Singularities induced by sine-Gordon solitons coupled with dilaton gravity (*)

H.-M. JOHNG (**), H.-S. SHIN (***) and K.-S. SOH ($*_*$*)
Department of Physics Education, Seoul National University - Seoul 151-742, Korea

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Summary. — Black-hole geometry induced by sine-Gordon solitons coupled with two-dimensional dilaton gravity are studied in the context of the CGHS model. We reviewed our previous work which only deals with solitons of one-kink and two-kink type. We extend the work to include general solitons of any number of kink and antikink, and a breather-type soliton.

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1. – Introduction

Many discussions on some problems of quantum gravity have been going on since Hawking's discovery of black-hole evaporation [1], in particular on the possibility of information loss in quantum gravity [2, 3]. A two-dimensional model of dilaton gravity was proposed by Witten and others [4] as a useful device to study the formation and evaporation of black holes and naked singularities without mathematical complexity of four-dimensional theories. Callan, Giddings, Harvey and Strominger (CGHS) [5] attempted to understand the dynamical formation and evaporation of black holes in two-dimensional space-time by coupling scalar matter fields to a dilaton field. It is exactly solvable classically, and it was expected that it could be successfully treated semiclassically including back reaction of Hawking radiation. Various attempts and improvements [6] have since been made without decisive results.

Integrable models of nonlinear partial differential equations, and especially their

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(**) E-mail: johng@power1.snu.ac.kr
(***) E-mail: hsshin@power1.snu.ac.kr
($*_*$*) E-mail: kssoh@phyb.snu.ac.kr

soliton solutions [7] have been objects of active research as field theories in two-dimensional space-time. The sine-Gordon theory of the interacting scalar fields has attracted much attention as a good example of solitons and their scatterings [8].

Recently, two-dimensional gravity coupled with sine-Gordon solitons [9] was introduced as a model of black-hole formation by solitons, and similar works [10, 11] followed. Vaz and Witten [12] studied in detail the nature of singularities produced by a kink-type soliton of sine-Gordon theory and examined their evaporation in the one-loop approximation.

In this work we extend our previous work [9] to include general soliton solutions of sine-Gordon nonlinear fields. In the previous work we studied the geometry only produced by one-kink– and two-kink–type solitons. Here we take N-soliton/anti-soliton solutions and obtain the corresponding metric fields as a general formula, and derived the previous results as special cases. In sect. **2** the action and field equations of the dilaton gravity model coupled with a sine-Gordon matter field are given, and in sect. **3** the geometry produced by a kink-type soliton is studied. We fix the constants of integration so that in the ultrarelativistic limit the shock wave solution of the CGHS model [5] is reproduced. We compare this particular choice of constants with that of Vaz and Witten [12], and notice that the detail geometry and nature of singularity depend on the integration constants. In sect. **4** we present a general solution of field equations in connection with N-soliton/antisoliton solutions, and in sect. **5** we take a particularly interesting case of a breather soliton (soliton-antisoliton bound state), and analyze the singularity structure of its space- time. In the last section we discuss briefly on the quantum treatment of the problem.

2. – Dilaton gravity coupled to general sine-Gordon solitons

The model we investigate is the dilaton gravity coupled to a scalar matter field of sine-Gordon type whose action in two space-time dimensions is

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}[R+4(\nabla\phi)^2+4\lambda^2]-\frac{1}{2}(\nabla f)^2+4\mu^2(\cos f-1)\,e^{-2\phi}\right], \tag{2.1}$$

where g, ϕ and f are the metric, dilaton, and matter fields, respectively, and λ^2 is a cosmological constant. The last sine-Gordon term is added to the CGHS action [5] in order to study formation of black holes by solitons.

