can be written in terms of energy density of photons,

$$\varrho_{\rm rad} = \frac{g_{\rm eff}}{2}\varrho_\gamma, \tag{2}$$

where the energy density of photons is given by

$$\varrho_\gamma = \frac{\pi^2}{15}T^4. \tag{3}$$

In the standard BBN, photons, electrons and light neutrinos are considered ([1]). For $T \sim 1$ MeV,

$$g_{\rm eff}^{\rm standard} = \frac{43}{4} + \frac{7}{4}(N_\nu - 3), \tag{4}$$

where N_ν is the number of light-neutrino species. The inclusion of new light particles and nonzero neutrino degeneracies [5] can be summarized in $g_{\rm eff}$ as

$$g_{\rm eff} = \frac{43}{4} + \frac{7}{4}(N_\nu^{\rm eff} - 3). \tag{5}$$

From eq. (1), one can see that the gravitational constant G and $N_\nu^{\rm eff}$ determine the expansion rate which controls the neutron-to-proton ratio at the beginning of nucleosynthesis and therefore modify the results of standard BBN [6]. The determination of G from BBN may provide the possible range of the parameters for the models with varying gravitational constant, for example Jordan-Brans-Dicke theory [7]. There have been analyses [8, 9] essentially using G and $N_\nu^{\rm eff}$ as free parameters. The contribution of neutrino degeneracy to $N_\nu^{\rm eff}$ can be written as

$$\begin{cases} N_\nu^{\rm eff} = N_\nu + \Delta N_\nu, \\ \Delta N_\nu = \dfrac{15}{7}\left(\dfrac{\xi}{\pi}\right)^4 + \dfrac{30}{7}\left(\dfrac{\xi}{\pi}\right)^2, \end{cases} \tag{6}$$

where $\xi = \mu_\nu / T$ for the degenarate neutrino with chemical potential μ_ν. As the magnitude of neutrino degeneracy (equivalently $N_\nu^{\rm eff}$) is increasing, it speeds up the expansion of the Universe [5]. As far as the expansion rate is concerned, the effects of nonvanishing neutrino degeneracy are equivalent to the change of gravitational constant. However, it has additional effects. The weak-interaction rates are also affected due to the nonzero degeneracy of electron neutrino, ξ_e. Since the weak-interaction freeze-out temperature T is determined not only by the expansion rate but also by the weak-interaction rate, the role of degenerate electron neutrinos cannot be simulated by considering only its effects on $N_\nu^{\rm eff}$.

The abundances of heavier elements like ^{9}Be and ^{11}B have also been of interest for the inhomogeneous BBN model [10]. It has been found that the abundances of ^{7}Li

([1]) Weakly coupled particles are decoupled earlier and they can be ignored as far as their temperatures are much less than T.

strongly constrain the inhomogeneous BBN such that the abundances of ^{9}Be and ^{11}B can hardly be distinguished from the standard BBN predictions [11]. However, with the possibility of nonzero neutrino degeneracy together with a variation in the gravitational constant for the abundances of light elements, it is an interesting question to ask what are the effects of those variations on heavier elements like ^{9}Be and ^{11}B. It is particularly more interesting because of the fact that it has become recently possible to observe ^{9}Be and ^{11}B abundances in extreme Pop. II stars with low Z [12, 13]. Although these can be understood as being due to the cosmic-ray spallation [14], the possibility for the new developments of observation of heavier-element abundances is expected.

