Renato Angelo Ricci

A life devoted to physics

⇨ Father of nuclear spectroscopy in Italy

Director, LNL (1968-1979)
Vice-President INFN (1980-82)
President, SIF (1981-1998)
President, EPS (1989-1991)

- Many contributions to nuclear spectroscopy, especially of $1f_{7/2}$ nuclei, which helped establishing the shell-model as an important tool to study medium-mass nuclei, and that has returned to the forefront of nuclear spectroscopy in recent years.

  R.A. Ricci and P.R. Maurenzig, The $1f_{7/2}$ Problem in Nuclear Spectroscopy, Rivista del Nuovo Cimento, Vol. 1, p. 291-354.

- Many contributions to the history of Physics, especially weak interactions.

  S. Focardi and R.A. Ricci, The beta-decay and the Fundamental Properties of Weak Interactions, Rivista del Nuovo Cimento, Vol. 6, p. 1-40.

I met Renato in 1974 at the International Nuclear Physics in Amsterdam, The Netherlands.

I was very close to him during the years 1987-1995 when I was Chairman of the Advisory Committee of the Tandem Accelerator at LNL, a facility that he realized and so to its completion.

The first heavy-ion electrostatic accelerator in Italy (16 MeV Tandem)!

I have enjoyed Renato's friendship for over 40 years.

To Renato, Happy 90th birthday
To Claudine, Marco and Françoise, Best wishes

DOUBLE BETA DECAY AND NEUTRINO MASSES

Francesco Iachello

Yale University

Varenna, July 17, 2017

Unanswered questions in neutrino physics (2017):

- What is the absolute mass scale of neutrinos?
- Are neutrinos Dirac or Majorana particles?
- How many neutrino species are there?

An answer to these questions can be obtained from neutrinoless double-beta decay (DBD)

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}Y_{N-2} + 2e^{-} $$

HISTORICAL INTRODUCTION

Double beta decay is a process in which a nucleus (A,Z) decays to another nucleus (A,Z±2) by emitting two electrons or positrons, and, usually, other light particles:

$$(A,Z) \rightarrow (A,Z \pm 2) + 2e^{\mp} + anything$$

The processes where two neutrinos (or antineutrinos) are emitted

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-} + 2\overline{\nu} \qquad (2\nu\beta^{-}\beta^{-})$$

$$(A,Z) \rightarrow (A,Z-2) + 2e^{+} + 2\nu \qquad (2\nu\beta^{+}\beta^{+})$$

are predicted by the standard model. Indeed, the study of this process was suggested by Maria Goeppert-Meyer[§] in 1935, shortly after the Fermi theory of beta decay[¶] appeared (1934).

¶ E. Fermi, Z. Phys. 88, 161 (1934).
§ M. Goeppert-Meyer, Phys. Rev. 48, 512 (1935).

Versuch einer Theorie der β-Strahlen. I[1]).

Von **E. Fermi** in Rom.

Mit 3 Abbildungen. (Eingegangen am 16. Januar 1934.)

Eine quantitative Theorie des β-Zerfalls wird vorgeschlagen, in welcher man die Existenz des Neutrinos annimmt, und die Emission der Elektronen und Neutrinos aus einem Kern beim β-Zerfall mit einer ähnlichen Methode behandelt, wie die Emission eines Lichtquants aus einem angeregten Atom in der Strahlungstheorie. Formeln für die Lebensdauer und für die Form des emittierten kontinuierlichen β-Strahlenspektrums werden abgeleitet und mit der Erfahrung verglichen.

1. Grundannahmen der Theorie.

Bei dem Versuch, eine Theorie der Kernelektronen sowie der β-Emission aufzubauen, begegnet man bekanntlich zwei Schwierigkeiten. Die erste ist durch das kontinuierliche β-Strahlenspektrum bedingt. Falls der Erhaltungssatz der Energie gültig bleiben soll, muß man annehmen, daß ein Bruchteil der beim β-Zerfall frei werdenden Energie unseren bisherigen Beobachtungsmöglichkeiten entgeht. Nach dem Vorschlag von W. Pauli kann man z. B. annehmen, daß beim β-Zerfall nicht nur ein Elektron, sondern auch ein neues Teilchen, das sogenannte „Neutrino" (Masse von der Größenordnung oder kleiner als die Elektronenmasse; keine elektrische Ladung) emittiert wird. In der vorliegenden Theorie werden wir die Hypothese des Neutrinos zugrunde legen.

Eine weitere Schwierigkeit für die Theorie der Kernelektronen besteht darin, daß die jetzigen relativistischen Theorien der leichten Teilchen (Elektronen oder Neutrinos) nicht imstande sind, in einwandfreier Weise zu erklären, wie solche Teilchen in Bahnen von Kerndimensionen gebunden werden können.

Es scheint deswegen zweckmäßiger, mit Heisenberg[2]) anzunehmen, daß ein Kern nur aus schweren Teilchen, Protonen und Neutronen, besteht. Um trotzdem die Möglichkeit der β-Emission zu verstehen, wollen wir versuchen, eine Theorie der Emission leichter Teilchen aus einem Kern in Analogie zur Theorie der Emission eines Lichtquants aus einem angeregten Atom beim gewöhnlichen Strahlungsprozeß aufzubauen. In der Strahlungstheorie ist die totale Anzahl der Lichtquanten keine Konstante: Lichtquanten entstehen, wenn sie von einem Atom emittiert werden, und verschwinden, wenn sie absorbiert werden. In Analogie hierzu wollen wir der β-Strahlentheorie folgende Annahmen zugrunde legen:

[1]) Vgl. die vorläufige Mitteilung: La Ricerca Scientifica **2**, Heft 12, 1933. —
[2]) W. Heisenberg, ZS. f. Phys. **77**, 1, 1932.

wir von *verbotenen β-Übergängen*. Man muß natürlich nicht erwarten, daß die verbotenen Übergänge überhaupt nicht vorkommen, da (32) nur eine Näherungsformel ist. Wir werden in Ziffer 9 etwas über diesen Typ von Übergängen sprechen.

7. Die Masse des Neutrinos.

Durch die Übergangswahrscheinlichkeit (32) ist die Form des kontinuierlichen β-Spektrums bestimmt. Wir wollen zuerst diskutieren, wie diese Form von der Ruhemasse μ des Neutrinos abhängt, um von einem Vergleich mit den empirischen Kurven diese Konstante zu bestimmen. Die Masse μ ist in dem Faktor p_σ^2/v_σ enthalten. Die Abhängigkeit der Form der Energieverteilungskurve von μ ist am meisten ausgeprägt in der Nähe des Endpunktes der Verteilungskurve. Ist E_0 die Grenzenergie der β-Strahlen, so sieht man ohne Schwierigkeit, daß die Verteilungskurve für Energien E in der Nähe von E_0 bis auf einen von E unabhängigen Faktor sich wie

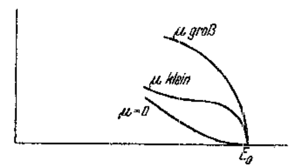


Fig. 1.

$$\frac{p_\sigma^2}{v_\sigma} = \frac{1}{c^3}(\mu c^2 + E_0 - E)\sqrt{(E_0 - E)^2 + 2\mu c^2 (E_0 - E)} \qquad (36)$$

verhält.

In der Fig. 1 ist das Ende der Verteilungskurve für $\mu = 0$ und für einen kleinen und einen großen Wert von μ gezeichnet. Die größte Ähnlichkeit mit den empirischen Kurven zeigt die theoretische Kurve für $\mu = 0$.

Wir kommen also zu dem Schluß, daß die Ruhemasse des Neutrinos entweder Null oder jedenfalls sehr klein in bezug auf die Masse des Elektrons ist[1]). In den folgenden Rechnungen werden wir die einfachste Hypothese $\mu = 0$ einführen. Es wird dann (30)

$$v_\sigma = c; \quad K_\sigma = p_\sigma c; \quad p_\sigma = \frac{K_\sigma}{c} = \frac{W - H_s}{c}. \qquad (37)$$

Die Ungleichungen (33), (34) werden jetzt:

$$H_s \leqq W; \quad W \geqq m c^2. \qquad (38)$$

Und die Übergangswahrscheinlichkeit (32) nimmt die Form an:

$$P_s = \frac{8\pi^3 g^2}{c^3 h^4}\left|\int v_m^* u_n \, d\tau\right|^2 \tilde{\psi}_s \psi_s (W - H_s)^2. \qquad (39)$$

[1]) In einer kürzlich erschienenen Notiz kommt F. Perrin, C. R. **197**, 1625, 1933, mit qualitativen Überlegungen zu demselben Schluß.

Double Beta-Disintegration

M. GOEPPERT-MAYER, *The Johns Hopkins University*

(Received May 20, 1935)

From the Fermi theory of β-disintegration the probability of simultaneous emission of two electrons (and two neutrinos) has been calculated. The result is that this process occurs sufficiently rarely to allow a half-life of over 10^{17} years for a nucleus, even if its isobar of atomic number different by 2 were more stable by 20 times the electron mass.

1. Introduction

In a table showing the existing atomic nuclei it is observed that many groups of isobars occur, the term isobar referring to nuclei of the same atomic weight but different atomic number. It is unreasonable to assume that all isobars have exactly the same energy; one of them therefore will have the lowest energy, the others are unstable. The question arises why the unstable nuclei are in reality metastable, that is, why, in geologic time, they have not all been transformed into the most stable isobar by consecutive β-disintegrations.

