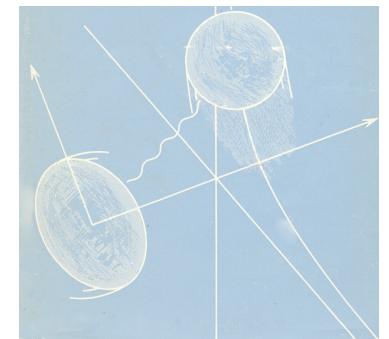


Coulomb excitation with radioactive ion beams



a tool to study nuclear collectivity and more

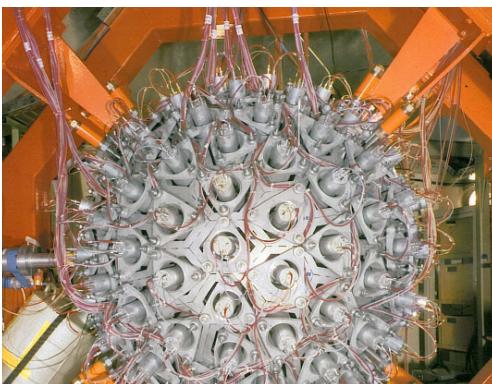
- Motivation and introduction
- Some theoretical aspects of Coulomb excitation
- Experimental considerations, set-ups and analysis techniques
- Recent experiments and future perspectives

Lectures given at the
International School of Physics "Enrico Fermi"
Varenna, July 2017
Wolfram KORTEN - CEA Paris-Saclay

My physics interest: nuclear spectroscopy

Crystal Ball

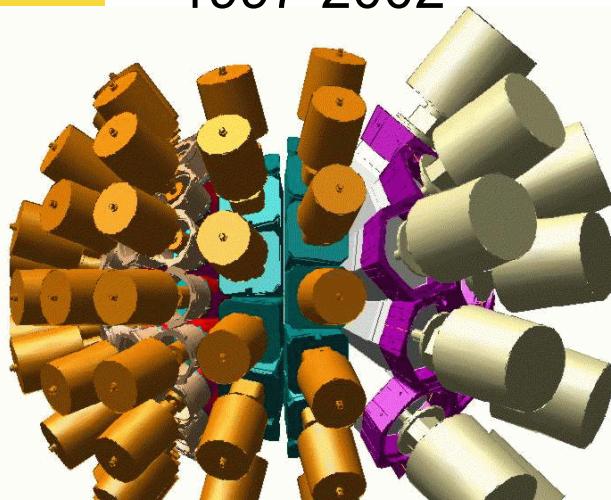
MPI-HD: 1983-1989



162 NaI detectors

EUROBALL

1997-2002



45 + 26x4+ 15x7 Ge

HERA/NORDBALL

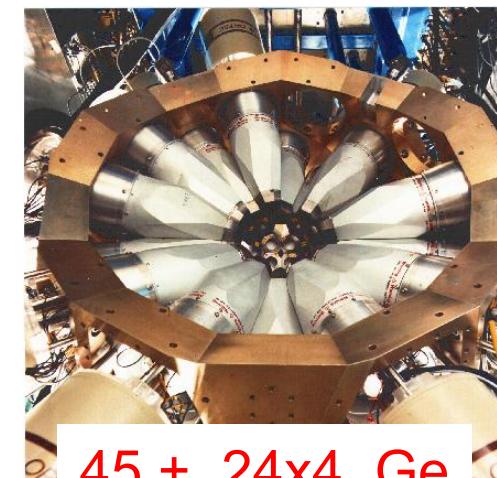
LBL/NBI: 1989-1992



20 Ge detectors

EUROGAM

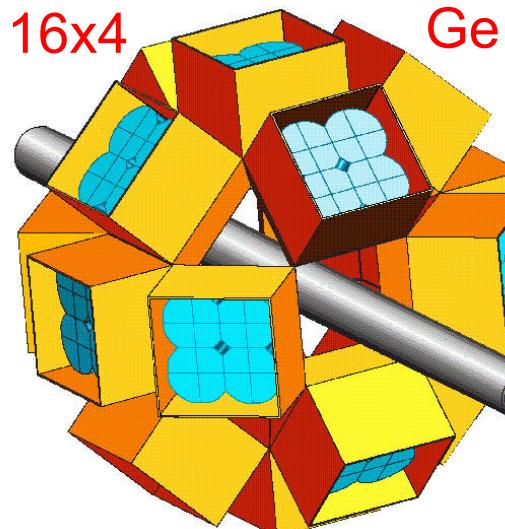
Bonn: 1992-1996



45 + 24x4 Ge

EXOGAM

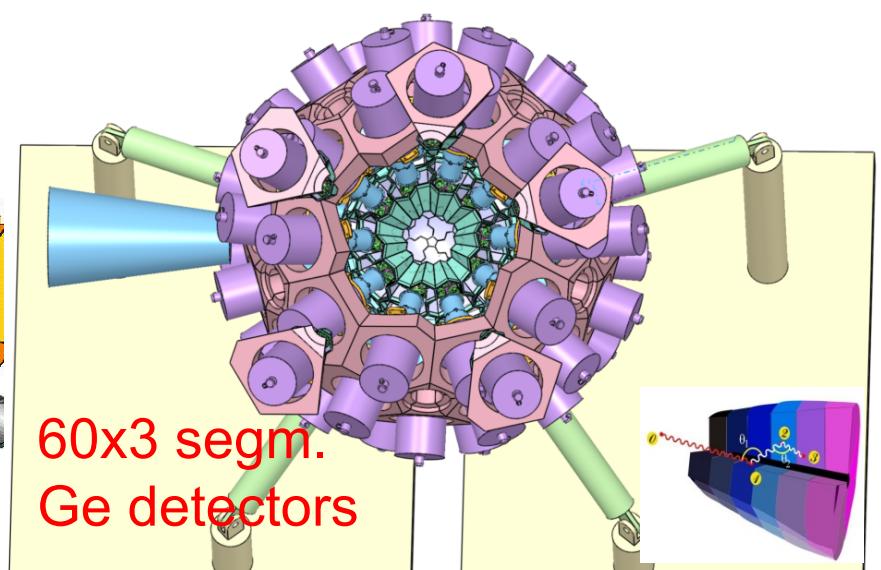
since 2002



16x4 Ge

AGATA

since 2009, today ~40 Ge



60x3 segm.
Ge detectors

Shell structure of atomic nuclei

Z, N = magic numbers

Closed shell = spherical shape

stable double-magic nuclei

^4He , ^{16}O , ^{40}Ca , ^{48}Ca , ^{208}Pb

↑ protons

50

82

28

20

50

8

2

neutrons

→

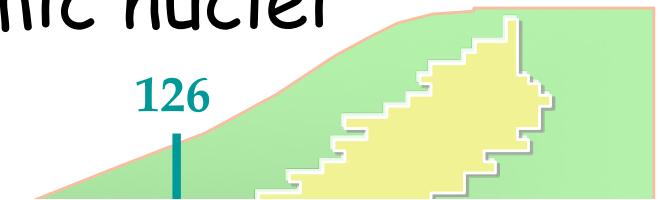
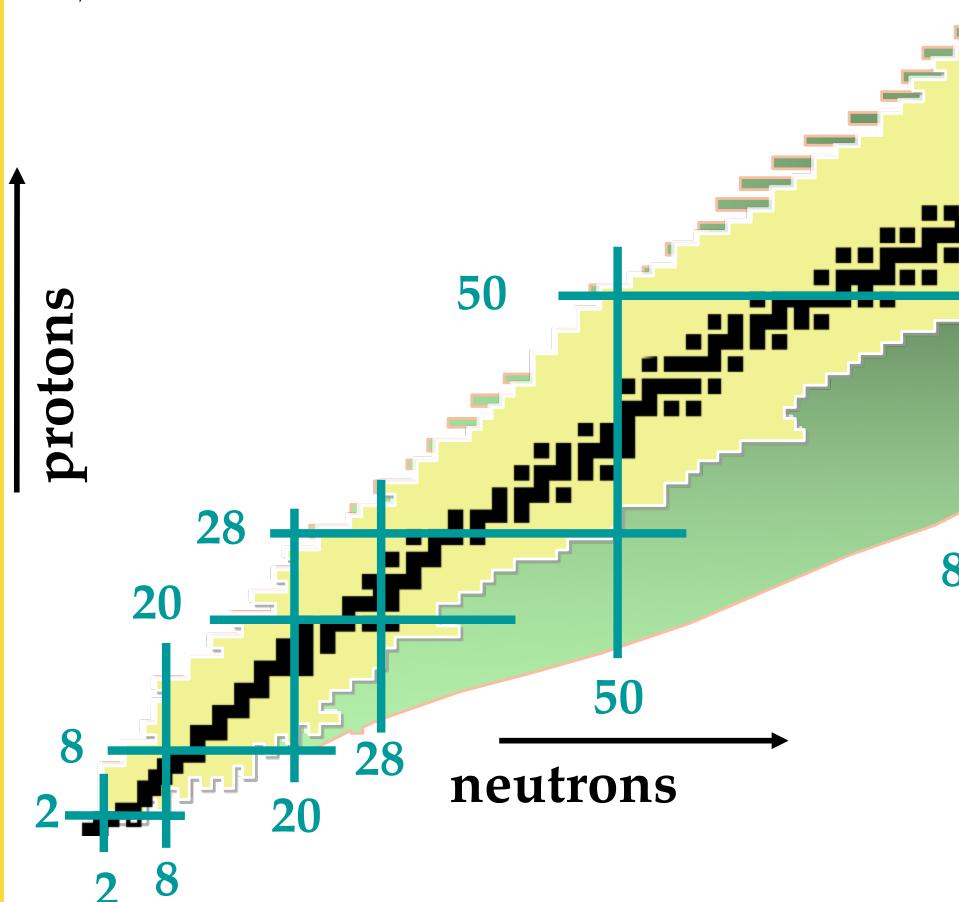
126

