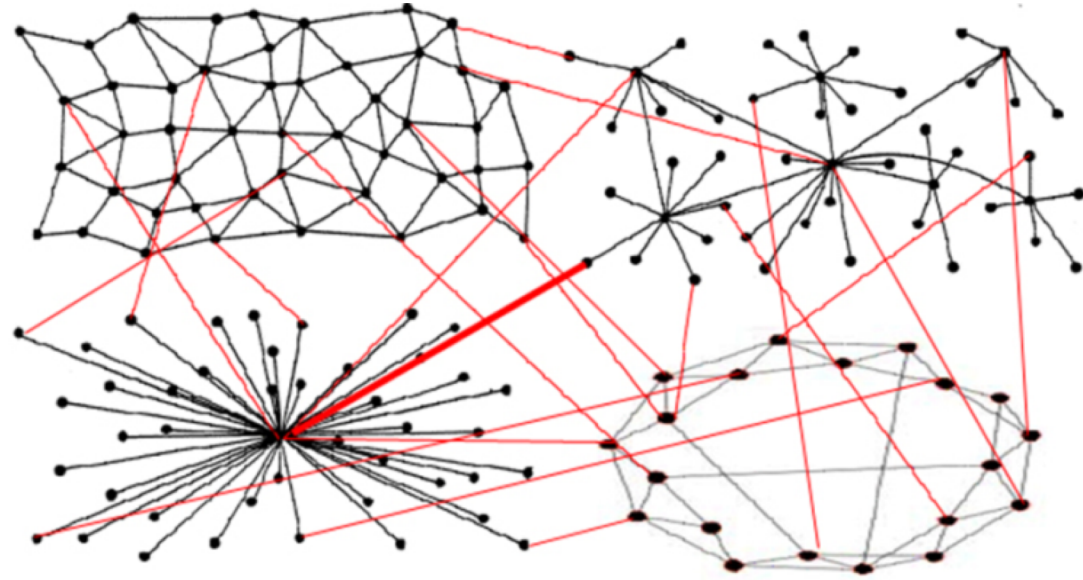
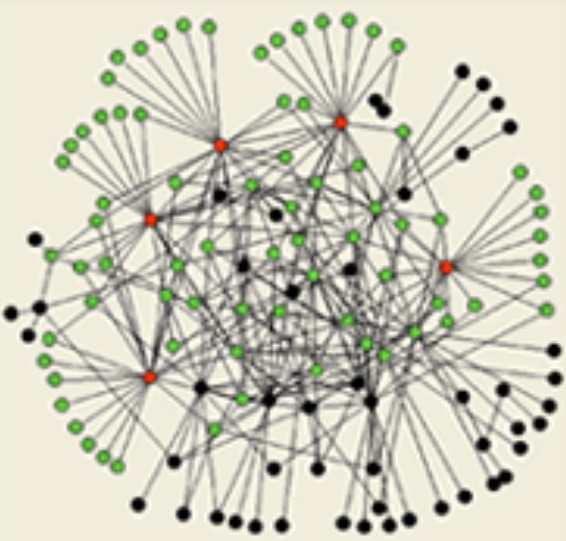


Spatio-Temporal Infrastructure Networks

From single networks to networks of networks

2000-Barabasi

2010



Shlomo Havlin
Bar-Ilan University

multilevel
multilayer
multiplex

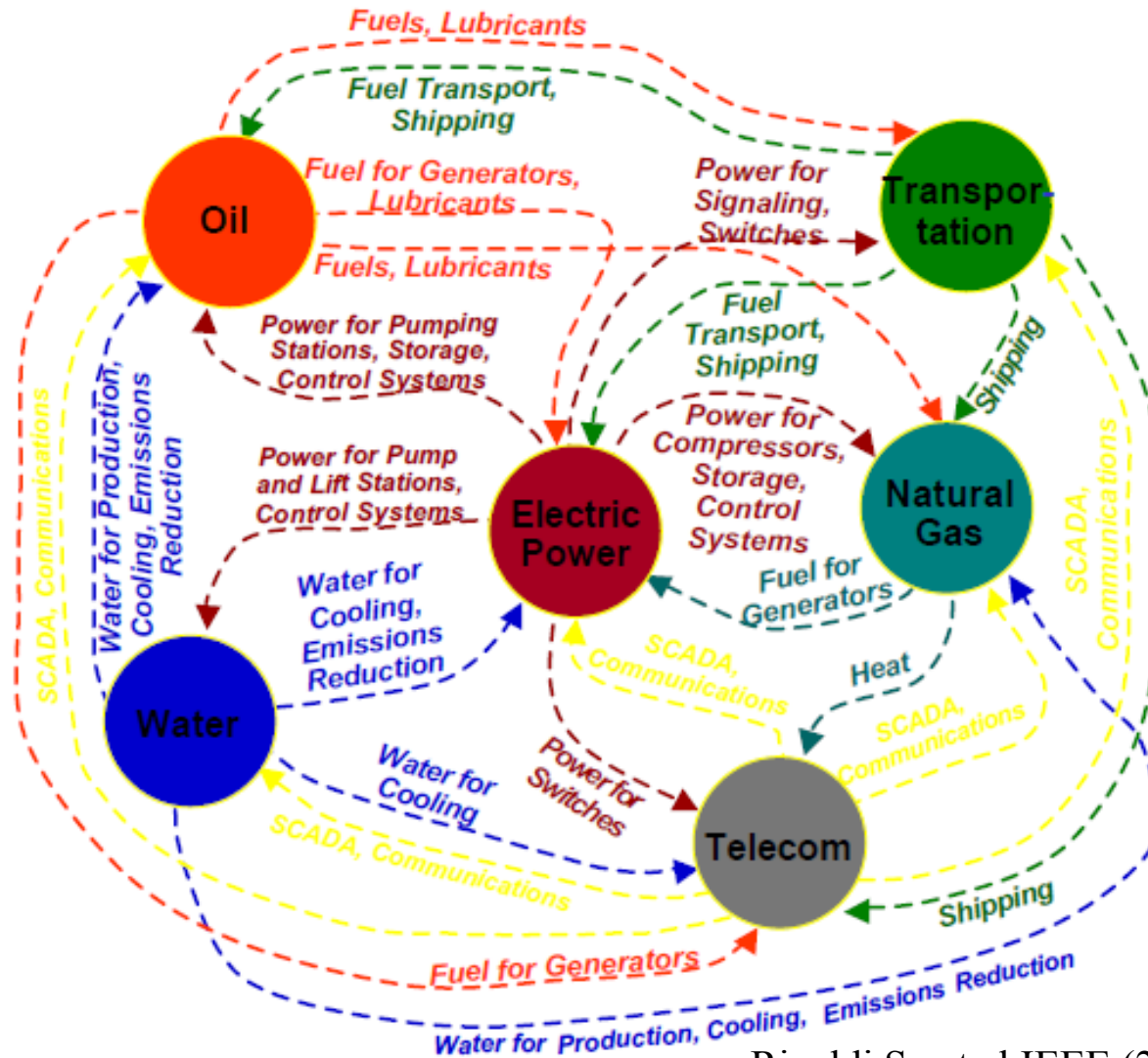
Electric grid,
Communication
Transportation
Services

Two types of links:
1. Connectivity
2. Dependency

Protein networks, brain networks-
Different functions can be regarded
as different networks
Dependency – nodes in one network
depend on nodes in another
network to function.

Cascading failures-abrupt transition

How interdependent are infrastructures?



