Duality symmetries in string theory (*)

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Summary. — We review the status of duality symmetries in superstring theories. These discrete symmetries mark the striking differences between theories of pointlike objects and theories of extended objects. They prove to be very helpful in understanding non-perturbative effects in string theories. We will also briefly discuss the strange role played by open strings and their solitons in the emerging scenario.

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1. – Introduction

In the mounting wave of interest in string theory a key role is played by discrete symmetries (dualities) [1]. This paper is intended to be an introduction for non-experts to these issues. In the discussion we will try to show how these dualities may prove useful in the study of non-perturbative effects in string theory. We will also emphasize the dramatic consequences implied by the presence of duality symmetries in the interpretation of low-energy effective field theories which are stringy generalizations of General Relativity.

Classical General Relativity is a very successful theory for the description of gravity at large distances. However, it is well known that at very short distances, *i.e.* at energy comparable with the Planck mass $M_{\rm P} = (hc/G_{\rm N})^{1/2}$, with $G_{\rm N}$ the Newton constant, the theory is inconsistent with quantum mechanics. Technically speaking, a quantum theory of gravity based on the Einstein-Hilbert action

$$S = \frac{1}{16\pi G_{\rm N}} \int R(g)\sqrt{g}\, {\rm d}^4x \tag{1.1}$$

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is not renormalizable since the coupling constant of the theory, G_N, has dimension of an inverse mass squared. In natural units indeed $G_N = M_P^{-2}$, and though there are no on-shell divergences in pure gravity at one loop [2], at two loops an on-shell counterterm is generated which signals the quantum inconsistency of the theory [3]. In almost any kind of coupling of gravity to matter fields things worsen (the counterterm which can be generated at one loop does not vanish on-shell any more), except when the coupling is supersymmetric. The "miraculous" cancellations between boson and fermion loops, which make (rigid) supersymmetry the only viable candidate for the resolution of the hierarchy problem, persist at low orders in locally supersymmetric theories, known as (extended) supergravity [4]. The resulting theories are still non-renormalizable and one expects divergencies to be generated at three loops or more [5].

In order to describe gravity at short distances one needs a more radical change of perspective. Thanks to its very soft ultraviolet behaviour ("Regge behaviour") string theory is the only known candidate to the unification of gravity with the other interactions in a consistent quantum theory (for a review see [6]). There are two classes of string theories: those with only closed oriented strings and those with open and closed unoriented strings. The string spectrum is encoded in the Regge trajectories which relate the masses M to the spins J of the states. For the closed-string states

$$\alpha' M_J^2 = J - 2 \tag{1.2}$$

which implies the existence of a massless tensor field ("graviton"). For the open-string states the relation becomes

$$\alpha' M_J^2 = J - 1 \tag{1.3}$$

and one may notice the presence of a massless vector boson ("photon"). In the early days of the hadronic string, in order to match the string states with the known resonances, the string tension $T = 1/2\pi\alpha'$ was supposed to be of the order of the proton mass $T = M_p^2$. In the unified string picture the string tension is taken to be comparable with the Planck mass squared $T = M_P^2$ and the tower of massive states decouples from the massless states at low energies, *i.e.* in the limit $\alpha' \to 0$. The infinite tower of massive modes, which at low energies can be neglected or rather "integrated out", may play a significant role in the early stages of evolution of the universe. The exponential growth of the degeneracy of string states with energy signals a phase transition at the Hagedorn temperature ($T_H \approx 1/\sqrt{\alpha'}$).

In order to quantize string theory and to display its symmetry principles, it is very convenient to proceed in analogy with the Feynman path-integral quantization of a point-particle. Instead of summing over "world-lines", Polyakov proposed to sum over "world-sheets", *i.e.* surfaces spanned by the string in its time evolution. The proper weight is the action of Brink, Di Vecchia and Howe (BDVH),

$$S[X, \gamma] = \frac{1}{4\pi\alpha'} \int_\Sigma G_{\mu\nu}(X)\, \partial_\alpha X^\mu \partial_\beta X^\nu \sqrt{\gamma}\, \gamma^{\alpha\beta}\, d^2\sigma\,, \tag{1.4}$$

where the string coordinates X^μ are maps from the world-sheet Σ to a "target space" $\mathcal{M}$ with metric $G_{\mu\nu}$. In order to compute amplitudes and get meaningful results one has to divide out the volume of the huge symmetry group of the classical action. The BDVH action is invariant under two-dimensional diffeomorphisms as well as under Weyl rescalings of the world-sheet metric $\gamma_{\alpha\beta}$. Using the freedom of world-sheet coordinate changes one can fix $\gamma_{\alpha\beta}$ to be conformally flat, $\gamma_{\alpha\beta} = e^{2\sigma}\delta_{\alpha\beta}$. At the classical level, the conformal factor σ decouples. At the quantum level, conformal anomalies are generated upon removing an explicit regularization scale. Following the standard Faddeev-Popov (FP) procedure, one finds that the conformal anomaly of the string coordinates X^μ is cancelled by the contribution of the FP ghosts when $D = 26$, which is to be considered as the critical dimension for the bosonic string. An analogous procedure shows that the critical dimension for the superstring, which requires the introduction of world-sheet fermionic partners Ψ^μ of the bosonic coordinates X^μ as well as the superconformal FP ghosts, is $D = 10$. Using conformal invariance the Polyakov integral may be reduced to a perturbative series in the topology of Riemann surfaces. Once the functional integration over the fields X^μ and Ψ^μ has been performed, one is left with a finite-dimensional integral over the Teichmüller parameters which describe the "shape" of a Riemann surface. The measure of integration descends to the moduli space of Riemann surfaces if invariance under "large" (disconnected) diffeomorphisms, generating the mapping class group (often called the modular group), is preserved at the quantum level. To summarize, conformal invariance and modular invariance are the guiding principles for the construction of (perturbatively) consistent string models.