We choose the conformal gauge such that the metric is simply

$$ds^2 = -e^{2\varrho}\,dx^+\,dx^- , \tag{2.2}$$

where $x^\pm = t \pm x$, and the action becomes

$$S = \frac{1}{\pi}\int d^2x\left[e^{-2\phi}(2\partial_+\partial_-\varrho - 4\partial_+\phi\partial_-\phi + \lambda^2 e^{2\varrho}) + \right.$$
$$\left. + \frac{1}{2}\partial_+ f\partial_- f + \mu^2(\cos f - 1)\,e^{2\varrho-2\phi}\right]. \tag{2.3}$$

The field equations are

$$T_{\pm\pm} = e^{-2\phi}(4\partial_\pm \varrho \partial_\pm \phi - 2\partial_\pm^2 \phi) + \frac{1}{2}(\partial_\pm f)^2 = 0\,, \tag{2.4}$$

$$T_{+-} = e^{-2\phi}(2\partial_+ \partial_- \phi - 4\partial_+ \phi \partial_- \phi) - (\lambda^2 + \mu^2(\cos f - 1))\, e^{2(\varrho - \phi)} = 0\,, \tag{2.5}$$

$$e^{-2\phi}(-4\partial_+ \partial_- \phi + 4\partial_+ \phi \partial_- \phi + 2\partial_+ \partial_- \varrho) + (\lambda^2 + \mu^2(\cos f - 1))\, e^{2(\varrho - \phi)} = 0\,, \tag{2.6}$$

$$\partial_+ \partial_- f + \mu^2 \sin f\, e^{2(\varrho - \phi)} = 0\,. \tag{2.7}$$

From eqs. (2.5) and (2.6) we have $\partial_+ \partial_- (\varrho - \phi) = 0$, and by fixing the subconformal gauge freedom we can let

$$\varrho - \phi = 0\,. \tag{2.8}$$

With the subconformal gauge-fixing we get the following simplified equations:

$$\partial_\pm^2 (e^{-2\phi}) + \frac{1}{2}(\partial_\pm f)^2 = 0\,, \tag{2.9}$$

$$\partial_+ \partial_- (e^{-2\phi}) + \lambda^2 + \mu^2(\cos f - 1) = 0\,, \tag{2.10}$$

$$\partial_+ \partial_- f + \mu^2 \sin f = 0\,, \tag{2.11}$$

where the last equation is the well-known sine-Gordon equation.

In the rest of this paper we will take known soliton solutions of the sine-Gordon equation, insert them in the rest of the field equations, and obtain ϕ and ϱ, which determines the geometry of space-time. We are particularly interested in black-hole formation by solitons. Black-hole formation in dilaton gravity is usually considered with matter fields without self-interactions [3]. Our work extends these studies to the case of nonlinearly interacting waves.

3. – Black-hole formation by a kink-type soliton

General N-soliton solutions of sine-Gordon equations are obtained through Backlünd transformations [13]. Before considering these general solutions we take the simplest illustrative case, namely, a soliton of kink type in order to investigate black-hole formation by solitons. The kink solution [14] is given by the following traveling wave:

$$f(x, t) = 4 \arctan \exp [2\mu\gamma((x - x_0) + v(t - t_0))]\,, \tag{3.1}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2}}\,. \tag{3.2}$$

Here $-v$ is the velocity of the soliton wave, and the center of the soliton moves along the line $x - x_0 = -v(t - t_0)$. It is somewhat more convenient to rewrite (3.1) in terms of

$x^{\pm} = t \pm x$ as

$$f(x^+, x^-) = 4 \arctan \exp [\mu_+ (x^+ - x_0^+) - \mu_- (x^- - x_0^-)] , \tag{3.3}$$

where

$$\mu_\pm = \mu \sqrt{\frac{1 \pm v}{1 \mp v}} . \tag{3.4}$$

After some algebraic manipulations we obtain

$$\cos f = 1 - \frac{2}{\cosh^2(\Delta - \Delta_0)} , \tag{3.5}$$

where

$$\Delta = \mu_+ x^+ - \mu_- x^- = 2\mu\gamma(x + vt) , \qquad \Delta_0 \equiv \mu_+ x_0^+ - \mu_- x_0^- . \tag{3.6}$$

The classical energy momentum tensor is concentrated around the center of the soliton. For instance, the T^f_{+-} is given by

$$T^f_{+-} = -\mu^2(\cos f - 1) = \frac{2\mu^2}{\cosh^2(\Delta - \Delta_0)} . \tag{3.7}$$

