Beta decay studies of the most exotic nuclei

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Outline of the lessons:

PARTI: General concepts

How to measure β decay in exotic nuclei

PARTII: β decay gross properties $T_{1/2}$ and P_n

PARTIII: High resolution vs TAS

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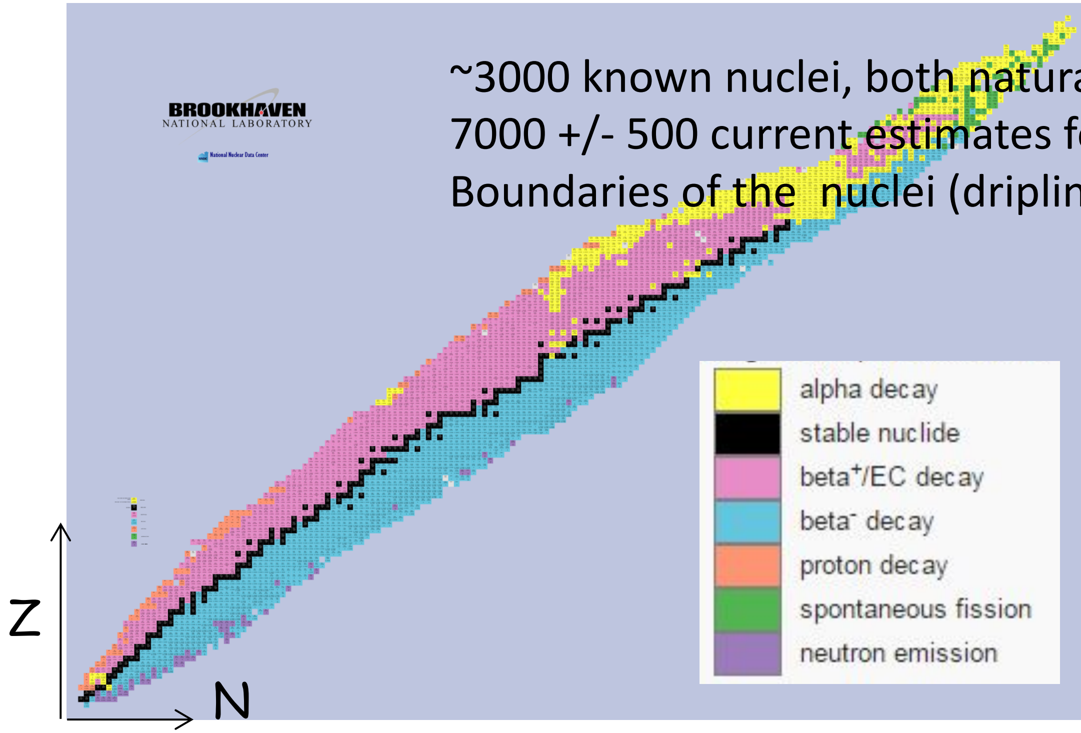
Introduction

~3000 known nuclei, both natural (288) and artificial
7000 +/- 500 current estimates for bound nuclei
Boundaries of the nuclei (driplines) are not defined yet



β decay dominated by weak interaction and its properties directly relate to fundamental questions:

- Properties of the neutrinos ➔ energy spectrum, double-β decay, sterile neutrinos
- Abundances of elements in universe ➔ half lives and competing decay modes

and applications:

- PET scanners
- Radioactive sources (science and medicine)
- Homeland security
- Nuclear waste

Introduction

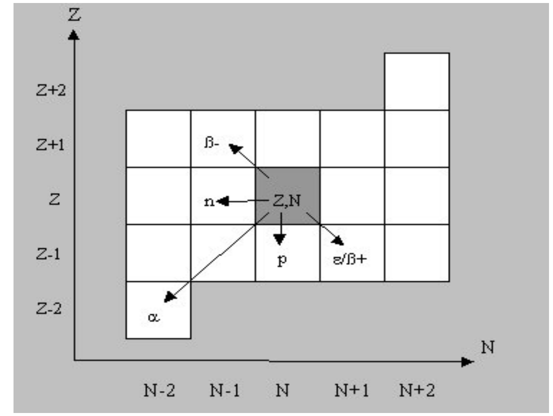
Decay modes

α decay: $${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$

β⁻ decay: $${}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + e^{-} + \bar{\upsilon}_e$$

β⁺ decay: $${}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y + e^{+} + \upsilon_e$$

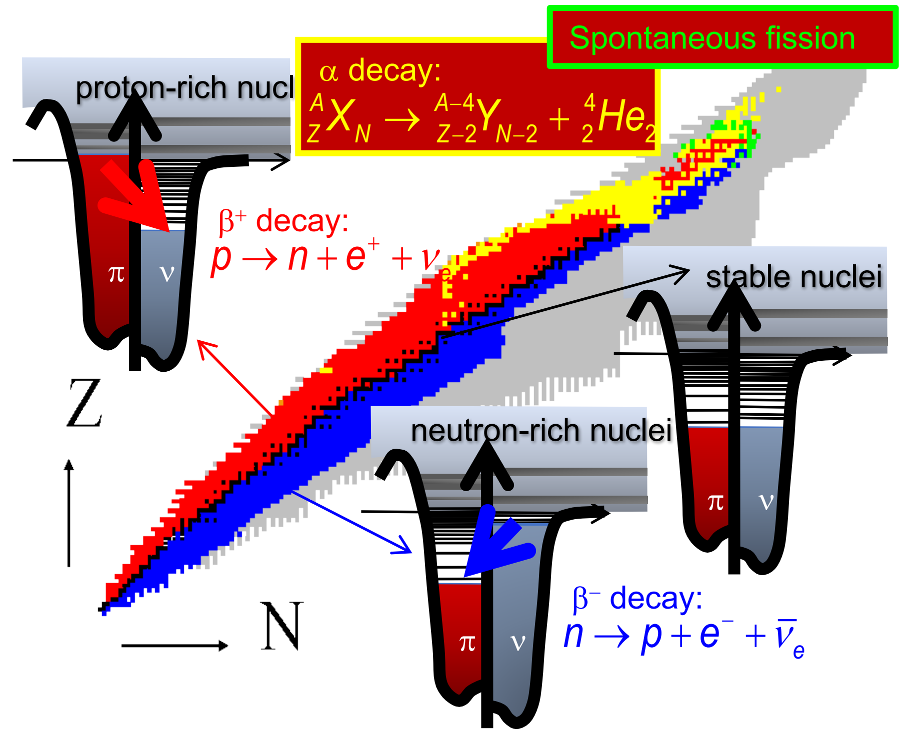
EC: $${}^{A}_{Z}X + e^{-} \rightarrow {}_{Z-1}^{A}Y + \upsilon_e$$



Spontaneous Fission

Exotic decay modes: n decay
p decay
cluster emission

β-mediated decays:
β-delayed n/p emission
β-delayed fission

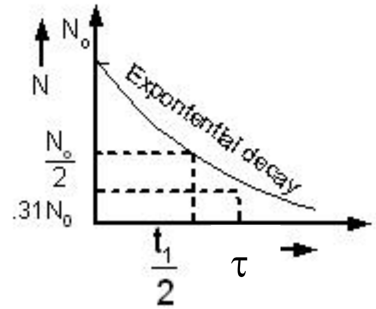


Introduction

Useful definitions for decays:

Law of radioactive decay:

$$N(t) = N_0 e^{-\lambda t}$$



λ decay rate

$\tau = 2\pi/\lambda$ meanlife

$T_{1/2} = \ln 2 * \tau$ half-life

$$T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda} \approx 0.693\tau$$

Radioactive half-life

Radioactive decay constant

Mean lifetime

$Q_{value} = M_{initial} - M_{final}$

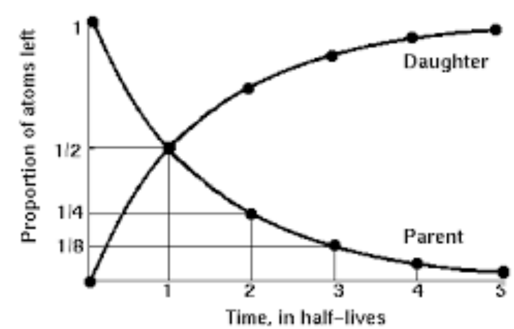
Branching ratio: decay probability of two or more competing processes

Mother/Precursor

➔ Initial/Emitting nucleus

Daughter/Successor

➔ Final nucleus



Introduction

Bateman equations: mathematical model describing abundances and activities in a **decay chain** as a function of time, based on the **decay rates and initial abundances**. If at a time t, there are $N_i(t)$ atoms of the i-th isotope which decays into the i+1 one with a decay rate λ_i, the amounts of isotopes in the k-th step of the decay chain evolves as:

$$\frac{dN_1(t)}{dt} = -\lambda_1 N_1(t)$$
$$\frac{dN_i(t)}{dt} = -\lambda_i N_i(t) + \lambda_{i-1} N_{i-1}(t)$$
$$\frac{dN_k(t)}{dt} = \lambda_{k-1} N_{k-1}(t)$$

This can be written in an explicit form as:

$$N_n(t) = \prod_{j=1}^{n-1} \lambda_j \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{N_i(0) e^{-\lambda_j t}}{\prod_{p=1, p\neq j}^{n} (\lambda_p - \lambda_j)} \right)$$

This can be extended to cases in which we have many branches

H. Bateman. "Solution of a System of Differential Equations Occurring in the Theory of Radio-active Transformations," Proc. Cambridge Phil. Soc. IS, 423 (1910) https://archive.org/details/cbarchive_122715_solutionofasystemofdifferentia1843

Introduction

Secular equilibrium:

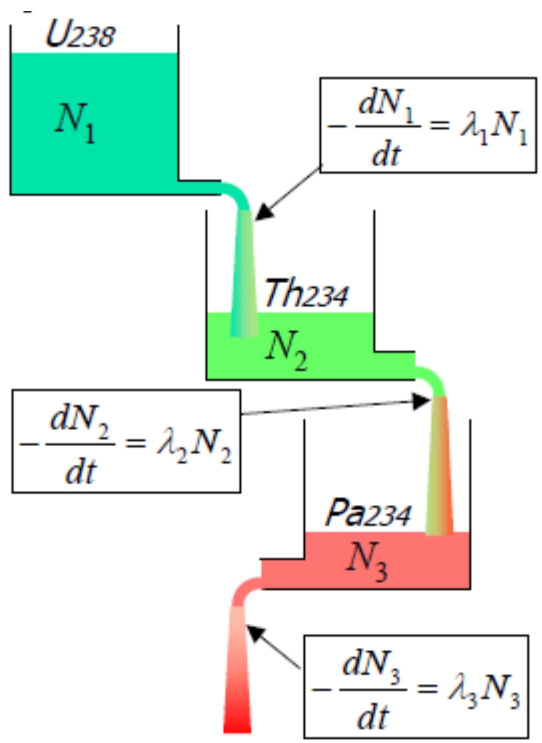
The rate of production and decay of an element is constant

It can be achieved in a decay chain if the daughter B has a much shorter half-life than the mother A

The decay rate of A, which is translated into the production rate of B is constant

The quantity of B accumulates since the number of nuclei which decay in 1 s if equal to that produced in 1 s.