2. – Superstrings in $D = 10$

The known supersymmetric and, as such, tachyonic free string theories can all be naturally formulated in $D = 10$. According to the number of (Majorana-Weyl) supersymmetries one has:

Type-I superstring: a theory of open and closed unoriented superstrings whose low-energy limit is the chiral $N = (1, 0)$ supergravity coupled to $N = (1, 0)$ super Yang-Mills theory with gauge group $SO(32)$;

Type-IIA superstring: a theory of closed oriented superstrings, L-R asymmetric on the world-sheet, whose low-energy limit is the non-chiral $N = (1, 1)$ supergravity;

Type-IIB superstring: a theory of closed oriented superstrings, L-R symmetric on the world-sheet, whose low-energy limit is the chiral $N = (2, 0)$ supergravity;

Heterotic string: an L-R asymmetric combination of bosonic and fermionic strings whose low-energy limit is the chiral $N = (1, 0)$ supergravity coupled to $N = (1, 0)$ super Yang-Mills theory with gauge group $Spin(32)/Z_2$ or $E(8) \times E(8)$.

The first and the last of these theories, though chiral, are anomaly free thanks to the Green-Schwarz mechanism which involves the antisymmetric tensor field $B_{\mu\nu}$ present in the massless spectrum. The type-IIB theory, though chiral, is anomaly free thanks to a remarkable cancellation among the contributions of two left-handed gravitini, two right-handed dilatini and a four-form potential $A^{(+)}_{\mu\nu\varrho\sigma}$ whose field strength is to be taken self-dual. All the above theories contain a metric $G_{\mu\nu}$, an antisymmetric tensor $B_{\mu\nu}$ and a scalar (dilaton) Φ as massless particles in their bosonic spectra. In addition, in the open-string spectrum of the type-I theory and in the Neveu-Schwarz (NS) spectrum of the heterotic theory massless vector bosons A^a_μ appear. In the type-II theories the massless NS-NS spectrum is accompanied by a rich spectrum of tensor fields coming from the massless Ramond-Ramond (R-R) sector. The type IIA contains an Abelian vector field A_μ and a three-form potential $C_{\mu\nu\varrho}$, while the type-IIB theory, apart from the above-discussed $A^{(+)}_{\mu\nu\varrho\sigma}$, contains another antisymmetric tensor $B'_{\mu\nu}$ and a second dilaton Φ'. In the Polyakov approach, the vacuum expectation value of the dilaton field plays the role of string coupling constant, *i.e.* of string-loop expansion parameter.

3. – Superstring compactifications and T-duality

The superstring theories formulated in $D = 10$ are not very appealing as models for particle physics. One would like to find a mechanism to compactify the unwanted six dimensions in order to remain with only four physical dimensions. Unfortunately, string theory admits so far only a perturbative formulation and a dynamical principle to infer compactification is still lacking. All one can do is to study the condition for perturbatively consistent compactifications and to deduce the spectrum and interactions of the resulting lower-dimensional theories. The stringy generalization of the dimensional reduction *à la* Kaluza-Klein reveals some unexpected features. Indeed apart from the usual Kaluza-Klein (KK) modes with quantized momenta, there exist "winding" modes corresponding to the string wrapping around the compact dimensions. The presence of these extra modes is responsible for the so-called Halpern-Frenkel-Kac (HFK) mechanism of symmetry enhancement. It is well known that in KK theories the components of the metric with mixed indices $G_{\mu i}$ behave as vector bosons which gauge the isometry group of the internal manifold. For instance, the KK gauge symmetry arising from k Abelian isometries of a k-torus $T^k = S^1 \times S^1 \ldots S^1$ can be enlarged in string theory to a non-Abelian group, for special values of the compactification radii. More specifically, the spectrum of the string states corresponding to a one-dimensional compactification on a circle of radius R is given by

$$M^2_\pm = M^2_J + \frac{1}{2}\left(\frac{m}{R} \pm \frac{nR}{\alpha'}\right)^2, \tag{3.1}$$

where $\pm$ denote L(R)-moving modes, m/R is the quantized momentum and n is the winding number. The spectrum shows a striking symmetry under the exchange $R \to \alpha'/R$ and at the fixed point of the transformation, *i.e.* $R = \sqrt{\alpha'}$, new massless fields appear in the spectrum (those with $m = \pm n = \pm 1$) that enlarge the $U(1)$ symmetry to $SU(2)$. In fact in closed oriented string theories there is a doubling of modes and both the Abelian and enhanced symmetries get doubled. In the low-energy description the vector fields which arise from the mixed components of the antisymmetric tensor $B_{\mu i}$ are responsible for this doubling. In a more complicated situation the above symmetry between large and small radii becomes an infinite-dimensional discrete non-Abelian symmetry which acts on the vacuum expectation values (v.e.v.'s) of the internal components of the metric G_{ij} and of the antisymmetric tensor B_{ij} in such a way as to preserve the string spectrum (3.1). From the low-energy point of view, G_{ij} and B_{ij} correspond to massless scalars with only derivative interactions. Their v.e.v.'s parametrize the space of classical vacua usually called the moduli space of the compactification. These moduli spaces are (locally) coset manifolds which admit very large group of isometries. Due to non-perturbative world-sheet effects (such as the windings) the global continuous symmmetries are reduced to discrete groups of so-called "T-duality".