Given the soliton solution, it is straightforward to solve the field equations (2.9), (2.10) as

$$e^{-2\varrho} = e^{-2\phi} = C + ax^+ + bx^- - \lambda^2 x^+ x^- - 2 \ln [1 + \exp [2\Delta - 2\Delta_0]] , \tag{3.8}$$

where a, b, C are constants. To fix these constants we need to impose some reasonable physical conditions. For this purpose we notice that in the ultrarelativistic limit ($v \to 1$) the incoming solition resembles an f-shock wave traveling in the x^--direction with the magnitude A. To be more precise, in the limit

$$v \to 1 , \qquad \mu \to 0 , \qquad \mu_+ = \text{finite} , \tag{3.9}$$

the energy momentum tensor becomes

$$\begin{cases} T^f_{+-} \to 0 , \qquad T^f_{--} \to 0 , \\ T^f_{++} = \dfrac{1}{2} (\partial_+ f)^2 = \dfrac{2\mu_+^2}{\cosh^2[\mu_+ x^+ - \mu_- x^- - \Delta_0]} \to 4\mu_+ \delta(x^+ - x_0^+) . \end{cases} \tag{3.10}$$

For a shock wave of magnitude A traveling in the x^--direction its only nonvanishing stress tensor component is given by

$$T^f_{++} = A\delta(x^+ - x_0^+) , \tag{3.11}$$

and the CGHS black hole formed by the shock wave is described by

$$e^{-2\varrho} = -A(x^+ - x_0^+) \, \Theta(x^+ - x_0^+) - \lambda^2 x^+ x^- . \tag{3.12}$$

We find that the choice of integration constants of eq. (3.8)

$$C = a = b = 0 \tag{3.13}$$

gives, in the ultrarelativistic limit, the CGHS metric as

$$e^{-2\varrho} \to \begin{cases} -\lambda^2 x^+ x^-, & x^+ - x_0^+ < 0, \\ -\lambda^2 x^+ x^- - 4\mu_+ (x^+ - x_0^+), & x^+ - x_0^+ > 0. \end{cases} \tag{3.14}$$

Our choice of the constants ($C = a = b = 0$) reproduces the CGHS geometry of a shock wave with magnitude $A = 4\mu_+$. So our solution is simply

$$e^{-2\varrho} = -\lambda^2 x^+ x^- - 2\ \ln[1 + \exp[2\Delta - 2\Delta_0]]. \tag{3.15}$$

The geometry of the space-time with a kink-type soliton is qualitatively analyzed by examining the asymptotic regions: $\Delta - \Delta_0 \ll -1$, and $\Delta - \Delta_0 \gg 1$. In both of the regions the stress tensor vanishes exponentially as we can see:

$$T^f_{++} \simeq 8\mu_+^2 e^{-2|\Delta - \Delta_0|}, \qquad |\Delta - \Delta_0| \gg 1. \tag{3.16}$$

Since the soliton is moving in the x^--direction, we expect that the left asymptotic region ($\Delta - \Delta_0 \ll -1$) approaches the linear dilaton vacuum while the right one ($\Delta - \Delta_0 \gg +1$) approaches the geometry of a black hole. The solution (3.15) indeed

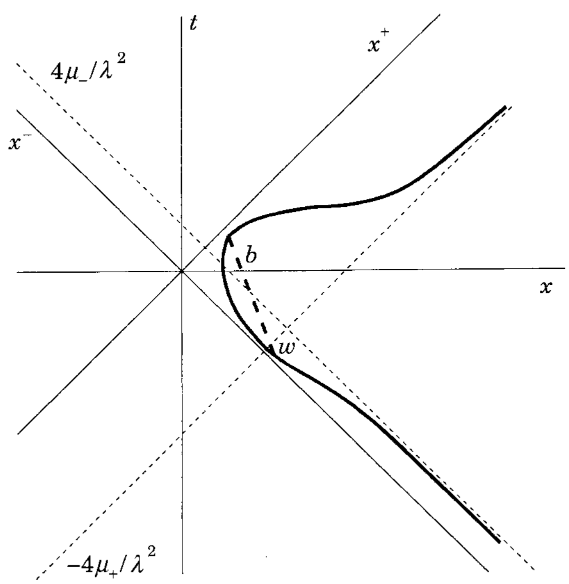

Fig. 1. – The space-time induced by a kink-type soliton with parameters $D = \Delta_0^2 - (8\mu^2/\lambda^2)\ln 2 > 0$. The soliton is produced at the point w by a white hole and absorbed at the point b by a black hole. A white hole, timelike singularity, and a black hole are joined smoothly across the soliton center. The thick broken line, the thick curve, and the dotted lines represent the soliton center, the singularity, and the event horizons, respectively.