This depends on $N_A(t=0)$, λ_A and λ_B



- Each member of the family is a dewar. They are connected and the fill each other in chain
- The rate of emptying (-dN/dt) are function of the level in the dewar N and the decay rate λ
- At equilibrium the rate of evacuation in each dewar is equal
- In nuclear physics rate of decay is the Activity and depend on the decay constant λ
- At equilibrium all activities are equal

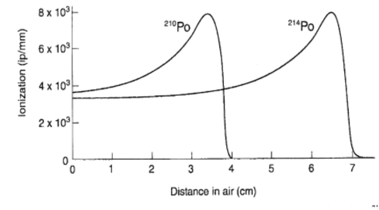
Introduction

Detecting α β γ n particles: reminder of basic properties

α particle: massive, charged

➔ interested in energy, angular distribution

➔ spectrometers, Si, plastic detectors

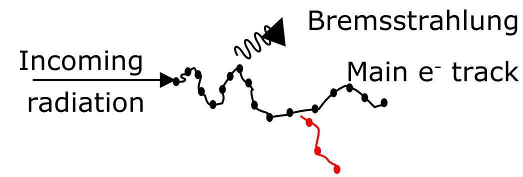


β particles: light, charged

➔ interested in energy and correlations

➔ e^+ annihilates into two 511-keV γ rays

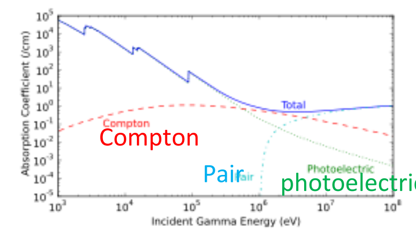
➔ spectrometers, Si, plastic detectors



γ particles: massless, neutral

➔ interested in energy and angular distributions

➔ HPGe detectors, inorganic scintillators

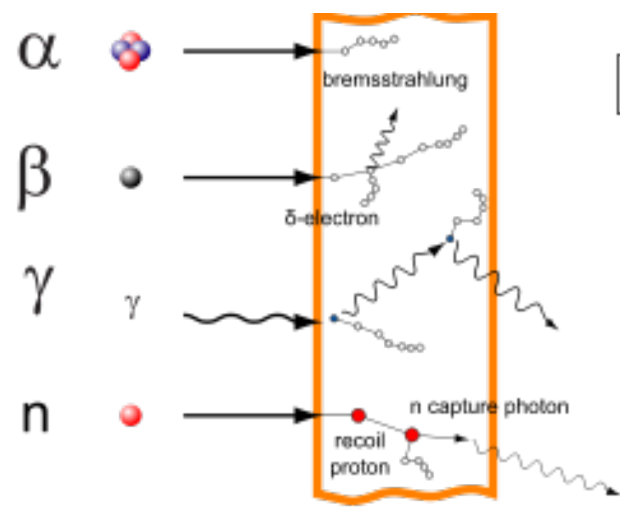


Neutrons: massive, neutral

➔ interested in energy and correlations

➔ measured indirectly via capture reactions

➔ liquid scintillators, inorganic scintillators



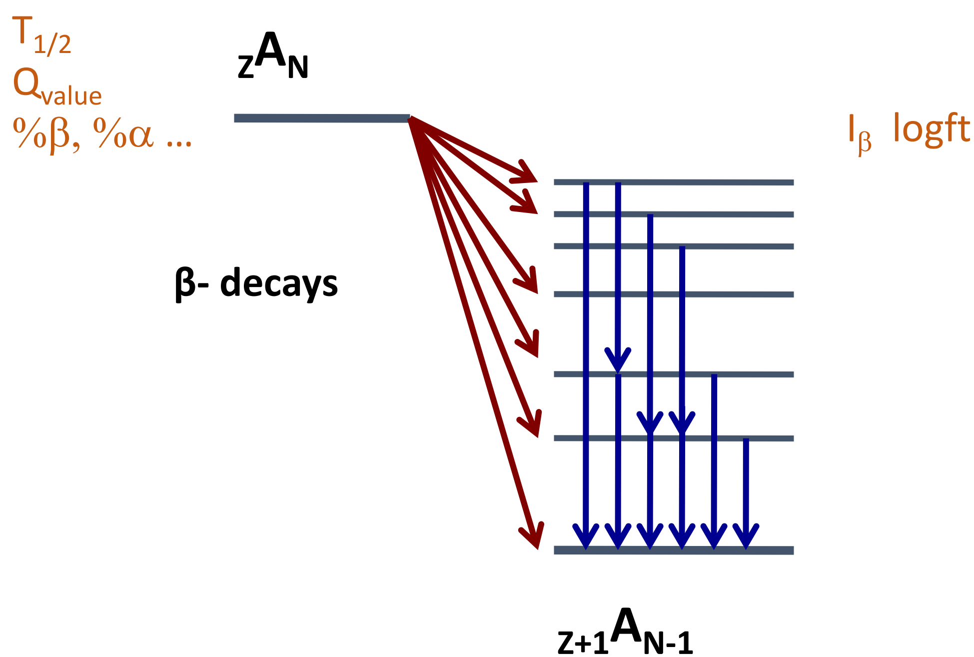
Introduction

Example of detailed study of a decay:

1) Produce and Identify PARENT nucleus: reactions/ISOL/fragmentation

2) Detect the decay: identify the emitted particle α/β
identify competing mechanisms
identify decaying state, g.s. or excited isomeric state
measure half-life

3) Study properties of DAUGHTER nucleus:
which levels are populated
Branching Ratio btw levels
subsequent decays

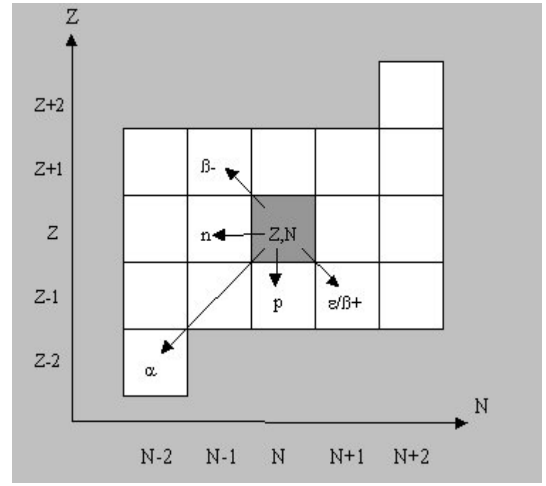


General Properties of β decay

$${}^{A}_{Z}X \rightarrow {}_{Z+1}^{\;\;A}Y + e^{-} + \upsilon_e$$ β⁻ decay

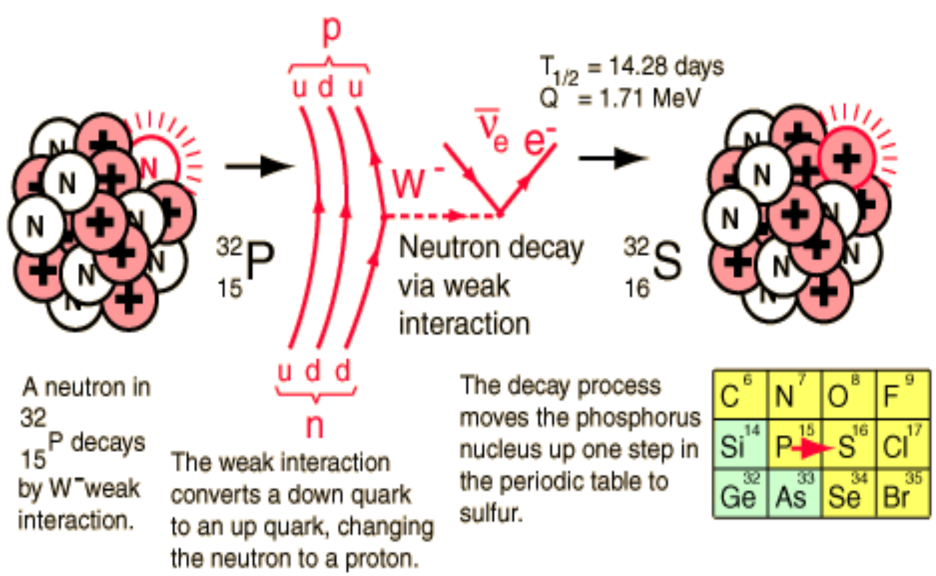
$${}^{A}_{Z}X \rightarrow {}_{Z-1}^{\;\;A}Y + e^{+} + \upsilon_e$$ β⁺ decay

$${}^{A}_{Z}X + e^{-} \rightarrow {}_{Z-1}^{\;\;A}Y + \upsilon_e$$ EC



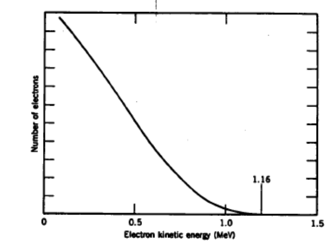
β decay is a weak interaction "semi-leptonic" decay

The quark level Feynman diagram for β⁻ decay is shown here:



General Properties of β decay

- The Q value in β decay is effectively shared between the electron and antineutrino.
- This is the case of gs-> gs decay



·e 9.1 The continuous electron distribution from the β decay of ^{210}Bi (also l RaE in the literature).