The massless spectrum of type-II theories compactified on a flat six-dimensional torus reproduces $N=8$ supergravity. It includes 28 Abelian vector bosons, called graviphotons (12 from the NS-NS sector and 16 from the R-R sector), and 70 scalars which parametrize the coset $E(7,7)/SU(8)$. The T-duality group is $SO(6,6;Z)$. For heterotic theories one has $N=4$ supergravities coupled to non-Abelian vector multiplets. However, at generic points of the moduli space, $SO(6,22)/SO(6)\times SO(22)$, the non-Abelian group is broken to $U(1)^{28}$ and the T-duality group is $SO(6,22;Z)$. For the type-I superstring, which leads to $N=4$ low-energy effective field theories, one would expect a behaviour similar to the heterotic-string case, but in the perturbative string spectrum part of the T-duality symmetry is absent. The asymmetric origin of the scalar fields, which are the internal components of the metric G_{ij} (from the closed-string NS-NS sector), the internal component of the antisymmetric tensor B_{ij} (from the closed-string R-R sector) and the internal components of the gauge fields A_i^a (from the open-string sector) prevents the existence of any obvious transformation among them. Supergravity considerations lead one to suspect that the inclusion of solitonic states, charged with respect to the R-R vector bosons, may restore the full T-duality symmetry.

4. – Buscher's duality and stringy gravitational instantons

The T-duality symmetry between large and small radius in toroidal compactifications has an analogue in any background with continuous isometries. Indeed Buscher [7] has shown that if a two-dimensional non-linear σ-model, describing string propagation in a non-trivial background with metric $G_{ij}(X)$, antisymmetric tensor $B_{ij}(X)$ and dilaton $\Phi(X)$, admits an Abelian isometry, then it is equivalent to a dual σ-model on a background with metric $\widetilde{G}_{ij}$, antisymmetric tensor $\widetilde{B}_{ij}$ and dilaton $\widetilde{\Phi}(X)$. In adapted coordinates, such that the Killing vector is $\xi=\partial/\partial X^0$, the relation between the two backgrounds reads

$$(4.1)\quad \begin{cases} \widetilde{G}_{00}=\dfrac{1}{G_{00}}\,, \quad \widetilde{G}_{0i}=\dfrac{B_{0i}}{G_{00}}\,, \quad \widetilde{G}_{ij}=G_{ij}-\dfrac{1}{G_{00}}(G_{0i}G_{0j}-B_{0i}B_{0j})\,,\\ \widetilde{B}_{0i}=\dfrac{G_{0i}}{G_{00}}\,, \quad \widetilde{B}_{ij}=B_{ij}-\dfrac{1}{G_{00}}(G_{0i}B_{0j}-B_{0i}G_{0j})\,, \quad \widetilde{\Phi}=\Phi-\dfrac{1}{2}\log G_{00}\,. \end{cases}$$

The above transformations have dramatic consequences for the geometric and even for the topological interpretation of string backgrounds [8]. For example, in black-hole–type geometries, Buscher's duality (4.1) exchanges singularity and horizon.

The very consistency of string propagation on a non-trivial background imposes the vanishing of the β-functions of the corresponding σ-model. They play the role of string equations of motion and are equivalent to extremizing an action functional for the effective low-energy field theory. Non-trivial supersymmetric solutions of the lowest-order (in α') equations of motion may be found by setting to zero the fermion fields together with their supersymmetric variations (for a review see [9]). In $D=4$ a supersymmetric ansatz for the solution of the heterotic-string equations of motion is

$$F_{\mu\nu} = \widetilde{F}_{\mu\nu}, \qquad H_{\mu\nu\varrho} = \sqrt{G}\varepsilon_{\mu\nu\varrho}{}^{\sigma}\partial_\sigma \phi , \qquad G_{\mu\nu} = e^{2\phi}\widehat{g}_{\mu\nu}, \tag{4.2}$$

where $F_{\mu\nu}$ is the field strength of the gauge fields A_μ, $H_{\mu\nu\varrho}$ is the (modified) field strength of the antisymmetric tensor $B_{\mu\nu}$ and $\widehat{g}_{\mu\nu}$ is a self-dual metric. The generalized spin-connection with torsion $\Omega^{ab}_{\mu\pm} = \omega^{ab}_\mu \pm H^{ab}_\mu$ deriving from (4.2) is self-dual. The ansatz in (4.2) must be supplemented with the (modified) Bianchi identity: $\mathrm{d}H = \alpha'\{\mathrm{tr}\, R(\Omega_-)\wedge R(\Omega_-) - \mathrm{tr}\, F(A)\wedge F(A)\}$. To simplify matters it is convenient to impose the "standard embedding" of the gauge connection in the $SU(2)$ spin group: $A = \Omega_-$. Then there are two options. The first corresponds to conformally flat "axionic instantons" with $\widehat{g}_{\mu\nu} = \delta_{\mu\nu}$ [10, 11]. The second requires a constant dilaton and vanishing torsion and it is completely specified by the choice of a self-dual metric $\widehat{g}_{\mu\nu}$, *i.e.* a gravitational instanton [12].

An interesting class of self-dual metrics is given by the Gibbons-Hawking multi-centre (GHMC) ansatz (see, *e.g.*, [13])

$$\mathrm{d}s^2 = V^{-1}(\vec{x})(\mathrm{d}\tau + \vec{\omega}\cdot \mathrm{d}\vec{x})^2 + V(\vec{x})\, \mathrm{d}\vec{x}\cdot \mathrm{d}\vec{x} \tag{4.3}$$

with $\vec{\nabla} V = \vec{\nabla}\times\vec{\omega}$ and $V(\vec{x}) = \varepsilon + 2m\sum_{i=1}^{k+1} 1/|\vec{x} - \vec{x}_i|$. The choice $\varepsilon = 0$, $m = (1/2)$ corresponds to metrics which are asymptotically locally Euclidean (ALE). ALE instantons are smooth resolutions of singular varieties in $\boldsymbol{C}^3$ which admit (non-compact) Ricci-flat hyper-Kähler metrics of $SU(2)/\Gamma$ holonomy, Γ being any Kleinian subgroup of $SU(2)$ [14].