radioactive: ^{56}Ni , ^{132}Sn ,
magic ? ^{48}Ni , ^{78}Ni , ^{100}Sn

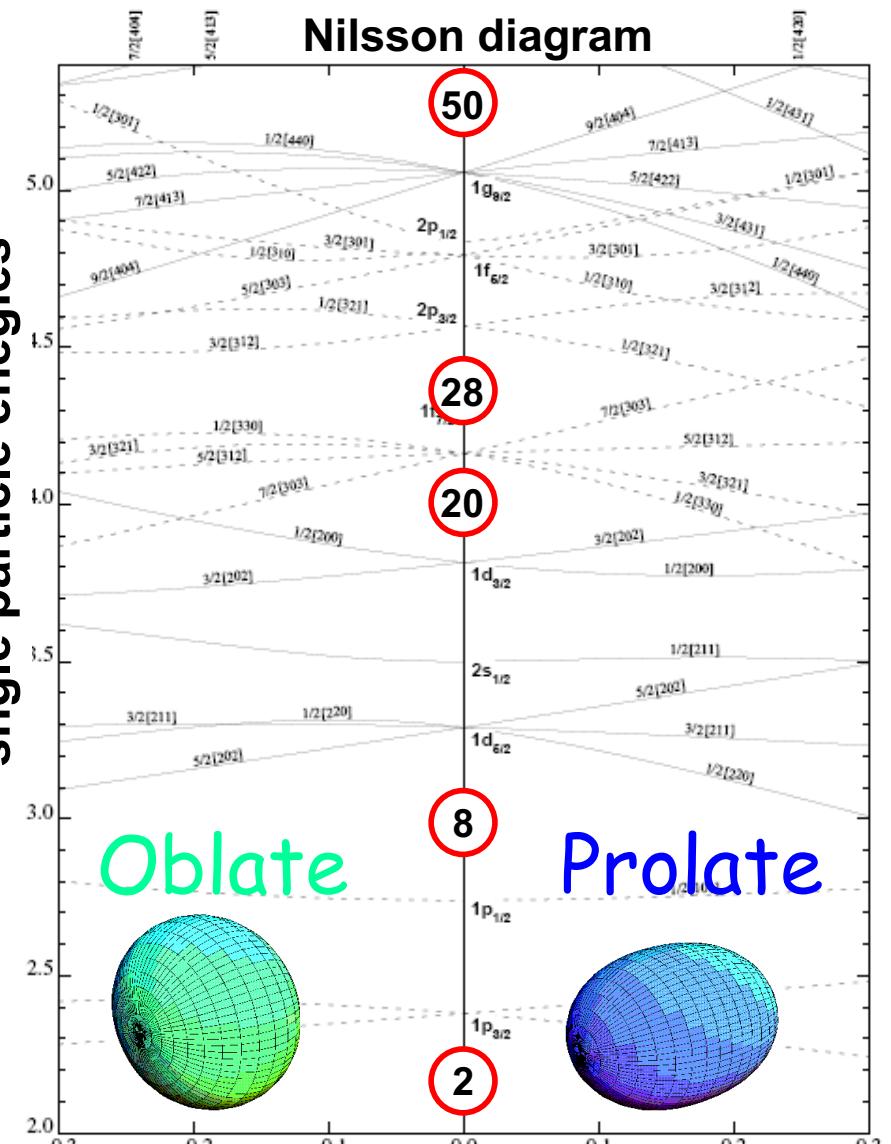


Shapes and shells of atomic nuclei

$Z, N = \text{magic numbers}$



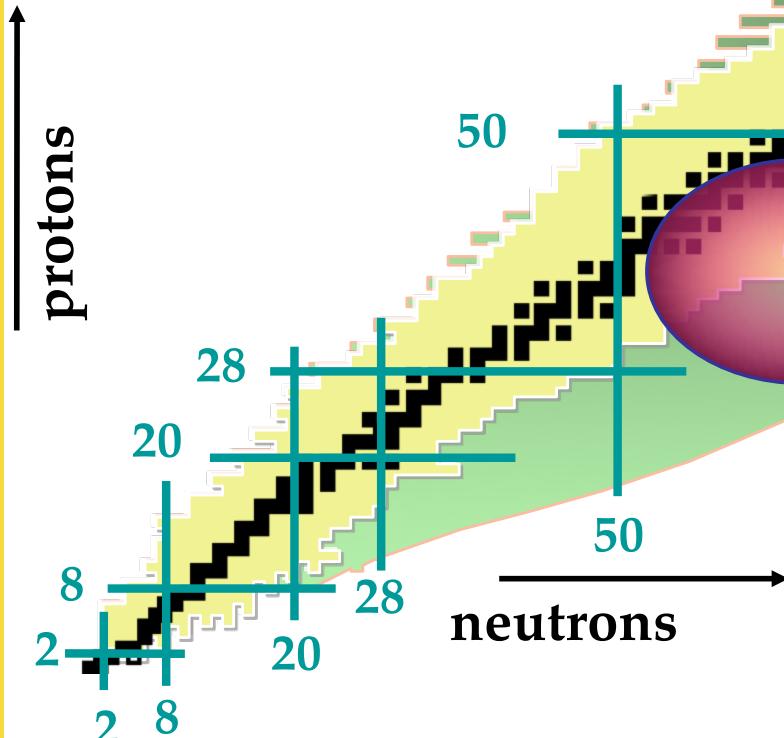
Closed shell = spherical shape



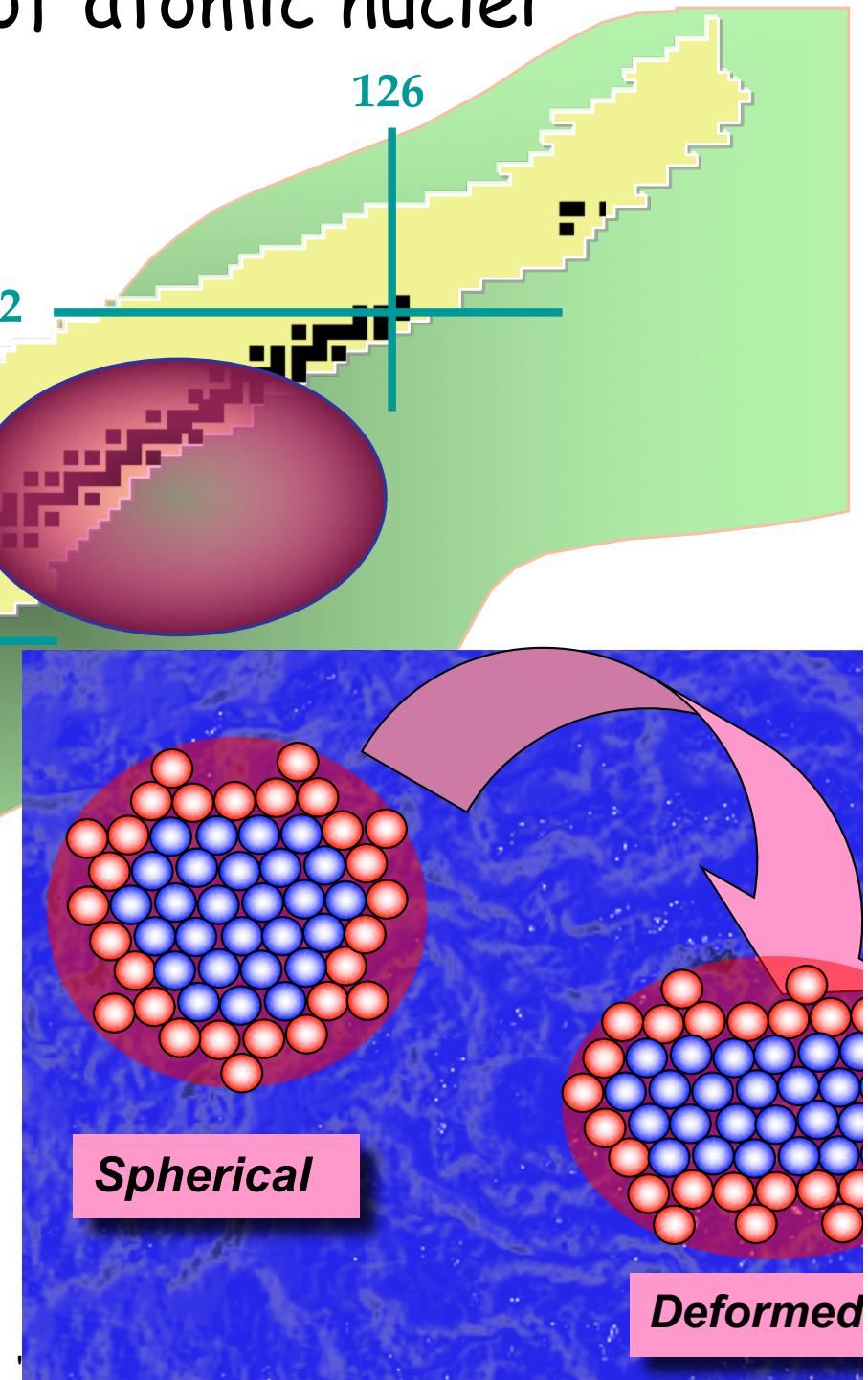
Shapes and shells of atomic nuclei

$Z, N = \text{magic numbers}$

Closed shell = spherical shape



The vast majority of all nuclei shows a non-spherical mass distribution





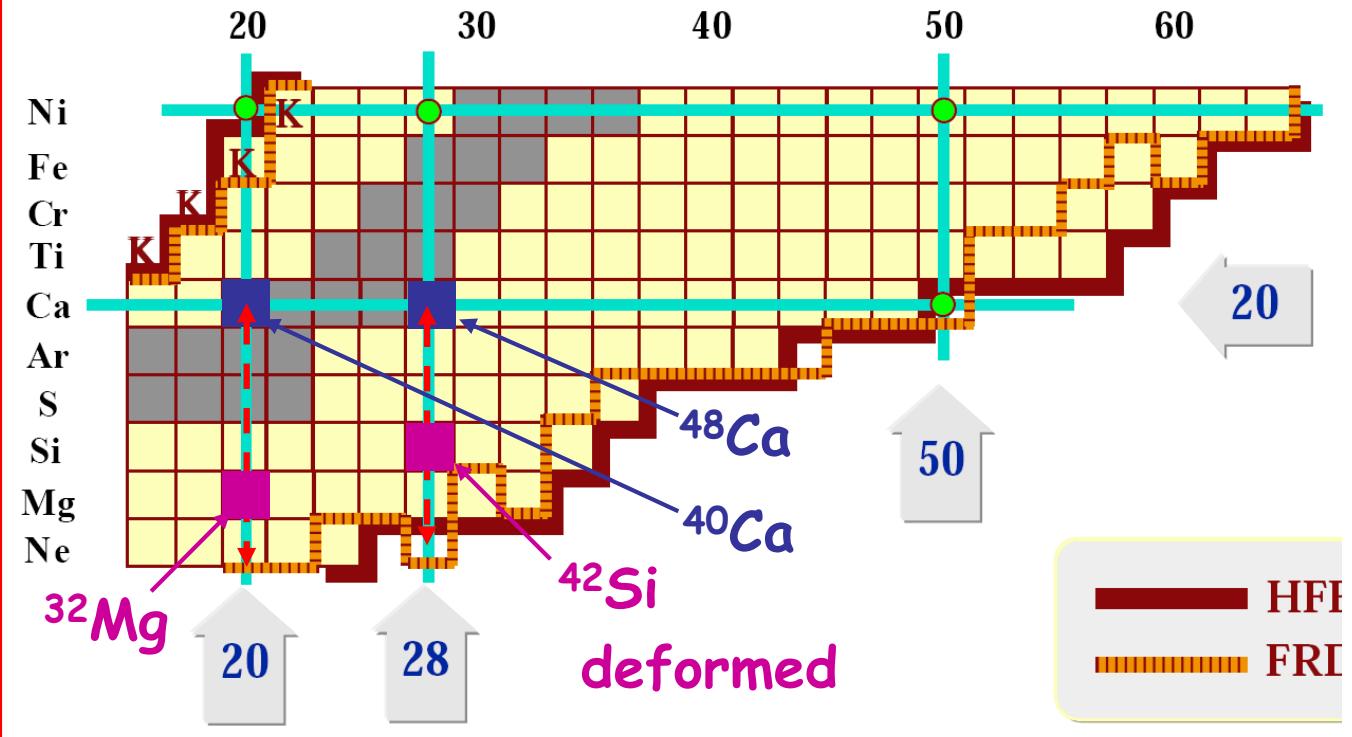
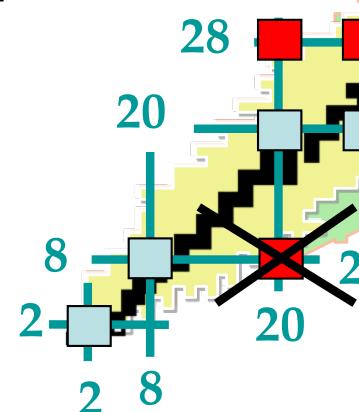
Shapes and shells of atomic nuclei