Rinaldi S, et al IEEE (2001)

Extensive Studies Since 2000 -- Single Networks

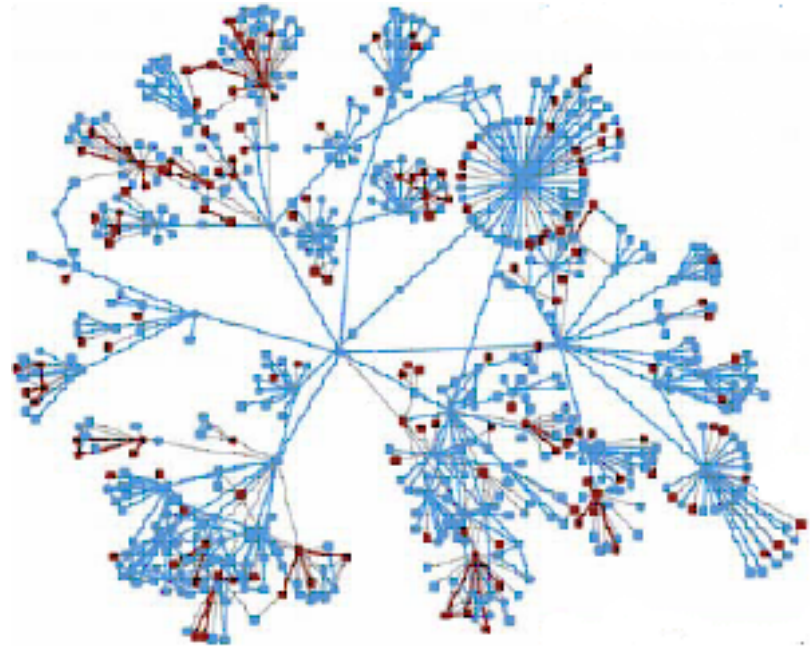
- A **Network** is a structure of N **nodes** and M **edges** (or $2M$ **links**)
- Called usually **graph** – in Mathematics
- Complex systems can be described and understood using networks

Internet: nodes represent computers
links the connecting cables

Airline systems: nodes represent
airports links their destinations

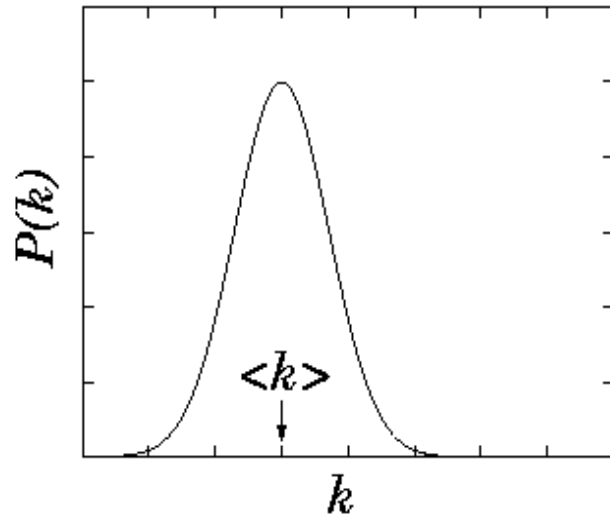
Climate system: nodes represent locations
links similar climate

Successful results: Efficient immunization strategies, identifying key players, robustness, climate, physiology, protein networks-function

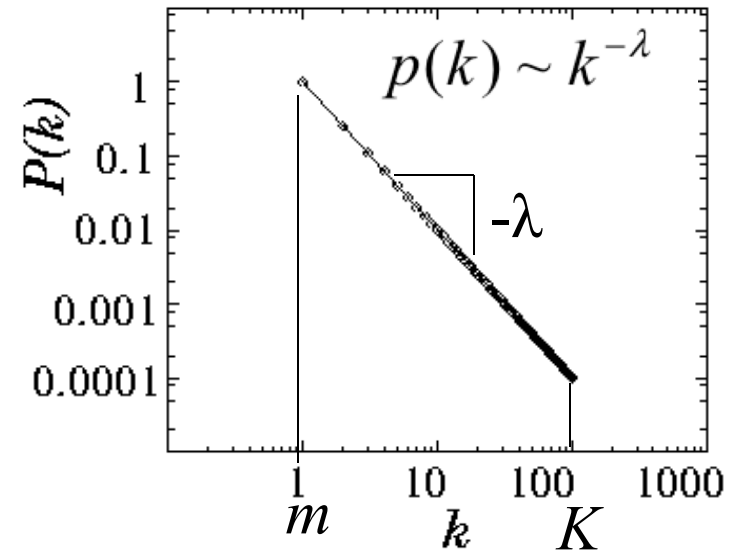


Complex Single Networks- Since 2000

Poisson distribution
(ER - 1959)



Scale-free distribution
(Barabasi –Albert 1999)

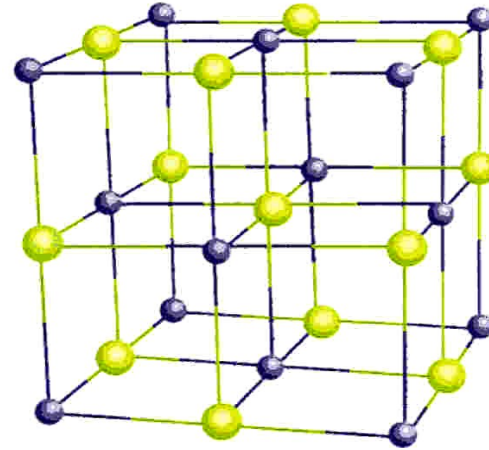
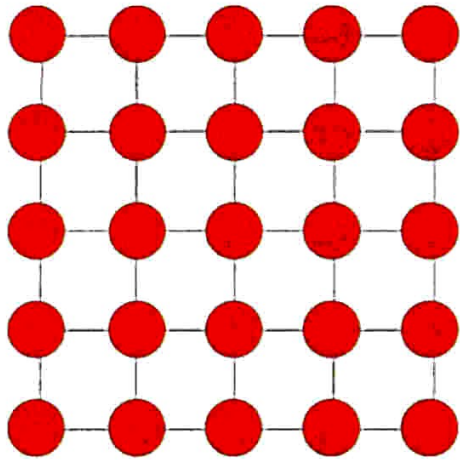


Erdős-Rényi Network

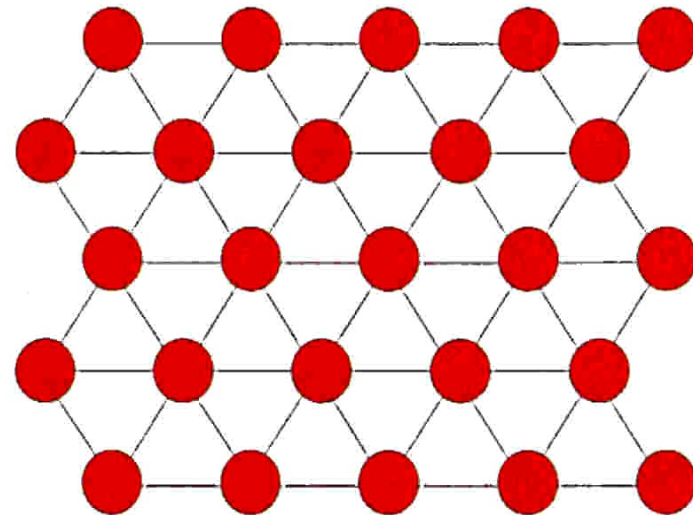
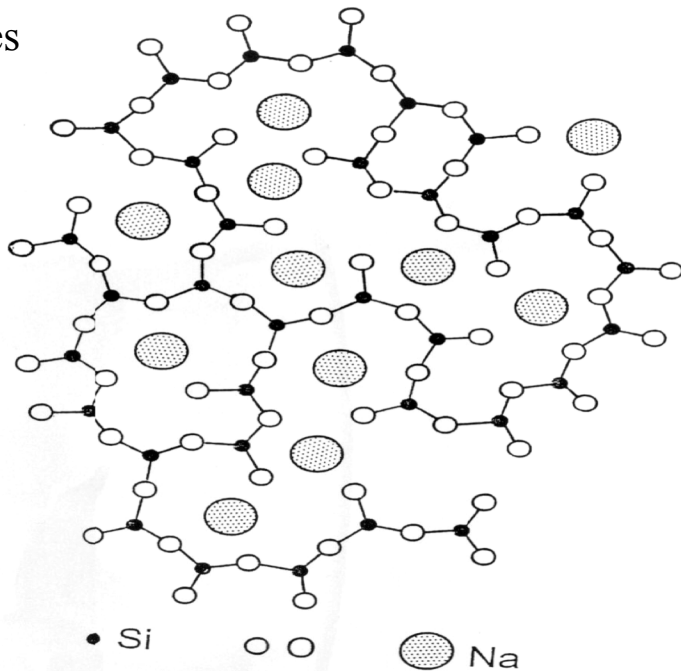