Q value for β^- decay is

$$Q_{\beta^-} = \left(M(A,Z) - M(A,Z+1)\right)c^2$$

Q_β Sum of the energy of the electron (positron) and antineutrino (neutrino)

$$Q_{\beta^-} = T_{M(A,Z+1)^+} + T_e + T_\nu \approx T_e + T_\nu \qquad \left(\text{since } T_{M(A,Z+1)^+} < keV\right)$$

$$Q_{\beta^-} = (T_e)_{\max} = (T_\nu)_{\max}$$ If the other term is null

$$Q_{\beta^+} = M(A,Z) - M(A,Z-1) - 2m_e c^2$$

For electron capture:

$$Q_{EC} = \left(M(A,Z) - M'(A,Z-1)\right)c^2 - B_n$$

$$B_n \ \textit{binding energy of } n - \textit{shell electron}$$

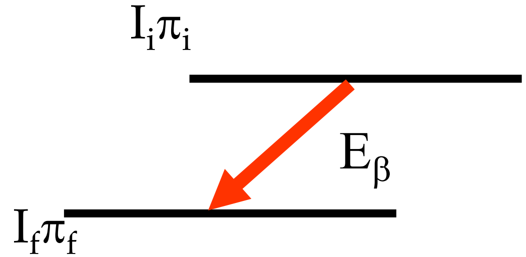
NB: EC and β+ are not always competing:

if β+ , EC is possible, the contrary is not guaranteed

Note these are atomic masses

General Properties of β decay

$$I_i = I_f + L_\beta + S_\beta$$



$$L_\beta = l_{\beta-(\beta+)} + l_{\tilde{\nu}(\nu)}$$

$$\pi_i \pi_f = (-1)^{L_\beta}$$

$$S_\beta = s_{\beta-(\beta+)} + s_{\tilde{\nu}(\nu\nu} = \begin{cases} 0 & \uparrow\downarrow \\ 1 & \downarrow\downarrow \text{or} \uparrow\uparrow \end{cases}$$

$L_\beta = n$ defines the degree of forbiddenness (n)

allowed

when $L_\beta = n = 0$

and $\pi_i\pi_f = +1$

Electron and neutrino do not carry angular momentum

forbidden

when the angular momentum conservation requires that $L_\beta = n > 0$ and $\pi_i\pi_f = (-1)^{L\beta}$

$$\Delta I = \left| I_i - I_f \right| \equiv 0,1$$

General Properties of β decay

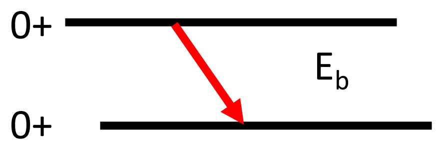
Selection rules for possible decays

Type of transition	Order of forbiddenness	ΔI	$\pi_i\pi_f$
Allowed		0,+1	+1
Forbidden unique	1	∓2	-1
	2	∓3	+1
	3	∓4	-1
	4	∓5	+1
	.	.	.
Forbidden	1	0, ∓1	-1
	2	∓2	+1
	3	∓3	-1
	4	∓4	+1
	.	.	.

Classification of **allowed decays**

$$(\pi_i \pi_f = +1)$$

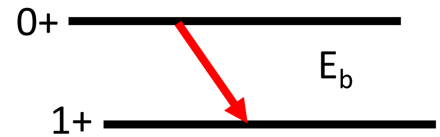
Fermi



$$\Delta I = \left| I_i - I_f \right| \equiv 0$$

$$L_\beta = 0 \quad S_\beta = 0 \downarrow\uparrow$$

Gamow-Teller



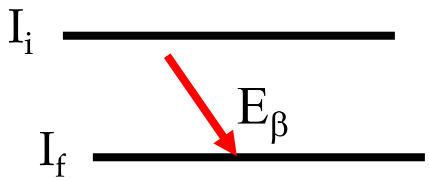
$$\Delta I = \left| I_i - I_f \right| \equiv 1$$

$$L_\beta = 0 \;\; S_\beta = 1 \uparrow\uparrow or \downarrow\downarrow$$

Useful empirical rules

The fifth power beta decay rule:

the speed of a β transition increases approximately in proportion to the fifth power of the total transition energy



$$\frac{1}{\tau} \propto \left[\left(M(Z) - M(Z \pm 1)\right)c^2\right]^5$$

- depends on spin and parity changes between the initial and final state
- additional hindrance due to nuclear structure effects :

  isospin, “l-forbidden”, “K-forbidden”, etc.

Fermi Golden Rule

- Treat beta decay as a transition that depends upon the strength of coupling between the initial and final states
- Decay constant is given by Fermi's Golden Rule

$$\lambda_\beta = \frac{2\pi}{\hbar}\left|M\right|^2 \rho(E_f); M = \int \psi_f V \psi_i dv$$

- ➔Electron and neutrino do not pre-exist in atom but are formed at the time of decay
- ➔The decay is the result of the interaction btw the nucleon and the field produced by the electron-antineutrino couple ➔ weak interaction
- Perturbation theory can be applied since the interaction is "weak"
- M matrix element which couples the initial and final states
- Rate proportional to the strength of the coupling between the initial and final states factored by the density of final states available to the system electron-antineutrino

General Properties of β decay

$$t \equiv T_{1/2}^{\beta_i} = \frac{T_{1/2}^{\exp}}{P_{\beta_i}}$$

partial half-life of a given β^- (β^+,EC) decay branch (*i*)

$$\frac{\ln 2}{T_{1/2}^n} = \frac{g^2}{2\pi^3} \int_1^W p_e W_e (W_0 - W_e)^2 F(Z, W_e) C_n dW_e$$

g – weak interaction coupling constant

p_e – momentum of the β particle

W_e – total energy of the β particle

W_0 – maximum energy of the β particle

$F(Z,W_e)$ – Fermi function – distortion of the β particle wave function by the nuclear charge

C_n – shape factor

Z – atomic number

$$\boxed{\lambda = \frac{\ln 2}{t_{1/2}} = K \left| M_{if} \right|^2 f_o}$$

$$K = 64\pi^4 m_o^5 c^4 g^2 / h^7$$

$$f_o = \int_1^{W_o} F(Z, W) W (W^2 - 1)^{1/2} (W_o - W)^2 dW$$

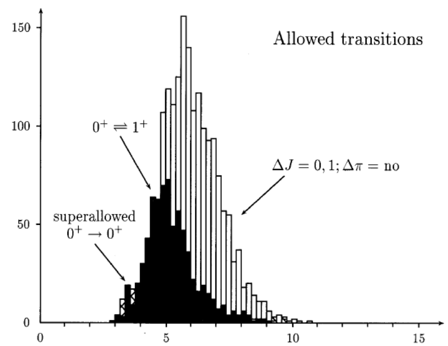
General Properties of β decay

Comparative Half Lives

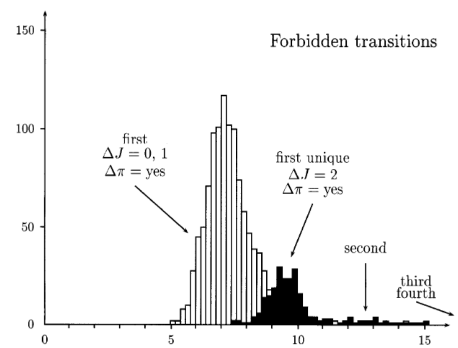
- Based on probability of electron energy emission coupled with spectrum and the Coulomb correction $f_o t_{1/2}$ is called the comparative half life of a transition

$$f\tau = \frac{2\pi^3 \hbar^7 c^3}{g^2 |M_{if}| 2}$$

- Assumes matrix element is independent of energy (true for allowed transitions) Yields $f\tau$ (or $f_o t_{1/2}$), comparative half-life may be thought of as the half life corrected for differences in Z and energy
- ALLOWED transitions second term is independent on nucleus,
- ➔ ft has the same value for all allowed transitions
- For forbidden decays ft increases with degree of forbiddenness



Logft < 6.0



Logft > 6.0

Review Of Log*ft* Values In β Decay*

Nuclear Data Sheets 84, 487 (1998)
Article No. DS980015

Measuring logft

Nature 486, 341 (2012)

doi:10.1038/nature11116

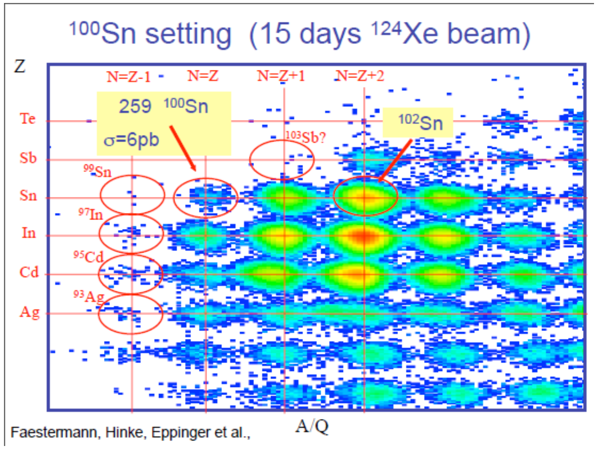
Superallowed Gamow–Teller decay of the doubly magic nucleus ^{100}Sn

^{100}Sn N=Z=50

Heaviest N=Z nucleus

Relativistic Fragmentation reaction

^{124}Xe beam on ^{9}Be target



~260 ^{100}Sn nuclei produced (0.75/h)

~ 126 fully reconstructed decay chains

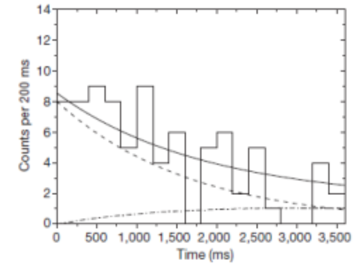
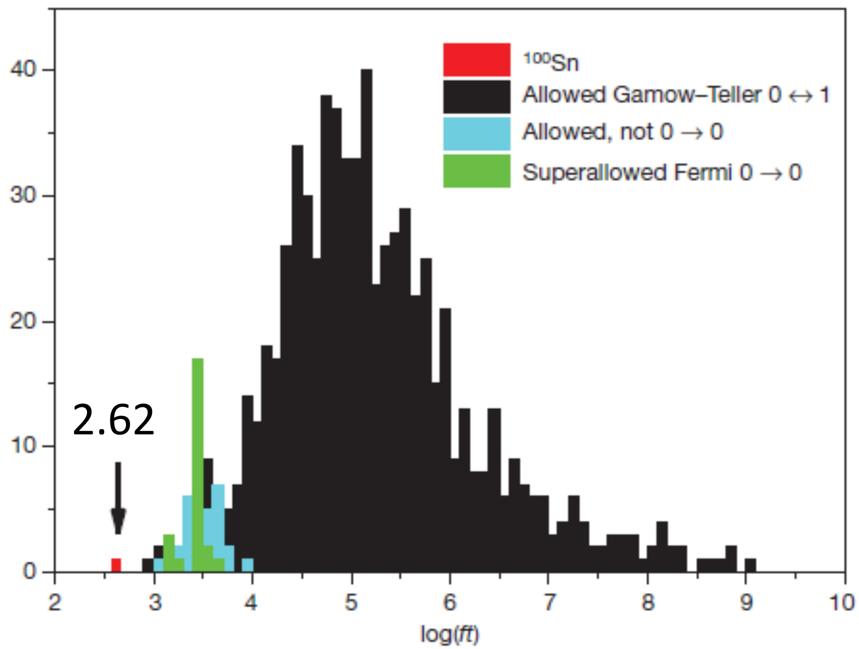


Figure 2 | Time distribution of first decay events. The histogram shows the observed time distribution of all first decay events in the nearest-neighbouring pixels after implantation of ^{100}Sn nuclei. Decay curves resulting from the MLH analysis are shown individually for ^{100}Sn (dashed) and its daughter nucleus ^{100}In (dash–dot). The solid line shows the sum of these decay curves and takes into account a small amount of random background.