The explanation has been given by Heisenberg[1] and lies in the fact that the energies of nuclei of fixed atomic weight, plotted against atomic number, do not lie on one smooth curve, but, because of the peculiar stability of the α-particle are distributed alternately on two smooth curves, displaced by an approximately constant amount against each other (the minimum of each curve is therefore at, roughly, the same atomic number). For even atomic weight the nuclei of even atomic number lie on the lower curve, those with odd atomic number on the higher one. One β-disintegration then brings a nucleus from a point on the lower curve into one of the upper curve, or *vice versa*. The nuclei on the upper curve are all of them unstable. But it may happen that a nucleus on the lower curve, in the neighborhood of the minimum, even though it is not the most stable one, cannot emit a single β-particle, since the resultant isobar, whose energy lies on the upper curve, has higher energy. This nucleus would then be metastable, since it cannot go over into a more stable one by consecutive emission of two electrons. This explanation is borne out by the fact that almost only isobars of even difference in atomic number occur.

A metastable isobar can, however, change into a more stable one by simultaneous emission of two electrons. It is generally assumed that the frequency of such a process is very small. In this paper the propability of a disintegration of that kind has been calculated.

The only method to attack processes involving the emission of electrons from nuclei is that of Fermi[2] which associates with the emission of an electron that of a neutrino, a chargeless particle of negligible mass. Thereby it is possible to explain the continuous β-spectrum and yet to have the energy conserved in each individual process by adjusting the momentum of the neutrino. In this theory the treatment of a β-disintegration is very similar to that of the emission of light by an excited atom.

A disintegration with the simultaneous emission of two electrons and two neutrinos will then be in strong analogy to the Raman effect, or, even more closely, to the simultaneous emission of two light quanta,[3] and can be calculated in essentially the same manner, namely, from the second-order terms in the perturbation theory. The process will appear as the simultaneous occurrence of two transitions, each of which does not fulfill the law of conservation of energy separately.

The following investigation is a calculation of the second-order perturbation, due to the interaction potential introduced by Fermi between neutrons, protons, electrons and neutrinos. As far as possible the notation used is that of Fermi. For a more detailed discussion and justification of this mathematical form and the assumptions involved reference must be made to Fermi's paper.

[1] W. Heisenberg, Zeits. f. Physik **78**, 156 (1932).

[2] E. Fermi, Zeits. f. Physik **88**, 161 (1934).

[3] M. Goeppert-Mayer, Ann. d. Physik (V) **9**, 273 (1931).

It took however more than 50 years to observe it (Elliott *et al.*, 1987)[§] in view of its very long half-life

$$\tau_{1/2}^{2\nu}(^{100}Mo) = \left(7.1 \pm 0.4\right) \times 10^{18}\, yr$$

Now (2015) $2\nu\beta^-\beta^-$ has been observed in 10 nuclei[¶] .
[The positron emitting and related processes $2\nu\beta^+\beta^+$, $2\nu\beta^+EC$, $2\nu ECEC$ has been observed only in 1 nucleus (^{130}Ba).]
The measured half-lives are

$$\tau_{1/2}^{2\nu} \sim (10^{18} - 10^{21})\, yr$$

[§] S. R. Elliott, A.A. Hahn, and M.K. Moe, Phys. Rev. Lett. 59 (1987) 2020.

[¶] A review of all observed $2\nu\beta\beta$ decays is given in:
A.S. Barabash, Nucl. Phys. A935, 52 (2015).

The processes where no neutrinos are emitted

$$(A,Z) \rightarrow (A,Z+2) + 2e^- \qquad (0\nu\beta^-\beta^-)$$

$0\nu\beta^-\beta^-$, and $0\nu\beta^+\beta^+$,$0\nu\beta^+EC$,$0\nu ECEC$, are forbidden by the standard model, and, if observed, will provide evidence for physics beyond the standard model, in particular will determine whether or not the neutrino is a Majorana particle and will measure its (average) mass.

Majorana[§] (1937) suggested that neutral particles could be their own antiparticles and Racah[¶] (1937) pointed out that the neutron cannot be its own antiparticle since it has a magnetic moment, while the neutrino could be such a particle.

§ E. Majorana, Nuovo Cimento 14, 171 (1937).
¶ G. Racah, Nuovo Cimento 14, 322 (1937).

E. Majorana, Nuovo Cimento 14, 171 (1937).

TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di ETTORE MAJORANA

Sunto. - *Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di* DIRAC *ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di « antiparticelle » corrispondenti ai « vuoti » di energia negativa.*

L'interpretazione dei cosidetti « stati di energia negativa » proposta da DIRAC (1) conduce, come è ben noto, a una descrizione sostanzialmente simmetrica degli elettroni e dei positroni. La sostanziale simmetria del formalismo consiste precisamente in questo, che fin dove è possibile applicare la teoria girando le difficoltà di convergenza, essa fornisce realmente risultati del tutto simmetrici. Tuttavia gli artifici suggeriti per dare alla teoria una forma simmetrica che si accordi con il suo contenuto, non sono del tutto soddisfacenti; sia perchè si parte sempre da una impostazione asimmetrica, sia perchè la simmetrizzazione viene in seguito ottenuta mediante tali procedimenti (come la cancellazione di costanti infinite) che possibilmente dovrebbero evitarsi. Perciò abbiamo tentato una nuova via che conduce più direttamente alla meta.

Per quanto riguarda gli elettroni e i positroni, da essa si può veramente attendere soltanto un progresso formale; ma ci sembra importante, per le possibili estensioni analogiche, che venga a cadere la nozione stessa di stato di energia negativa. Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi.

(1) P. A. M. DIRAC, « Proc. Camb. Phil. Soc. », **30**, 150, 1924. V. anche W. HEISENBERG, « ZS. f. Phys. », **90**, 209, 1934.

SULLA SIMMETRIA TRA PARTICELLE E ANTIPARTICELLE

Nota di GIULIO RACAH

Sunto. - *Si mostra che la simmetria tra particelle e antiparticelle porta alcune modificazioni formali nella teoria di* FERMI *sulla radioattività* β, *e che l'identità fisica tra neutrini ed antineutrini porta direttamente alla teoria di* E. MAJORANA.

Nella prima parte del presente lavoro si pone in rilievo una certa arbitrarietà che ancora sussiste nella trasformazione delle autofunzioni di DIRAC associata a un cambiamento di assi nello spazio-tempo, e si mostra come si possa eliminare questa arbitrarietà aggiungendo al postulato dell'invarianza relativistica quello della simmetria tra particelle e antiparticelle. Si perviene così ad una legge di trasformazione che differisce in alcuni casi da quella generalmente ammessa (1), e ad una conseguente modificazione dell'interazione proposta da FERMI nella sua teoria dei raggi β (2). Gli effetti di tale modificazione non sono verificabili sperimentalmente, perchè tendono a zero con la massa del neutrino, ma hanno una certa importanza teorica, perchè eliminano una dissimmetria che era stata rilevata da KONOPINSKI e UHLENBECK (3).

Nella seconda parte si considera l'ipotesi (che dovrà essere un giorno verificata sperimentalmente) che nel caso particolare dei neutrini non si abbia una semplice simmetria, ma addirittura una identità fisica tra neutrini ed antineutrini, e si mostra come questa ipotesi porti automaticamente al formalismo di E. MAJORANA (4). Si rende così evidente il contenuto fisico assolutamente nuovo della teoria di E. MAJORANA, e si indica come l'esperienza potrà decidere della sua validità.

(1) W. PAULI, « Handbuch der Physik », vol. XXIV/1, pp. 220-224.
(2) E. FERMI, « Nuovo Cimento », **11**, 1, 1934.
(3) E. J. KONOPINSKI e G. E. UHLENBECK, « Phys. Rev. », **48**, 7, 1935.
(4) E. MAJORANA, « Nuovo Cimento », **14**, 171, 1937.

Da un punto di vista più fisico possiamo riassumere queste considerazioni dicendo che la teoria di E. MAJORANA equivale a identificare le particelle con le antiparticelle, e che se tale identificazione può farsi per i neutrini, essa non sembra possibile per i neutroni, perchè l'antineutrone dovrebbe differire dal neutrone e per il segno del momento magnetico e per la capacità di trasformarsi per processo β in antiprotone anzichè in protone. Ricordando l'ipotesi di WICK (1) sull'origine del momento magnetico del neutrone, si vede che le due difficoltà non sono indipendenti.

Desidero ringraziare il prof. W. PAULI per interessanti e proficue discussioni sugli argomenti di questa nota.

Firenze, Istituto Fisico di Arcetri, Luglio 1937-XV.

(1) G. C. WICK, « Rend. Lincei », **21**, 170, 1935.

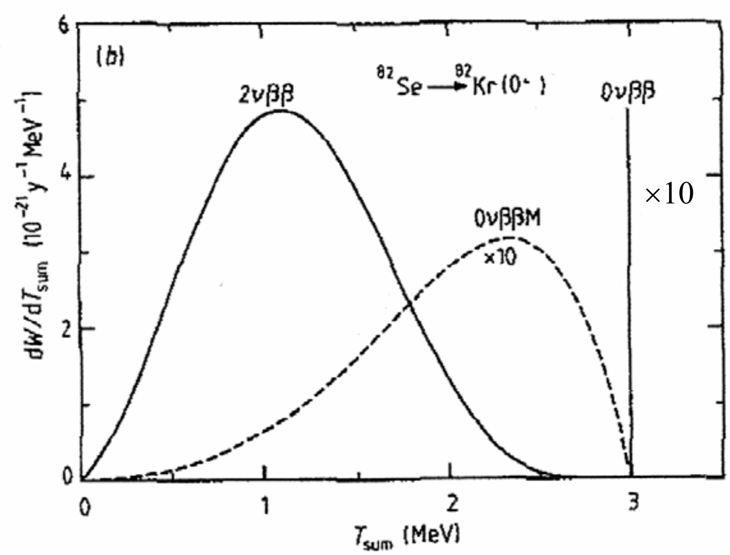
G. Racah, Nuovo Cimento 14, 322 (1937).