Scale-free Network

Networks in Physics



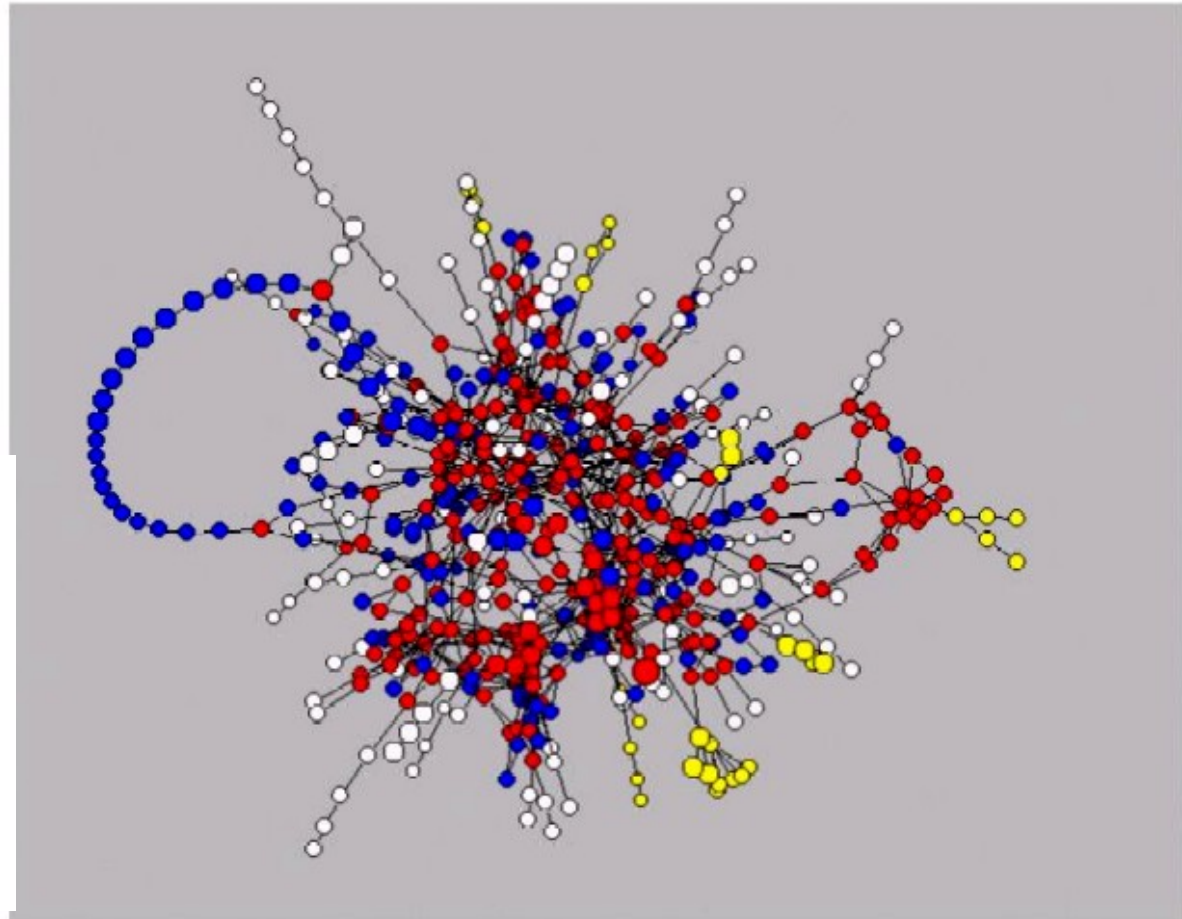
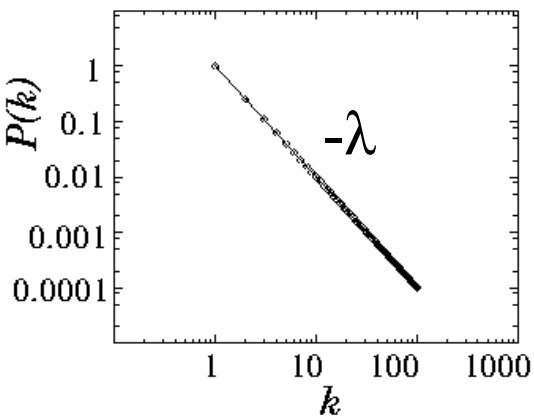
glasses



Metabolic Network

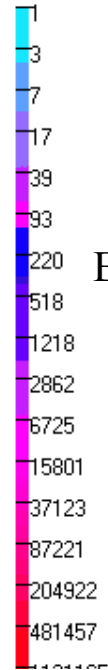
Nodes: chemicals (substrates)
Links: bio-chemical reactions

$$p(k) \sim k^{-\lambda}$$



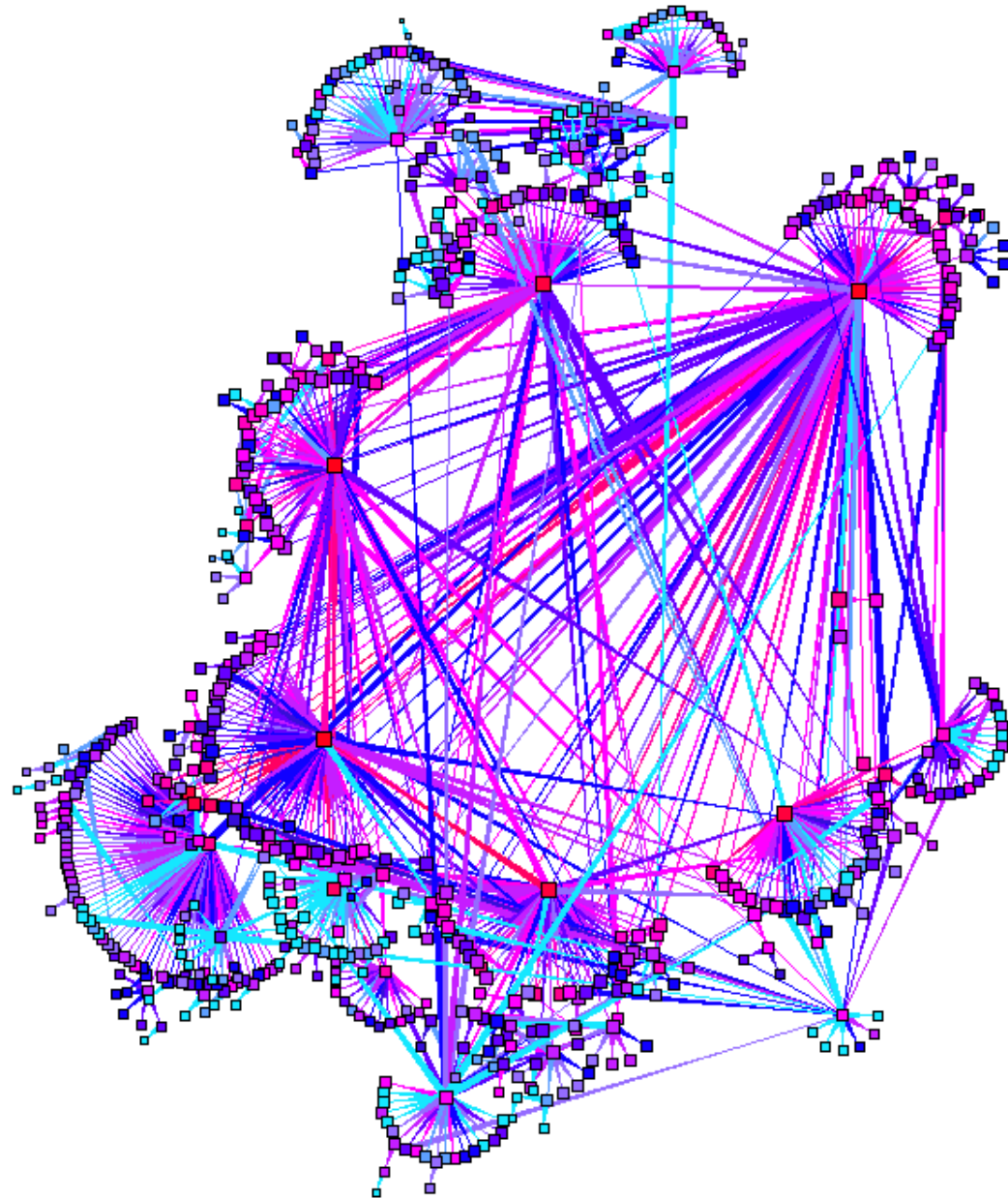
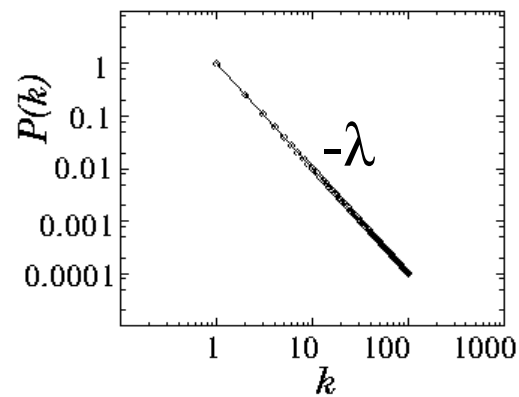
Jeong, Tombor, Albert, Barabasi, Nature (2000)