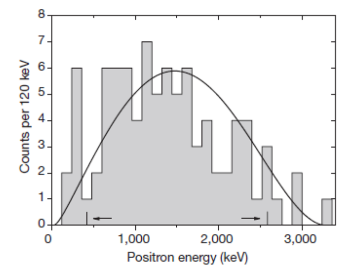
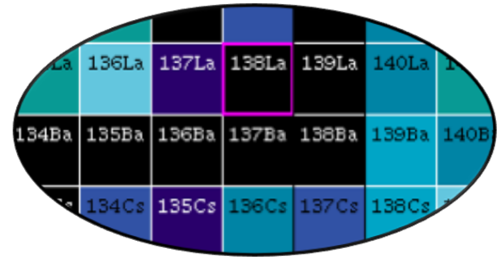


Figure 5 | Distribution of the positron energies emitted in the β-decay of ^{100}Sn. The spectrum contains only decay events that can be assigned to ^{100}Sn decays with a probability of at least 75%. The MLH fit was applied to the region between 400 and 2,600 keV, which is indicated with markers. The solid curve illustrates the shape of the best-fitting single-component β-decay phase space function determined by MLH analysis.

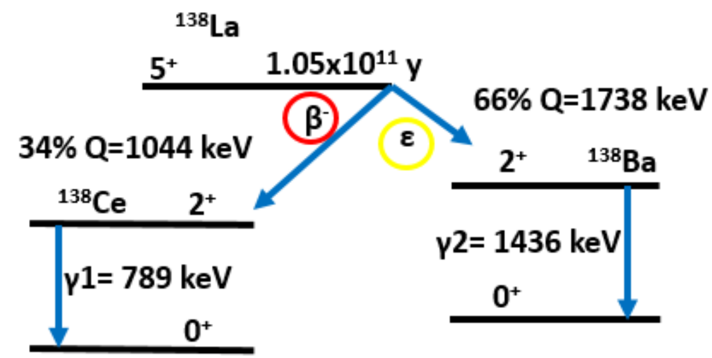
Highly forbidden decays

β Decay of ^{138}La

Lantanum has two isotopes
- ^{139}La, stable
- **^{138}La, radioactive**:
  - $T_{1/2}$= **1.05*10^{11}** years
  - abundance **0.09%**.

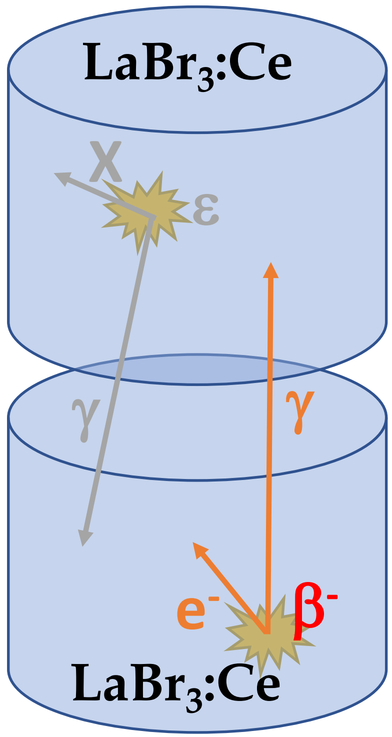


^{138}La has two decay branches
- β⁻ towards ^{138}Ce, BR= 33.6%
- EC towards ^{138}Ba BR=66%



Both decays are **second forbidden decays**
5⁺ ➔ 2⁺

A.Giaz et al. EPL 110 (2015) 42002



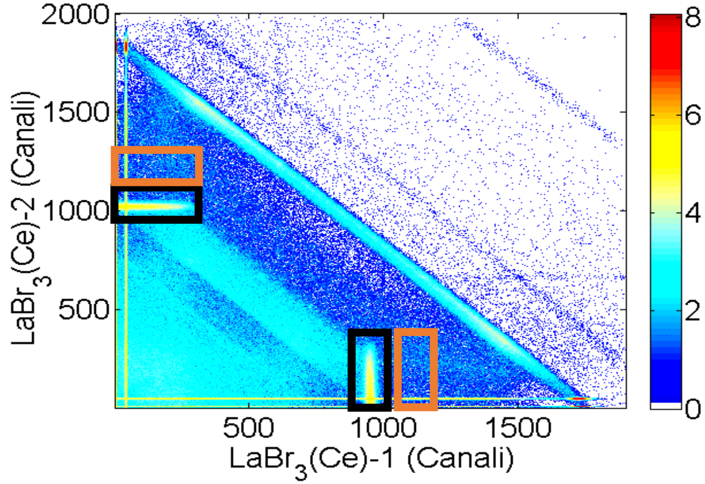
Experiment: measuring the BR and the electron spectra
Using LaBr3(Ce) detectors as **source AND as detectors**

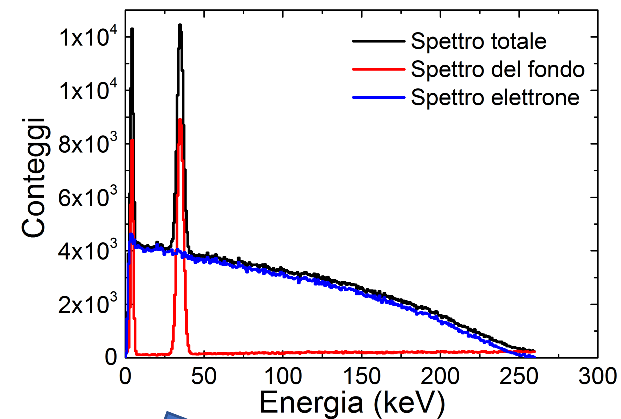
- β⁻ decay: e⁻ in the det. which β decays, 789 keV γ ray in the other;
- EC: X ray in the det. which decays, 1436 keV γ ray in the other

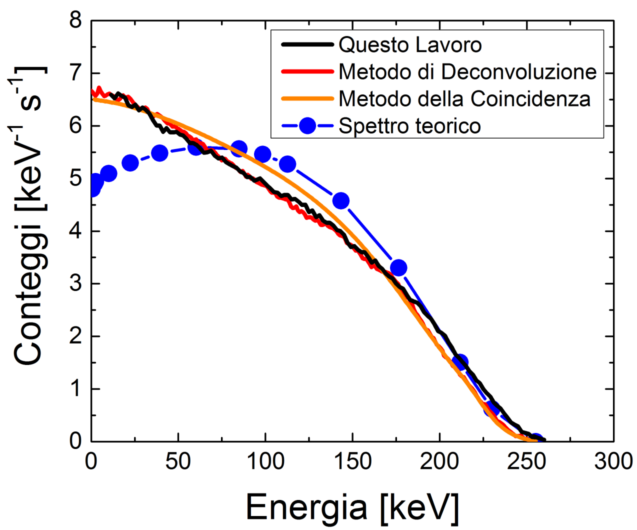
Highly forbidden decays

Background subtraction to get e⁻ spectrum:

- Coincidence with 789 keV (black box) **total spectrum**
- **Compton bg** from 1436 keV (red box)
- Bg subtraction cancel peaks at **37.44 and 5.6 keV** in coincidence with 1436 keV.







Theory has hard time reproducing low-energy shape of β-electron spectrum

Interaction btw Coulomb field of the nucleus and electron

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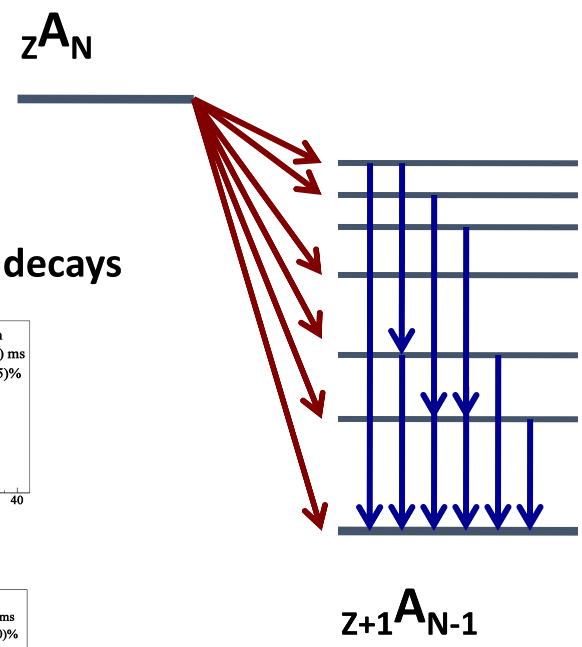
General Properties of β decay

Quantities that can be extracted in a β decay experiment

$T_{1/2}$
$P_{n/p}$
%β, %α ..

β- decays



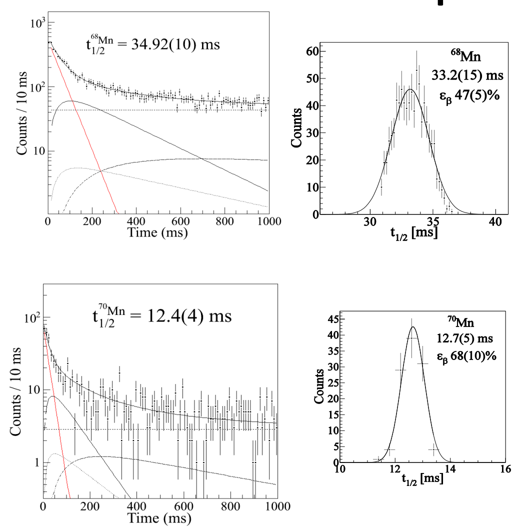
γ transitions

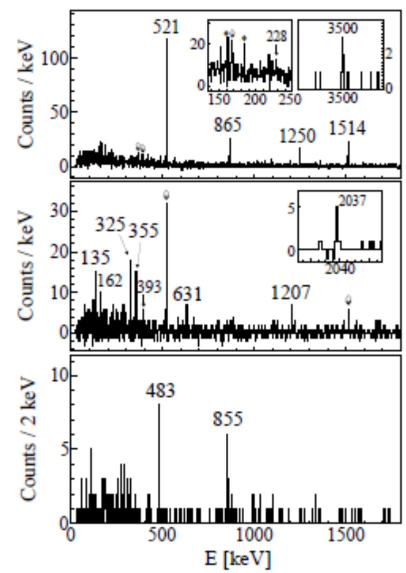
Possible existence of many decaying states