A major experimental effort started a few years ago to detect neutrinoless DBD. All experiments so far have given negative results, with exception of Klapdor- Kleingrothaus *et al*., 2004. This result has however been very recently (2013) disproved.

Neutrino less DBD remains therefore one of the most fundamental problems in physics today. Its detection will be crucial for understanding whatever physics is beyond the standard model (SM) and is currently the subject of many experiments.

In addition to the fact that the expected half-life is very long, a major problem is the concomitance of the 2ν process

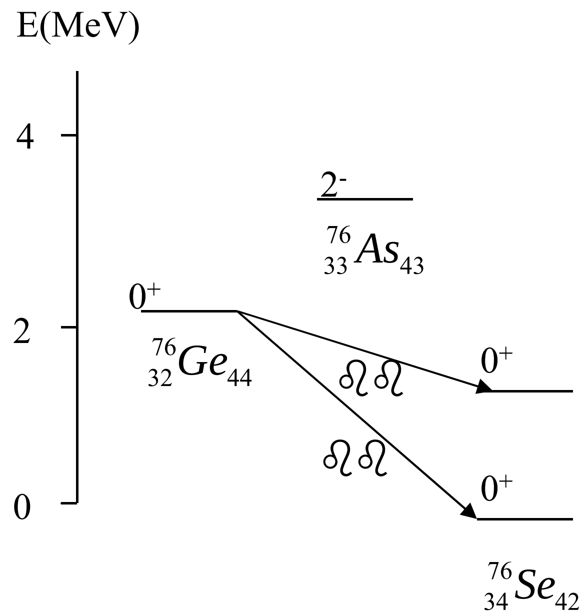
Summed energy spectra of the two emitted electrons →



In order to be able to extract the neutrino mass if DBD is observed, or to put a limit on its value if it is not observed, one needs a theory of $0\nu\beta\beta$ and of its concomitant process $2\nu\beta\beta$.

DOUBLE BETA DECAY

$${}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z\pm 2}Y_{N\mp 2} + 2e^{\mp} + anything$$



Half-life for processes not allowed by the standard model:

$$\left[\tau_{1/2}^{0\nu\beta\beta}(0^{+} \rightarrow 0^{+})\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^{2}\left|f(m_i, U_{ei})\right|^{2}$$

Beyond the standard model
(Particle physics)

Phase-space factor
(Atomic physics)
PSF

Matrix elements
(Nuclear physics)
NME

For processes allowed by the standard model, the half-life can be, to a good approximation, factorized in the form

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}\left|M_{2\nu}\right|^2$$

Phase-space factor (Atomic Physics) PSF

Matrix elements (Nuclear Physics) NME

A special case is 0νECEC, which is forbidden by energy and momentum conservation, but can occur under resonance conditions. In this case the inverse half-life is given by

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2 \frac{(m_e c^2)\Gamma}{\Delta^2 + (\Gamma^2/4)}$$

Resonance factor

Prefactor (Atomic Physics) PF

Matrix elements (Nuclear Physics) NME

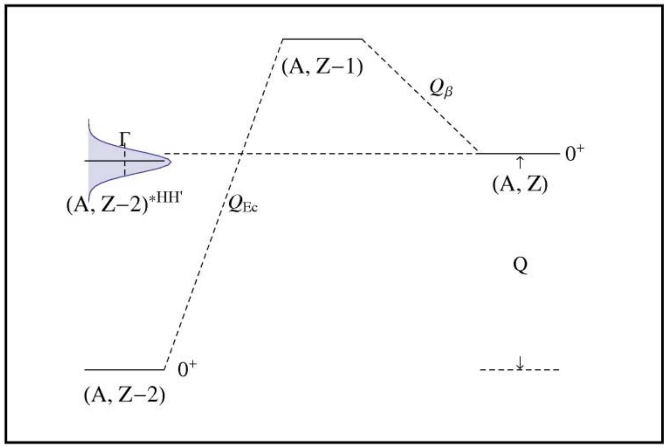
Beyond the SM (Particle Physics)

$$\Delta = \left|Q - B_{2h} - E\right|$$

$$\Gamma = \Gamma_{e_1} + \Gamma_{e_2}$$

↑ Two-hole width (Atomic Physics)

Degeneracy parameter (Atomic and Nuclear Physics)



For all processes and to extract physics beyond the standard model one needs to calculate the phase space factors (PSF) and the nuclear matrix elements (NME).

NUCLEAR MATRIX ELEMENTS (NME)

NME can be written as:

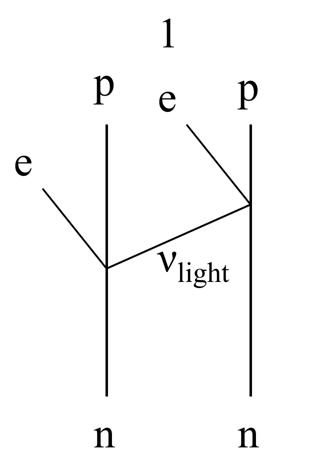
$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} \equiv M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

Several methods have been used to evaluate $M_{0\nu}$:
QRPA (Quasiparticle Random Phase Approximation)
ISM (Shell Model)
IBM-2 (Interacting Boson Model)
DFT (Density Functional Theory)

Calculations of NME in IBM-2 for all processes have been completed (2015) and are available upon request.
A list of references is given in Appendix A.

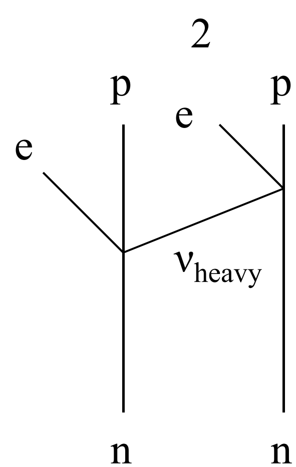
For 0ν processes two scenarios have been considered:
(i) Emission and re-absorption of a light ($m_{light} \ll 1MeV$) neutrino.
(ii) Emission and re-absorption of a heavy ($m_{heavy} \gg 1GeV$) neutrino.



$$f = \frac{\langle m_\nu \rangle}{m_e}$$

$m_{\nu light} \ll 1keV$

Long range



$m_{\nu heavy} \gg 1GeV$

Short range

$$f_h = m_p \left\langle \frac{1}{m_{\nu_h}} \right\rangle$$

Scenario 1: LIGHT NEUTRINO EXCHANGE

Dependence on the average neutrino mass

$$f = \frac{\langle m_\nu \rangle}{m_e}$$

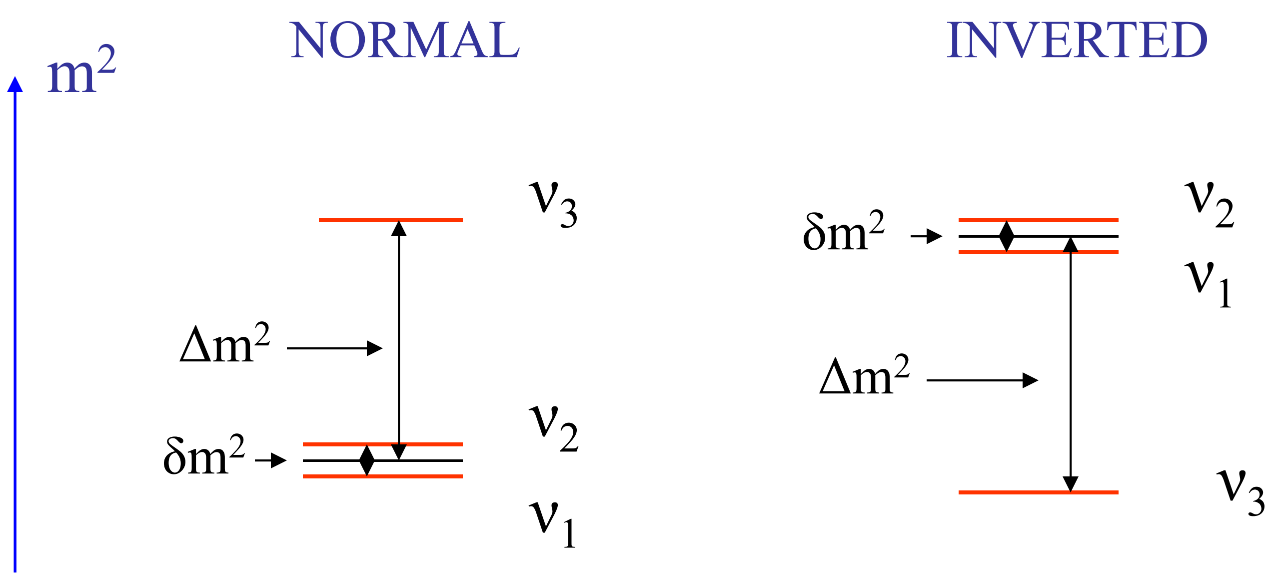
$$\langle m_\nu \rangle = \sum_{k=ligth} (U_{ek})^2 m_k$$

Fourier transform of the neutrino "potential"

$$v(p) = \frac{2}{\pi} \frac{1}{p\left(p + \tilde{A}\right)}$$

$\tilde{A}$=closure energy=1.12A$^{1/2}$(MeV)

In the last few years atmospheric, solar, reactor and accelerator neutrino oscillation experiments have provided information on light neutrino mass differences and their mixings. Two possibilities, normal and inverted hierarchy, are consistent with experiment.