HTTP Requests

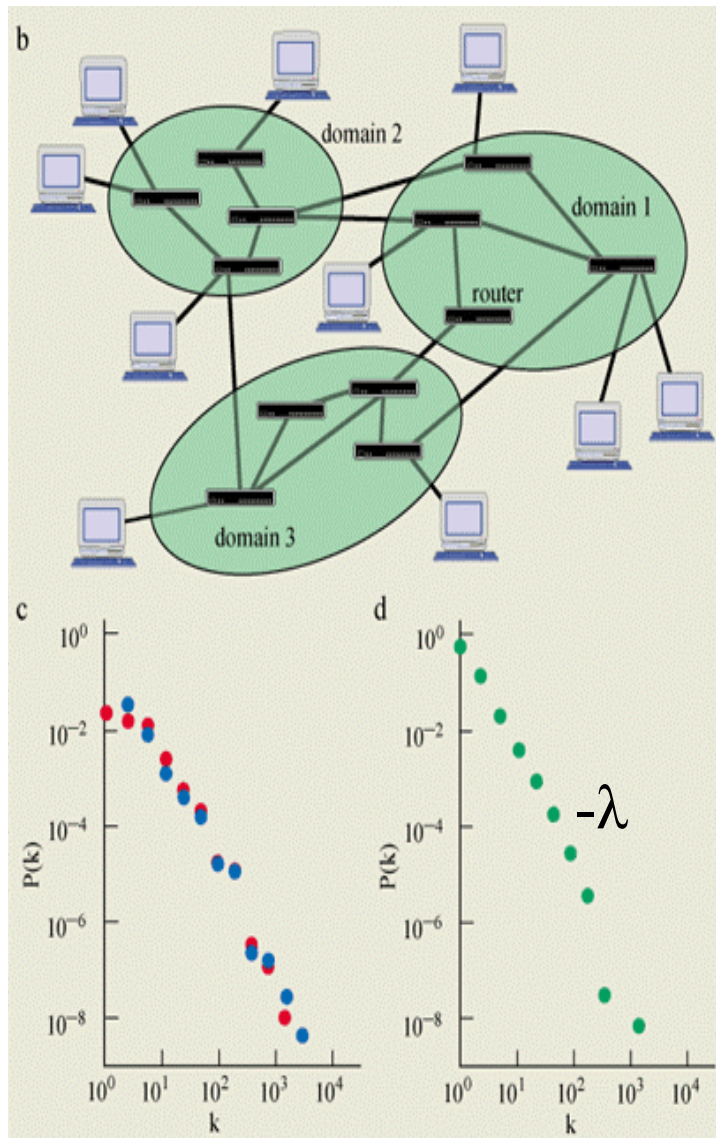
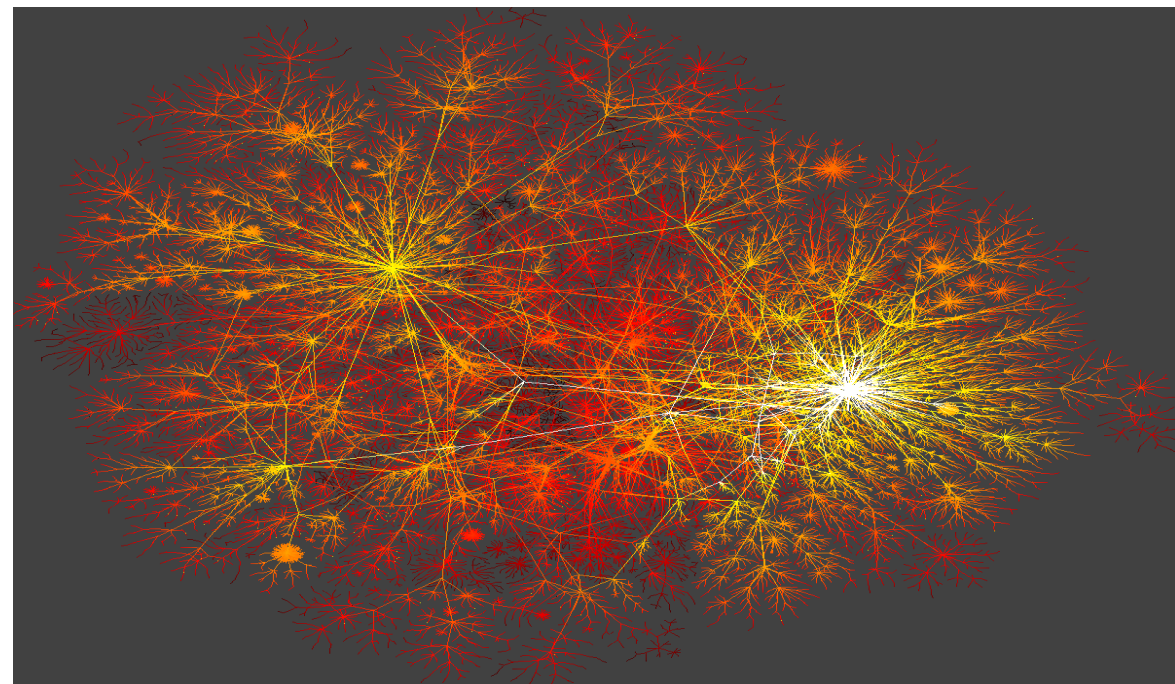
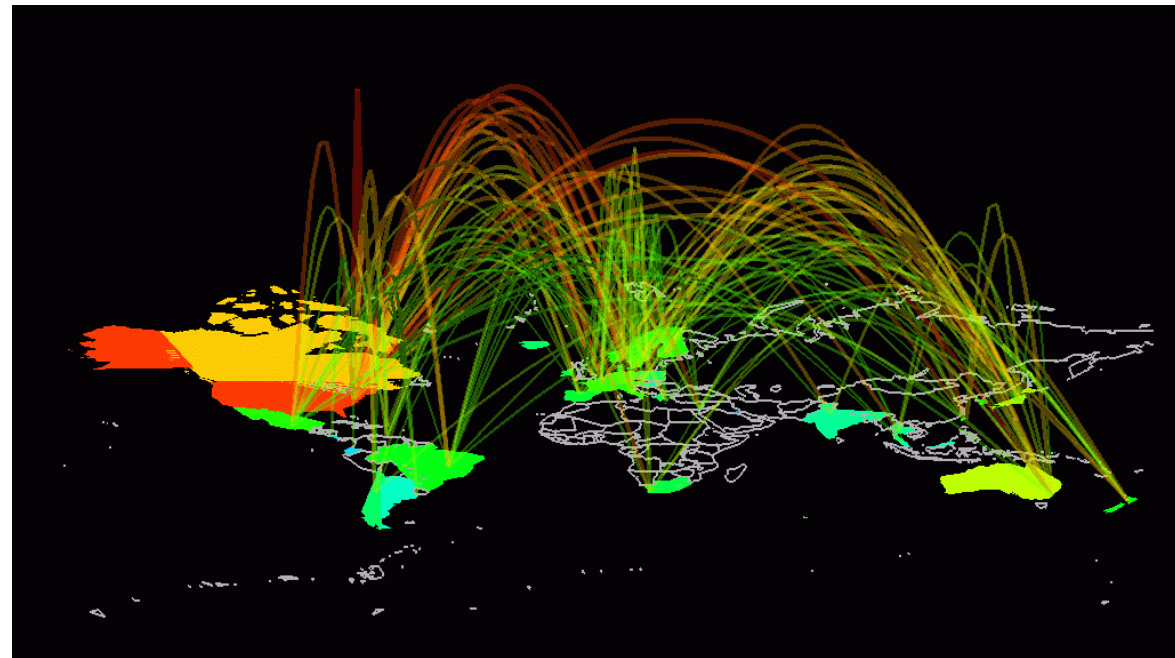