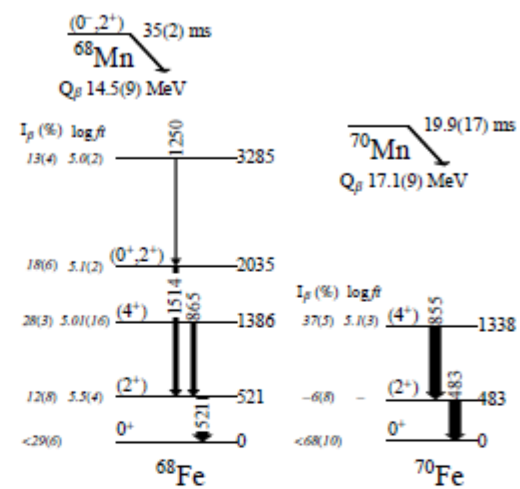
γ–γ coincidences ➔ level schemes

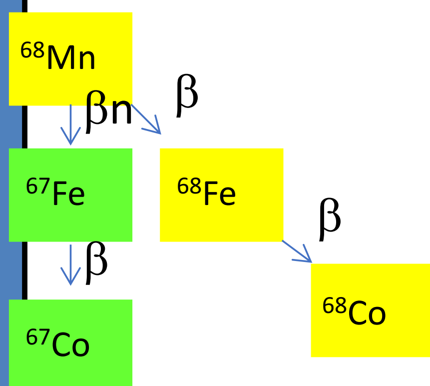
Relative intensities I_β

logft









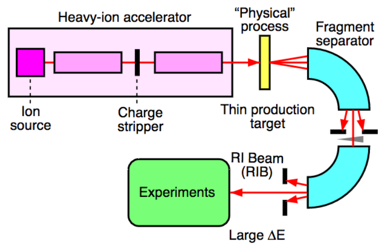
Selection rules allow to suggest/define I^π in both mother and daughter

"pandemonium effect" – exotic nuclei – log *ft* is a just upper limit

Introduction

Producing radioactive beams i.e. exotic nuclei

IN-FLIGHT



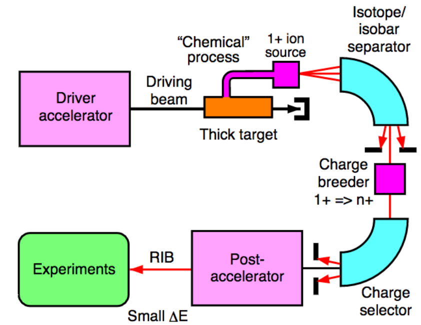
relativistic fragmentation/fission of heavy nuclei on thin targets

- > 50 MeV/u ➔ production of cocktail beams of many nuclei
- Use of spectrometers to transport/separate nuclei of interest➔ Relatively long decay paths Δt > 150-300 ns
- Nuclei are brought to rest in final focal plane and let decay

+ cocktail beam: many nuclei at once
+ both short and long-living species
+ get information already with few ions

- Low cross sections
- Limitation on rate to distinguish contribution from each species

ISOL method



spallation/fission/fragmentation on thick targets, followed by chemical/physical processes to extract desired nuclei

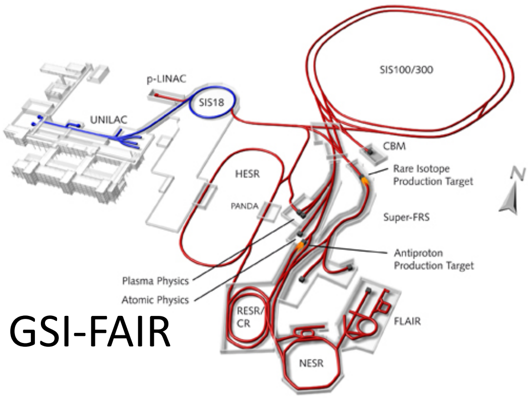
- beams produced at very low energies (~60 keV)
- Mono-isotopic beams sometimes achieved. Impurities due to few contaminant species ➔ usually long-living though

+ high cross section
+ no need to re-accelerate beams
+ high rates accepted

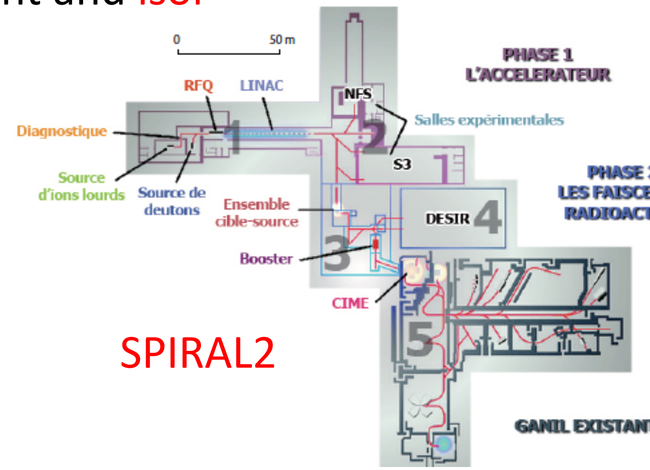
- short-living species might not be accessed easily
- Refractory elements
- Presence of long-living impurities (isobaric contamination)

Examples of facilities all over the world…in-flight and isol

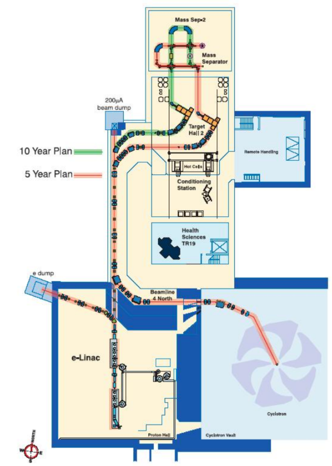
GSI-FAIR



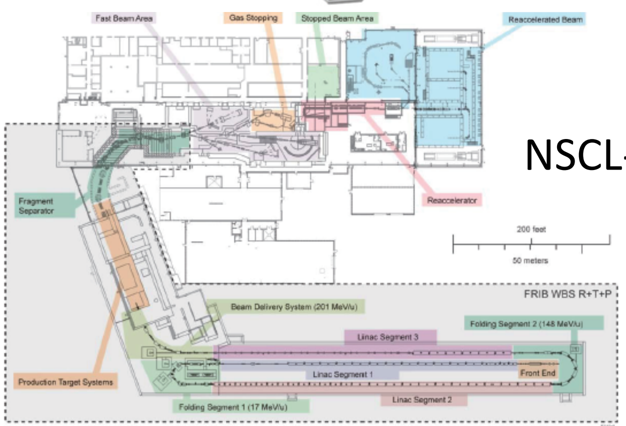
SPIRAL2



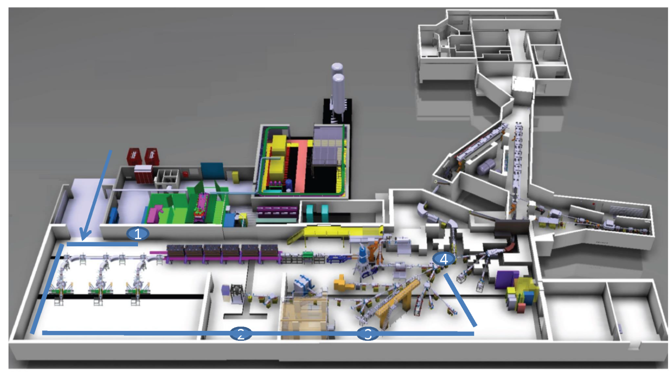
TRIUMF



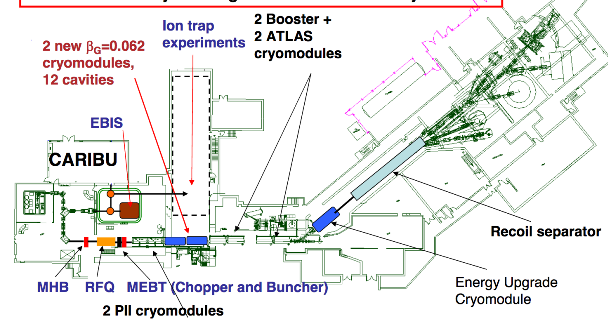
NSCL-FRIB



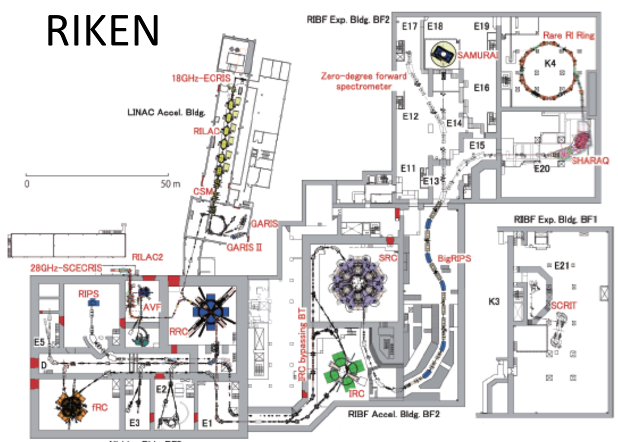
HIE-ISOLDE



Increase intensity of stable beams from ATLAS by factor >10
Increase intensity of CARIBU reaccelerated beams by > 5-10
Increase intensity of in-flight radioactive beams by >100



RIKEN



ALTO

S
P
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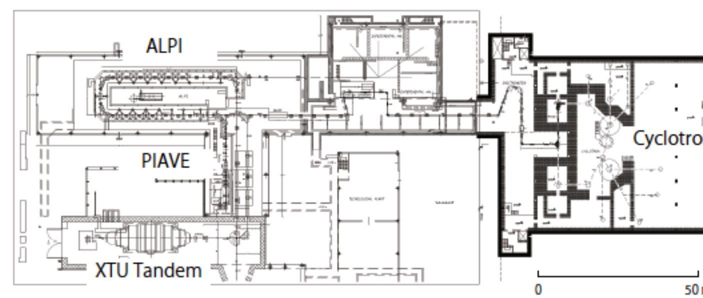
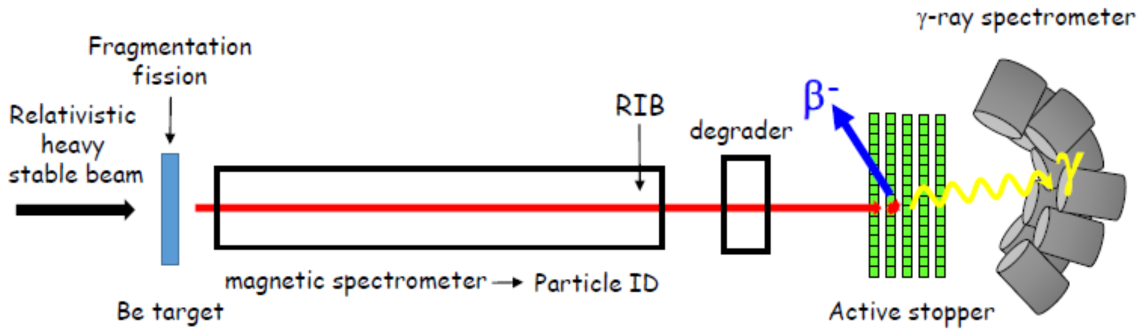
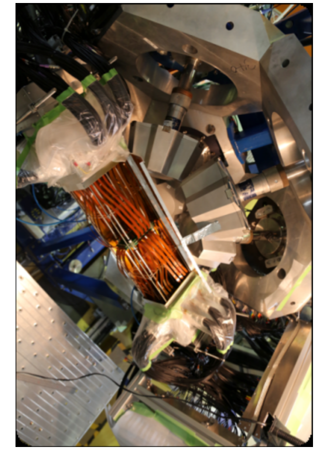