bears out this expectation as can be seen by the asymptotic expression

$$e^{-2\varrho} \simeq \begin{cases} -\lambda^2 x^+ x^- , & \Delta - \Delta_0 \ll -1 , \\ 4\left(\Delta_0 - \dfrac{4\mu^2}{\lambda^2}\right) - \lambda^2 \left(x^+ - \dfrac{4\mu_-}{\lambda^2}\right)\left(x^- + \dfrac{4\mu_+}{\lambda^2}\right), & \Delta - \Delta_0 \gg +1 , \end{cases} \tag{3.17}$$

where the second line represents the geometry of a black hole of mass $4\lambda(\Delta_0 - 4\mu^2/\lambda^2)$ (we take $\Delta_0 > 4\mu^2/\lambda^2$) after shifting x^+ by $4\mu_-/\lambda^2$, and x^- by $-4\mu_+/\lambda^2$.

The curvature has a singularity at $e^{-2\varrho} = 0$. The space-time displayed in fig. 1 shows the singularity along with the trajectory of the soliton center. The spacelike singularity of a white hole approaches the asymptote $x^+ = 4\mu_-/\lambda^2$, and creates the soliton at the point $w(x^+ = (\Delta_0 - \sqrt{D})/2\mu_+ , x^- = (-\Delta_0 - \sqrt{D})/2\mu_-)$, where

$$D \equiv \Delta_0^2 - \frac{4\mu^2}{\lambda^2}(2 \ln 2) > 0 . \tag{3.18}$$

The spacelike singularity turns smoothly to a timelike singularity proceeding to the point $b\,(x^+ = (\Delta_0 + \sqrt{D})/2\mu_+ , x^- = (-\Delta_0 + \sqrt{D})/2\mu_-)$ where the soliton is absorbed by the black hole whose asymptote is the line $x^- = -4\mu_+/\lambda^2$. Here we assumed the constant D is positive, but if we choose the parameters λ, μ, and Δ_0 such that D is negative, the trajectory of the soliton will be entirely to the left of the singularity curve. The space-time of our case is essentially the same as that of Vaz and Witten [12] with a positive cosmological constant.

4. – N-soliton/antisoliton solutions

Perring and Skyrme [15] found a two-kink solution which shows that kinks emerge unscathed from collision, suffering only a phase shift. The solution is

$$f_{\mathrm{KK}}(x, t) = 4 \arctan\left[\frac{v \sinh(2\mu\gamma x)}{\cosh(2\mu\gamma v t)}\right], \tag{4.1}$$

where $\gamma = 1/\sqrt{1 - v^2}$. The geometry induced by these colliding kinks was obtained as [9]

$$e^{-2\varrho} = e^{-2\phi} = C - \lambda^2 x^+ x^- - 2 \ln[\cosh^2(2\mu\gamma v(t - t_0)) + v^2 \sinh^2(2\mu\gamma(x - x_0))], \tag{4.2}$$

where C is a constant to be determiend by boundary conditions.

It is even possible to compute exact multi-soliton solutions by the methods of the inverse spectral transformation or the Backlünd transformation. The N-soliton/anti-soliton solution is given by the following simple expression [13]:

$$\cos f_N = 1 + \frac{2}{\mu^2} \partial_+ \partial_- \ln |\det M| , \tag{4.3}$$

where the matrix M is given by

$$M_{ij} = \frac{2}{a_i + a_j} \cosh\left[\frac{\theta_i + \theta_j}{2}\right], \tag{4.4}$$

$$a_i^2 = \frac{1-v_i}{1+v_i} \;, \qquad |v_i| < 1 \;, \qquad v_i \neq v_j \;, \tag{4.5}$$

$$\theta_i = \pm 2\mu\gamma_i (x - v_i t - \xi_i) = \pm(\Delta_i - \Delta_{0i}) \;, \tag{4.6}$$

where the positive (negative) sign in the last line corresponds to a kink (antikink) soliton. Here v_i is the speed parameter of the i-th soliton, ξ_i is the constant defining the trajectory of the i-th soliton center.