ALE instantons have played a fundamental role in Euclidean Quantum (Super)-Gravity since instanton-dominated correlators give rise to gravitino condensation which may trigger the dynamical breaking of local supersymmetry [15]. In the context of low-energy effective field theories for the heterotic or type-I superstrings, the combined effects of ALE and Yang-Mills instantons may be studied [16] thanks to the possibility of generalizing the ADHM construction [17, 18].

Since (4.3) admits a Killing vector $\xi = \partial/\partial\tau$, one can perform a Buscher's duality to a new background with

$$\begin{cases} \widetilde{G}_{00} = V , & \widetilde{G}_{0i} = 0 , \quad \widetilde{G}_{ij} = V\delta_{ij}, \\ \widetilde{B}_{0i} = \omega_i , & \widetilde{B}_{ij} = 0 , \quad \widetilde{\Phi} = \Phi_0 + \dfrac{1}{2}\log V . \end{cases} \tag{4.4}$$

The new background is singular and conformally flat ([1]) and has non-vanishing torsion related to the dilaton through $H = *\,\mathrm{d}\Phi$. This is precisely the condition for a supersymmetric axionic instanton. Manifest supersymmetry has been preserved in (4.4) since the relevant Killing vector is triholomorphic, *i.e.* it preserves the hyper-Kähler structure of (4.3).

Buscher's dualities with respect to non-triholomorphic isometries yield string backgrounds in which supersymmetry is non-locally realized [19]. This may force one to

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([1]) The canonical Einstein metric is related to the string σ-model metric through a Weyl rescaling $g_{\mu\nu} = e^{-4\Phi/(D-2)}G_{\mu\nu}$; in this case $D = 4$ and $g_{\mu\nu}$ is flat.

reconsider the relation between space-time and world-sheet supersymmetry in superstring compactifications [20].

5. – Calabi-Yau compactifications and mirror symmetry

Toroidal compactifications of superstring theories bring too many supersymmetries in $D = 4$. Although supersymmetry is phenomenologically well motivated as the only known solution of the hierarchy problem, chiral fermions are compatible only with $N = 1$ supersymmetry. Compactifications of the heterotic or type-I superstring which preserve only one supersymmetry in $D = 4$ require six-dimensional backgrounds which admit only one covariantly constant (Killing) spinors. The relevant compact manifolds are known as Calabi-Yau spaces (CYS), which are complex Kähler manifolds of $SU(n)$ holonomy, *i.e.* vanishing first Chern class. In the case of interest for superstring phenomenology the number of complex dimensions is $n = 3 = 6/2$. (Lower-dimensional cases are very tightly restricted. For $n = 1$, only the flat torus T^2 would do the job. For $n = 2$, manifolds of $SU(2)$ holonomy are hyper-Kähler and, in the compact case, they are topologically equivalent to Kümmer's third surface $K3$ while, in the non-compact case, they belong to the class of ALE instantons encountered previously.) Simply connected manifolds of $SU(3)$ holonomy, $\mathscr{M}$, are topologically characterized by only two independent Hodge numbers: $h_{11} = \dim H^{(1,1)}$ (the number of harmonic (1, 1)-forms on $\mathscr{M}$) and $h_{21} = \dim H^{(2,1)}$ (the number of harmonic (2, 1)-forms $\mathscr{M}$). The other non-vanishing Hodge numbers are $h_{00} = h_{33} = 1$, related to the constant scalar and to the volume top-form, and $h_{30} = h_{03} = 1$, related to the existence of one (anti)-holomorphic nowhere vanishing 3-form. The Euler characteristic $\chi(\mathscr{M}) = 2(h_{11} - h_{21})$ turns out to be twice the net number of generations of chiral multiplets in the **27** ($\mathbf{26}_{(2)} + \mathbf{1}_{(-4)}$) of the low-energy gauge group $E_6 \times E_8$ ($SO(26) \times U(1)$) resulting from the standard embedding of the $SU(3)$ spin connection in the $D = 10$ gauge group.

The massless fluctuations of the metric and of the antisymmetric tensor correspond to h_{11} complex parameters of deformations of the (complexified) Kähler class and to h_{21} complex parameters of deformations of the complex structure. From the low-energy $D = 4$ viewpoint these are complex scalars with only derivative interactions, known as "moduli" of the CYS [20, 21]. The same scalars (accompanied by others arising from the R-R sector) would appear in type-II string compactification on CYS. Compatibility with $N = 2$ supersymmetry, then implies that the non-linear σ-model for the moduli fields ϕ is based on a "special Kähler manifold" $\mathscr{S}$ whose Kähler potential admits an analytic prepotential $\mathscr{F}(\phi)$. Furthermore, the σ-model manifold has locally a product structure $\mathscr{S} = \mathscr{S}_{11} \times \mathscr{S}_{21} \times SU(1,1)/U(1)$ (2), with $\dim_C(\mathscr{S}_{11}) = h_{11}$ and $\dim_C(\mathscr{S}_{21}) = h_{21}$. The metric on $\mathscr{S}_{21}$ is neither renormalized by quantum perturbative corrections in α' nor by world-sheet non-perturbative effects. At the classical level, the metric on $\mathscr{S}_{11}$ is completely determined by the intersection form among three (1, 1)-forms, which is a topological invariant and as such receives no perturbative corrections in α'. However, world-sheet instantons do correct it non-perturbatively in α' ($\sim e^{-R^2/\alpha'}$, where R^2 is the typical size of the CYS). Fortunately, "mirror symmetry" comes to rescue us.