The average light neutrino mass can be written as

$$\langle m_\nu \rangle = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\varphi_2} + s_{13}^2 m_3 e^{i\varphi_3} \right|$$

$$c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}, \varphi_{2,3} = [0, 2\pi]$$

$$\left(m_1^2, m_2^2, m_3^2\right) = \frac{m_1^2 + m_2^2}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm\Delta m^2\right)$$

A fit to oscillation experiments gives §

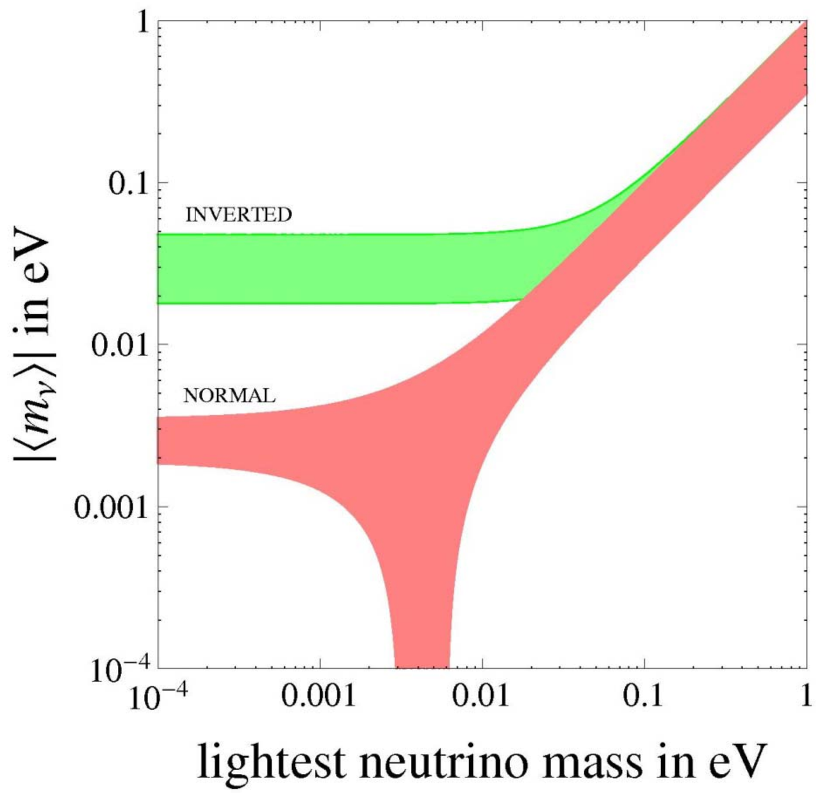
$$\sin^2\theta_{12} = 0.312, \sin^2\theta_{13} = 0.016, \sin^2\theta_{23} = 0.466$$

$$\delta m^2 = 7.67\times 10^{-5} eV^2, \Delta m^2 = 2.39\times 10^{-3} eV^2$$

§ G.L. Fogli *et al.*, Phys. Rev. D75, 053001(2007); D78, 033010 (2008).

[A recent result from Daya Bay, Phys. Rev. Lett. 108, 171803 (2012) gives $\sin^2\theta_{13}$=0.024±0.005, which slightly modifies the fit.]

Variation of the phases φ_2 and φ_3 from 0 to 2π gives the values of $<m_\nu>$ consistent with oscillation experiments (constraints on the neutrino masses)



Vissani-Strumia plot ¶

¶ F. Vissani, J. High Energy Phys. 06, 022 (1999)

Scenario 2: HEAVY NEUTRINO EXCHANGE

Dependence on the average neutrino mass

$$f = m_p \left\langle \frac{1}{m_{\nu_h}} \right\rangle$$

$$\left\langle m_{\nu_h}^{-1} \right\rangle = \sum_{k=heavy} \left(U_{ek_h}\right)^2 \frac{1}{m_{k_h}}$$

Fourier transform of the neutrino "potential"

$$v(p) = \frac{2}{\pi} \frac{1}{m_p m_e}$$

Constraints on the average inverse heavy neutrino mass are model dependent. V. Tello *et* al. ¶ have recently (2011) worked out constraints from lepton flavor violating processes and (potentially LHC experiments). In this model

$$f \equiv \eta = \frac{M_W^4}{M_{WR}^4} \sum_{k=heavy} \left(V_{ek_h}\right)^2 \frac{m_p}{m_{k_h}} \equiv \frac{M_W^4}{M_{WR}^4} \frac{m_p}{\left\langle m_{\nu_h} \right\rangle}$$

$$M_W = 80.41 \pm 0.10 GeV; M_{WR} = 3.5 TeV$$

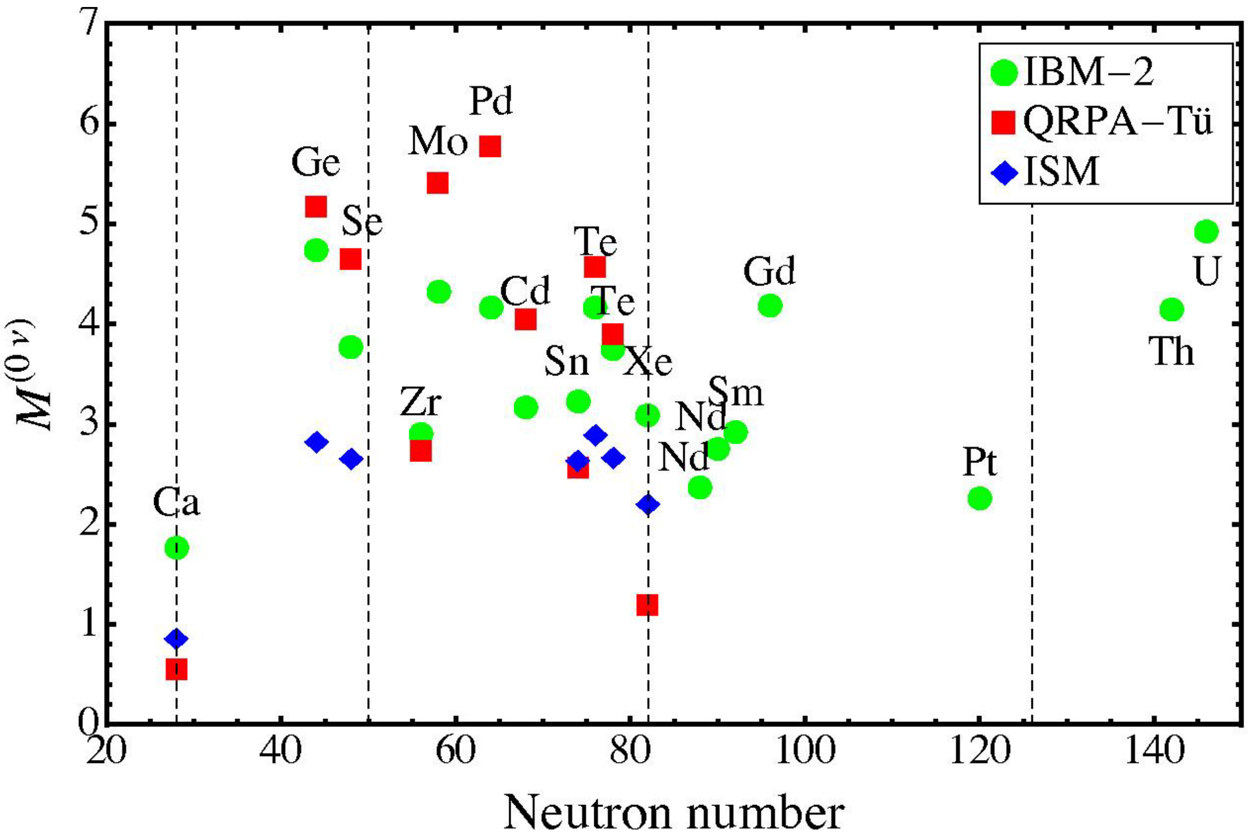
η=lepton violating parameter.

Constraints on η can then be converted into constraints on the average heavy neutrino mass as

$$\left\langle m_{\nu_h} \right\rangle = m_p \left(\frac{M_W}{M_{WR}} \right)^4 \frac{1}{\eta}$$

¶ V. Tello, M. Nemevšek, F. Nesti, G. Senjanović, and F. Vissani, Phys. Rev. Lett. 106, 151801 (2011).