$Z, N = \text{magic numbers}$

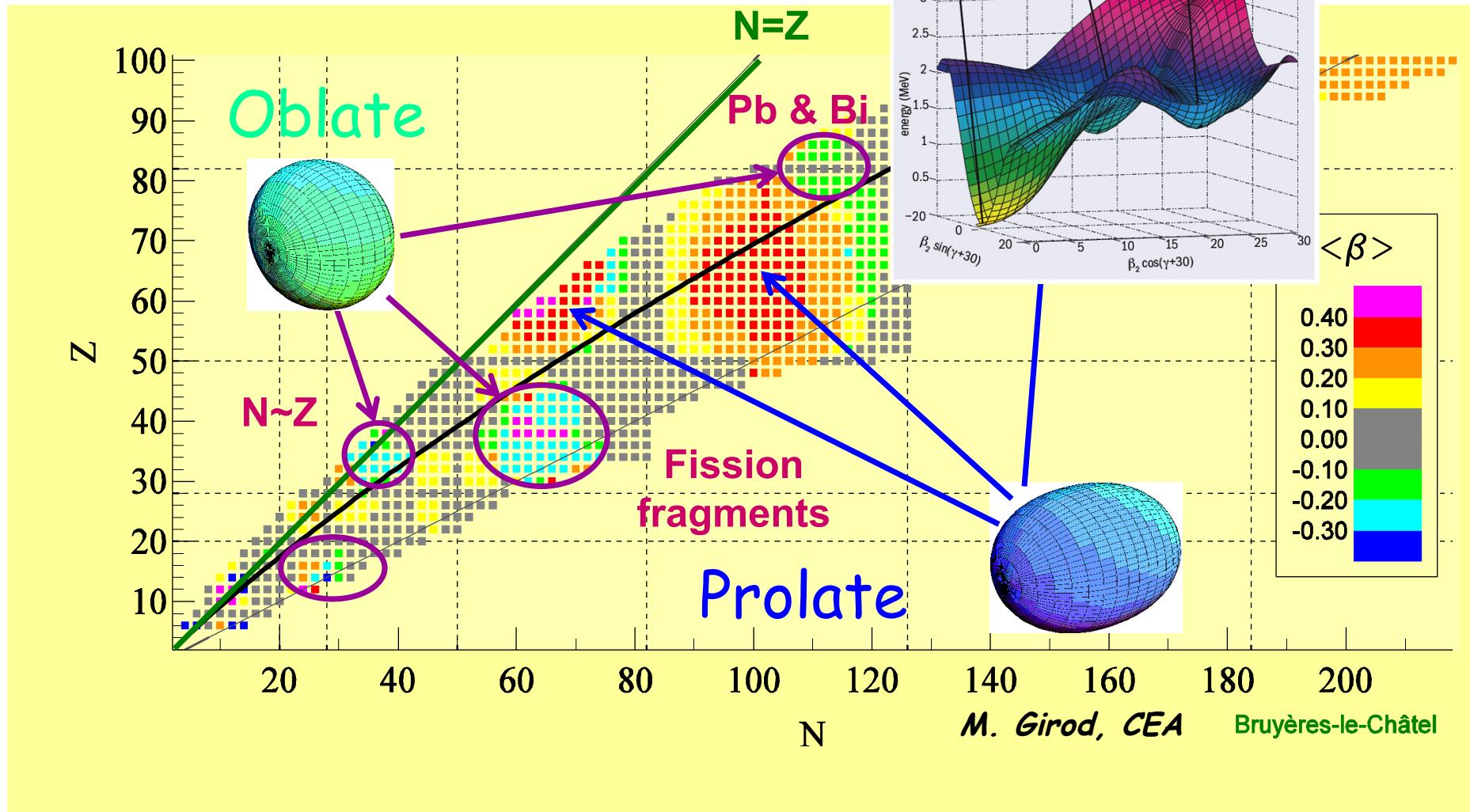
Closed shell = spherical shape ^{82}Se

stable double-magic nuclei
 $^4\text{He}, ^{16}\text{O}, ^{40}\text{Ca}, ^{48}\text{Ca}, ^{208}\text{Pb}$

↑ protons

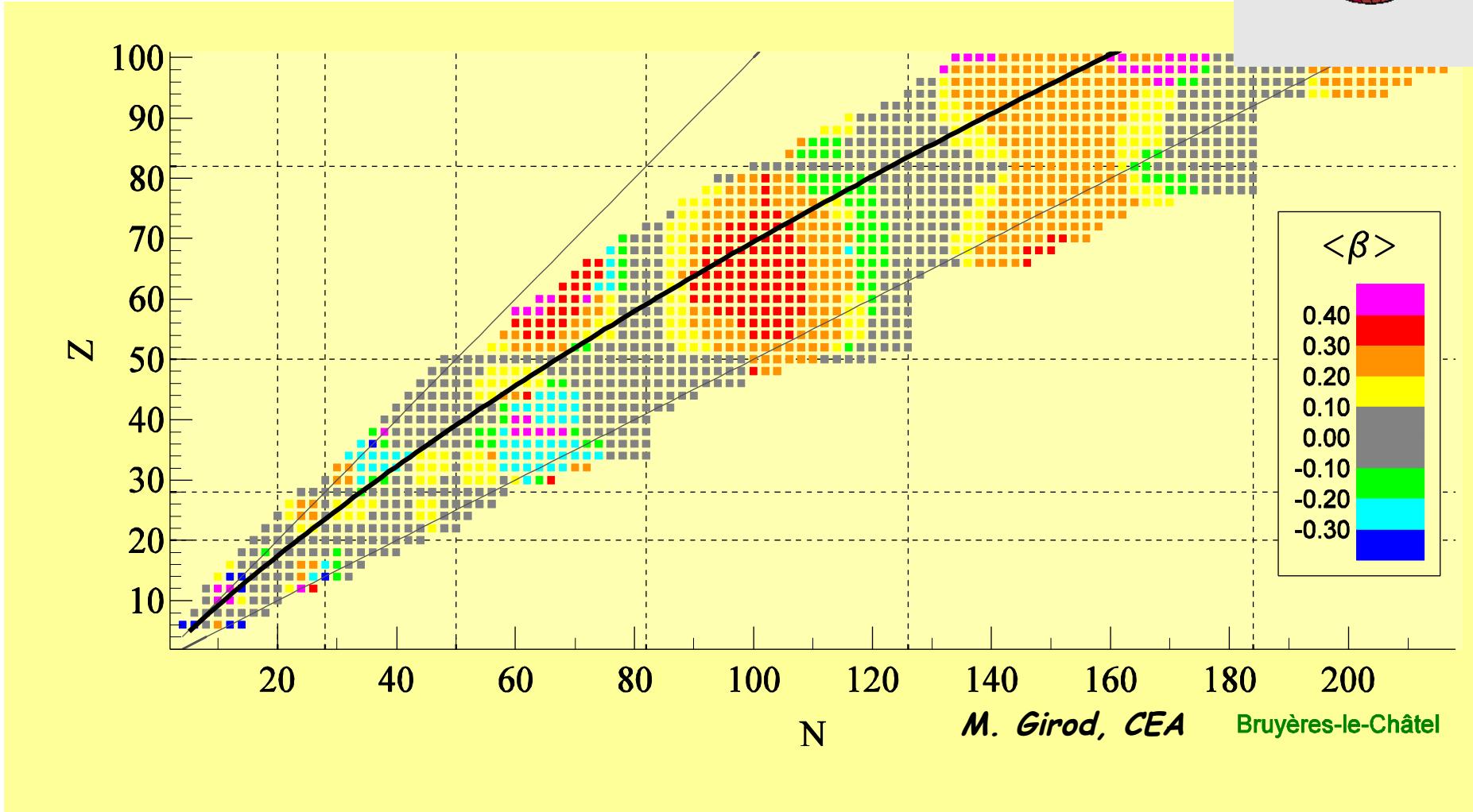
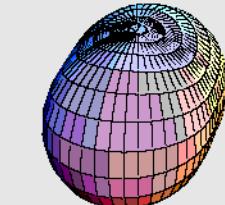


Quadrupole deformation of nuclei



Oblate deformed nuclei are far less abundant than prolate nuclei
Shape coexistence possible for certain regions of N & Z

Quadrupole deformation of nuclei



Coulomb excitation can, in principal, map the shape of all atomic nuclei

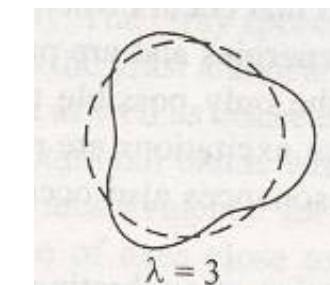
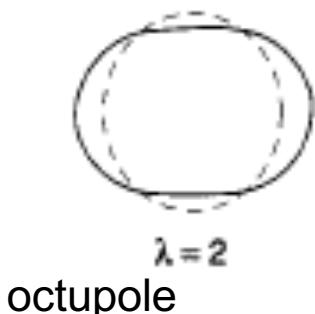
Nuclear shapes and deformation parameters

Generic nuclear shapes can be described by a development of spherical harmonics

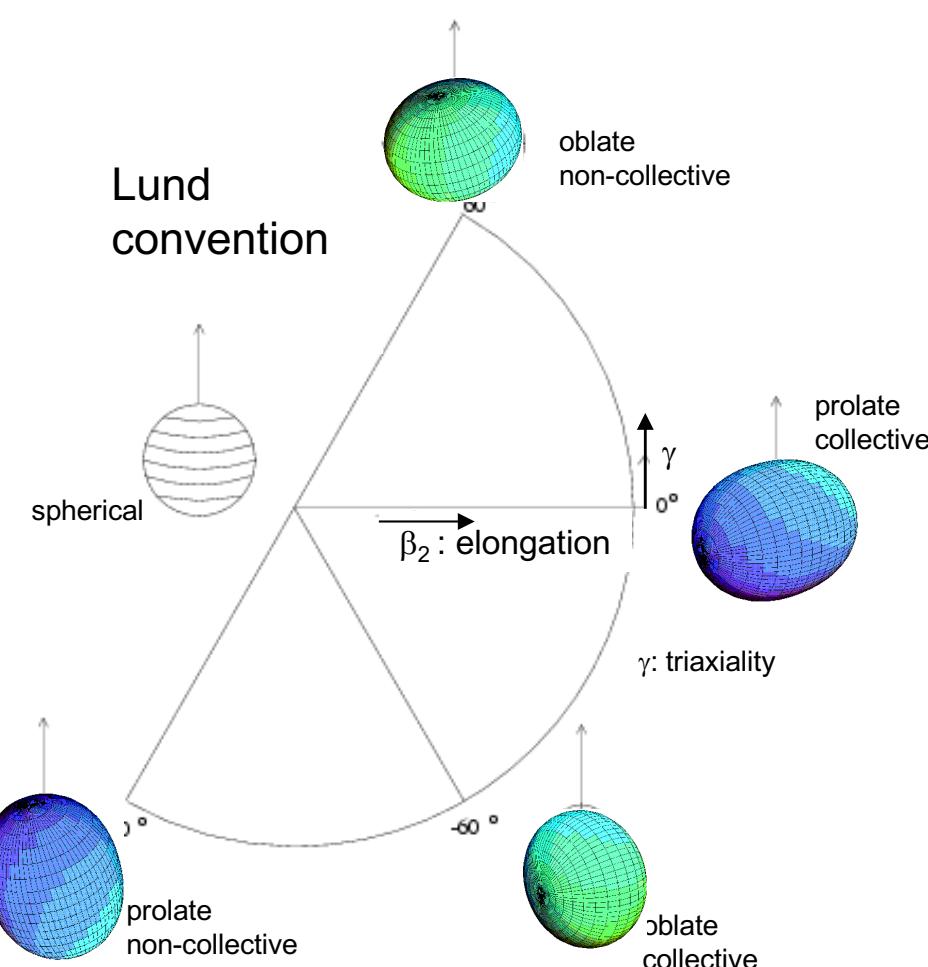
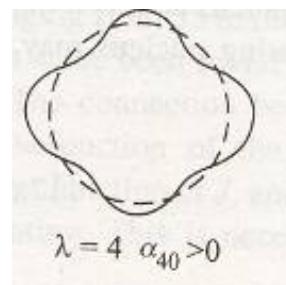
$$R(t) = R_0 \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} a_{\lambda\mu}(t) Y_{\lambda\mu}(\vartheta, \varphi) \right]$$

quadrupole $a_{20} = \beta_2 \cos \gamma$ $a_{22} = a_{2-2} = \frac{1}{\sqrt{2}} \beta_2 \sin \gamma$

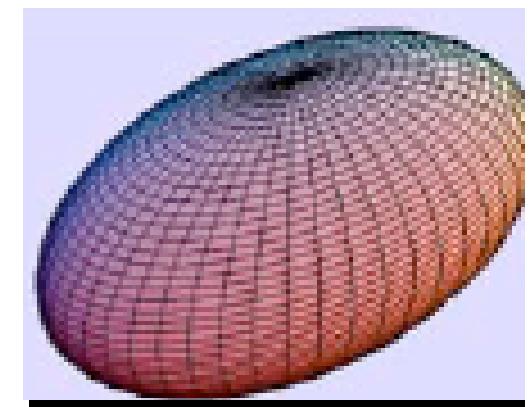
$\alpha_{\lambda\mu}$: deformation parameters