One can easily check that in the single-soliton case ($N = 1$)

$$(\cos f)_{1\text{-soliton}} = 1 - \frac{2}{\cosh^2(\Delta - \Delta_0)} \;, \tag{4.7}$$

which agrees with the previous solution (3.5). The two-kink solution of Perring and Skyrme is also derived from the general one by taking

$$\theta_1 = 2\mu\gamma(x + vt) \;, \qquad \theta_2 = -2\mu\gamma(x - vt) \;, \tag{4.8}$$

and

$$a_1 = \sqrt{\frac{1+v}{1-v}} \;, \qquad a_2 = -\sqrt{\frac{1-v}{1+v}} \;. \tag{4.9}$$

In this case the matrix M is given by

$$M = \begin{bmatrix} \dfrac{1}{a_1}\cosh\theta_1 & \dfrac{2}{a_1+a_2}\cosh\left[\dfrac{\theta_1+\theta_2}{2}\right] \\ \dfrac{2}{a_1+a_2}\cosh\left[\dfrac{\theta_1+\theta_2}{2}\right] & \dfrac{1}{a_2}\cosh\theta_2 \end{bmatrix} , \tag{4.10}$$

and its determinant is

$$\det M = \frac{-1}{v^2}\left[\cosh^2(2\mu\gamma vt) + v^2 \sinh^2(2\mu\gamma x)\right] , \tag{4.11}$$

from which we get

$$(\cos f)_{2\text{-soliton}} = 1 - \frac{8v^2\cosh^2(2\mu\gamma vt)\,\sinh^2(2\mu\gamma x)}{[\cosh^2(2\mu\gamma vt) + v^2\sinh^2(2\mu\gamma x)]^2} \;. \tag{4.12}$$

One can check that the last equation is in agreement with the Perring and Skyrme solution (4.1).

Turning back to the general situation we insert the general soliton solution (4.3) into the dilaton field equations (2.9), (2.10), and we get the solution

$$e^{-2\varrho} = e^{-2\phi} = C + ax^+ + bx^- - \lambda^2 x^+ x^- - 2\,\ln|\det M| \;, \tag{4.13}$$

where the constants C, a, and b are to be determined by suitable boundary conditions. Intuitively the region of space-time which is not affected by the stress tensor of solitons must be a linear dilaton vacuum, which could be used to fix the constants. A detailed analysis of the geometry and curvature singularity is quite complicated but physically not so illuminating. We refer to our previous work [9] for the two-kink case, and turn to the less complicated yet physically interesting case of bound solitons.

5. – Breather solitons

Perring and Skyrme [15] found two other interesting solutions using analytic-continuation methods. One is the so-called kink-antikink scattering solution described by the function

$$f_{K\bar{K}}(x, t) = 4 \arctan\left[\frac{1}{v} \frac{\sinh(2\mu\gamma vt)}{\cosh(2\mu\gamma x)}\right]. \tag{5.1}$$

Performing the analytic continuation $v \to iv$ (5.1) can be transformed into another solution:

$$f_B(x, t) = 4 \arctan\left[\frac{1}{v} \frac{\sin(2\mu\Gamma vt)}{\cosh(2\mu\Gamma x)}\right] \tag{5.2}$$

with

$$\Gamma = \frac{1}{\sqrt{1+v^2}} . \tag{5.3}$$

This localized solution is known as a breather [13], or bion (a bound state of a soliton and an anti-soliton). Since v is no longer a velocity in (5.2), it is rather more natural to introduce new variables:

$$\Omega = 2\mu\Gamma v , \qquad \eta = \frac{1}{v} = \frac{\sqrt{(2\mu)^2 - \Omega^2}}{\Omega} . \tag{5.4}$$

In terms of the new variables the breather takes the form

$$f_B(x, t) = 4 \arctan\left[\eta \frac{\sin(\Omega t)}{\cosh(\eta\Omega x)}\right], \tag{5.5}$$

where Ω is a breathing frequency. The energy-momentum tensor of this breather is given by

$$T_{\pm\pm} = 2\eta^2\Omega^2 \frac{[\cos\Omega t \cosh\eta\Omega x \mp \eta \sin\Omega t \sinh\eta\Omega x]^2}{[\cosh^2\eta\Omega x + \eta^2 \sin^2\Omega t]^2} , \tag{5.6}$$

$$T_{+-} = 2\eta^2\Omega^2(1+\eta^2) \frac{\sin^2\Omega t \cosh^2\eta\Omega x}{[\cosh^2\eta\Omega x + \eta^2\sin^2\Omega t]^2} , \tag{5.7}$$

which shows that the energy-momentum density is oscillating with a frequency Ω, and is concentrated in the region $|x| < (\eta\Omega)^{-1}$.