In this paper, we will discuss in detail the role of varying G and nonzero neutrino degeneracies on the primordial abundances of light elements including ^{9}Be and ^{11}B and the possible range of η, using the extended reaction network [15]. The reaction network has been extended to include the relevant reactions for ^{9}Be and ^{11}B, for example, ^{7}Li$(t, n)^{9}$Be [16], ^{8}Li$(d, n)^{9}$Be [10], ^{9}Li$(p, n)^{9}$Be [11] and ^{9}Be$(t, n)^{11}$B [17]. The new reaction network with 28 nuclei is composed of 103 reactions. We assume that the neutrinos are effectively massless for $T \sim 1$ MeV. In this work, we fix the number of neutrino species to be three [3, 18]. The neutron mean lifetime which determines weak-interaction rate is taken to be 889.1 ± 2.1 s [19]. Since the product $G(t)\varrho_{\rm rad}$ enters into the expression for the expansion rate, one might think that the effect of changing $\varrho_{\rm rad}$ due to the degeneracies of muon- and tau-neutrino is equivalent to redefining G. However, the degeneracy of electron neutrino not only affects the energy density but also affects strongly the neutron-to-proton density ratio [20]. Therefore, we have essentially two independent parameters: the gravitational constant G, which might be different [7] from the present value G_0 and possibly accommodates the effects on the energy density due to the muon- and tau-neutrino degeneracy, and the electron neutrino degeneracy $\xi_{\rm e} = \mu_{\nu_{\rm e}}/T$. One additional parameter which is crucial in determining the nuclear reaction rate in nucleosynthesis is the baryon-to-photon ratio η. In this work, we have three parameters η, G, and $\xi_{\rm e}$, whereas only one parameter η is used in the standard BBN. Using the extended nuclear reaction network [21], we calculate the possible ranges of the three parameters G, $\xi_{\rm e}$, and η which are consistent with the inferred primordial abundances of light elements up to ^{7}Li in ref. [22]:

$$(7) \qquad \begin{cases} 0.21 \leqslant Y_{\rm p} \leqslant 0.24\,, \\ 1.8 \cdot 10^{-5} \leqslant ({\rm D/H})_{\rm p}\,, \\ [({\rm D} + {}^{3}{\rm He})/{\rm H}]_{\rm p} \leqslant 9.0 \cdot 10^{-5}\,, \\ 1.1 \cdot 10^{-10} \leqslant ({}^{7}{\rm Li/H})_{\rm p} \leqslant 2.3 \cdot 10^{-10}\,, \end{cases}$$

where $Y_{\rm p}$ is the mass fraction of ^{4}He. The possible ranges for the parameters which are consistent with the abundances of light elements up to ^{7}Li are calculated for various values of η_{10} $(\eta_{10} \equiv \eta \times 10^{10}) = 2.8$, 3.5, 5.0, 7.0, and 10.0. The results are shown in table I.

It is found that for higher baryon number density (η_{10} greater than ~ 4) a larger gravitational constant than the presently observed value is required. For larger η the nuclear reactions become faster. Then, for a given freeze-out temperature of nuclear reactions, the abundances become smaller than the inferred values. Therefore, a faster expansion rate, equivalently the larger gravitational constant, eq. (1), is required to increase the freeze-out temperature such that the final abundances are kept within the observed ranges. We can also see that a smaller gravitational constant is required for the relatively low baryon number density. From table I, one can see that wide ranges of parameters are possible if the variations are allowed. Particularly the possiblity of higher baryon number density (η_{10} up to 10) compared to the standard BBN should be noted.

Within these five sets of possible ranges of parameters, the maximum and minimum abundances of ^{9}Be and ^{11}B were calculated and are presented in tables II and III, respectively. In the case of the standard BBN with $\xi_e = 0.0$ and $G = G_0$, the allowed ranges of η_{10} are found to be $2.4 \leqslant \eta_{10} \leqslant 3.3$. Within this permitted range of η_{10}, the abundance of ^{9}Be has a maximum yield of 2.7×10^{-18}, and that of ^{11}B a maximum of 2.2×10^{-17}.

From these tables one can see that for η_{10} less than ~ 4 the abundances for ^{9}Be and ^{11}B are not distinguishable from the standard BBN. The abundances both of ^{9}Be and ^{11}B are observed to increase as η increases. For ^{9}Be, the rate of increase is very small, which means that abundance of ^{9}Be is not so sensitive to the parameters ξ_e, G, or η within the ranges which are consistent with the abundances of light elements. On the other hand, ^{11}B abundances are rapidly increasing as η increases. This difference in

TABLE I. – *Allowed ranges of G/G_0 and ξ_e.*

η_{10}	G/G_0(min)	G/G_0(max)	ξ_e(min)	ξ_e(max)
2.8	0.311	1.34	− 0.482	0.119
3.5	0.51	2.13	− 0.2358	0.285
5.0	1.10	4.49	0.086	0.509
7.0	2.25	9.02	0.336	0.684
10.0	4.82	19.0	0.536	0.846

TABLE II. – *Maximum and minimum abundances of* ^{9}Be.