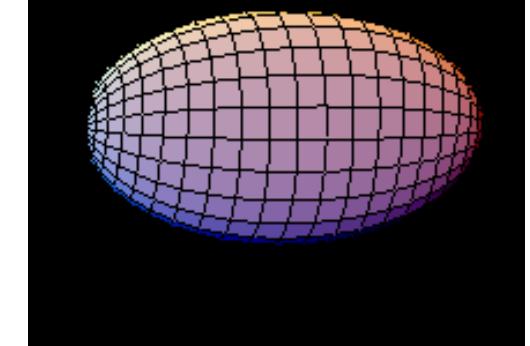
hexadecapole



Tetrahedron Y₂₂₃₂ deformation



Dynamic → vibration



Nuclear shapes and electric multipole moments

Deformation parameters of the nuclear mass distribution

$$R(t) = R_0 \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} a_{\lambda\mu}(t) Y_{\lambda\mu}(\vartheta, \varphi) \right]$$

can be linked to the electric multipole moments of the charge distribution

$$M(E\lambda, \mu) \equiv Q_{\lambda, \mu} = \sqrt{\frac{2\lambda+1}{16\pi}} \int_0^R \rho(r) r^\lambda Y_{\lambda\mu}(\theta, \varphi) r^2 dr d\Omega$$

For **axially symmetric shapes** ($\beta_\lambda = \alpha_{\lambda 0}$) and a homogenous density distribution ρ
the quadrupole, octupole and hexadecupole moments (Q_2, Q_3, Q_4) become:

$$Q_2 = \sqrt{\frac{3}{5\pi}} Z R_0^2 \left(\beta_2 + 0.360\beta_2^2 + 0.336\beta_3^2 + 0.328\beta_4^2 + 0.967\beta_2\beta_4 + O(\beta^3) \right) [fm^2]$$

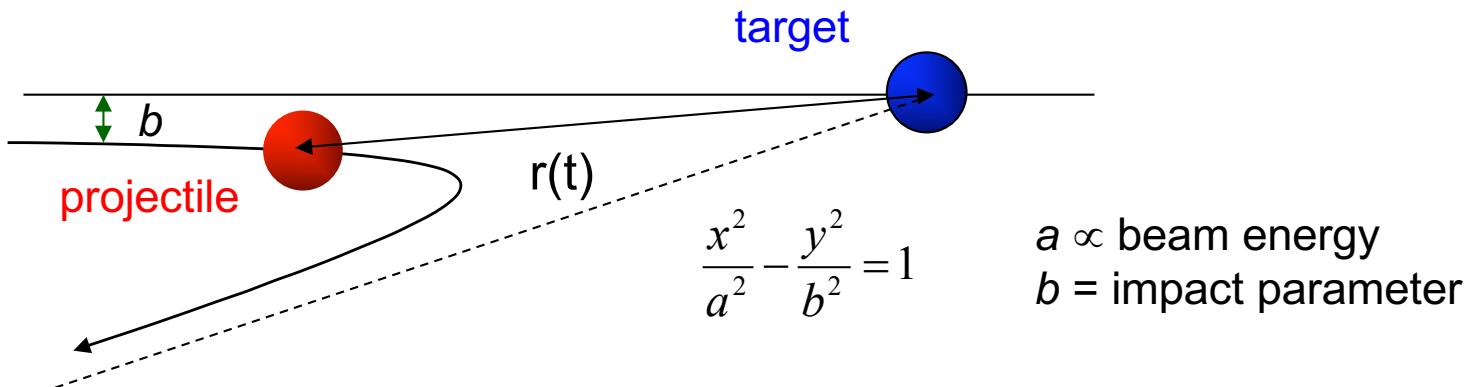
$$Q_3 = \sqrt{\frac{3}{7\pi}} Z R_0^3 \left(\beta_3 + 0.841\beta_2\beta_3 + 0.769\beta_3\beta_4 + O(\beta^3) \right) [fm^3]$$

$$Q_4 = \sqrt{\frac{1}{\pi}} Z R_0^4 \left(\beta_4 + 0.725\beta_2^2 + 0.462\beta_3^2 + 0.411\beta_4^2 + 0.983\beta_2\beta_4 + O(\beta^3) \right) [fm^4]$$

$$Q_1 = C_{LD} Z A \left(\beta_2\beta_3 + 1.46\beta_3\beta_4 + O(\beta^3) \right) [fm]$$

Coulomb excitation - an introduction

Rutherford scattering - some reminders

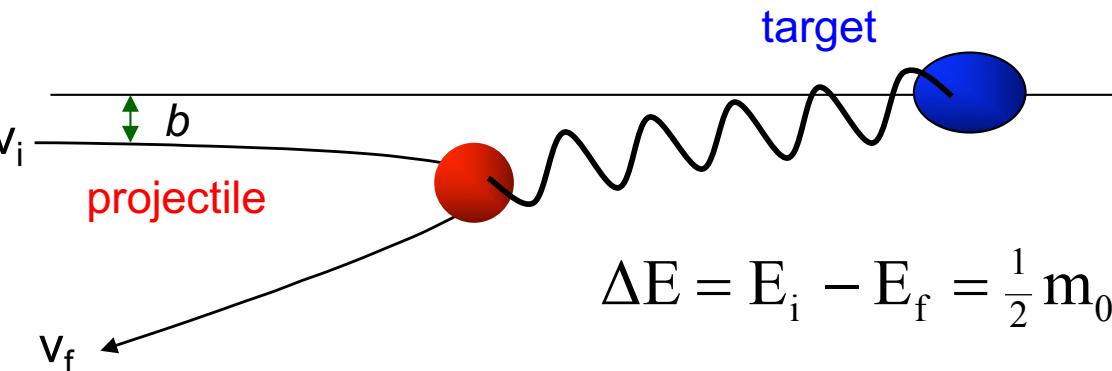


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a \propto$ beam energy
 b = impact parameter

- Elastic scattering of charged particles (point-like → monopoles) under the influence of the Coulomb field $F_C = Z_1 Z_2 e^2 / r^2$ with $r(t) = |\mathbf{r}_1(t) - \mathbf{r}_2(t)|$
→ hyperbolic relative motion of the reaction partners
- Rutherford cross section
 $d\sigma/d\theta = Z_1 Z_2 e^2 / E_{cm}^2 \sin^{-4}(\theta_{cm}/2)$
valid as long as $E_{cm} = m_0 v^2 = \frac{m_p \cdot m_T}{m_p + m_T} v^2 \ll V_c = Z_1 Z_2 e^2 / R_{int}$

Coulomb excitation - the principal process

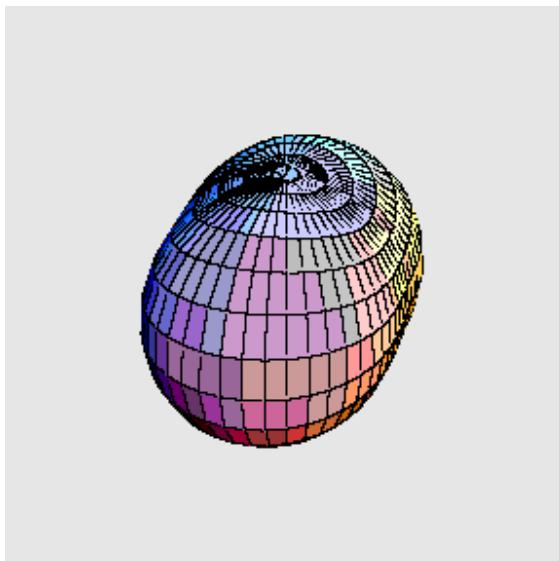


$$\Delta E = E_i - E_f = \frac{1}{2} m_0 (v_i^2 - v_f^2) \text{ with } m_0 = \frac{m_p \cdot m_T}{m_p + m_T}$$

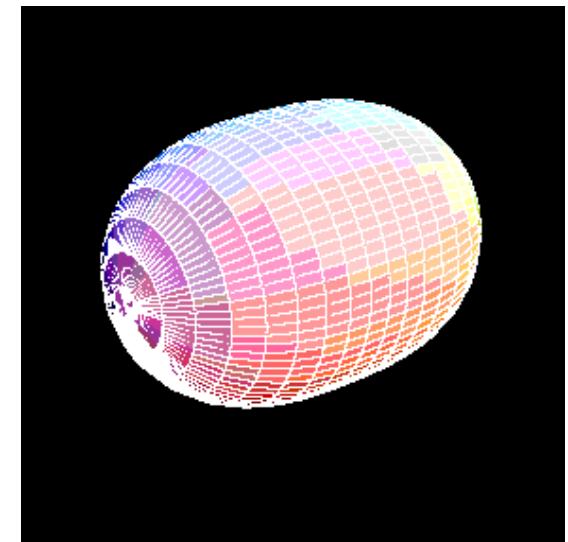
Inelastic scattering: kinetic energy is transformed into nuclear excitation energy

e.g. rotation

vibration

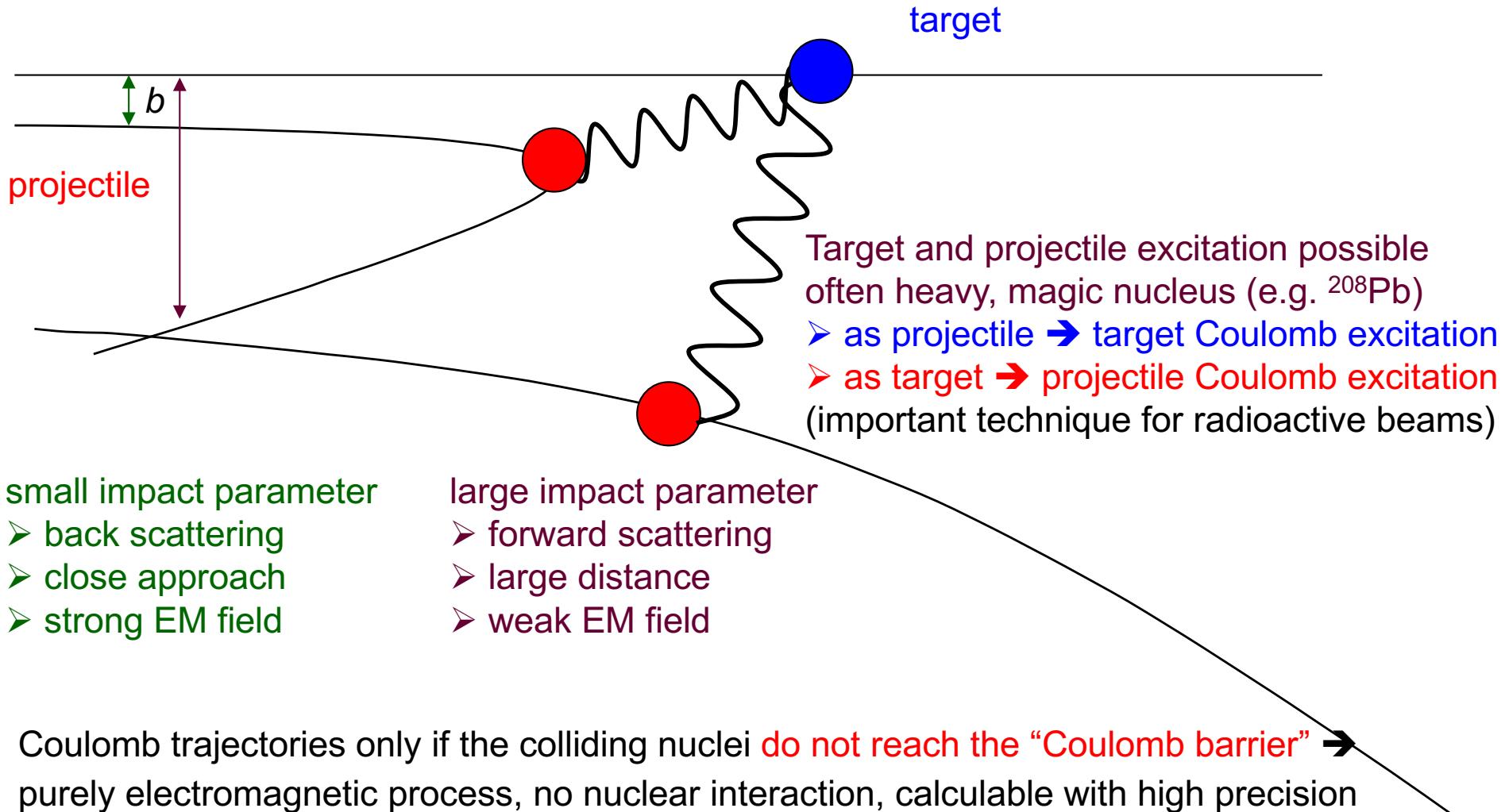


Excitation probability (or cross section) is a **measure of the collectivity** of the nuclear state of interest
 → complementary to, e.g., transfer reactions