(2) The last factor includes the dilaton and the axion, related to $B_{\mu\nu}$ through $\partial_\mu b = \varepsilon_\mu{}^{\nu\varrho\sigma} \partial_\nu B_{\varrho\sigma}$.

Mirror symmetry emerges from two rather different observations. The first is the "experimental" observation that the known CYS tend to come in pairs with opposite Euler characteristic (*i.e.* with h_{11} and h_{21} exchanged). The second is a result from two-dimensional $N=(2,2)$ superconformal field theory: the projection of the spectrum on states of integer $U(1)$ charges, necessary to achieve space-time supersymmetry [20, 21], allows to revert the $U(1)$ charges in one of the two sectors [22]. This operation turns out to interchange deformations of the complex structure with deformations of the Kähler class. One is thus led to conjecture that any reasonable Calabi-Yau manifold $\mathcal{M}$ should admit a "mirror image" $\mathcal{W}$ with $h_{11}(\mathcal{W}) = h_{21}(\mathcal{M})$ and $h_{21}(\mathcal{W}) = h_{11}(\mathcal{M})$. The appropriate setting to discuss mirror symmetry is "toric geometry". For CYS embedded in toric varieties mirror symmetry should correspond to the exchange of the "fan" defining the toric variety with its dual ([3]). Using mirror symmetry one can exactly compute the metric on $\mathcal{S}_{11}$ for $\mathcal{M}$ by relating it to the metric on $\mathcal{S}_{21}$ for the mirror manifold $\mathcal{W}$ [23]. The determination of the exact mirror map between the two sets of parameters relies on the solution of Picard-Fuchs equations for the periods of the holomorphic three-form along a symplectic basis of three-cycles on CYS [23]. Matching the asymptotic behaviours and the "classical monodromies" one can finally sum up the α' non-perturbative contributions arising from the world-sheet instantons, *i.e.* rational curves holomorphically embedded in the CYS. The resulting "quantum-corrected" moduli spaces admit the action of discrete groups of isometries which are the by-product of the monodromy group of the Picard-Fuchs equations and may serve as exact symmetries to constrain the string-loop corrections to the low-energy effective action. In order to get some information about string non-perturbative effects the above arguments are not sufficient. A (discrete) symmetry between strong and weak coupling would certainly be very helpful.

6. – S-duality and H-monopoles

The Maxwell equations for the electro-magnetic field *in vacuo* are manifestly invariant under the interchange of the electric and magnetic fields. In fact one can expose a continuous invariance under phase transformations of the complex vector $\mathbf{E} + i\mathbf{B}$. This symmetry is clearly broken by the presence of electric charges or, equivalently, by the absence of magnetic charges. In order to restore the symmetry, one should introduce magnetic charge and current densities. Magnetic monopoles in an Abelian theory are singular objects and the motion of electric charges in their presence is inconsistent unless one imposes the Dirac quantization condition $eg = 2n\pi$. In non-Abelian theories (such as $SU(2)$) with scalars, however, 't Hooft and Polyakov have shown that non-singular monopole solutions exist with quantized magnetic charge. Montonen and Olive [24] have conjectured the existence of a symmetry which exchanges an "electric" theory with coupling e and a "magnetic" theory with coupling $g = 2\pi/e$. This strong-weak coupling duality is substantiated by an observation made by Manton concerning the scattering of magnetic monopoles. Goddard, Nyuts and Olive have generalized the conjecture to larger non-Abelian group [24], for which one needs the introduction of dual non-Abelian groups, *e.g.* for $SU(2)$ the dual group is $SO(3) = SU(2)/Z_2$. Moreover, in the presence of a non-vanishing vacuum angle θ the effective electric charge is shifted. The strong-weak coupling duality $g^2 \to 1/g^2$ is then

([3]) Claudio Procesi, private communication.

modified to an infinite-dimensional discrete group of $SL(2, Z)$ transformations called "S-duality", which in terms of the complex coupling constant $\tau = 4\pi i/g^2 + \theta/2\pi$ read

$$\tau \to \frac{a\tau + b}{c\tau + d} \tag{6.1}$$

with $a, b, c, d \in Z$ such that $ad - bc = 1$. S-duality invariance requires the existence of stable dyon solutions, *i.e.* solitons with both electric and magnetic charges, subject to the Schwinger-Zwanziger quantization condition (see, *e.g.*, [9]). The discovery of dyon solutions by Julia and Zee seems to support the conjecture. However, the electrically charged quanta are vector bosons while the magnetically charged solitons are scalars. Moreover, if proper account is taken of quantum corrections, the masses of monopoles and dyons are renormalized and their stability is jeopardized. Supersymmetry can come to rescue the situation. Indeed in all N-extended supersymmetric Yang-Mills (SYM) theories the scalar fields ϕ in the adjoint representation appear in the vector multiplets and magnetic charges appear in the central extensions Z of the supersymmetry algebra. States which saturate the Bogomolny-Prasad-Sommerfield (BPS) bound [25] $M \geqslant |Z|$, between mass M and central charge Z, lie in (super)short multiplets and are stable against quantum (perturbative) corrections due to supersymmetry. In Yang-Mills theory with global $N = 4$ supersymmetry the situation is expected to persist even non-perturbatively. Explicit instanton calculations and the determination of the free energy in a box with twisted boundary conditions seem to support this expectation [26].