A moving breather with velocity $-u$ can simply be obtained by the Lorentz boost of (5.5)

$$f_B(x, t) = 4 \arctan\left[\eta \frac{\sin\Omega\gamma(t+ux)}{\cosh\eta\Omega\gamma(x+ut)}\right], \qquad \gamma = \frac{1}{\sqrt{1-u^2}} . \tag{5.8}$$

Given a breather solution we can evaluate the dilaton field and the metric in a similar way as in the previous section. After some calculations we get

$$\begin{aligned} e^{-2\varrho} = {} & C - \lambda^2 x^+ x^- - \\ & -2 \ln[\cosh^2\eta\Omega\gamma[(x+ut) - (x_0+ut_0)] + \eta^2\sin^2\Omega\gamma[(t+ux) - (t_0+ux_0)]] , \end{aligned} \tag{5.9}$$

where the origin of the coordinate is chosen such that linear terms in x^+ or x^- are zero.

We only consider a simple illustrative case with $u=0$ to study the geometry induced by the breather soliton. We determine the integration constant C by requiring that the space-time be that of a linear dilaton vacuum in the vanishing $T_{\mu\nu}$ limit $\mu \to 0$ ($\eta \to 0$, $\Omega \to 0$). Then the metric becomes

$$e^{-2\varrho} = \left(4\eta\Omega x_0 + \frac{4\eta^2\Omega^2}{\lambda^2} - 4\ln 2\right) - \lambda^2 x^+ x^- - \\ - 2\ln\left[\cosh^2 \eta\Omega(x-x_0) + \eta^2 \sin^2 \Omega(t-t_0)\right], \tag{5.10}$$

whose asymptotic expressions are

$$e^{-2\varrho} \simeq \begin{cases} -\lambda^2\left(x^+ + \dfrac{2\eta\Omega}{\lambda^2}\right)\left(x^- - \dfrac{2\eta\Omega}{\lambda^2}\right), & \eta\Omega(x-x_0) \ll -1, \\ 8\eta\Omega x_0 - \lambda^2\left(x^+ - \dfrac{2\eta\Omega}{\lambda^2}\right)\left(x^- + \dfrac{2\eta\Omega}{\lambda^2}\right), & \eta\Omega(x-x_0) \gg +1, \end{cases} \tag{5.11}$$

where the last line represents a combination of a white hole and black hole with event horizons at $x^- = -2\eta\Omega/\lambda^2$, and $x^+ = 2\eta\Omega/\lambda^2$, respectively, and its ADM (Arnowitt-Deser-Misner) mass is $8\lambda\eta\Omega x_0$. The singularity of the curvature scalar is given by the equation $e^{-2\varrho} = 0$. One difference from the previous kink-soliton case is that it may have an oscillation of the singularity near the breather center due to the oscillation of the breather, but it is not clear whether it gives any observable effect to the observer at I_R^+ because the singularity is behind the event horizon.

6. – Discussion

We have studied the geometry of two-dimensional dilaton gravity coupled with incoming solitons of one kink, two scattering kinks, a breather, and arbitrary numbers of kink/antikinks. The singularities of a white hole and black hole are joined smoothly with a linear dilaton vacuum across a soliton. In a relativistic limit the geometry of the soliton approaches that of a shock wave described in the CGHS model [5].

We confined our studies to classical treatments, and the Hawking radiation and semiclassical nature of the problem are left for future work. However, as Vaz and Witten [12] had already pointed out, the classical stress-energy tensor of the sine-Gordon field is exponentially vanishing at infinity and the Hawking radiation is dominant there. Following Vaz and Witten we may impose reasonable boundary conditions on the energy-momentum tensor in order to examine the Hawking evaporation of the singularities, and the essential character of the radiation would be similar to that produced by an incoming shock wave in the CGHS model [16]. In spite of this similarity we expect that there are qualitative differences in the quantum treatment of the soliton-induced geometry. First of all, we need to quantize not only the radiation but also the solitons, which will require distribution of soliton's trajectories, which, in turn, introduce probabilistic distribution of singularities. Another point is that there may exist bound states of soliton and radiation which have not yet been analyzed in connection with the Hawking radiation.