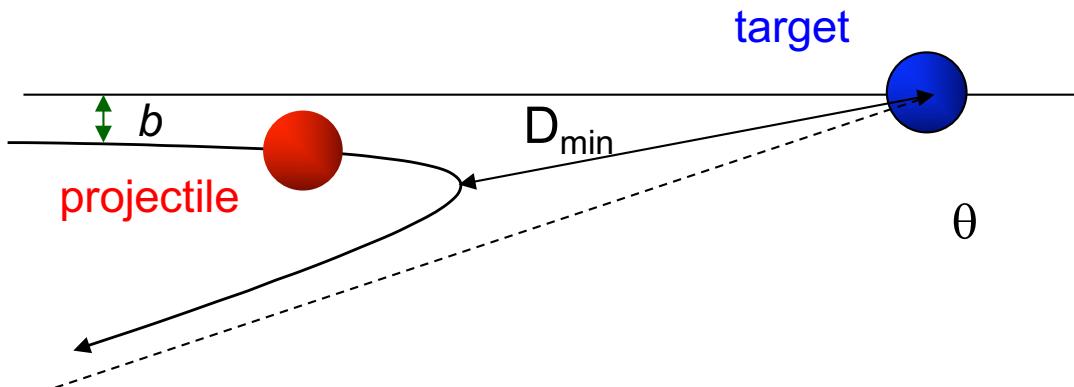


Coulomb excitation - some basics

Nuclear excitation by the **electromagnetic interaction** acting between two colliding nuclei.



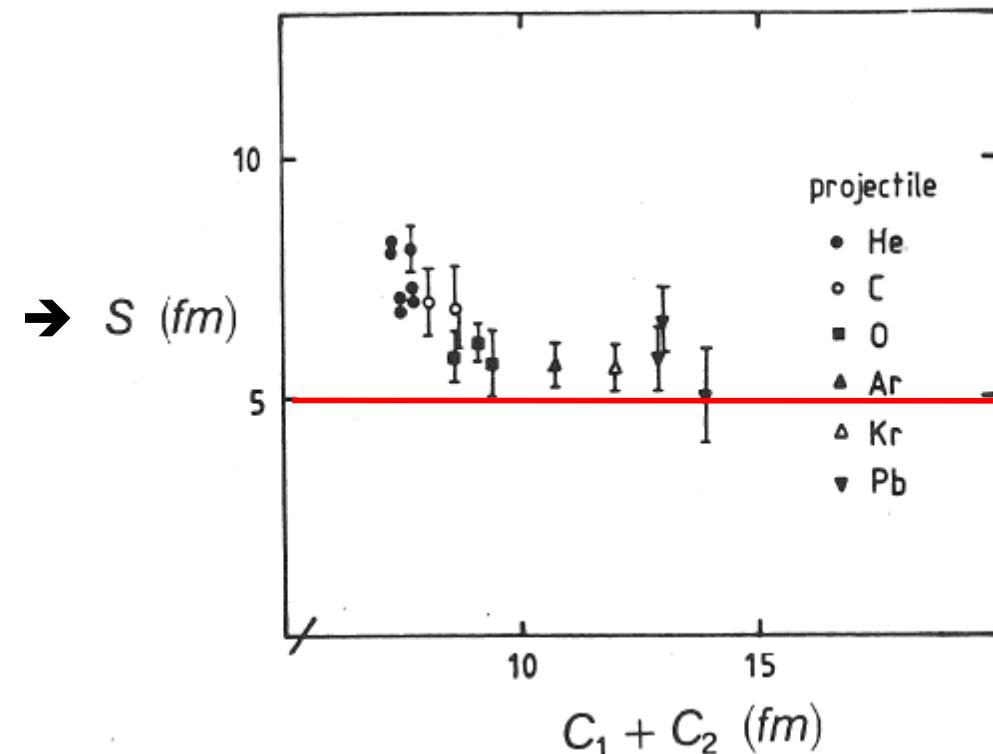
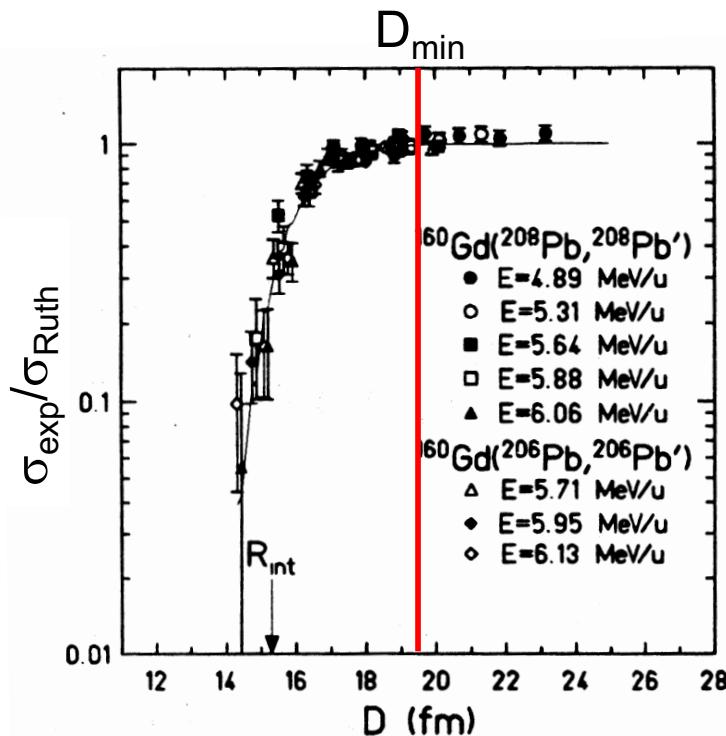
„Safe“ energy requirement



$$E_{\text{cm}} = \frac{Z_P Z_T e^2}{D_{\min}}$$

- Rutherford scattering is only valid as long as the distance of closest approach (D_{\min}) is large compared to the **nuclear radii plus surfaces**:
„Simple“ approach using the liquid-drop model
$$D_{\min} \geq r_s = [1.25 (A_1^{1/3} + A_2^{1/3}) + 5] \text{ fm}$$
- More realistic approach using the **half-density radius** of a **Fermi mass distribution** of the nuclei :
 $C_i = R_i(1-R_i^{-2})$ with $R = 1.28 A^{1/3} - 0.76 + 0.8 A^{-1/3}$
 $\rightarrow D_{\min} \geq r_s = [C_1 + C_2 + S] \text{ fm}$

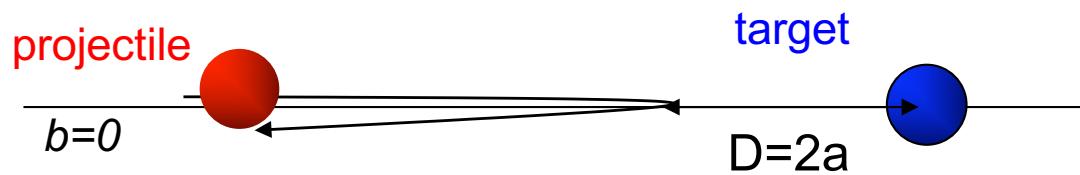
„Safe“ energy requirement



Empirical data on surface distance S as function of half-density radii C_i require **distance of closest approach $S > 5 - 8 \text{ fm}$**

- choose **adequate beam energy ($D > D_{\min}$ for all θ)**
- low-energy Coulomb excitation
- limit scattering angle, i.e. select impact parameter $b > D_{\min}$,
- high-energy Coulomb excitation

Validity of classical Coulomb trajectories

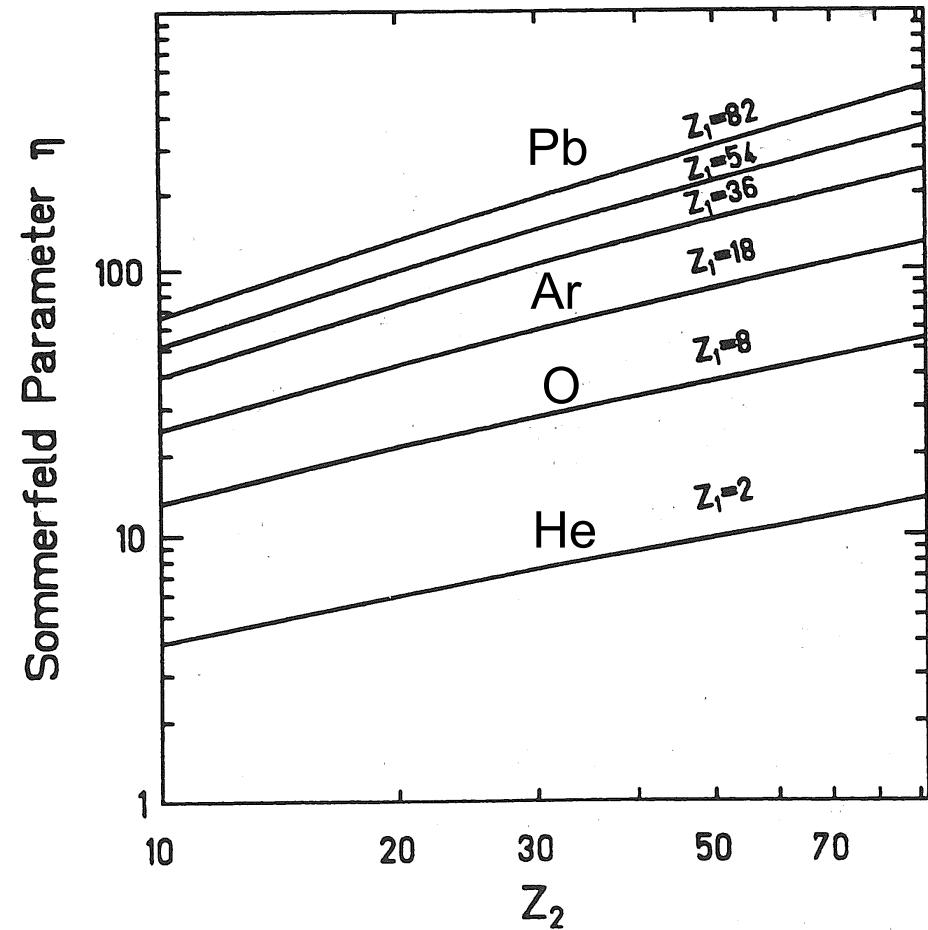


Sommerfeld parameter

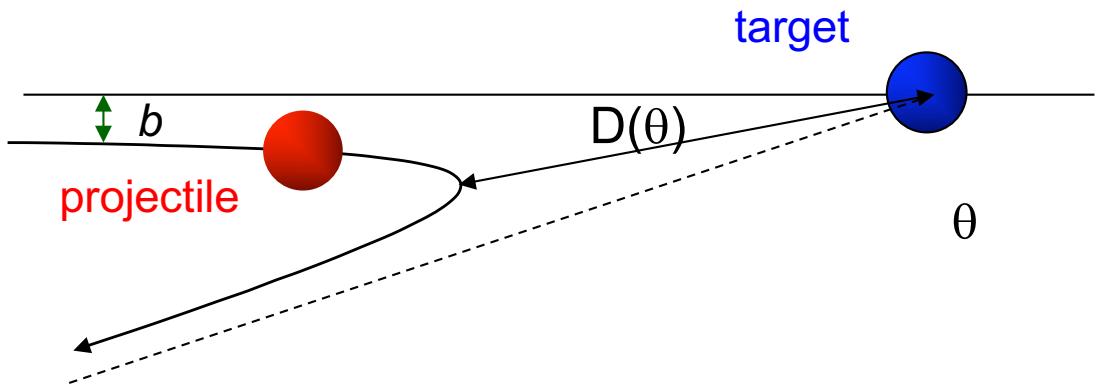
$$\eta = \frac{a}{\lambda} = \frac{Z_P Z_T e^2}{\hbar v_\infty} \gg 1$$