General Properties of β decay

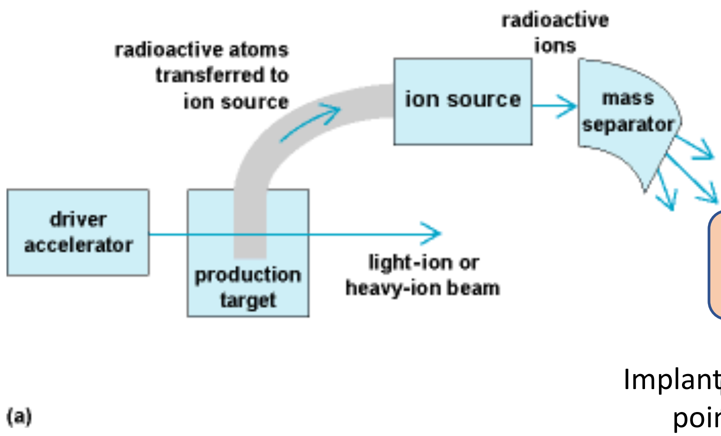
Measuring β decay



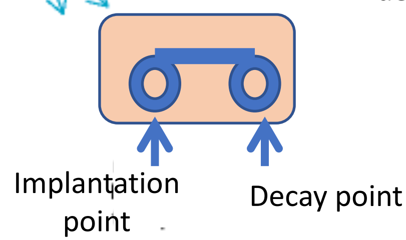


Eurica@RIKEN

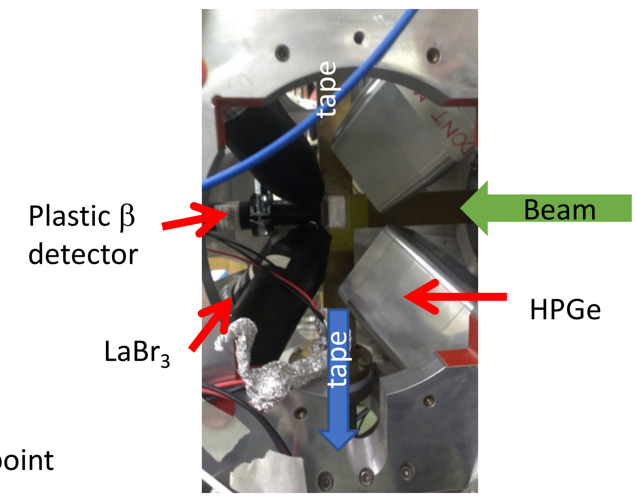
IN-Flight facility



ISOL facility

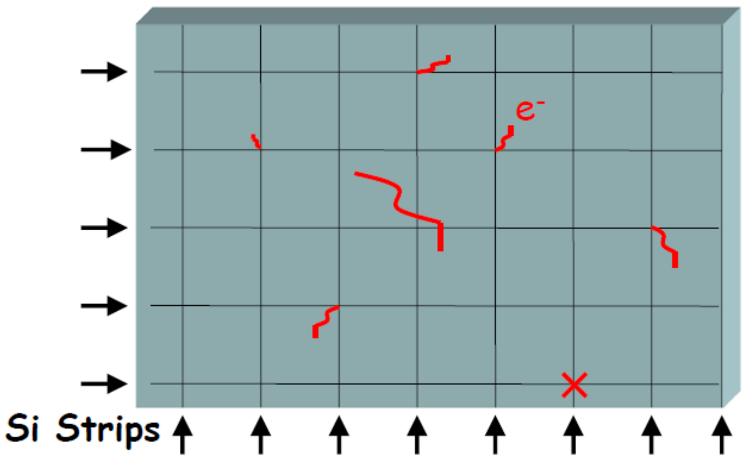


Tape decay stations



IDS @ CERN

Ion-beta correlation techniques:
distinguish implantation and decay within same detector



The waiting time between particle implantation and β-particle (or i.c. electron) emission is a measure of the decay half-life. Gamma rays emitted following these decays are detected by the RISING array.

Implantation-decay correlations with large background
(half lifes similar to the implantation rate):

✓ implant-decay time correlation: active catcher
✓ implant-decay position correlation: granularity
✓ implant of several ions:
thickness and area
✓ energy of the implanted ion and the emitted β

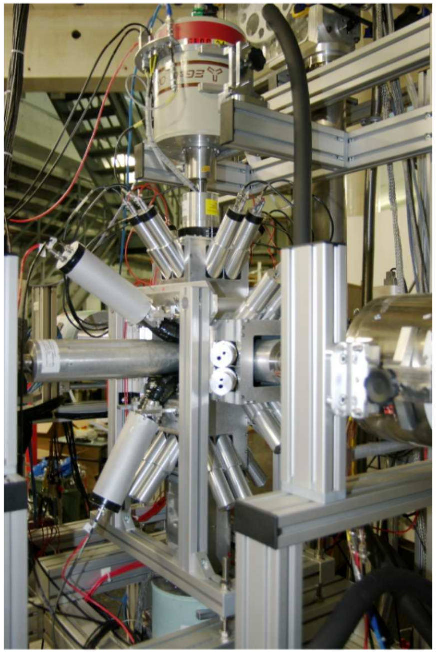
GSI

* Dual gain pre-amps on DSSSD to get energies of implanted ion and b-particle
* All events time stamped with MHz clock.

RIKEN

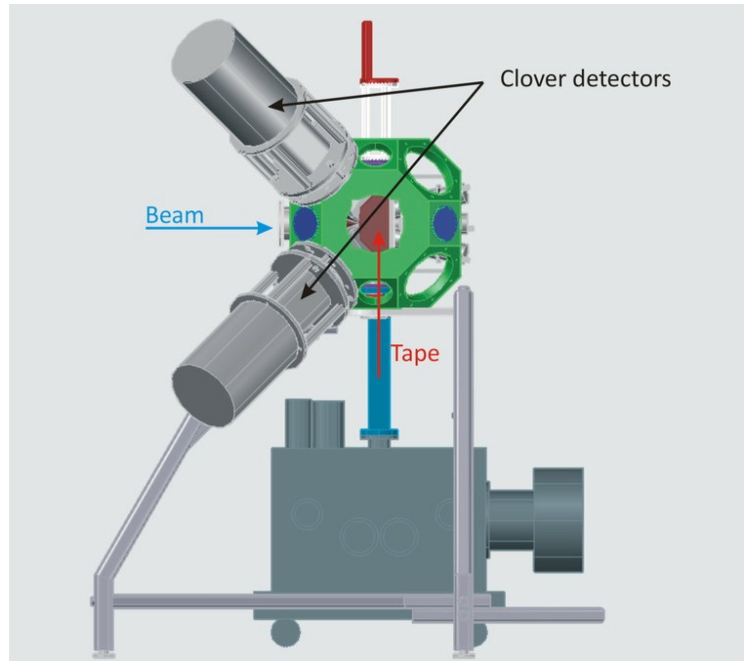
* Low gain branch for implanted ions
* High-gain branch for β and α decays

TAPE Station systems



BEDO@ALTO

Principle of operation

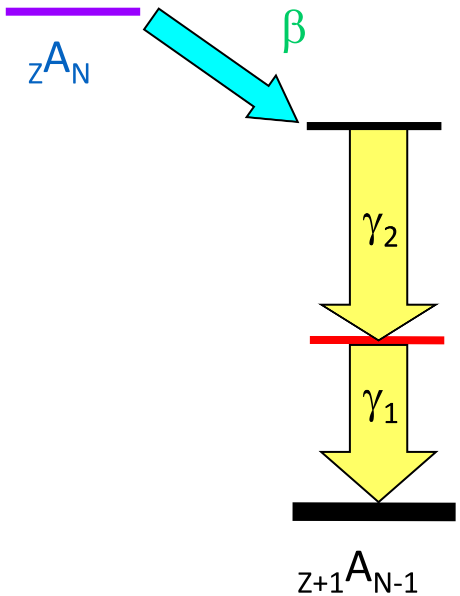


IDS@ISOLDE

- Implantation and decay in the same measuring point
- Equipped with egs. Ge detectors and Plastic (or Si detectors) for β particles
- Trigger given by proton arrival and β signal
- Long-living activity is removed by moving away the tape
- NB: no direct signal of implantation