Most recent (2015) results for $0\nu\beta^-\beta^-$ (light neutrino exchange)

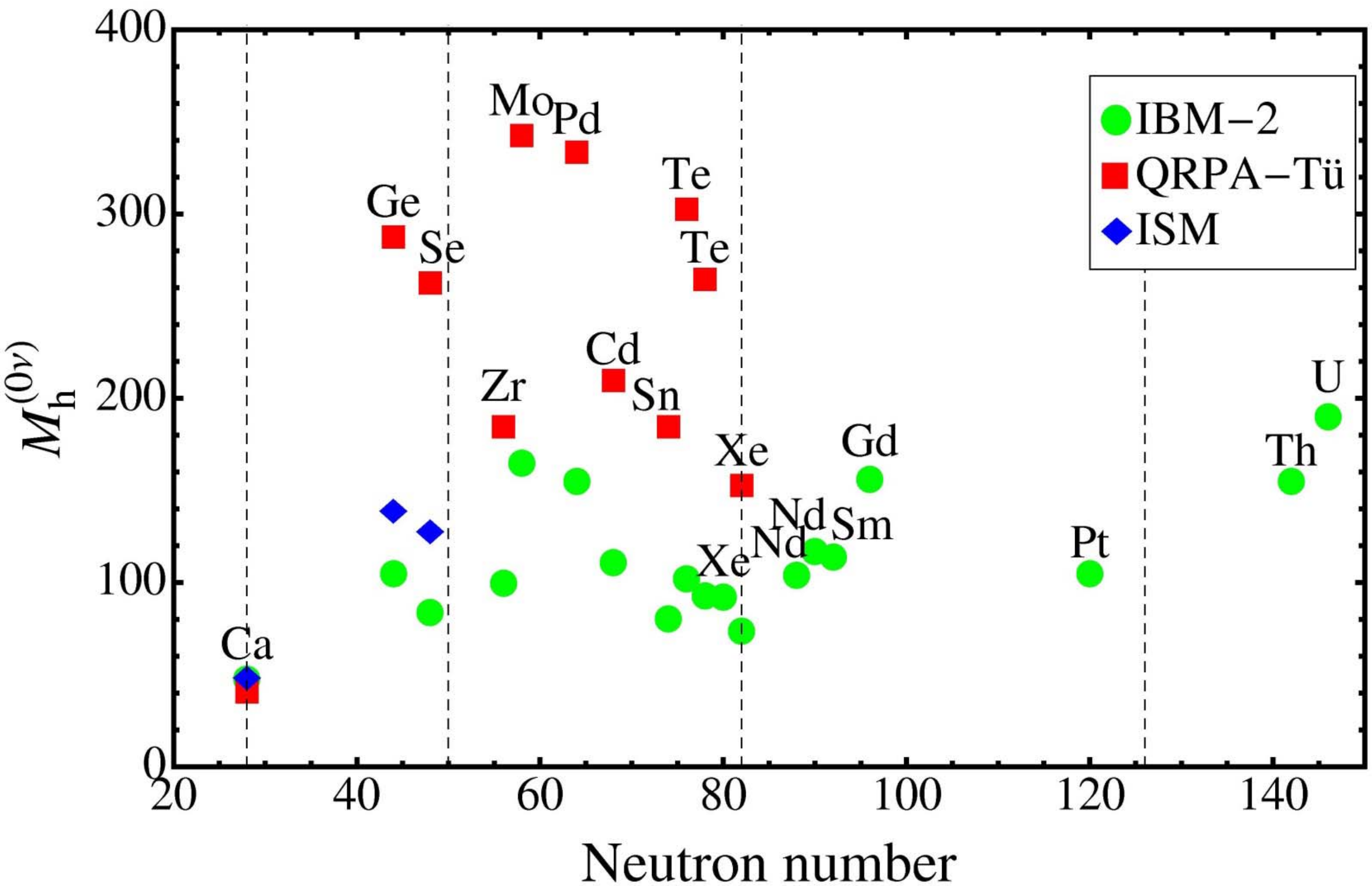


g_A=1.269

IBM-2 *: J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C 91, 034304 (2015).
QRPA-Tu *: F. Simkovic, V. Rodin, A. Faessler, and P. Vogel, Phys. Rev. C 87, 045501 (2013).
ISM: J. Menendez, A. Poves, E. Caurier, and F. Nowacki, Nucl. Phys. A 818, 139 (2009).

* With isospin restoration and Argonne SRC

Most recent (2015) results for $0\nu\beta^-\beta^-$ (heavy neutrino exchange)



g_A=1.269

* With isospin restoration and Argonne SRC

PHASE SPACE FACTORS (PSF)

PSF were calculated in the 1980's by Doi *et al.* [*]. Also, a calculation of phase-space factors is reported in the book of Boehm and Vogel [§]. These calculations use an approximate expression for the electron wave functions at the nucleus.

PSF have been recently recalculated [**] with exact Dirac electron wave functions and including screening by the electron cloud.

These new PSF are available from jenni.kotila@yale.edu and are on the webpage nucleartheory.yale.edu

[*] M. Doi, T. Kotani, N. Nishiura, K. Okuda and E. Takasugi, Prog. Theor. Phys. 66 (1981) 1739.

[§] F. Bohm and P. Vogel, *Physics of massive neutrinos*, Cambridge University Press, 1987.

[**] J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

QUENCHING OF g_A

Results in the previous slides are obtained with g_A=1.269.

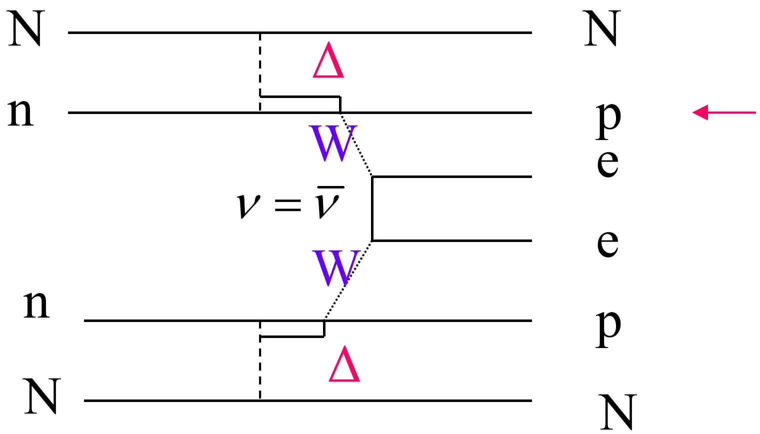
It is well-known from single β-decay/EC [¶] and from 2νββ that g_A is renormalized in models of nuclei. Two reasons:

(i) Limited model space

(ii) Omission of non-nucleonic degrees of freedom (Δ,…)

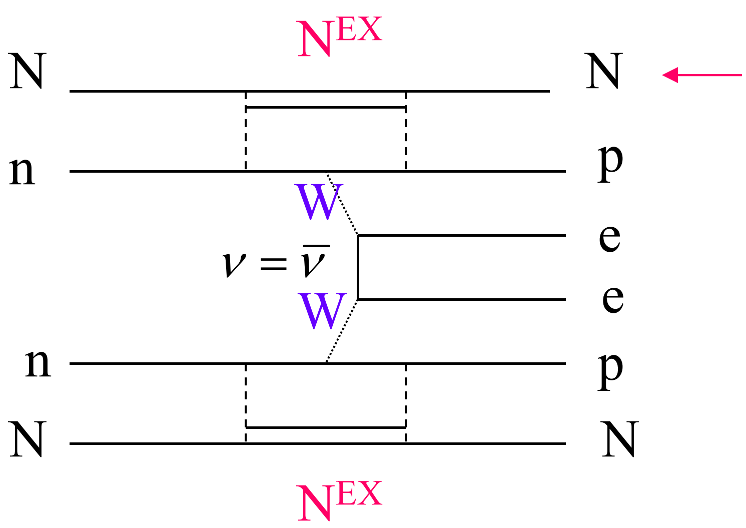
[¶] J. Fujita and K. Ikeda, Nucl. Phys. 67, 145 (1965).
D.H. Wilkinson, Nucl. Phys. A225, 365 (1974).

ORIGIN OF QUENCHING OF g_A IN DBD



Quenching factor $q_\Delta \cong 0.7$

(Δ means excited states of the nucleon)



Quenching factor $q_{N^{EX}} \cong 0.7$
(nuclear model dependent)

(N^{EX} means excited states of the nucleus not included explicitly)

Maximal quenching:

$$Q = q_\Delta q_{N^{EX}} \cong 0.5$$

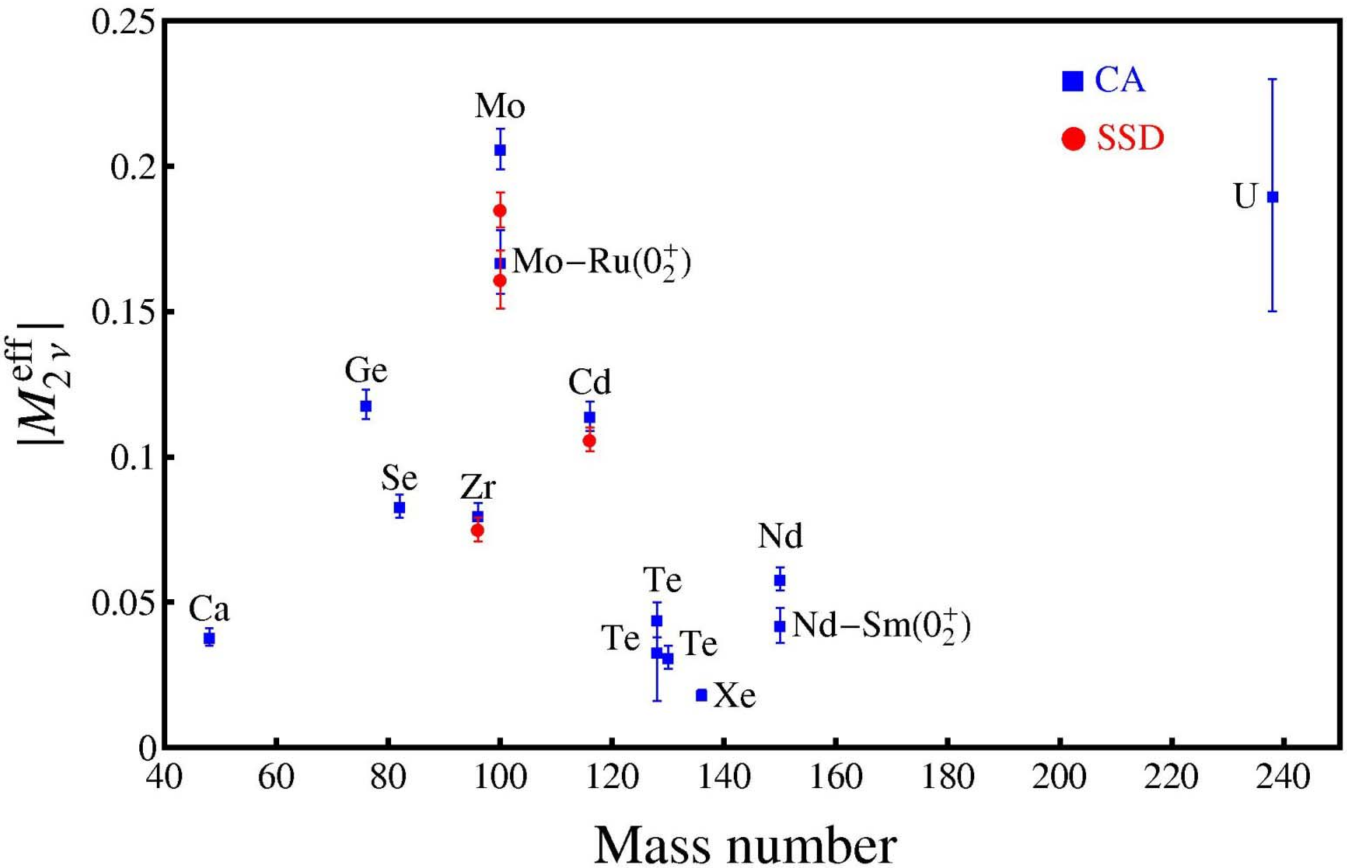
For each model (ISM/QRPA/IBM-2) one can define an effective $g_{A,eff}$ by writing

$$M_{2\nu}^{eff} = \left(\frac{g_{A,eff}}{g_A} \right)^2 M_{2\nu}$$

$$M_{\beta/EC}^{eff} = \left(\frac{g_{A,eff}}{g_A} \right) M_{\beta/EC}$$

The value of $g_{A,eff}$ in each nucleus can then be obtained by comparing the calculated and measured half-lives for β/EC and for 2νββ.

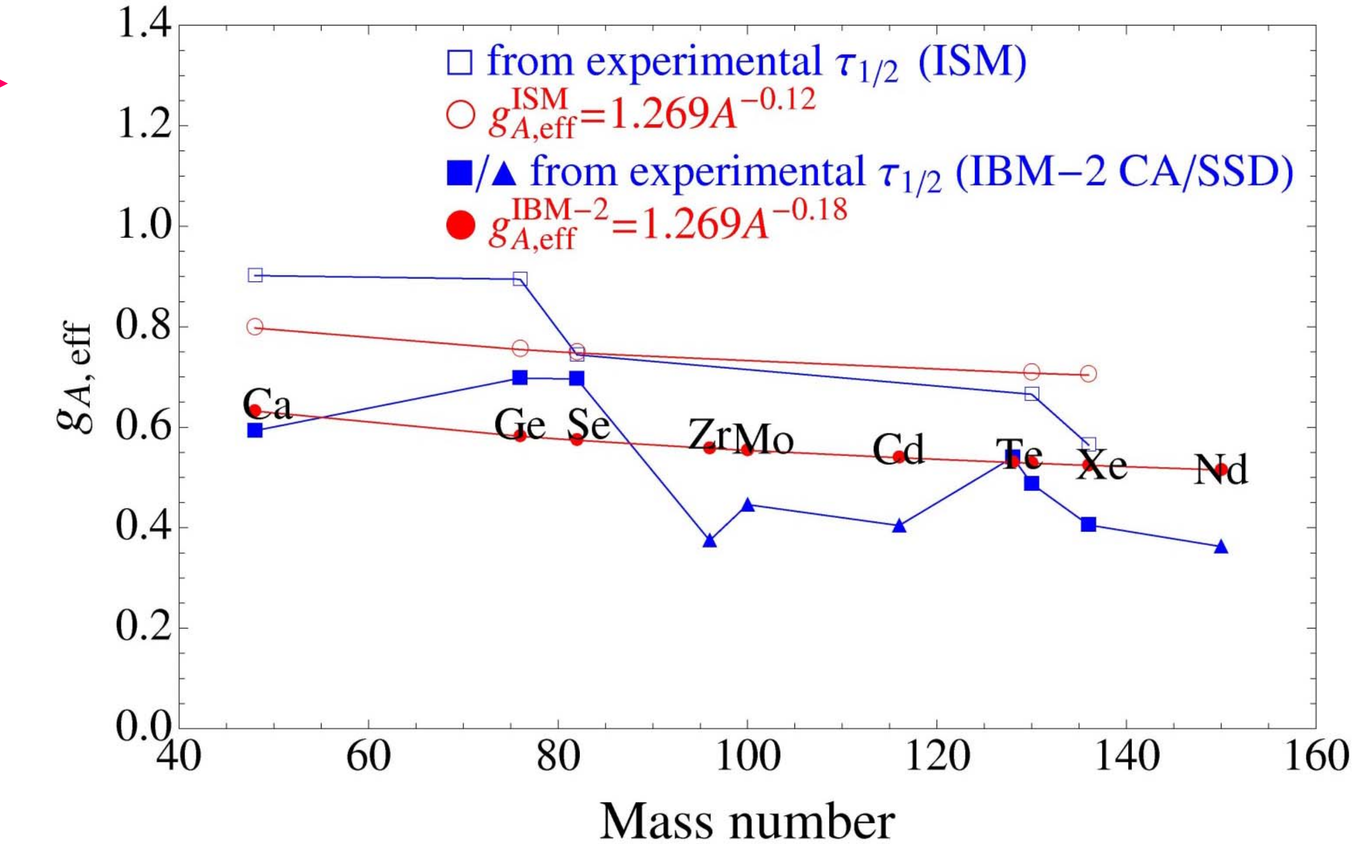
Values of $|M_{2\nu}^{eff}|$ obtained from experimental half-lives ¶



¶ From a compilation by A.S. Barabash, Phys. Rev. C 81, 035501 (2010). For ^{136}Xe, N. Ackerman *et al.* (EXO Collaboration), Phys. Rev. Lett. 107, 212501 (2011).

Effective axial vector coupling constant in nuclei from $2\nu\beta\beta$ ¶

Free value



One obtains $g_{A,eff}^{IBM\text{-}2}\sim0.6\text{-}0.5$.

The extracted values can be parametrized as

$$g_{A,eff}^{IBM2} = 1.269A^{-0.18}$$

A similar analysis can be done for the ISM for which $g_{A,eff}^{ISM}\sim0.8\text{-}0.7$.

$$g_{A,eff}^{ISM} = 1.269A^{-0.12}$$

¶ J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013).

$g_{A,eff}$, has been extracted also from single β/EC in QRPA, very recently by Suhonen and Civitarese (QRPA-Jy), $g_{A,eff}^{QRPA} \sim 0.8$-$0.4$ [§], and a few years ago by Faessler *et al*. (QRPA-Tü) ~ 0.7 [*].

[In some earlier (1989) QRPA papers[¶], it is claimed that no renormalization of g_A is needed. However, this claim is based on results where the renormalization of g_A is transferred to a renormalization of the free parameter g_{pp} used in the calculation and adjusted to the experimental 2νββ half-life.]

[§] J. Suhonen and O. Civitarese, Phys. Lett. B 725, 153 (2013).
[*] A. Faessler, G.L. Fogli, E. Lisi, V. Rodin, A.M. Rotunno, and F. Šimkovic, J. Phys. G: Nucl. Part. Phys. 35, 075104 (2008).

[¶] K. Muto, E. Bender, H.V. Klapdor, Z. Phys. A334, 177 (1989); 187 (1989).

IMPACT OF THE RENORMALIZATION

The axial vector coupling constant, g_A, appears to the second power in the NME

$$M_{2\nu} = g_A^2 M^{(2\nu)}$$

$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

and hence to the fourth power in the half-life!

Therefore, the results of the previous slides should be multiplied by 6-34 to have realistic estimates of expected half-lives. [See also, H. Robertson ¶, and S. Dell'Oro, S. Marcocci, F. Vissani#.]

¶ R.G.H. Robertson, Modern Phys. Lett. A 28, 1350021 (2013).

S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D90, 033005 (2014).

The question of whether or not g_A in $0\nu\beta\beta$ is renormalized as much as in $2\nu\beta\beta$ is of much debate. In $2\nu\beta\beta$ only the 1^+ (GT) multipole contributes. In $0\nu\beta\beta$ all multipoles 1^+, 2^-,…; 0^+, 1^- … contribute. Some of these could be unquenched. However, even in $0\nu\beta\beta$, 1^+ intermediate states dominate. Hence, our current understanding is that g_A is renormalized in $0\nu\beta\beta$ as much as in $2\nu\beta\beta$.

This problem is currently being addressed from various sides. Experimentally by measuring the matrix elements to and from the intermediate odd-odd nucleus in $2\nu\beta\beta$ decay [§]. Theoretically, by using effective field theory (EFT) to estimate the effect of non-nucleonic degrees of freedom (two-body currents) [¶].