$\eta \gg 1$ requirement for a semi classical treatment of equations of motion

- measures the **strength of the monopole-monopole interaction**
- equivalent to the **number of exchanged photons** needed to force the nuclei on a hyperbolic orbit



Coulomb trajectories - some more definitions



$$\begin{aligned} r(w) &= a (\varepsilon \sinh w + 1) \\ t(w) &= a/v_\infty (\varepsilon \cosh w + w) \\ \text{with } a &= Z_p Z_t e^2 E^{-1} \end{aligned}$$

Principal assumption $\eta \gg 1$ → classical description of the relative motion of the center-of-mass of the two nuclei → hyperbolic trajectories

- distance of closest approach (for $w=0$): $D(\theta_{cm}) = a(1+\varepsilon) = a \left[1 + \sin\left(\frac{\theta_{cm}}{2}\right)^{-1} \right]$
- impact parameter: $b = \sqrt{D^2 - 2aD} = a \cdot \cot\left(\frac{\theta_{cm}}{2}\right)$
- angular momentum : $L = \hbar \eta \sqrt{\varepsilon^2 - 1} = \hbar \eta \cot\left(\frac{\theta_{cm}}{2}\right)$

Coulomb excitation - "sudden impact"

Excitation occurs only if nuclear time scale is long compared to the collision time:
„sudden impact“ if $\tau_{\text{nucl}} \gg \tau_{\text{coll}} \sim a/v \approx 10 \text{ fm} / 0.1c \approx 2-3 \cdot 10^{-22} \text{ s}$
with $\tau_{\text{nucl}} \sim \hbar/\Delta E \rightarrow$ adiabatic limit for (single-step) excitations

$$\xi = \frac{\Delta E}{\hbar} \cdot \tau_{\text{coll}} = \frac{\Delta E}{\hbar} \frac{a}{v_\infty} = \frac{Z_1 Z_2 e^2}{\hbar} \left(\frac{1}{v_f} - \frac{1}{v_i} \right)$$

ξ : adiabaticity parameter

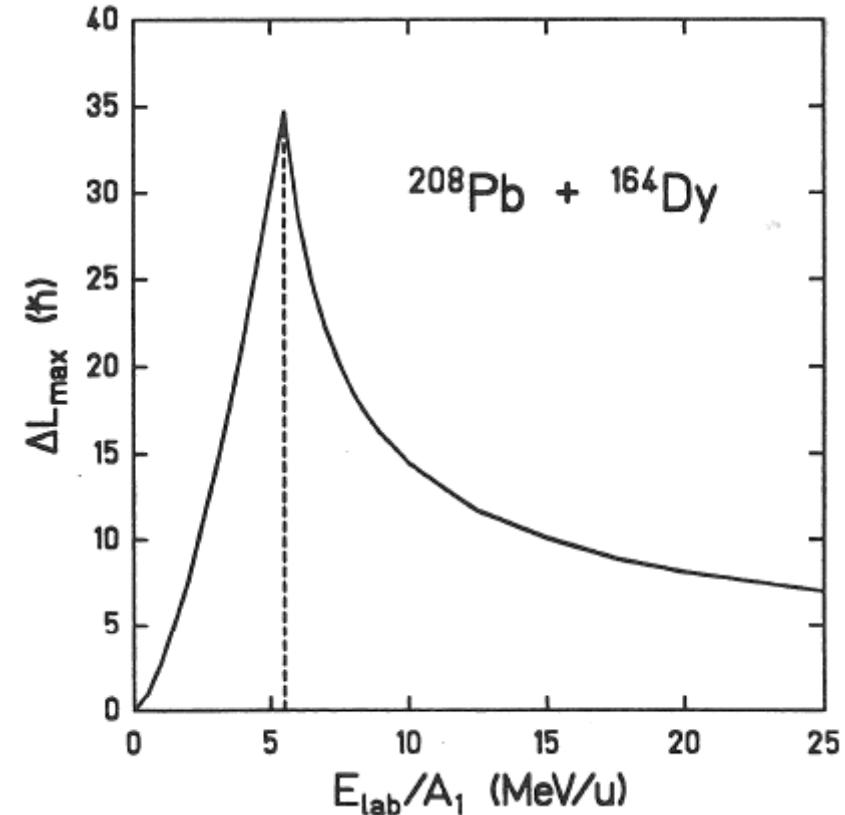
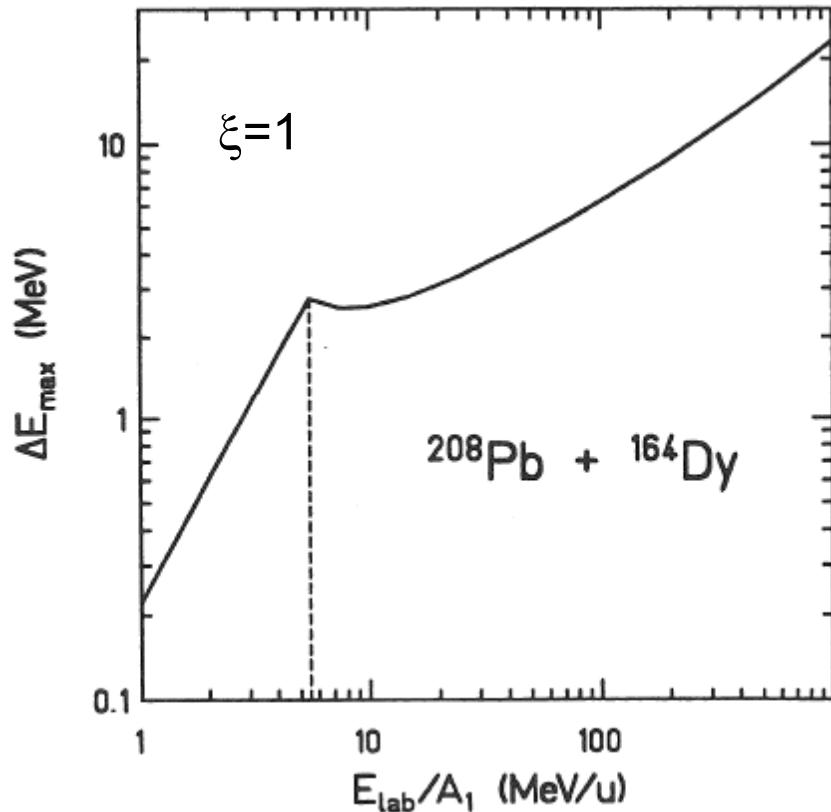
sometimes also $\xi(\theta)$ with $D(\theta)$ instead of a

$$\Rightarrow \Delta E_{\max} (\xi = 1) = \frac{\hbar v_\infty}{a}$$

Limitation in the excitation energy ΔE
for single-step excitations in particular
for low-energy reactions ($v < c$)

Coulomb excitation - first conclusions

Maximal transferable **excitation energy** and **spin** in heavy-ion collisions



$$\Delta E_{\max}(\xi = 1) = \frac{\hbar c}{a} \frac{v_i}{c} \quad \text{for } v_i/c \ll 1$$

$$\Delta E_{\max}(\xi = 1) = \frac{\hbar c}{a} \beta \gamma$$

$$L_{\max} = \frac{Z_1 e^2 Q_{\text{rot}}}{\hbar v_{\infty} D^2} (1 - \cos \theta_{\text{cm}})$$

$$L_{\max} = \hbar \eta \cot\left(\frac{\theta_{\text{gr}}}{2}\right) \quad E_{\text{CM}} > V_c$$

Coulomb excitation - the different energy regimes

Low-energy regime (< 5 MeV/u)

Energy cut-off $\Delta E_{\max} = \frac{\hbar v_{\infty}}{a \epsilon} \approx 2 \text{ MeV}$

Spin cut-off: ΔL up to $30\hbar$

High-energy regime (>>5 MeV/u)

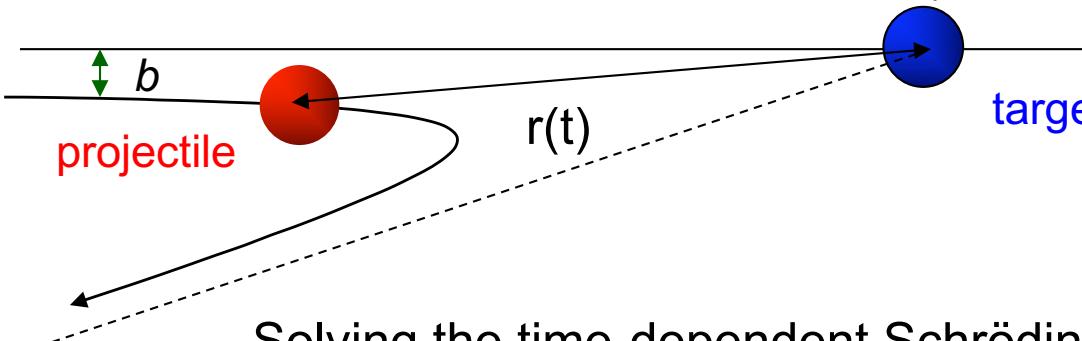
$\Delta E_{\max} = \hbar c \frac{\beta \gamma}{a \epsilon} \approx 10 \text{ MeV} (\beta = 0.4)$

$\Delta L \sim n\lambda$ with $n \sim 1$

Summary (1)

- Coulomb excitation is a **purely electro-magnetic excitation** process of nuclear states due to the Coulomb field of two colliding nuclei.
- Coulomb excitation is a very precise tool to measure **the collectivity of nuclear excitations** and in particular **nuclear shapes**.
- Coulomb excitation **appears in all nuclear reactions** (at least in the incoming channel) and can lead to doorway states for other excitations.
- Pure electro-magnetic interaction (which can be readily calculated without the knowledge of optical potentials etc.) requires “safe” distance between the partners at all times.