However, in theories with $N = 2$ supersymmetry Seiberg and Witten [27] have shown that non-perturbative effects (instantons) do correct the analytic prepotential $\mathcal{F}(\phi)$ and renormalize the mass formula $M = |q\phi + p\partial\mathcal{F}/\partial\phi|$. The Seiberg-Witten solution for $\mathcal{F}(\phi)$ shows some very peculiar features. First of all, the non-Abelian $SU(2)$ gauge symmetry is never restored in the quantum moduli space, *i.e.* there is no place where $\langle\phi\rangle = 0$. New massless hypermultiplets (monopoles or dyons) appear in the spectrum at two special points where $\mathcal{F}$ is singular. Moreover, introducing a mass perturbation, which explicitly breaks $N = 2$ supersymmetry to $N = 1$, causes the monopole (or dyon) to condense and provides a model for the realization of confinement (in the resulting $N = 1$ SYM theory) as a dual Meissner effect. The algebraic solution of Seiberg and Witten, based on the introduction of a "dynamical elliptic curve", whose periods determine the analytic prepotential, has been generalized to other non-Abelian theories, through the study of periods of "hyperelliptic curves", and to $N = 2$ Super QCD [27]. In $N = 2$ theories S-duality exchanges different low-energy descriptions of the same microscopic theory which is thus not self-dual. On the other hand, $N = 4$ theories as well as other superconformal theories seem to live in a self-dual phase.

Since $N = 4$ SYM theories coupled to $N = 4$ supergravity are the low-energy limit of heterotic string compactified on a six-torus, it is natural to conjecture that S-duality should hold or even originate in these theories [28]. Schwarz and Sen have shown that at generic points of the moduli space where the gauge group is Abelian, *i.e.* $U(1)^{28}$, the equations of motion are indeed invariant under $SL(2, R)$ trans-formations,

$$\lambda \to \frac{a\lambda + b}{c\lambda + d}\,, \qquad F^{(a)}_{\mu\nu} \to (c\lambda_1 + d)\, F^{(a)}_{\mu\nu} + c\lambda_2\, G^{(a)}_{\mu\nu}, \tag{6.2}$$

with $\lambda = \lambda_1 + i\lambda_2 = \Psi + ie^{-\Phi}$ parametrizing the dilaton-axion system and $G^{(a)}_{\mu\nu} = (R)^a{}_b \widetilde{F}^{(a)}_{\mu\nu}$ with $R \in SO(6, 22; R)$. In order for the discrete subgroup $SL(2, Z)$ to be effectively realized in the theory, Schwarz and Sen have conjectured the existence of a spectrum of BPS saturated states invariant under S-duality [28]. This spectrum of solitons includes H-monopoles and dyonic black-holes, which may become massless at special points of the quantum moduli space [29]. Some of the required solitons have been explicitly found and their stability has been argued on the basis of mass non-renormalization in extended supersymmetric theories. A strong argument in favour of the existence and stability of the needed BPS saturated states is still lacking and may require a deeper understanding of S-duality.

7. – String-string dualities and extended objects

In $D = 4$ duality between charges and monopoles is a natural symmetry since the dual of a two-form (the electromagnetic field strength $F_{\mu\nu}$) is a two-form (the dual field strength $\widetilde{F}_{\mu\nu} = (1/2)\,\varepsilon_{\mu\nu}{}^{\varrho\sigma} F_{\varrho\sigma}$). In diverse dimensions things look quite differently. First of all, while a vector potential A_μ naturally couples to "world-lines" and thus to pointlike objects, a $(p+1)$-form potential $B_{\mu_1 \ldots \mu_{p+1}}$ naturally couples to the "world-volume" of an extended object with p dimensions, *i.e.* a p-brane, through the interaction term [30, 9]

$$S_p = \int \mathrm{d}^{p+1}\xi \varepsilon_{\alpha_1\alpha_2\ldots\alpha_{p+1}} B_{\mu_1\mu_2\ldots\mu_{p+1}} \partial_{\alpha_1} X^{\mu_1} \partial_{\alpha_2} X^{\mu_2} \ldots \partial_{\alpha_{p+1}} X^{\mu_{p+1}}. \tag{7.1}$$

The field strength of a $(p+1)$-form is a $(p+2)$-form and in D dimensions its dual is a $(D-p-2)$-form which is the field strength of a $(D-p-3)$-form potential naturally coupled to a $(D-p-4)$-brane. Electro-magnetic strong-weak coupling duality in D dimensions thus exchanges p-branes and $(D-p-4)$-branes [9].

It has been known for a while that strings are dual to penta-branes in $D = 10$ and to strings in $D = 6$. In the latter case, a candidate dual pair is formed by the heterotic string compactified on a four-torus and the type-IIA superstring compactified on a K3 surface, the topologically unique compact manifold of $SU(2)$ holonomy [29, 31]. The basic argument to support this string-string duality is the coincidence of the low-energy spectrum at generic points of the 80-dimensional moduli space $SO(4, 20)/SO(4) \times SO(20)/SO(4, 20; Z)$. At the points where the Abelian gauge symmetry $U(1)^{24}$ is enhanced through the HFK mechanism in the heterotic string, one expects the appearence of new massless vector multiplets in the type-IIA string. These states should be charged with respect to the vector fields of the R-R sector of the spectrum and cannot arise from perturbative Kaluza-Klein modes, *i.e.* momentum or winding states. They are intrinsically non-perturbative solitons which may correspond to R-R charged black holes [29]. Upon further compactification $D = 6$ to $D = 4$ both theories become $N = 4$ supersymmetric and the heterotic-string S-duality gets mapped to type-II string T-duality.