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REFERENCES

[1] HAWKING S. W., *Commun. Math. Phys.*, **43** (1975) 199.
[2] BARDEEN J., *Phys. Rev. Lett.*, **46** (1981) 392; PAGE D. N., *Phys. Rev. D*, **25** (1982) 1499; HOWARD K. W. and CANDELAS P., *Phys. Rev. Lett.*, **53** (1984) 403; VAZ C., *Phys. Rev. D*, **39** (1988) 1776; BARRIOS F. A. and VAZ C., *Phys. Rev. D*, **40** (1989) 1340.
[3] HAWKING S. W., *Phys. Rev. D*, **14** (1976) 2460; PAGE D. N., *Phys. Rev. D*, **9** (1974) 3292; *Black hole information*, hep-th 9305040, 1993; BANKS T., *Lectures on black holes and information loss*, hep-th 9412131, 1994; THORLACIUS L., *Nucl. Phys. Proc. Suppl.*, **41** (1995) 245; GIDDINGS S. B., *Quantum mechanics of black holes*, hep-th 9412138, 1994.
[4] WITTEN E., *Phys. Rev. D*, **44** (1991) 314; MANDAL G, SENGUPTA M. M. and WADIA S. R., *Mod. Phys. Lett. A*, **6** (1991) 1685.
[5] CALLAN C. G., GIDDINGS S. B., HARVEY J. A. and STROMINGER A., *Phys. Rev. D*, **45** (1992) R1005.
[6] HAWKING S. W., *Phys. Rev. Lett.*, **69** (1992) 406; BIRNIR B., GIDDINGS S. B., HARVEY J. A. and STROMINGER A., *Phys. Rev. D*, **46** (1992) 638; BANKS T., DABHOLKAR A., DOUGLAS M. R. and O'LOUGHLIN M., *Phys. Rev. D*, **45** (1992) 3607: MIKOVIC A., *Phys. Lett. B*, **291** (1992) 19; RUSSO J. G., SUSSKIND L. and THORLACIUS L., *Phys. Lett. B*, **292** (1992) 13; *Phys. Rev. D*, **46** (1992) 3444; **47** (1993) 533; LOWE D. A., *Phys. Rev. D*, **47** (1993) 2446; PIRAN T. and STROMINGER A., *Phys. Rev. D*, **48** (1993) 4729; NAVARRO-SALAS J., NAVARRO M. and ALDAYA V., *Nucl. Phys. B*, **403** (1993) 291.
[7] JACKIW R., *Rev. Mod. Phys.*, **49** (1977) 681; RAJARMAN R., *Solitons and Instantons* (North-Holland, Amsterdam) 1982; ABDALLA E., ABDALLA M. C. and ROTHER K., *Nonperturbative Methods in Two-Dimensional Quantum Field Theory* (World Scientific, Singapore) 1991.
[8] ABLOWITZ M. J., KAUP D. J., NEWELL A. C. and SEGUR H., *Phys. Rev. Lett.*, **30** (1973) 1262; DODD R., EILBECK J., GIBBON J. and MORRIS H., *Solitons and Nonlinear Wave Equations* (Academic Press, New York, N.Y.) 1982.
[9] SHIN H. S. and SOH K. S., *Phys. Rev. D*, **52** (1995) 981.
[10] STOETZEL B., *Phys. Rev. D*, **52** (1995) 2192.
[11] JOHNG H. M., SHIN H. S. and SOH K. S., *Int. J. Mod. Phys. A*, **10** (1995) 4681.
[12] VAZ C. and WITTEN L., *Class. Quantum Grav.*, **12** (1995) 2607.
[13] FELSAGER B., *Geometry, Particles and Fields* (Odense University Press, Odense) 1983.
[14] TABOR M., *Chaos and Integrability in Nonlinear Dynamics* (John Wiley & Sons, New York, N.Y.) 1989.
[15] PERRING K. K. and SKYRME T. H. R., *Nucl. Phys.*, **31** (1962) 550.
[16] KIM S. W. and KIM W. T., *Phys. Lett. B*, **361** (1995) 38.