Pandemonium effect

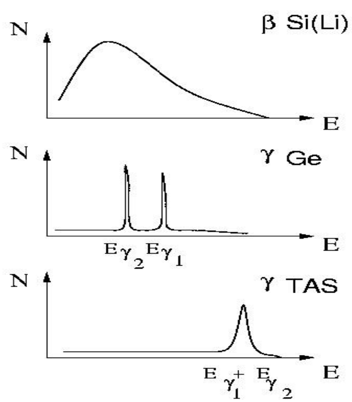
Real situation



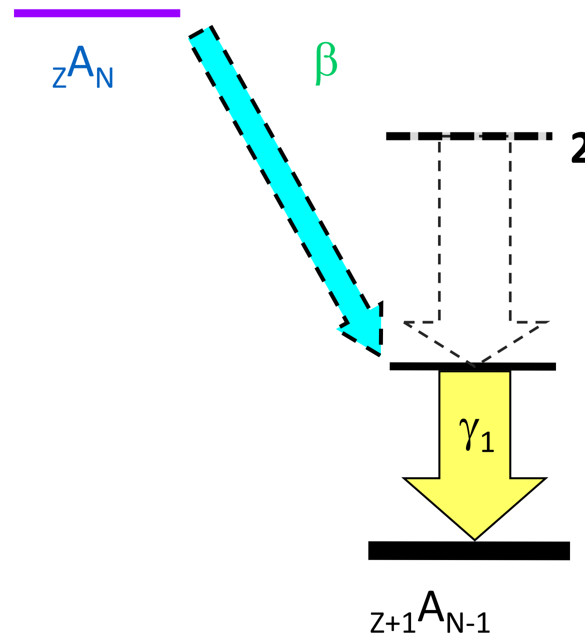
$$f_2 = I_{\gamma_2}$$

$$f_1 = 0$$

$$(I_{\gamma_2} = I_{\gamma_1})$$

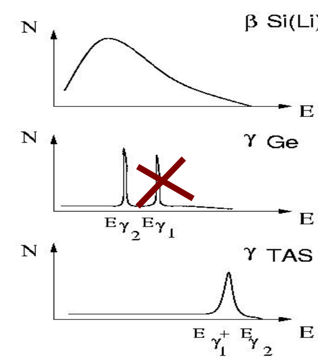


Apparent situation

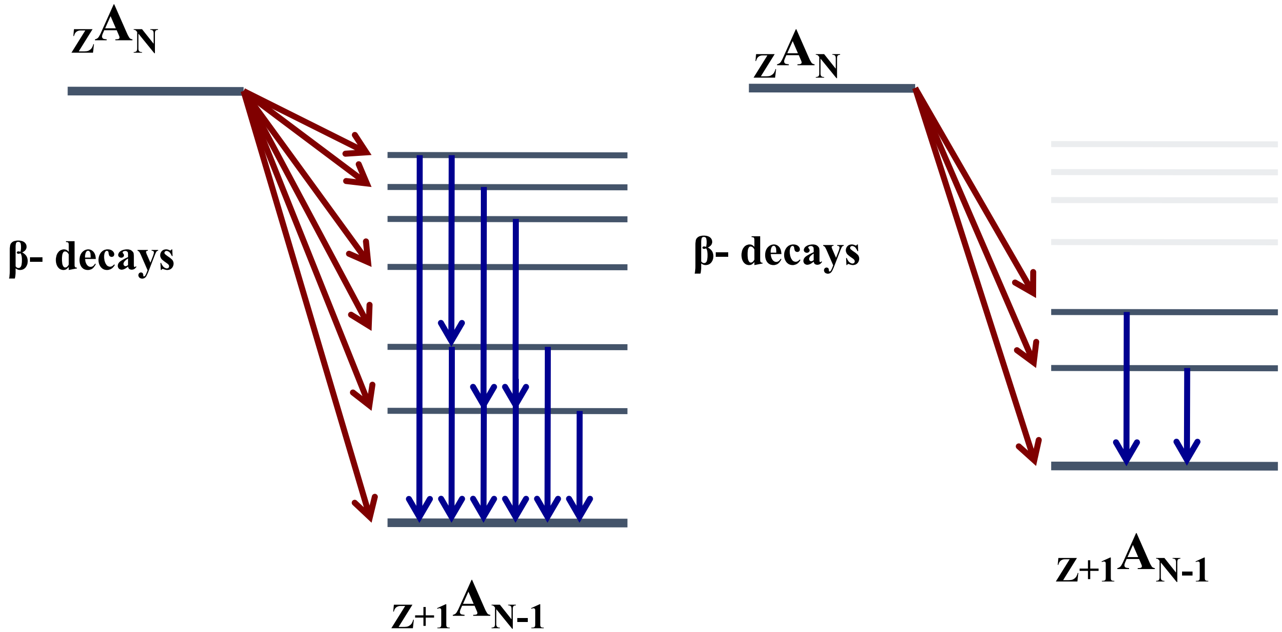


$$f_2 = 0$$

$$f_1 = I_{\gamma_1}$$



General Properties of β decay



- HPGe detectors are conventionally used to construct the level scheme populated in the decay

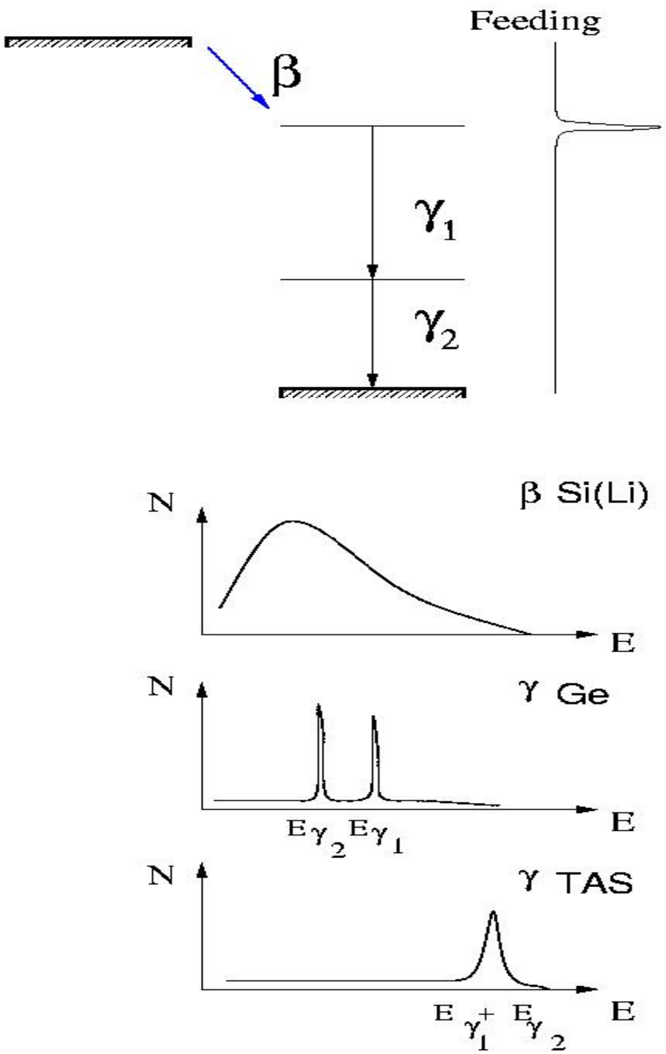
➔ Higher Q_{value} higher possibility of missing feeding owing to high-energy γ ray emission or emission of large number of γ rays

- From the γ intensity balance we deduce the β-feeding

Pandemonium effect implies

➔ Wrong definition of gamma feeding and branching ratios I_{β} and logft

Total Absorption Gamma Spectrometer measurements



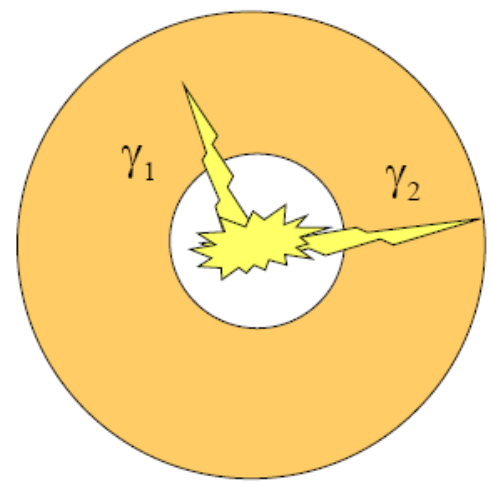
Since the gamma detection is the only reasonable way to solve the problem, we need a highly efficient device:

A TOTAL ABSORPTION SPECTROMETER

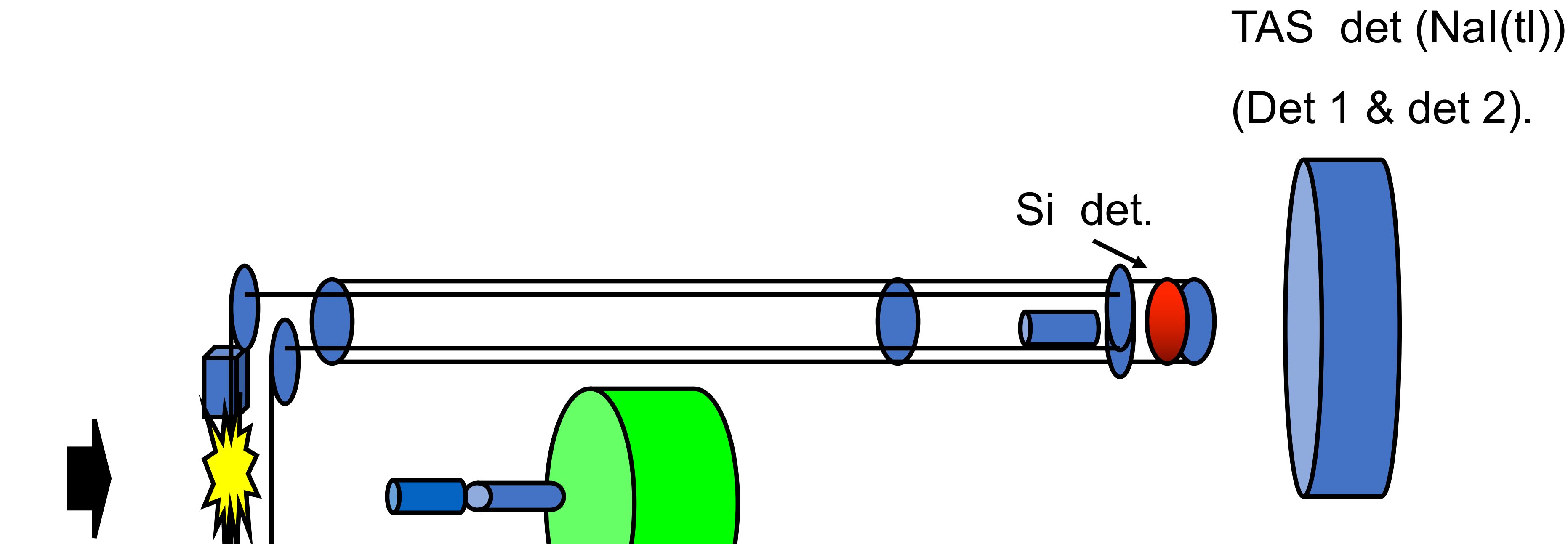
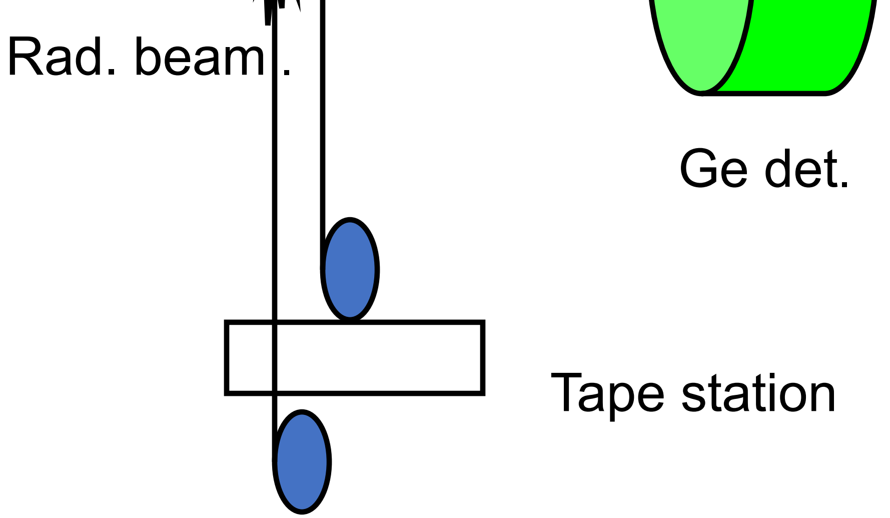
Instead of detecting the individual gamma rays we sum the energy deposited by the gamma cascades in the detector.