η_{10}	G/G_0	ξ_e	^{9}Be/H(min)	G/G_0	ξ_e	^{9}Be/H(max)
2.8	0.311	− 0.393	$2.081 \cdot 10^{-19}$	1.32	0.092	$2.801 \cdot 10^{-18}$
3.5	0.51	− 0.145	$2.225 \cdot 10^{-19}$	2.09	0.25	$2.882 \cdot 10^{-18}$
5.0	1.1	0.167	$2.429 \cdot 10^{-19}$	4.385	0.471	$2.994 \cdot 10^{-18}$
7.0	2.245	0.42	$2.475 \cdot 10^{-19}$	8.85	0.646	$3.095 \cdot 10^{-18}$
10.0	4.82	0.639	$2.585 \cdot 10^{-19}$	18.6	0.805	$3.158 \cdot 10^{-18}$

TABLE III. – *Maximum and minimum abundances of* ^{11}B.

η_{10}	G/G_0	ξ_e	^{11}B/H(min)	G/G_0	ξ_e	^{11}B/H(max)
2.8	1.34	0.119	$4.920 \cdot 10^{-18}$	0.4	− 0.407	$1.981 \cdot 10^{-17}$
3.5	2.13	0.285	$7.419 \cdot 10^{-17}$	0.56	− 0.2358	$4.614 \cdot 10^{-17}$
5.0	4.485	0.509	$1.165 \cdot 10^{-17}$	1.2	0.086	$1.391 \cdot 10^{-16}$
7.0	9.02	0.683	$1.699 \cdot 10^{-17}$	2.45	0.336	$3.003 \cdot 10^{-16}$
10.0	19.0	0.846	$2.320 \cdot 10^{-17}$	5.35	0.535	$5.503 \cdot 10^{-16}$

increasing rates of abundances has also been observed even in the standard BBN [21]. However for larger η, one can notice that it becomes larger by an order of magnitude than in standard BBN as shown in table III.

We have calculated the primordial abundances of light elements including ^{9}Be and ^{11}B to investigate the effects of the gravitational constant and a nonzero neutrino degeneracy. Both of them essentially control the expansion rate of the Universe which competes with the weak- and the nuclear-reaction rates. The effects of possible muon- and tau-neutrino degeneracies are considered to be equivalent to redefining the gravitational constant, which is assumed to vary with the evolution of the Universe [7]. However, the degeneracy of electron neutrino affects also the weak-interaction rate which determines the neutron-to-proton ratio. Hence, it cannot be simply absorbed into G, and we consider it as an independent parameter ξ_e. In this work, therefore, we have effectively three independent parameters η_{10}, G/G_0, and ξ_e to investigate the possibility of nonzero neutrino degeneracy [5] together with the variation of the gravitational constant [9]. It is demonstrated that rather wide ranges of η_{10} and G/G_0 for the abundances of light elements, D, ^{3}He, ^{4}He and ^{7}Li are possible [20]. It is observed that $G/G_0 \leqslant 1$ is possible only for η_{10} less than ~ 4. Within the permitted range of parameters, primordial abundances of ^{9}Be and ^{11}B are calculated. It is found that the abundance of ^{9}Be is almost the same order of magnitude as the value in the standard BBN. However, the ^{11}B abundances can be considerably enhanced [21] as η_{10}, G/G_0, and ξ_e increased, compared to that of the standard BBN. Hence, these results imply that the observation of the primordial abundances of ^{9}Be and ^{11}B are very important since ^{11}B abundances are sensitive enough to constrain both the possible variations of the gravitational constant and the neutrino degeneracies.

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