Coulomb excitation theory - the general approach



target

$$r(w) = a (\varepsilon \sinh w + 1)$$

$$t(w) = a/v_\infty (\varepsilon \cosh w + w)$$

$$a = Z_p Z_t e^2 E^{-1}$$

Solving the time-dependent Schrödinger equation:

$$i\hbar d\psi(t)/dt = [H_P + H_T + V(r(t))] \psi(t)$$

with $H_{P/T}$ being the free Hamiltonian of the projectile/target nucleus and $V(t)$ being the time-dependent electromagnetic interaction
(remark: often only target or projectile excitation are treated)

Expanding $\psi(t) = \sum_n a_n(t) \phi_n$ with ϕ_n as the eigenstates of $H_{P/T}$
leads to a set of coupled equations for the
time-dependent excitation amplitudes $a_n(t)$

$$i\hbar da_n(t)/dt = \sum_m \langle \phi_n | V(t) | \phi_m \rangle \exp[i/\hbar (E_n - E_m) t] a_m(t)$$

The transition amplitude b_{nm} are calculated by the (action) integral
 $b_{nm} = i\hbar^{-1} \int \langle a_n \phi_n | V(t) | a_m \phi_m \rangle \exp[i/\hbar (E_n - E_m) t] dt$

Finally leading to the excitation probability

$$P(I_n \rightarrow I_m) = (2I_n + 1)^{-1} b_{nm}^2$$

Coulomb excitation theory - the general approach

The coupled equations for $a_n(t)$ are usually solved by a **multipole expansion** of the **electromagnetic interaction $V(r(t))$**

$$\begin{aligned} V_{P-T}(r) = & Z_T Z_P e^2 / r \\ & + \sum_{\lambda\mu} V_P(E\lambda,\mu) \\ & + \sum_{\lambda\mu} V_T(E\lambda,\mu) \\ & + \sum_{\lambda\mu} V_P(M\lambda,\mu) \\ & + \sum_{\lambda\mu} V_T(M\lambda,\mu) \\ & + O(\sigma\lambda, \sigma'\lambda' > 0) \end{aligned}$$

monopole-monopole (Rutherford) term
 electric multipole-monopole target excitation,
 electric multipole-monopole project. excitation,
 magnetic multipole project./target excitation
 (but small at low v/c)
 higher order multipole-multipole terms (small)

$$V_{P/T}(E\lambda,\mu) = (-1)^\mu Z_{T/P} e^{4\pi/(2\lambda+1)} r^{-(\lambda+1)} Y_{\lambda\mu}(\theta, \phi) \cdot M_{P/T}(E\lambda, \mu)$$

$$V_{P/T}(M\lambda,\mu) = (-1)^\mu Z_{T/P} e^{4\pi/(2\lambda+1)} i/c\lambda r^{-(\lambda+1)} dr/dt L Y_{\lambda,\mu}(\theta, \phi) \cdot M_{P/T}(M\lambda, \mu)$$

electric multipole moment:

$$M(E\lambda,\mu) = \int \rho(r') r^\lambda Y_{\lambda\mu}(r') d^3r'$$

magnetic multipole moment:

$$M(M\lambda,\mu) = -i/c(\lambda+1) \int j(r') r'^\lambda (i\mathbf{r} \times \nabla) Y_{\lambda,\mu}(r') d^3r'$$

- ➔ Coulomb excitation cross section is sensitive to **electric multipole moments of all orders**, while angular correlations give also access to magnetic moments

Transition rates in the Coulomb excitation process

- **1st order perturbation theory**

applicable if only one state is excited, e.g. $0^+ \rightarrow 2^+$ excitation,
and for small excitation probability (e.g. semi-magic nuclei)

→ 1st order transition probability for multipolarity λ

$$P_{i \rightarrow f}^{(1)}(\vartheta, \xi) = |\chi_{i \rightarrow f}^{(\lambda)}(\vartheta, \xi)|^2 = |\chi_{i \rightarrow f}^{(\lambda)}|^2 R_\lambda^2(\vartheta, \xi)$$

$$\chi_{i \rightarrow f}^\lambda = \frac{\sqrt{16\pi}(\lambda - 1)!}{(2\lambda + 1)!!} \left(\frac{Z_{T/P} e}{\hbar v_i} \right) \frac{\langle i | M(E\lambda) | f \rangle}{a^\lambda \sqrt{2I_i + 1}} \quad \text{Strength parameter}$$

$$R_\lambda^2(\vartheta, \xi) = \sum |R_{\lambda\mu}(\vartheta, \xi)|^2$$

Orbital integrals

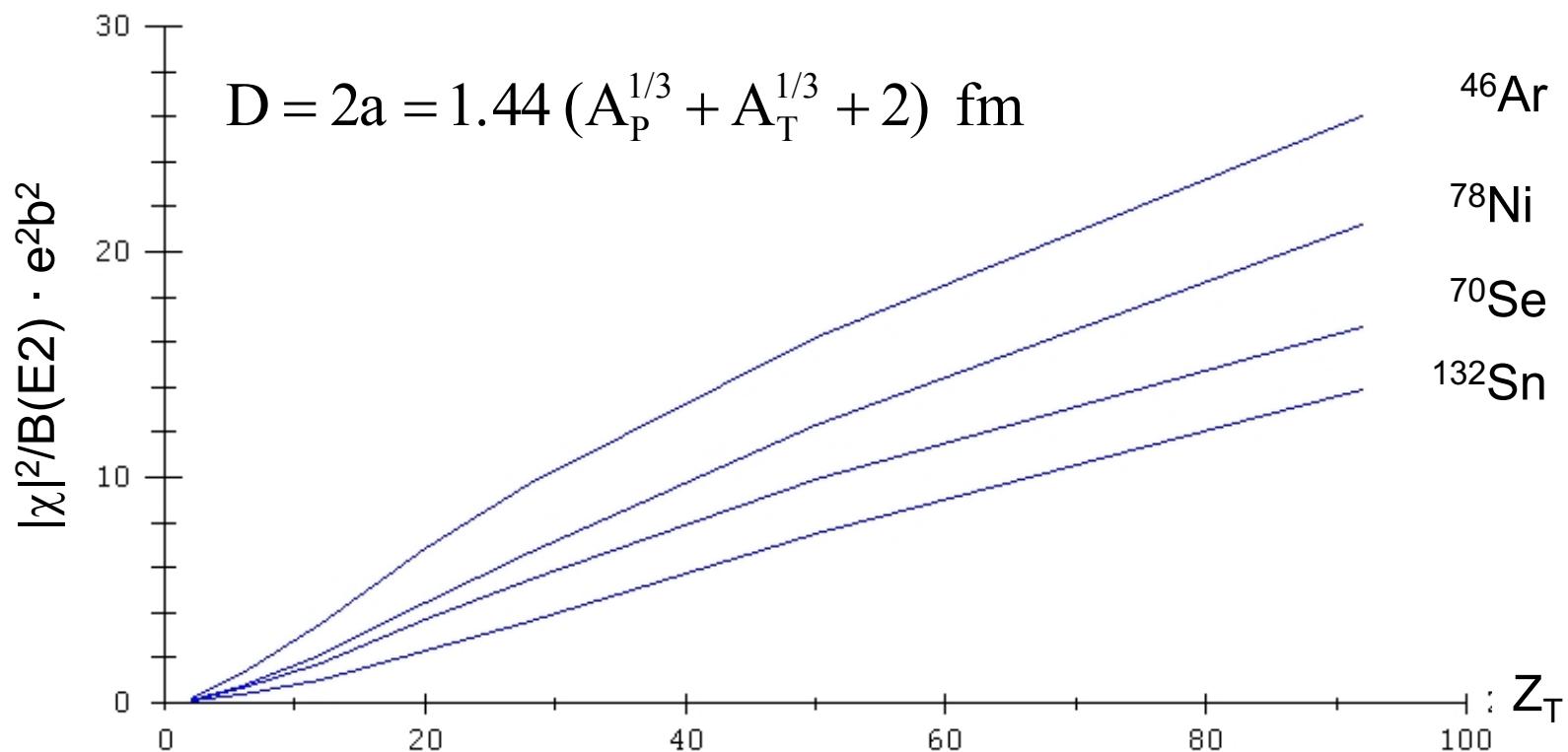
$$\xi = \xi_{if} = \frac{Z_1 Z_2 e^2}{\hbar} \left(\frac{1}{v_f} - \frac{1}{v_i} \right)^\mu$$

Adiabaticity parameter

Strength parameter χ_{E2} as function of (Z_p, Z_T)

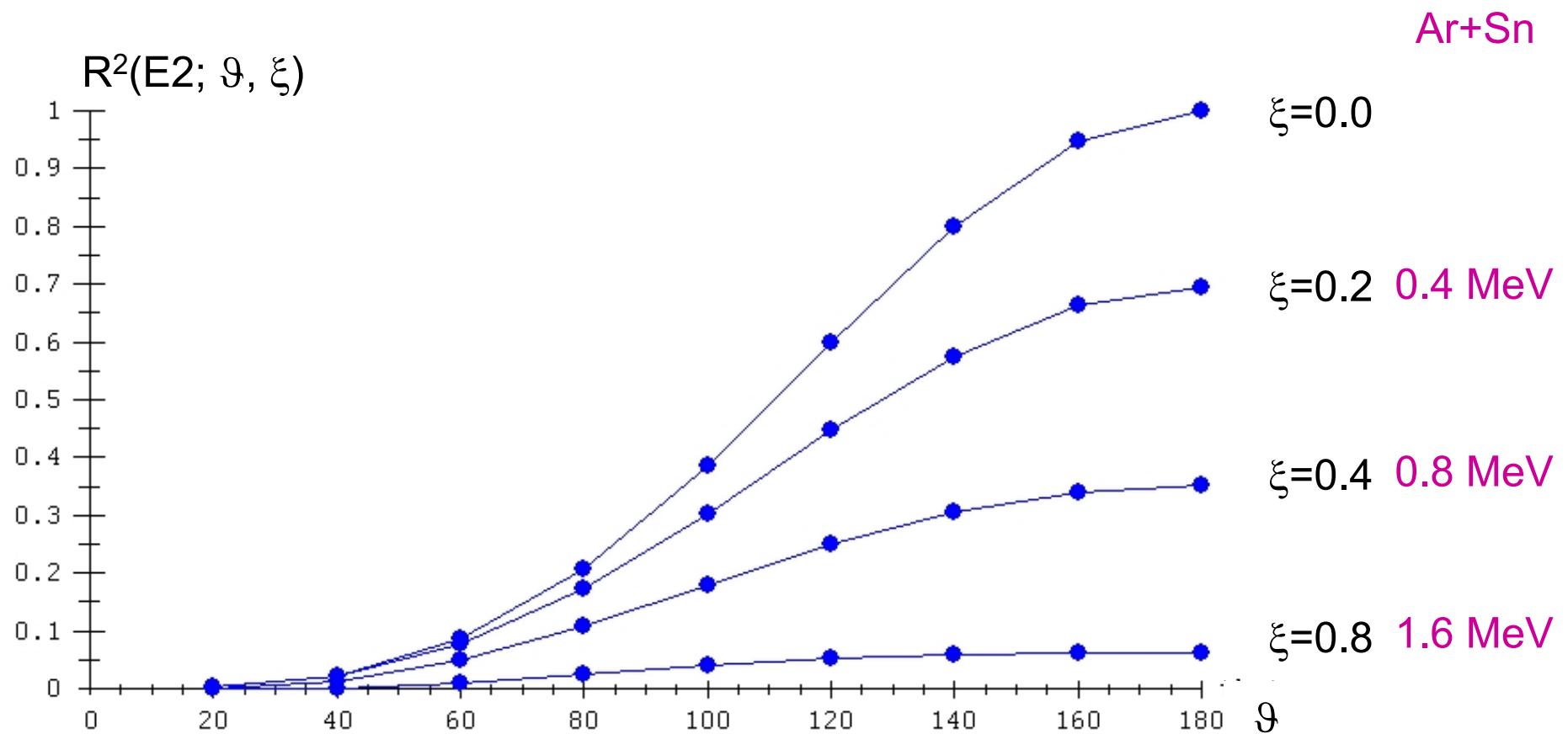
$$|\chi_{i \rightarrow f}^{E2}|^2 = \frac{16\pi}{15^2} \frac{Z_{T/P}^2 e^2}{\hbar^2 v_i v_f} \frac{B(E2; 0 \rightarrow 2)}{(D/2)^4}$$

$$P_{i \rightarrow f}^{E2}(\vartheta = 180^\circ, \xi = 0) = |\chi_{i \rightarrow f}^{(E2)}|^2$$



Orbital integrals $R(E2)$ as function of ϑ and ξ

$$R_\lambda^2(\vartheta, \xi) = \sum_{\mu} |R_{\lambda\mu}(\vartheta, \xi)|^2 \quad \xi = \xi_{\text{if}} = \frac{Z_1 Z_2 e^2}{\hbar} \left(\frac{1}{V_f} - \frac{1}{V_i} \right)$$



Cross section for Coulomb excitation

Differential and total cross sections

$$d\sigma = \frac{1}{4} a^2 \sin^{-4}(\vartheta/2) d\Omega \sum_{\substack{\sigma=E,M \\ \lambda=1,\infty}} |\chi_{i \rightarrow f}^{\sigma\lambda}|^2 R_\lambda^2(\vartheta, \xi)$$