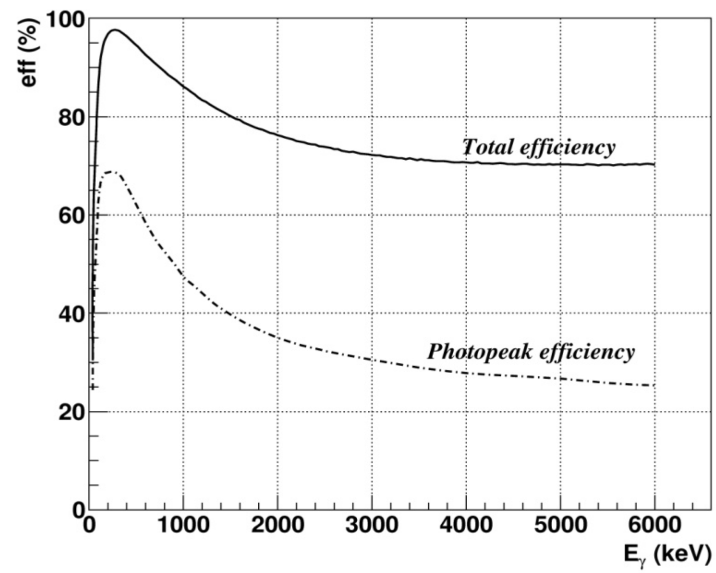
A TAS is a **calorimeter**!

Big crystal, 4π (BaF_2/NaI/HPGe)

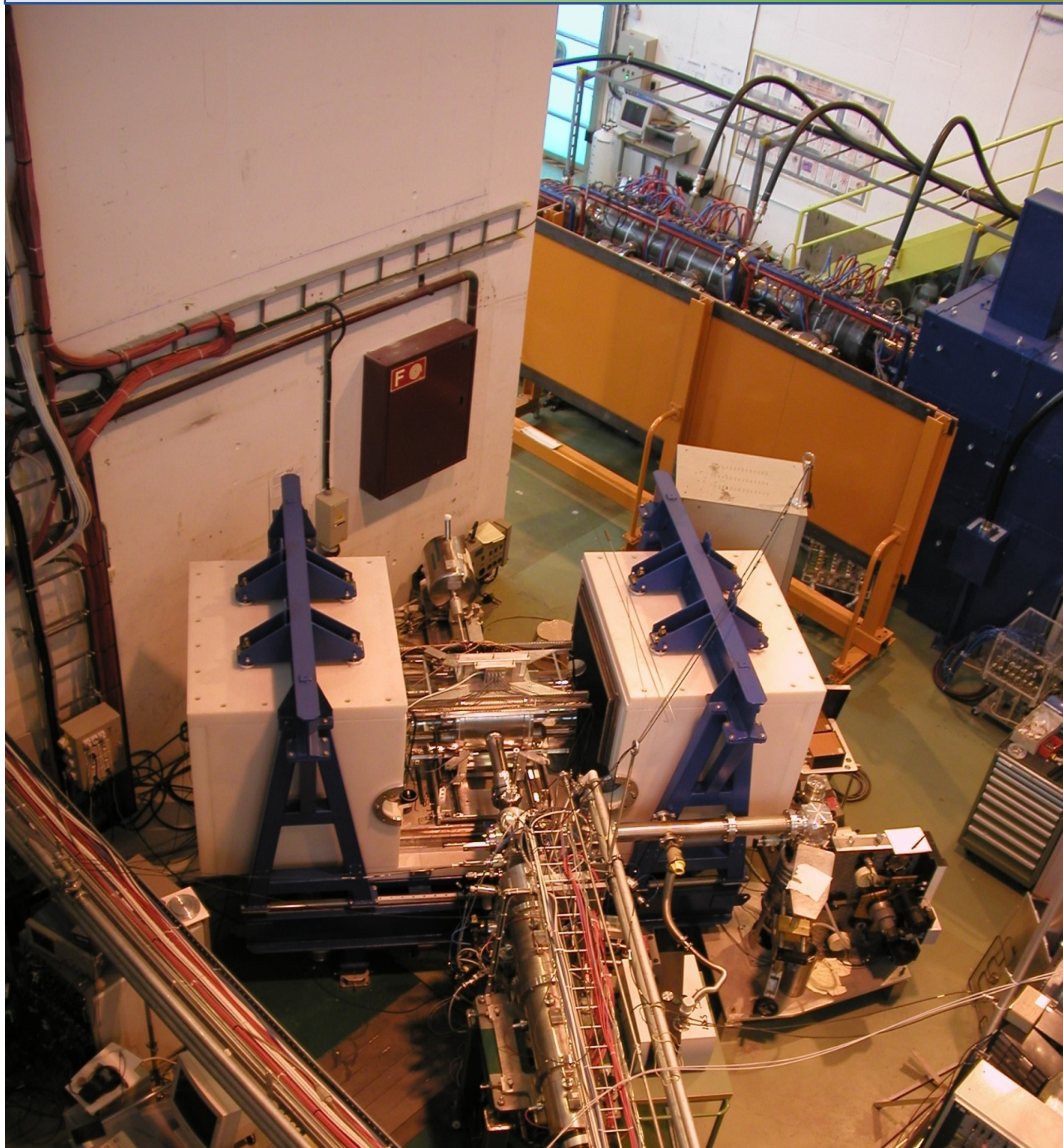


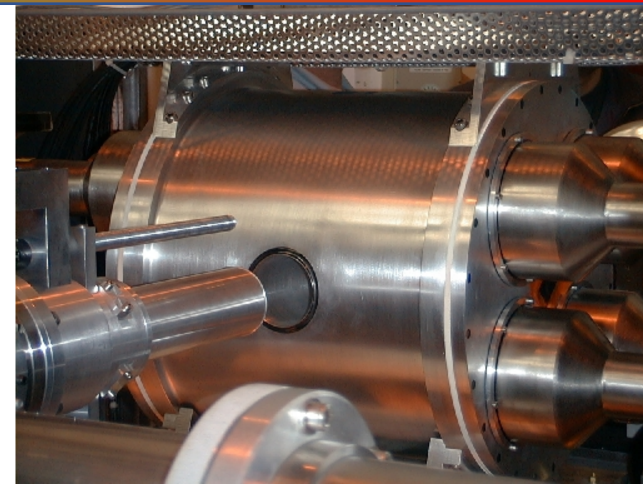
TAS experimental setup



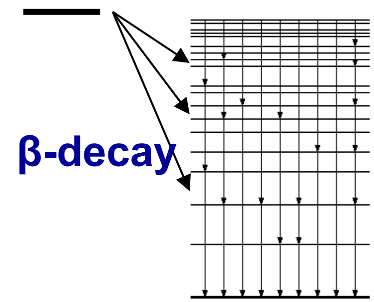
Lucrecia: the TAS at ISOLDE (CERN)
(Madrid-Strasbourg-Surrey-Valencia)





- A large NaI cylindrical crystal 38 cm Ø, 38cm length
- An X-ray detector (Ge)
- A β detector
- Possibility of collection point inside the crystal

TAGS analysis

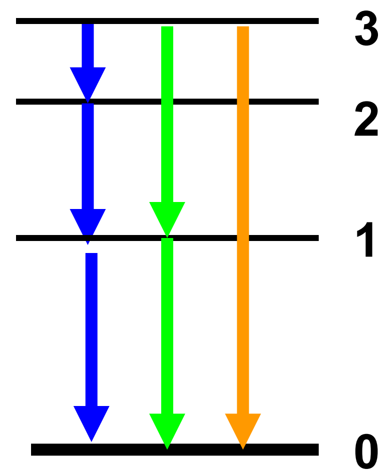


$$d_i = \sum_j R_{ij} f_j \quad or \quad \mathbf{d} = \mathbf{R} \cdot \mathbf{f}$$

R is the response function of the spectrometer, $\boldsymbol{R_{ij}}$ means the probability that feeding at a level ***j*** gives counts in data channel ***i** of the spectrum*

The response matrix $\mathbf{R}$ can be constructed by recursive convolution:

$$\mathbf{R_j} = \sum_{k=0}^{j-1} b_{jk} \mathbf{g_{jk}} \otimes \mathbf{R_k}$$



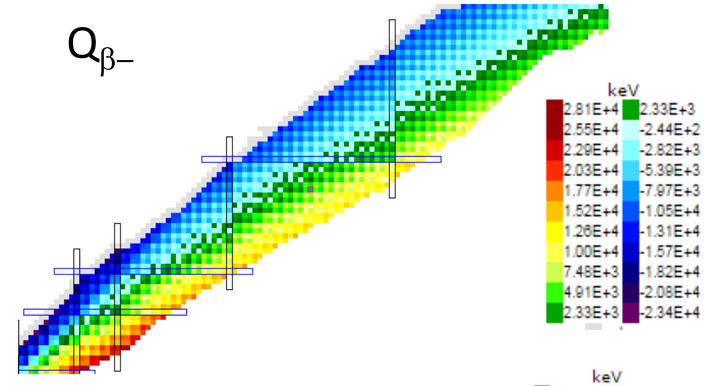
$\mathbf{g_{jk}}$: γ-response for *j* (*k* transition
$\mathbf{R_k}$: response for level *k*
b_{jk}: branching ratio for *j* (*k* transition

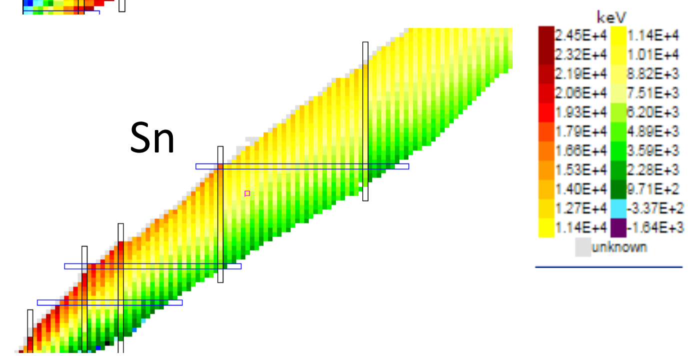
Mathematical formalization by Tain, Cano, et al.

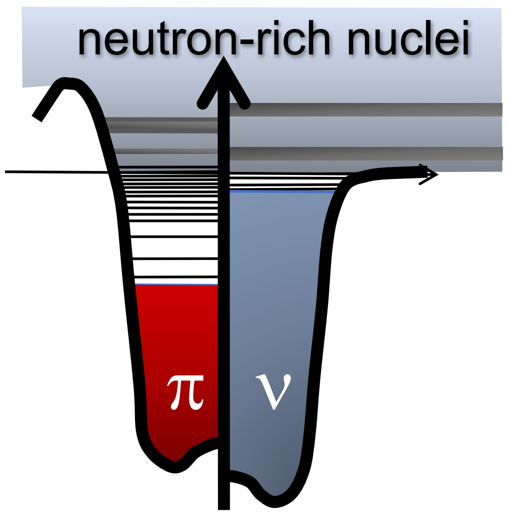
β Decay in Exotic Nuclei

Going away from stability

- Shortening of half-lives
- Increasing Q_{value}
- Decreasing S_n (on n-rich side)
- Higher possibilities of competing mechanisms
- GT unfavoured owing to large mismatch in wave functions btw mother/daughter
- Increasing role of forbidden decays







- Pandemonium effect