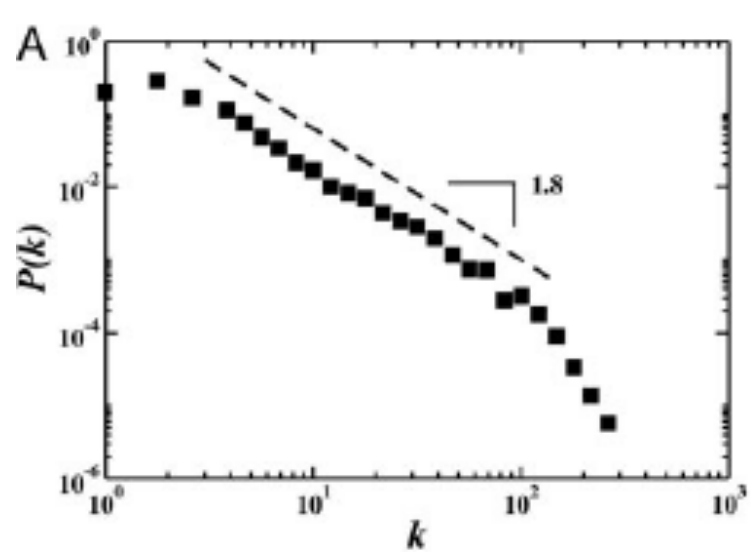
WWW-Network

Barabasi et al (1999)



Faloutsos et. al., SIGCOMM '99



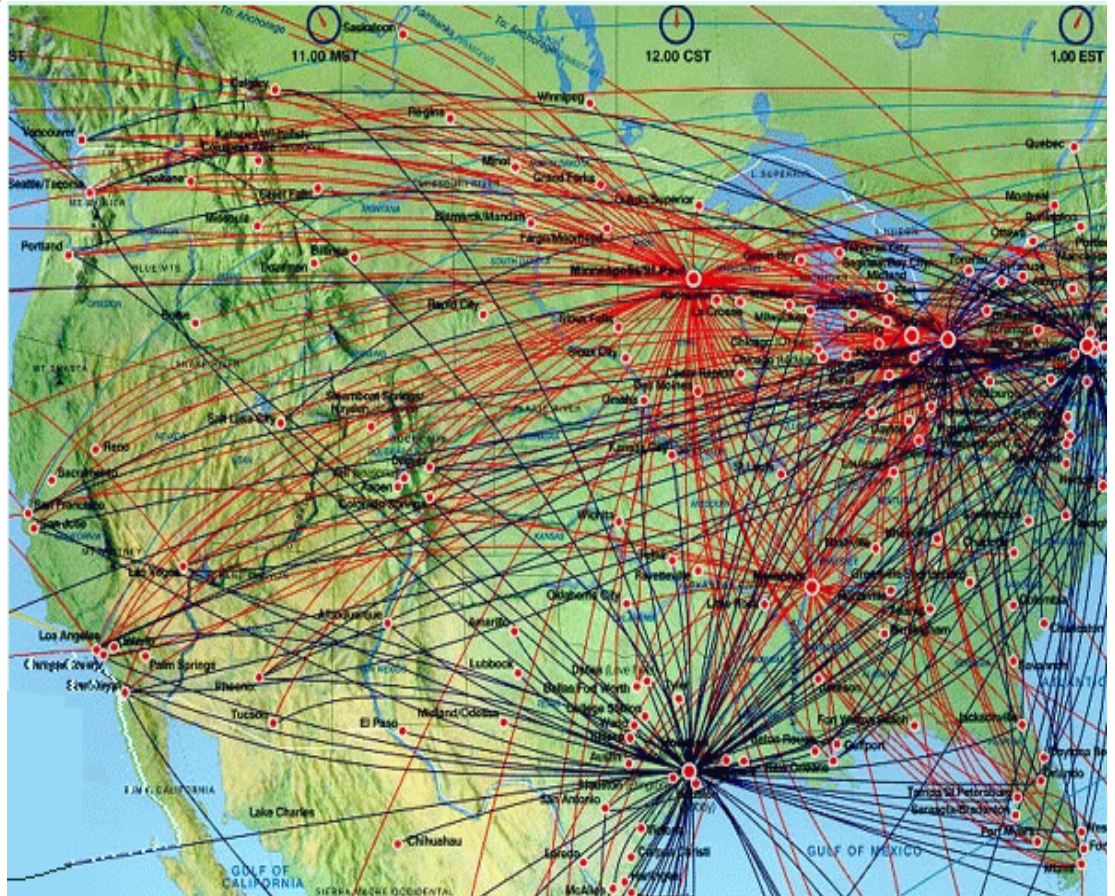


Global Airline Network

Colizza, Vespignani et al PNAS (2006)

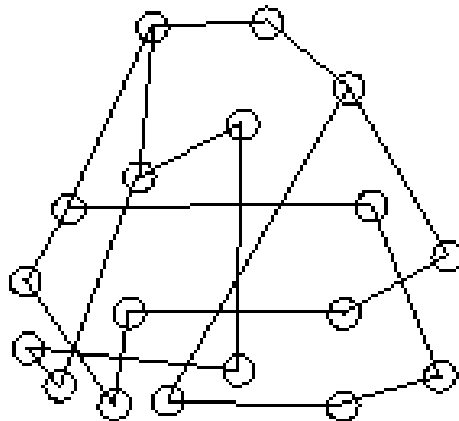
Scale free-degree distribution:

$$p(k) \sim k^{-\lambda}$$



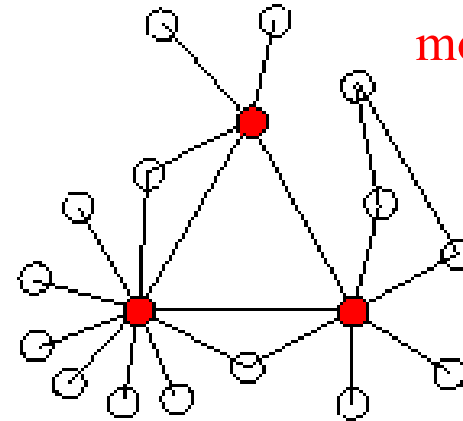
Many real networks are non-Poissonian

Exponential



$$P(k) = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

Scale-free



more robust!!

$$P(k) = \begin{cases} ck^{-\lambda} & m \leq k \leq K \\ 0 & \text{otherwise} \end{cases}$$

Classical Erdos-Renyi (1960)

Homogeneous, similar to lattices

$d \sim \log N$ -- Small world

$$p_c = 1 - q_c = 1 / \langle k \rangle$$

$$P_\infty = p[1 - \exp(-\langle k \rangle P_\infty)]$$

Barabasi-Albert (1999)

Heterogeneous-translational symmetry breaks!