Other candidate pairs of dual string theories are the heterotic string compactified on $K3 \times T^2$ and the type-II theories (both A and B) compactified on a restricted class of CYS which arise as $K3$ fibrations of the sphere S^2 [32]. If true this duality may shed some light on the locally supersymmetric extension of the Seiberg-Witten solution of $N = 2$ SYM theories. In the locally supersymmetric case the role of the dynamical Riemann surface is played by the dynamical CYS of compactification on the type-II side [33]. Relating the two low-energy effective Lagrangians through the appropriate mirror map one may in principle exactly solve the theory at least for the terms with two derivatives on the bosonic fields. Higher-derivative couplings such as $R^2 F^{2g-2}$ have been considered in [34] and are related to an holomorphic anomaly [35]. On the type-II side they are determined by a g-loop computation, on the heterotic side they correspond to one-loop amplitudes. From an effective Lagrangian they arise from non-perturbative (instanton) effects. Indeed chiral selection rules imply that interactions of the above type receive contribution from ALE gravitational instantons with topological charge (Hirzebruch signature) $\tau = g$. Matching the results for the model with low-rank gauge groups on the heterotic side lends support to Strominger's interpretation [36] of conifold singularities in Calabi-Yau moduli spaces as the appearence of massless hypermultiplets of R-R charged black holes on the type-II side [32, 34].

8. – U-duality: Ramond-Ramond charged black holes as string solitons

In order to understand the origin of the $SL(2, Z)$ S-duality symmetry in $N = 4$, $D = 4$ effective field theories for the superstrings, it may prove very helpful to observe that a similar symmetry is already present in the type-IIB theory in $D = 10$. Indeed, the two scalars present in the massless spectrum (one from the NS-NS sector, the other from the R-R sector) may be assembled into a complex scalar whose self-interactions are described by a non-linear σ-model on the coset manifold $SU(1, 1)/U(1)$. Though a covariant action for the type-IIB theory with self-dual field strength of the 4-form potential is still lacking, one may sacrifice manifest supersymmetry and work with a non self-dual theory, for which an action can be written, and impose the self-duality constraints only on the equations of motion. Then one can show that the theory is invariant under $SL(2, R)$ transformations, under which the four-form is inert. This continuous symmetry is expected to be broken to a discrete symmetry $SL(2, Z)$ [28, 37, 29]. After toroidal compactification the two type-II theories are equivalent and in $D = 4$ their common massless spectrum precisely fits that of $N = 8$ supergravity. This theory contains 70 scalars which parametrize the coset $E(7, 7)/SU(8)$. It is remarkable that the scalars which arise from the compactification ("moduli fields") and the dilaton (whose expectation value is the string-loop expansion parameter) can be transformed into one another. Since the massive perturbative states of the string break the continuous T-symmetry to a discrete T-duality, one expects the massive perturbative and non-perturbative states to break the continuous non-compact $E(7, 7; R)$ symmetry to a discrete symmetry group $E(7, 7; Z)$ which has been termed U-duality by Hull and Townsend [29].

The U-duality group includes the product of the T-duality group $SO(6, 6; Z)$ and of the S-duality group $SL(2, Z)$ as a "maximal" subgroup (recall that $SO(12) \times SU(2) \subset E_7$). In order for $E(7, 7; Z)$ to be a symmetry of the spectrum one has to assume the existence of solitonic states charged with respect to the 16 R-R vector bosons. Their masses scale as $M_{\mathrm{RR}} \approx 1/g$ [31]. There are two pathways for truncating the type-II theory to $N = 4$. Either through a target-space orbifold of the six-torus T^6 or through a "world-sheet" orbifold [38]. The latter leads to a type-I superstring which includes open-string states. For a complete low-energy description one needs to include all the solitonic states which may become massless at points corresponding to the enhanced symmetry points on the heterotic string moduli space [1]. From the four-dimensional point of view, these $N=4$ supersymmetric solitons are charged extremal black-holes [29, 39] and the very consistency of the theory forces one to treat them as elementary particles! From the ten-dimensional point of view these states may be pictured as penta-branes wrapped around internal dimensions. When some of the homology cycles of the internal compact manifold shrink to zero size, a conifold transition may take place and the solitonic state whose mass is proportional to the size of the cycle becomes massless [36]. If the soliton is charged, the $U(1)^{28}$ Abelian group may be enlarged to a non-Abelian group.

9. – Heterotization of the Chan-Paton group

In $N=4$, $D=4$ heterotic effective actions, there are points of the moduli space where the Abelian gauge group $U(1)^{28}$ gets enhanced to large non-Abelian groups (at most $SO(44)\times U(1)^6$). In type-II string theories one may expect a similar behavior only if at the corresponding points in the K3 moduli space some non-perturbative solitonic state ("R-R charged black holes") become massless. For type-I theories the situation is more intricate. According to Sagnotti [38], type-I theories can be considered as "parameter space orbifolds" of the type-IIB theory. It is not our purpose here to review the construction of open-string descendants of type-IIB superstring models which is the subject of Pradisi's contribution to these Proceedings [40], however it is necessary to recall some relevant facts.

First of all, in order to couple open strings to closed unoriented ones the standard Polyakov perturbative series must be supplemented with the inclusion of surfaces with boundaries and/or crosscaps. These may be considered as orbifolds of closed oriented surface (at particular values of their moduli) with respect to anticonformal involutions. The fixed points of the involution, if any, are the boundaries of the resulting "open" surface. Conformal field theories on surfaces of this kind are equivalent to conformal field theories on double-covering surfaces endowed with a Z_2-orbifold projection of the spectrum under the exchange of left-movers with right-movers. Roughly speaking, this procedure halves the world-sheet symmetries as well as their target space by-products. For instance, in $D=10$ identifying the type-IIB superstring states under the exchange of left- and right-movers effectively reduces the massless spectrum to $N=(1,0)$ supergravity. The truncation of the closed-string spectrum encoded in the torus partition function T is implemented by the introduction of the Klein-bottle projection K. These two contributions make up the "untwisted sector" of the parameter space orbifold