[§] P. Puppe *et al.*, Phys. Rev. C 86, 044603 (2012).

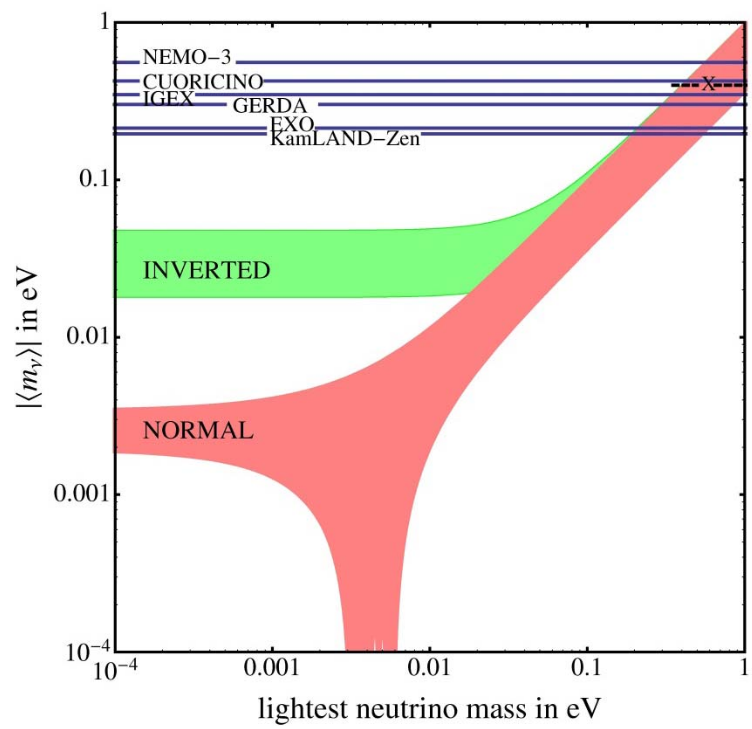
[¶] J. Menendez, D. Gazit, and A. Schwenk, Phys. Rev. Lett. 107, 062501 (2011).

Another question is whether or not the vector coupling constant, g_V, is renormalized in nuclei.
Because of CVC, the mechanism (ii) omission of non-nucleonic degrees of freedom cannot contribute.
However, the mechanism (i), limited model space, can contribute, and, if so, the ratio g_V/g_A may remain the same as the non-renormalized ratio 1/1.269.
No experimental information is available, but is could be obtained by measuring with (^{3}He,t) and (d,^{2}He) reactions the F matrix elements to and from the intermediate odd-odd nucleus.

CONCLUSIONS

Major progress has been made in the last few years to narrow down predictions of $0\nu\beta\beta$ decay to realistic values in *all* nuclei of interest.

Current (2015) limits on the neutrino mass from $0\nu\beta^-\beta^-$ (light neutrino exchange) with g_A=1.269, IBM-2 NME, and KI PSF:



x H.V. Klapdor-Kleingrothaus *et al.*, Phys. Lett. B586, 198 (2004).

With g_A=1.269:

For light neutrino exchange, only the degenerate region can be tested in the immediate future. The current best limit (with g_A=1.269) is from EXO/KamLAND-Zen, $m_\nu < 0.20$ eV.
Exploration of the inverted region >1 ton
Exploration of the normal region >>1 ton

For heavy neutrino exchange, the limit is model dependent. In the model of Tello *et al.* ¶, the current best limit from EXO/KamLAND-Zen is $m_{\nu h} > 257\ \text{GeV}(3.5/M_{WR})^4$.

¶ V. Tello, M. Nemevšek, F. Nesti, O. Senjanovic, and F. Vissani, Phys. Rev. Lett. 106, 151801 (2011).

The major remaining question is the value of g_A.

Three scenarios are[¶,§] :

$$g_A = 1.269$$ ← Free value

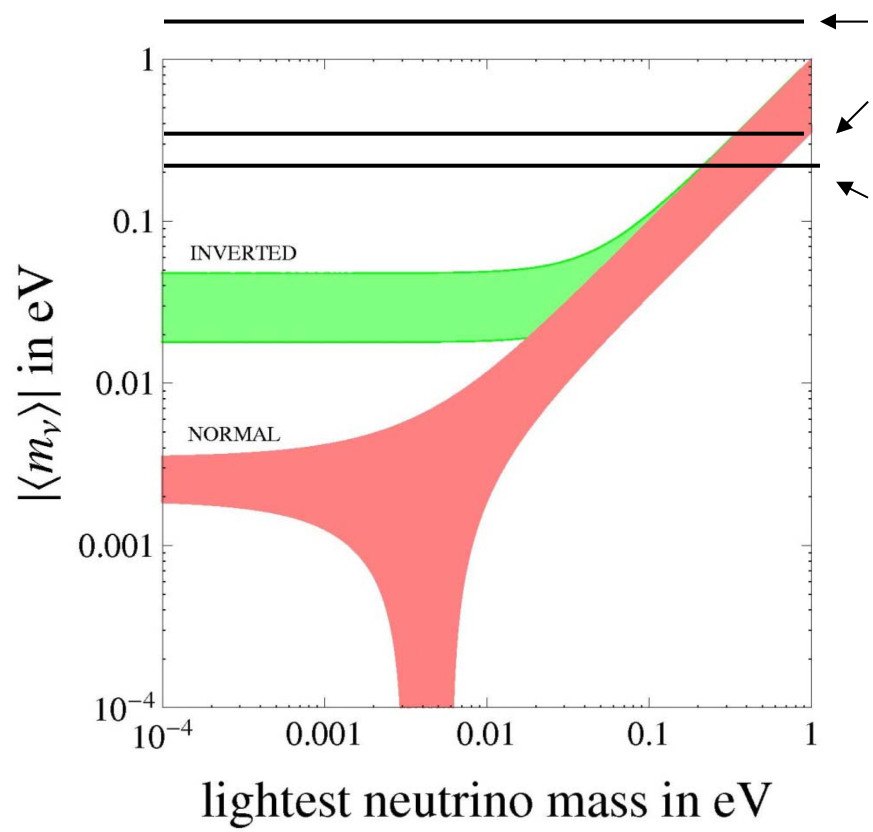
$$g_A = 1$$ ← Quark value

$$g_A = 1.269\,A^{-0.18}$$ ← Maximal quenching

[¶] J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C 87, 014315 (2013).

[§] S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D90, 033005 (2014).

If g_A is renormalized to ~0.8-0.5, all estimates for half-lives should be increased by a factor of ~6-34 and limits on the average neutrino mass should be increased by a factor ~2.5-6, making it impossible to reach in the foreseeable future even the inverted region.



$g_A = 0.5$ $\langle m_\nu \rangle < 1.3 eV$

$g_A = 1$ $\langle m_\nu \rangle < 0.3 eV$

$g_A = 1.269$ $\langle m_\nu \rangle < 0.2 eV$

Limits from EXO in ^{136}Xe decay

If g_A is renomalized to ~0.8-0.5, even the exploration of the inverted hierarchy will require a multiton large neutrino infrastructure.

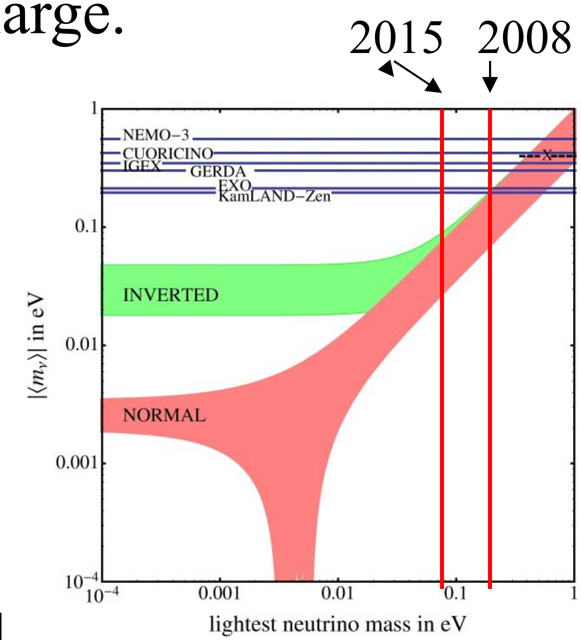
Possibilities to escape this negative conclusion are:
(1) Neutrino masses are degenerate and large.

This possibility will be in tension with the cosmological bound on the sum of the neutrino masses

$$\sum_i m_i \leq 0.6 eV \quad (2008)$$

$$\sum_i m_i \leq 0.230 eV \quad (2015) \text{ Planck } ¶$$

68% confidence level



¶ S. Matarrese for the Planck collaboration, Proc. XVI Int. Workshop NEUTEL 2015, in press.

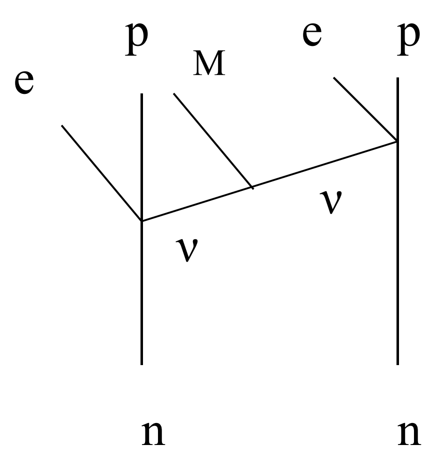
(2) Both mechanisms, light and heavy exchange, contribute simultaneously, are of the same order of magnitude, and interfere constructively.

$$[\tau_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu}\left| M_{0\nu,light}\frac{\langle m_\nu \rangle}{m_e} + M_{0\nu,heavy}\frac{m_p}{\langle m_{\nu_h} \rangle}\right|^2$$

This possibility requires a fine tuning which is quite unlikely.