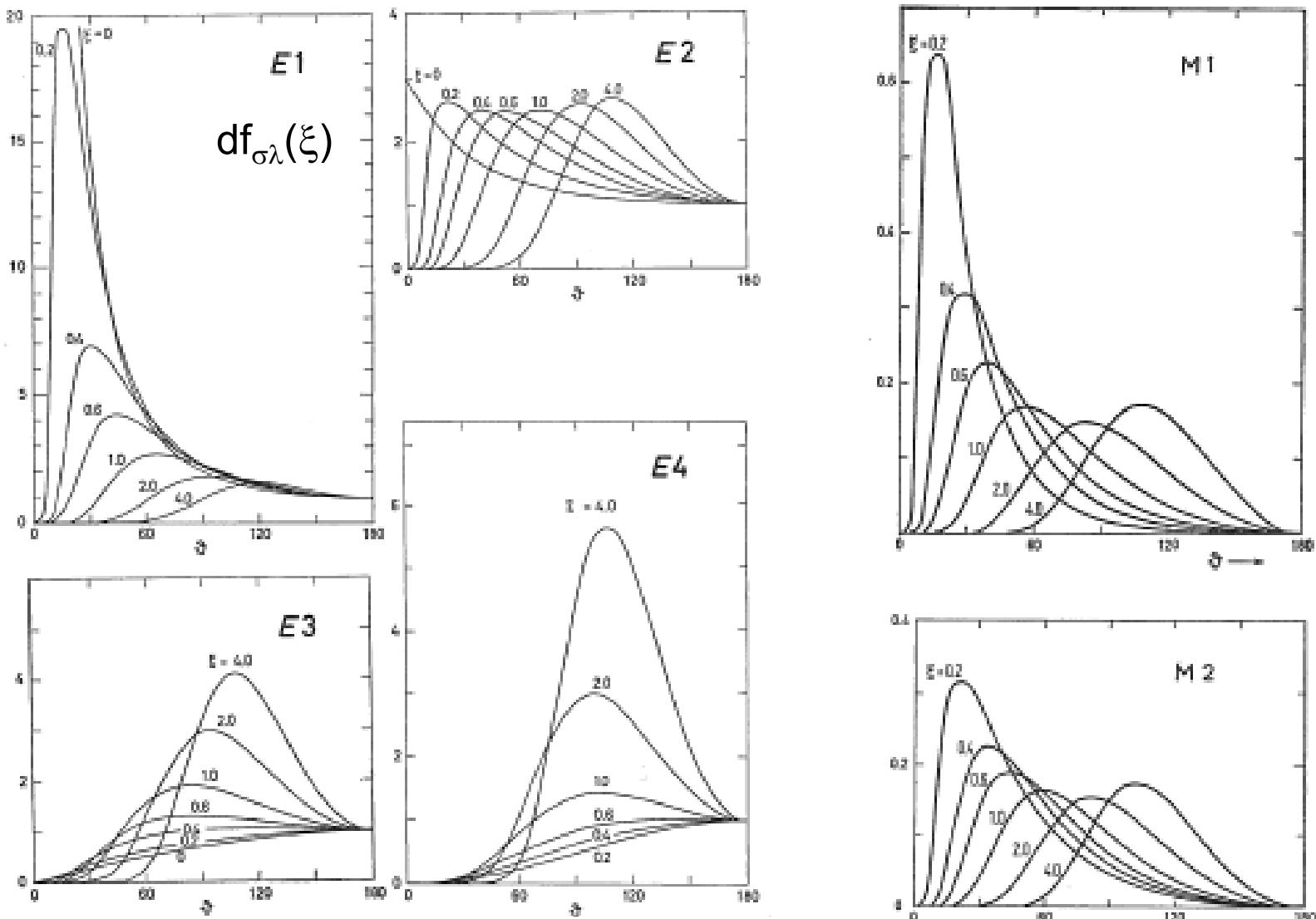
Rutherford

$$= \left(\frac{Z_1 e}{\hbar v} \right)^2 \sum_{\substack{\sigma=E,M \\ \lambda=1,\infty}} a^{-2(\lambda-1)} B(\sigma\lambda; I_i \rightarrow I_f) df_{\sigma\lambda}(\xi)$$

$$\sigma = \sum_{\substack{\sigma=E,M \\ \lambda=1,\infty}} \sigma_{\sigma\lambda} = a^2 \sum_{\substack{\sigma=E,M \\ \lambda=1,\infty}} |\chi_{i \rightarrow f}^{\sigma\lambda}|^2 \frac{|(2\lambda+1)!!|^2}{16\pi[(\lambda-1)!]^2} f_{\sigma\lambda}(\xi)$$

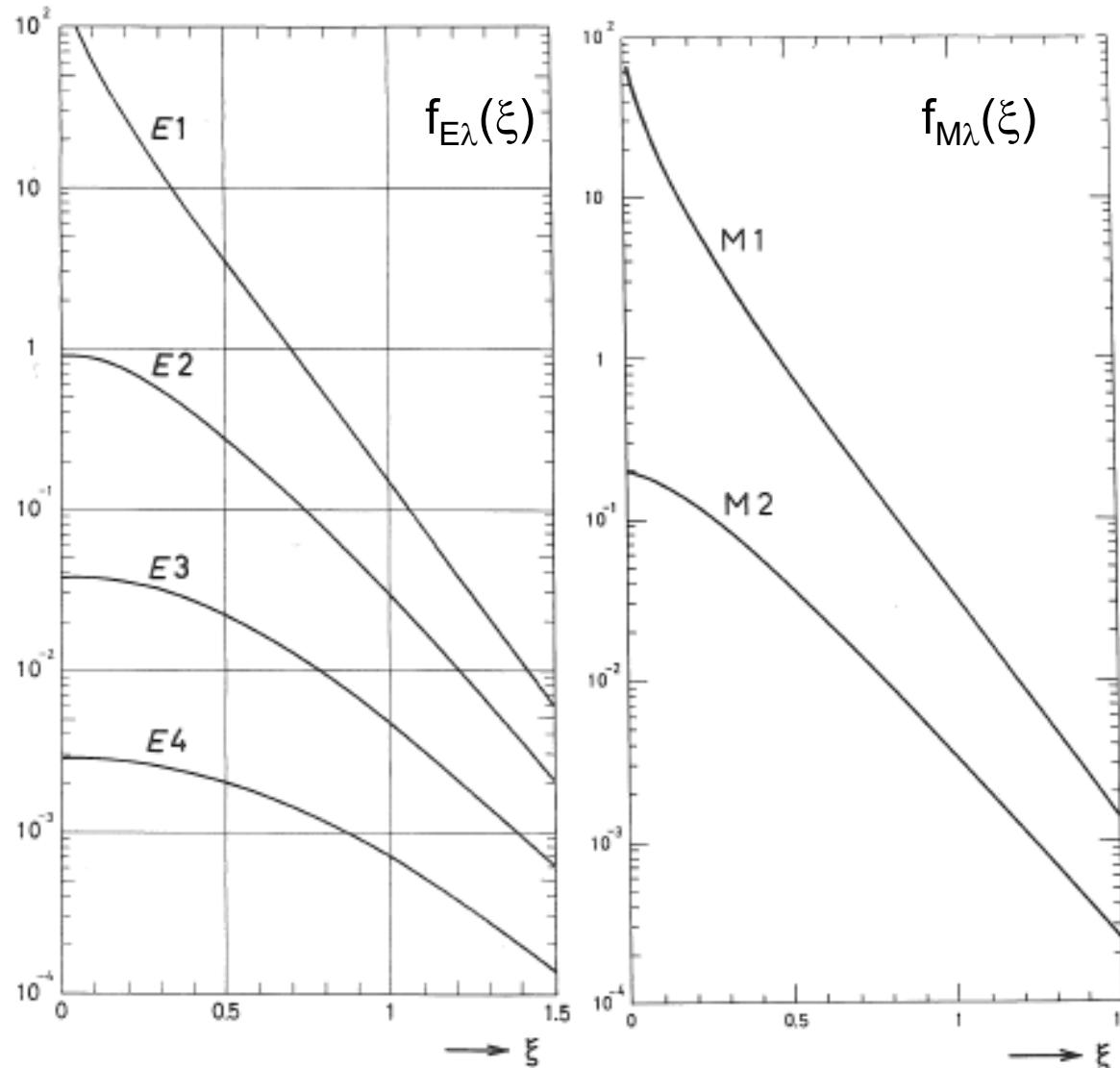
$$= \left(\frac{Z_1 e}{\hbar v} \right)^2 \sum_{\substack{\sigma=E,M \\ \lambda=1,\infty}} a^{-2(\lambda+1)} B(\sigma\lambda; I_i \rightarrow I_f) f_{\sigma\lambda}(\xi)$$

Angular distribution functions for different multipolarities



$$d\sigma_{\sigma\lambda}/d\Omega = \left(\frac{Z_1 e}{\hbar v} \right)^2 a^{-2(\lambda-1)} B(\sigma\lambda; I_i \rightarrow I_f) df_{\sigma\lambda}(\xi)$$

Total cross sections for different multipolarities



$B(\sigma\lambda)$ values for single particle like transitions (W.u.):

$$B_{sp}(\lambda) = (2\lambda+1) \frac{9e^2}{4\pi} (3+\lambda)^{-2} R^{2\lambda} \times 10 (\hbar c / M_p R_0)^2$$

$$B(\sigma\lambda) [e^2 b^\lambda] \quad {}^{208}\text{Pb}$$

$$\text{E1: } 6.45 \cdot 10^{-4} A^{2/3} \quad 2.3 \cdot 10^{-2}$$

$$\text{E2: } 5.94 \cdot 10^{-6} A^{4/3} \quad 7.3 \cdot 10^{-3}$$

$$\text{E3: } 5.94 \cdot 10^{-8} A^2 \quad 2.6 \cdot 10^{-3}$$

$$\text{E4: } 6.28 \cdot 10^{-10} A^{8/3} \quad 9.5 \cdot 10^{-4}$$

$$\text{M1: } 1.79$$

$$\text{M2: } 0.0594 A^{2/3} \quad 2.08$$

$$\sigma_{\sigma\lambda} = a^2 | \chi_{i \rightarrow f}^{\sigma\lambda} |^2 \frac{|(2\lambda+1)!!|^2}{16\pi[(\lambda-1)!]^2} f_{\sigma\lambda}(\xi) = \left(\frac{Z_1 e}{\hbar v} \right)^2 a^{-2(\lambda+1)} B(\sigma\lambda; I_i \rightarrow I_f) f_{\sigma\lambda}(\xi)$$

Coulomb excitation - the different energy regimes

Low-energy regime (< 5 MeV/u)

Energy cut-off $\Delta E_{\max} = \frac{\hbar v_\infty}{a \epsilon} \approx 2 \text{ MeV}$

Spin cut-off: L_{\max} : up to $30\hbar$

Cross section: $d\sigma/d\theta \sim \langle I_i | M(\sigma\lambda) | I_f \rangle$
differential

Luminosity: **low** mg/cm² targets
Beam intensity: high $>10^3$ pps

**Comprehensive study of
low-lying excitations**

High-energy regime (>>5 MeV/u)

Energy cut-off $\Delta E_{\max} = \hbar c \frac{\beta \gamma}{a \epsilon} \approx 10 \text{ MeV} (\beta = 0.4)$

mainly single-step excitations

$\sigma_\lambda \sim (Z_p e^2 / \hbar c)^2 B(\sigma\lambda, 0 \rightarrow \lambda)$
integral

high g/cm² targets
low a few pps

**First exploration of excited
states in very "exotic" nuclei**

Summary

- Coulomb excitation probability $P(I^\pi)$ increases with increasing strength parameter (χ), i.e. $Z_{P/T}$, $B(\sigma\lambda)$, $1/D$, θ_{cm} decreasing adiabacity parameter (ξ), i.e. ΔE , a/v_∞
- Differential cross sections $d\sigma(\theta)/d\Omega$ show varying maxima depending on multipolarity λ and adiabacity parameter ξ
→ allows to distinguish different multipolarities (E2/M1, E2/E4 etc.)
- Total cross section σ_{tot} decreases with increasing adiabacity parameter ξ and multipolarity λ is generally smaller for magnetic than for electric transitions

End of Lecture 1