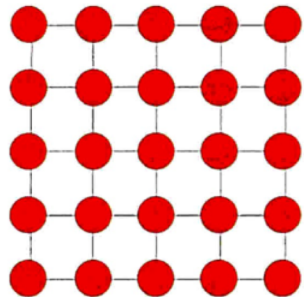
New universality class-many anomalous laws

$$\text{e.g., } d \sim \log \log N ; p_c = 0$$

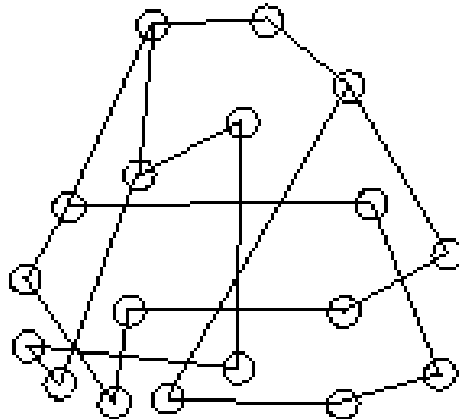
Ultra Small worlds (Cohen and SH, PRL (2003))

Breakthrough in understanding many problems!

Many real networks are non-Poissonian

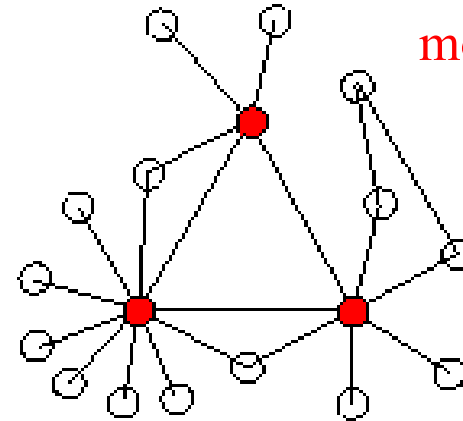


Exponential



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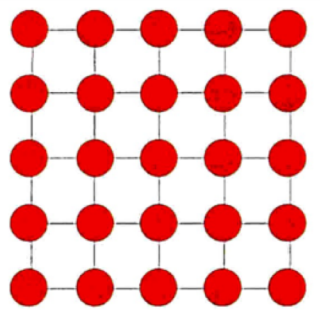
New universality class-many anomalous laws

$$\text{e.g., } d \sim \log \log N ; p_c = 0$$

Ultra Small worlds (Cohen and SH, PRL (2003))

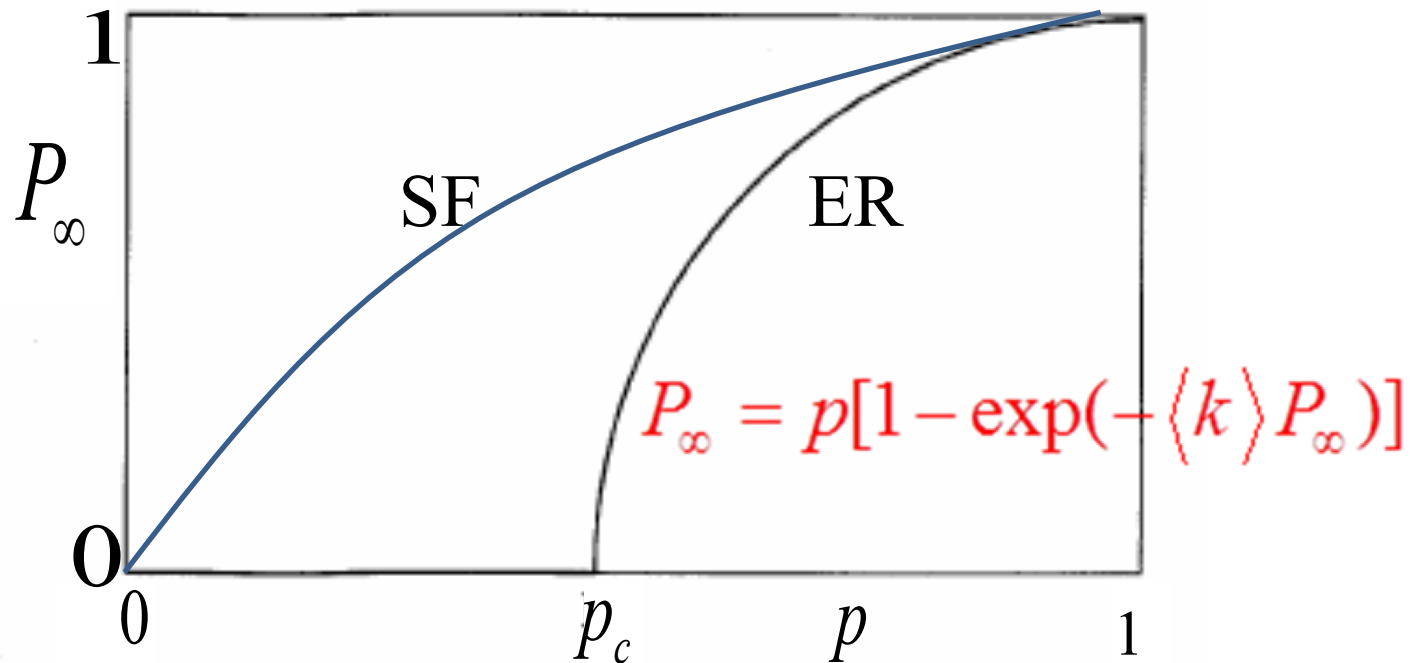
Breakthrough in understanding many problems!

Many real networks are non-Poissonian



$$p_c = 0.593$$

Long-range links
More robust!



SF more robust!!

Classical Erdos-Renyi (1960)

Homogeneous, similar to lattices

$d \sim \log N$ -- Small world

$$p_c = 1 - q_c = 1 / \langle k \rangle$$

$$P_\infty = p[1 - \exp(-\langle k \rangle P_\infty)]$$

Barabasi-Albert (1999)

Heterogeneous-translational symmetry breaks!

New universality class-many anomalous laws

$e.g., d \sim \log \log N ; p_c = 0$

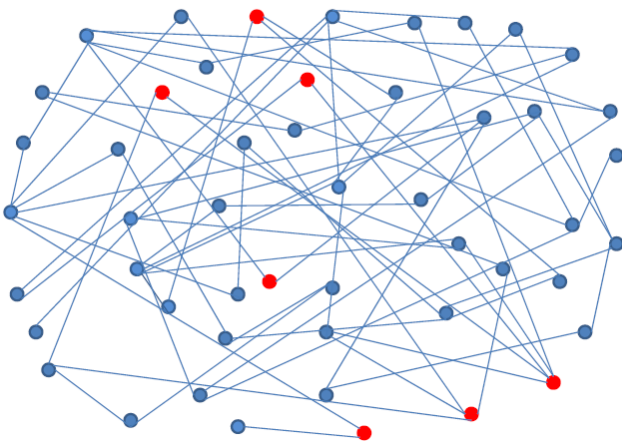
Ultra Small worlds (Cohen and SH, PRL (2003))

Breakthrough in understanding many problems!