$$Z_{\rm u}=\frac{1}{2}(T+K)\,. \tag{9.1}$$

The role of the "twisted sector" is played by the open-string spectrum encoded in the annulus partition function A and its projection, the Möbius strip M

$$Z_{\rm t}=\frac{1}{2}(A+M)\,. \tag{9.2}$$

In standard orbifold construction the twisted sectors come out with the multiplicity of the fixed points. Similarly, in the parameter-space orbifolds the open-string states may acquire multiplicities associated to their ends through the so-called Chan-Paton factors. Consistency requirements may be deduced transforming the above amplitudes to the transverse channel, where Klein bottle $\widetilde{K}$, annulus $\widetilde{A}$ and Möbius strip $\widetilde{M}$ are to be identified with closed-string amplitudes between boundary and/or crosscap states. Since "half" of the closed-string states have been projected out of the spectrum, it would be inconsistent if some of them coupled to the vacuum. The cancellation between boundary and crosscap contributions to these tadpoles constrains the Chan-Paton factors and the signs of the projections [40].

In order to explicitly solve these constraints, *i.e.* in order to linearize the dyophantine equations for the Chan-Paton factors, it proves very useful to exploit Cardy's proposal [41] of associating a boundary state to each sector of the spectrum. This amounts to expressing the annulus amplitude in terms of the fusion rule coefficients. This in turn translates in a transverse channel annulus amplitude which is a sum of perfect squares. The transverse-channel Möbius amplitude is then fixed for consistency in each sector of the spectrum as an appropriate "square root" of the product of Klein bottle and annulus amplitudes. Following this procedure one may construct a web of consistent models of type-I superstring in any dimension, including supersymmetric models in $D=6$ and $D=4$ as well as non-supersymmetric models in $D=10$ [42].

This is not, however, the whole story. Indeed the above construction assumed an almost unique K. Sewing constraints for conformal field theories on closed oriented Riemann surfaces can be extended to surfaces with boundaries and/or crosscaps. In particular, a crosscap constraint can be deduced [40]. In many interesting cases there are several solutions to this constraint which allow to deduce several different projections of the closed-string spectrum. The procedure is then reversed, the number of allowed boundary states is reduced and may be inferred from $\widetilde{M}$. Many new open-string descendants of (world-sheet) left-right symmetric models can be constructed systematically [40].

Recently, these ideas have attracted much attention in the string literature in connection with the observation that non-perturbative string corrections may be as large as $\exp[-1/g]$ rather than $\exp[-1/g^2]$ as usual in field theory with coupling constant g. Since a boundary contributes half a handle to the Euler characteristic of a surface, the open-string coupling constant is the square-root of the closed-string one. Thus non-perturbative effects of the above kind are naturally generated by the introduction of boundaries, *i.e.* by coupling open strings to the closed-string spectrum [43]. The way to achieve this coupling in a natural way is to observe the existence of solitonic solutions in type-IIB theories known as "orientifolds" or "D-branes", *i.e.* solutions with some of the string coordinates satisfying Dirichlet rather than Neumann boundary conditions [44]. The spectrum of these BPS saturated solitons, whose masses scale as $M_{\mathrm{RR}} \approx 1/g$, is expected to reflect the $SL(2, Z)$ multiplicity of the type-IIB strings constructed by Schwarz [37]. These strings, labelled by two integers (r, s), may be thought of as solitons of the fundamental type-IIB string (which corresponds to $r=1$ and $s=0$). To support the resulting formula for the solitonic string tensions

$$T_{(r,s)} = T_{(1,0)} \sqrt{(r - s\Psi)^2 + e^{-2\Phi} s^2}, \tag{9.3}$$

Schwarz has conjectured the existence of an underlying "M-theory" whose physical excitations are p-branes. Polchinski has suggested that p-brane solitons carrying R-R charges may be thought of as D-branes, *i.e.* p-branes with Dirichlet boundary conditions on the transverse coordinates [44]. In particular, for $p = 1$ one finds the D-strings which can be interpreted as the $(0, 1)$ strings above. Bound states of these D-strings and N-strings (*i.e.* strings with standard Neumann boundary conditions) can be formed to generate the other (r, s) dy-strings, which may interact via exchange of open dy-string states [44].

In order to properly couple open dy-string states, one has to solve the crosscap constraint for the conformal field theory describing D-branes and then fix the Chan-Paton multiplicities imposing the cancellation of the tadpoles for the unphysical states. The open dy-string states carry both Chan-Paton and R-R charges. When some of these states become massless a new mechanism of symmetry enhancement is thus expected to take place. It deserves the name of "heterotization" of the Chan-Paton group. A better understanding of this mechanism would shed some light on the strong-weak coupling duality between type-I and $SO(32)$ heterotic theory in $D = 10$ proposed in [44]. A thorough check of the conjectured duality could be provided by an explicit determination of the spectrum of dy-string states. Upon toroidal compactification to $D = 4$, new massless states should somewhere appear to allow the "heterosis" of the original Chan-Paton group $SO(32)$ with the R-R group $U(1)^6$.

10. – Outlook

String theory seems to undergo a new period of rapid proliferation of ideas and the fundamental issue of understanding non-perturbative effects may not be so out of reach. The idea that many different-looking theories can be unified in a common more fundamental description may require the introduction of extended object of different kinds. The enormous progress in the study of supersymmetric gauge theories in $D = 4$ after the work of Seiberg and Witten [27, 45] and Strominger's interpretation of the conifold transition [36] point in the direction of suggesting that any singularity in an otherwise physically consistent theory must be taken as a hint to some relevant phenomena. In this respect the strange role that open strings seem to play in closed-string theories deserves particular attention [43].

* * *

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