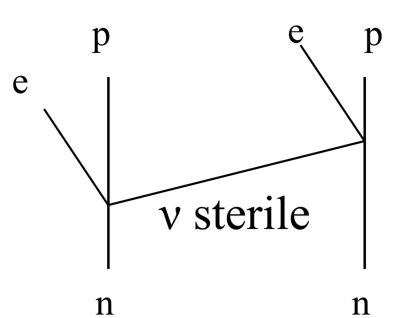
(3) Other scenarios (Majoron emission, …) and/or new mechanisms (sterile neutrinos, …) must be considered.

3



Majoron means a massless neutral boson

4



Sterile means no standard model interactions §

§B. Pontecorvo, Sov. Phys. JETP 26 (1968) 984

No matter what the mechanism of neutrinoless DBD is, its observation will answer the fundamental questions:

- What is the absolute neutrino mass scale?
- Are neutrinos Dirac or Majorana particles?
- How many neutrino species are there?

APPENDIX A: REFERENCES

PSF

$2\nu\beta^-\beta^-/0\nu\beta^-\beta^-$

J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

$2\nu\beta^+\beta^+/0\nu\beta^+\beta^+$

J. Kotila and F. Iachello, Phys. Rev. C 87, 024313 (2013).

NME

$2\nu\beta^-\beta^-/0\nu\beta^-\beta^-$

J. Barea and F. Iachello, Phys. Rev. C 79, 044301 (2009).

J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013).

J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 91, 034304 (2015).

$2\nu\beta^+\beta^+/0\nu\beta^+\beta^+$

J. Barea, J. Kotila and F. Iachello, Phys. Rev. C87, 057301 (2013).

R0νECEC

J. Kotila, J. Barea, and F. Iachello, Phys. Rev. C 89, 064319 (2014).

APPENDIX B
Scenario 3: MAJORON EMISSION

The inverse half-life for this scenario (0νββφ decay) is given by

$$\left[\tau_{1/2}^{0\nu\beta\beta\varphi}\left(0^+ \rightarrow 0^+\right)\right]^{-1} = G_{0\nu\varphi}\left|M_{0\nu}\right|^2 \langle g \rangle^2$$

effective Majoron coupling constant

NME are the same as for scenario 1 and 2.

PSF have been recalculated recently.

Best limit ¶ with IBM-2 NME, KBI PSF and g_A=1.269 from EXO/KamLAND-Zen

$$\langle g^2 \rangle < 6.2 \times 10^{-5}$$

¶ J. Kotila, J. Barea and F. Iachello, in preparation (2015).

This scenario was suggested by H.M. Georgi, S.L. Glashow, and S. Nussinov, Nucl. Phys. B193, 297 (1981).

Scenario 4: STERILE NEUTRINOS

Another scenario is currently being discussed, namely the mixing of additional "sterile" neutrinos.
[The question on whether or not "sterile" neutrinos exist is an active areas of research at the present time with experiments planned at FERMILAB and CERN-LHC.]

NME for sterile neutrinos of arbitrary mass can be calculated by using a transition operator as in scenario 1 and 2 but with

$$f = \frac{m_{\nu I}}{m_e}$$

$$v(p) = \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_{\nu I}^2}\left(\sqrt{p^2 + m_{\nu I}^2} + \tilde{A}\right)}$$

Effective mass of the sterile neutrinos

IBM-2 NME for this scenario have just been calculated (April 2015).
PSF are the same as in scenarios 1 and 2.

Several types of sterile neutrinos have been suggested.

Scenario 4a: HEAVY STERILE NEUTRINOS

Sterile neutrinos with masses $m_{\nu I} \gg 1eV$

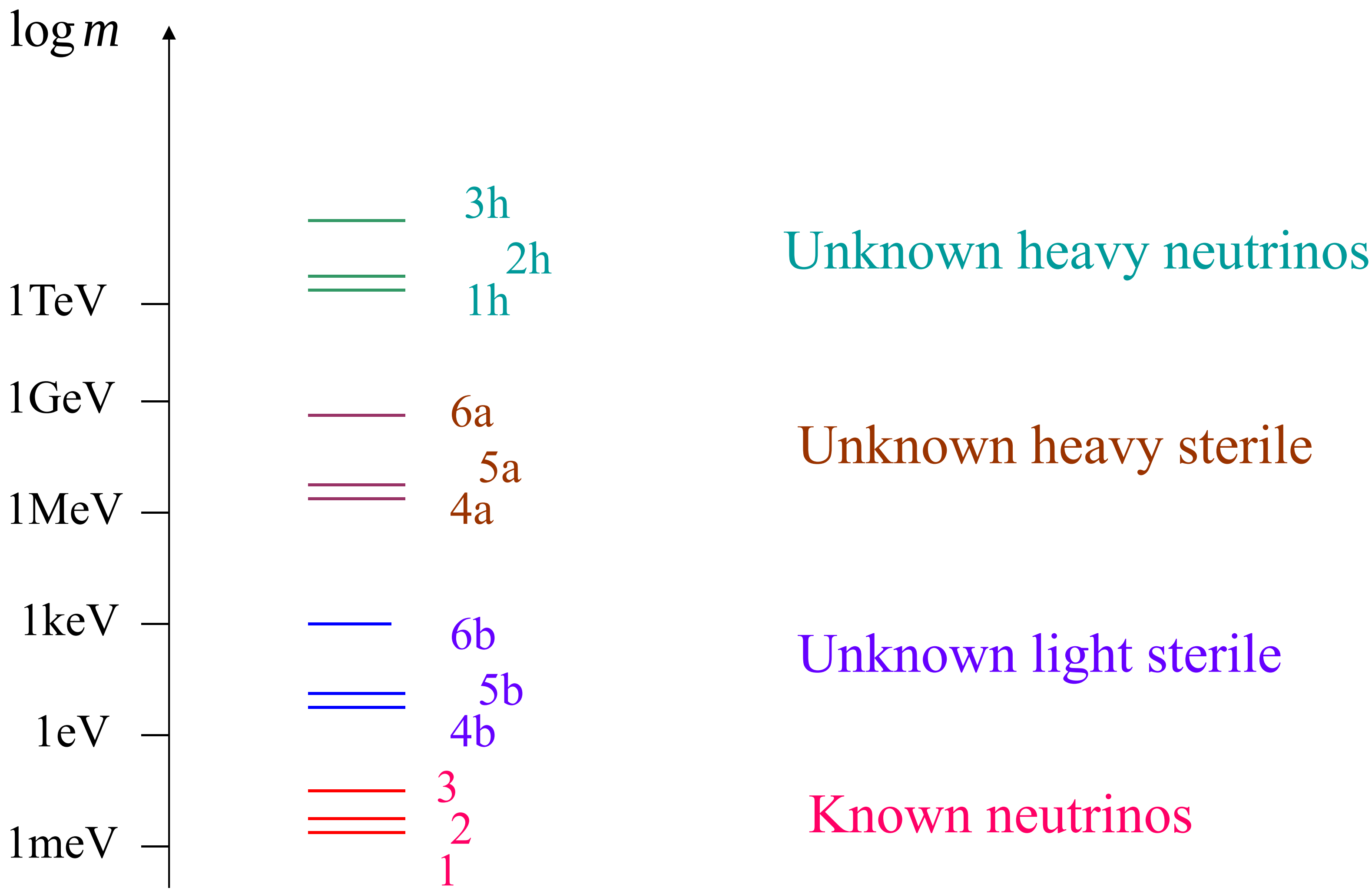
Possible values of the sterile neutrino, 4a,5a, 6a,…, masses in the keV-GeV range have been suggested by T. Asaka and M. Shaposhnikov, Phys. Lett. B620, 17 (2005) and T. Asaka, S. Blanchet, and M. Shaposhnikov, Phys. Lett. B631, 151 (2005).

Scenario 4b: LIGHT STERILE NEUTRINOS

Sterile neutrinos with masses $m_{\nu I} \sim 1eV$

Very recently C. Giunti and M. Laveder have suggested sterile neutrinos, 4b,…, with masses in the eV range to account for the reactor anomaly in oscillation experiments, G. Giunti, XVI International Workshop on Neutrino Telescopes, Venice, Italy, March 4, 2015.

HYPOTHETICAL NEUTRINO SPECTRUM



CONTRIBUTIONS OF HYPOTHETICAL NEUTRINOS ALL

$$\left[\tau_{1/2}^{0\nu\beta\beta}(0^+ \to 0^+)\right]^{-1} = G_{0\nu} g_A^4 M \left| \frac{m_p}{B} \sum_{k=1}^{3} U_{ek}^2 m_k + m_p \sum_{i_b} \frac{U_{ei_b}^2}{m_{i_b}} + \frac{m_p}{B} \sum_{i_a} U_{ei_a}^2 m_{i_a} + m_p \sum_h \frac{U_{eh}^2 m_h}{B + m_h^2} \right|^2$$

Known neutrinos

Unknown heavy sterile

Unknown light sterile

Unknown heavy neutrinos

The values of M and B in IBM-2 have been just calculated ¶

¶ J. Barea, J. Kotila and F. Iachello, paper in preparation (2015).