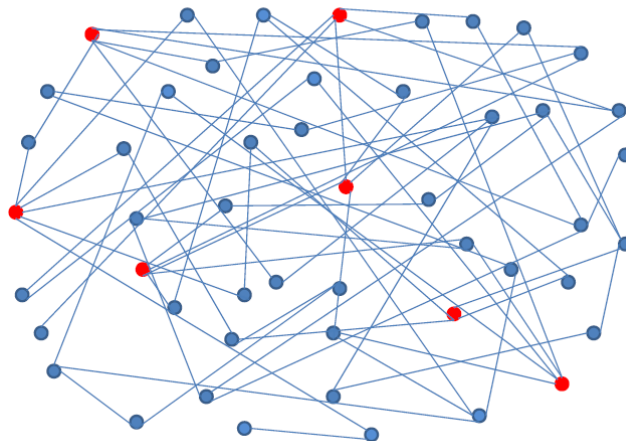
Comparing random, targeted, and equal graph partition strategies

Random scale-free network: $N=50$; $\lambda=2.5$ – immunizing 7 nodes

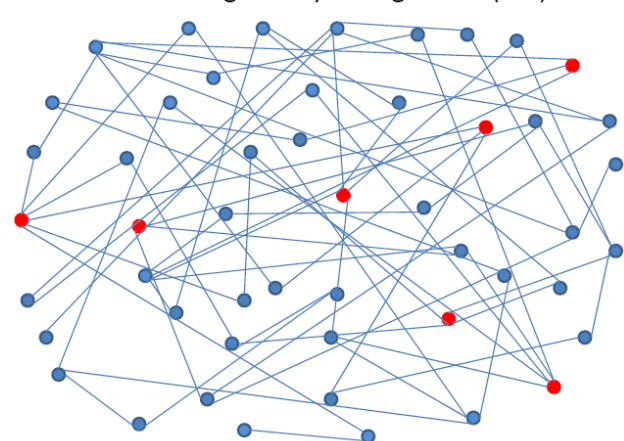
Nodes targeted by random (red)



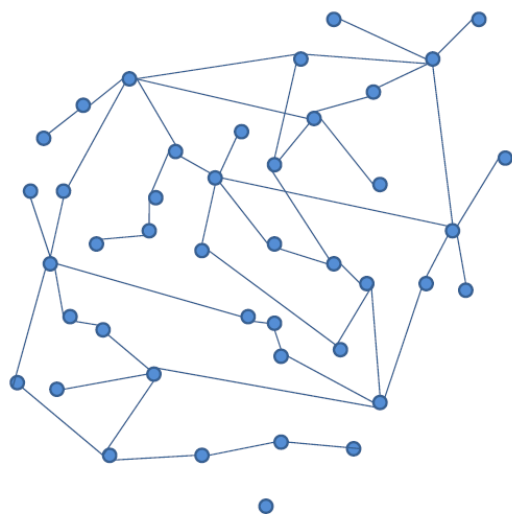
Nodes targeted by High-Degree method (red)



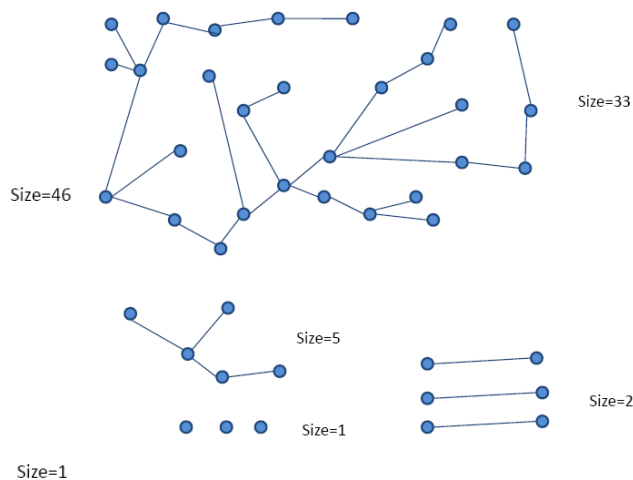
Nodes targeted by GP algorithm (red)



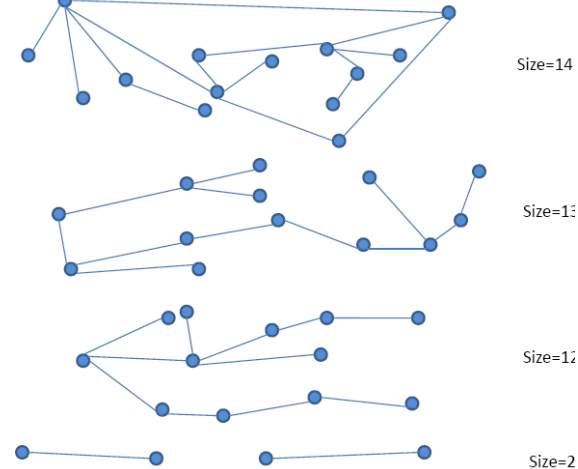
Result (rearranged clusters)



Result (rearranged clusters)



Result (rearranged clusters)



WHAT IS DIFFERENT?

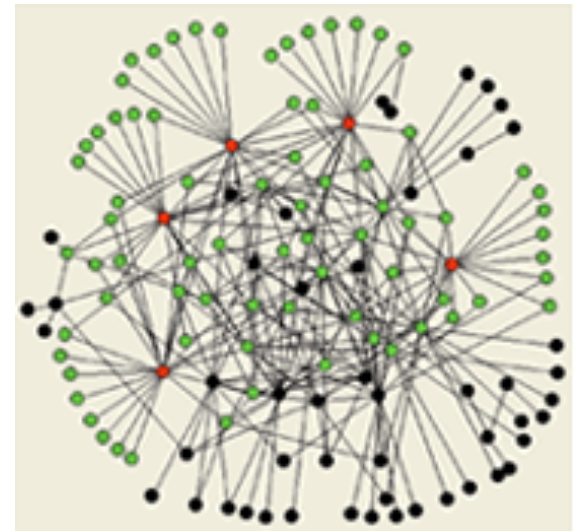
Known values of epidemic thresholds:

Infectious disease	Critical Threshold $q_c = 1 - p_c$
Malaria	99%
Measles	90-95%
Whooping cough	90-95%
Fifths disease	90-95%
Chicken pox	85-90%
Internet	more than 99%

Such immunization thresholds were **not understood** since they were well above the expected value of percolation in classical random networks:

$$q_c = 1 - p_c = 1 - 1 / \langle k \rangle$$

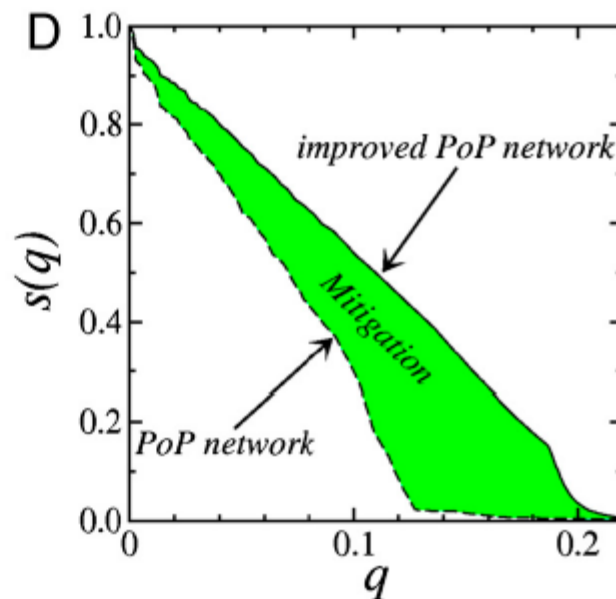
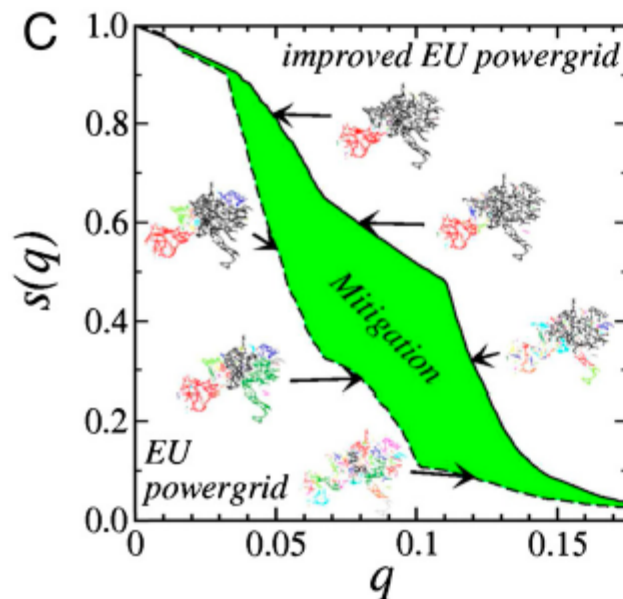
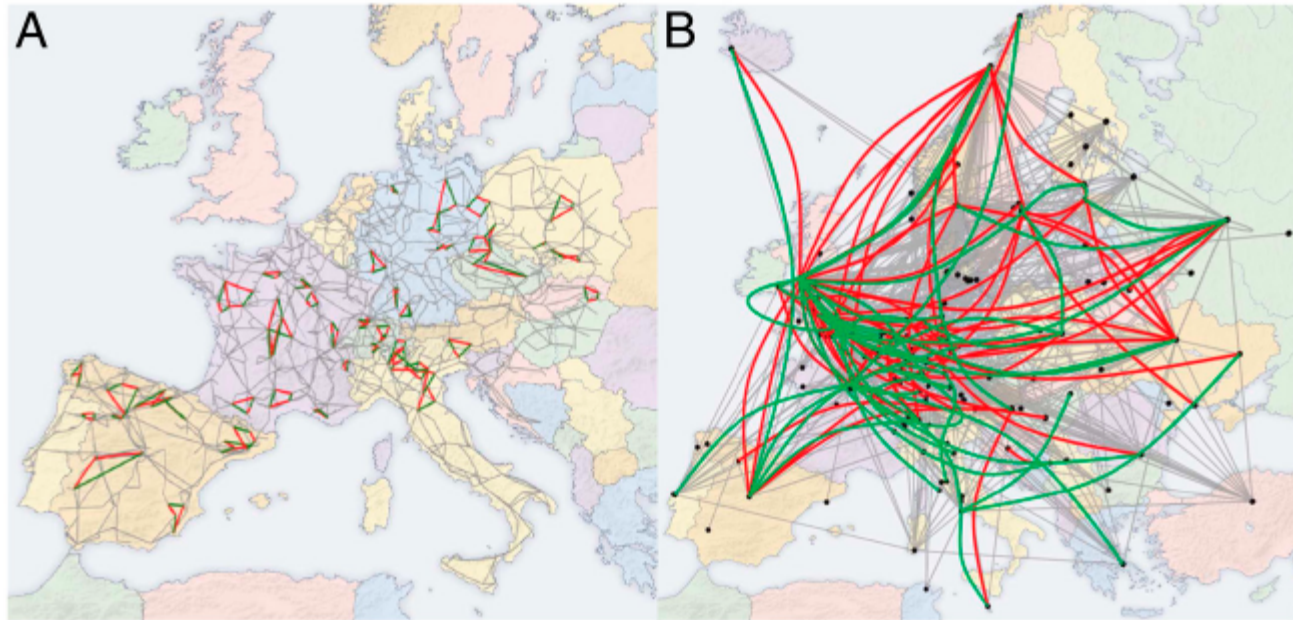
This puzzle is solved due to the broad degree distribution (HUBS) of social networks which does not occur in random graphs!



Mitigation of malicious attacks on networks

Power Grid

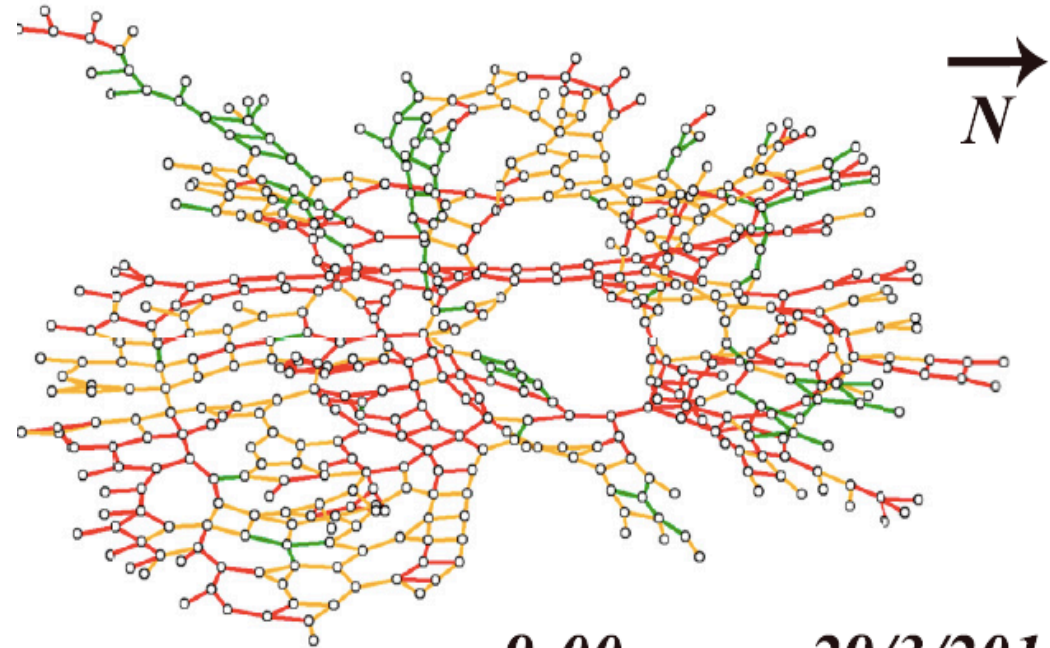
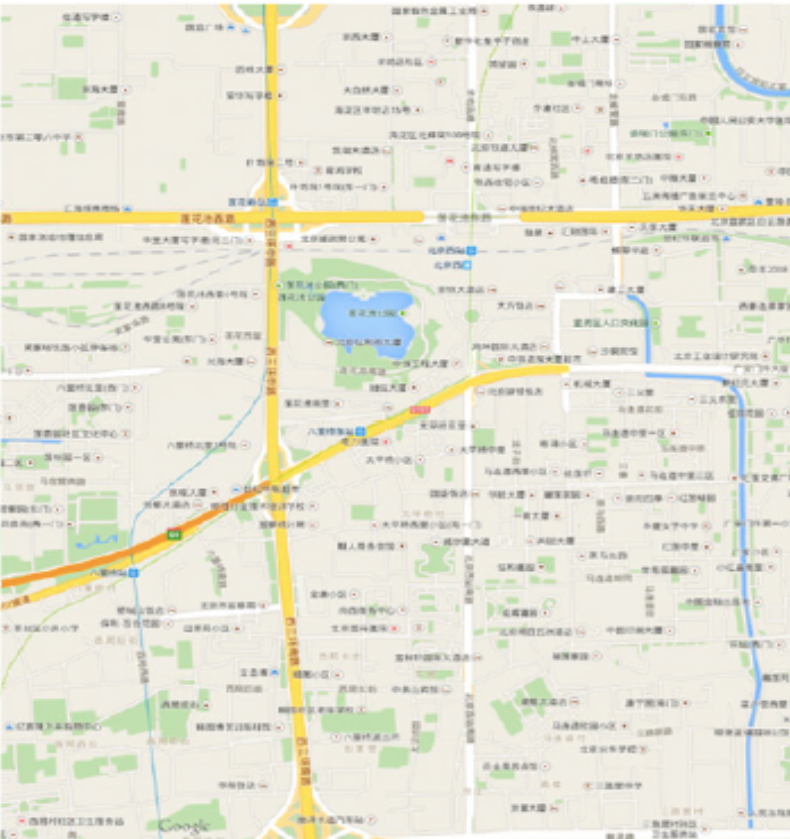
Internet



Schneider, Moreira,
Andrade, SH
and Herrmann
PNAS (2011)

Traffic and Network Theory

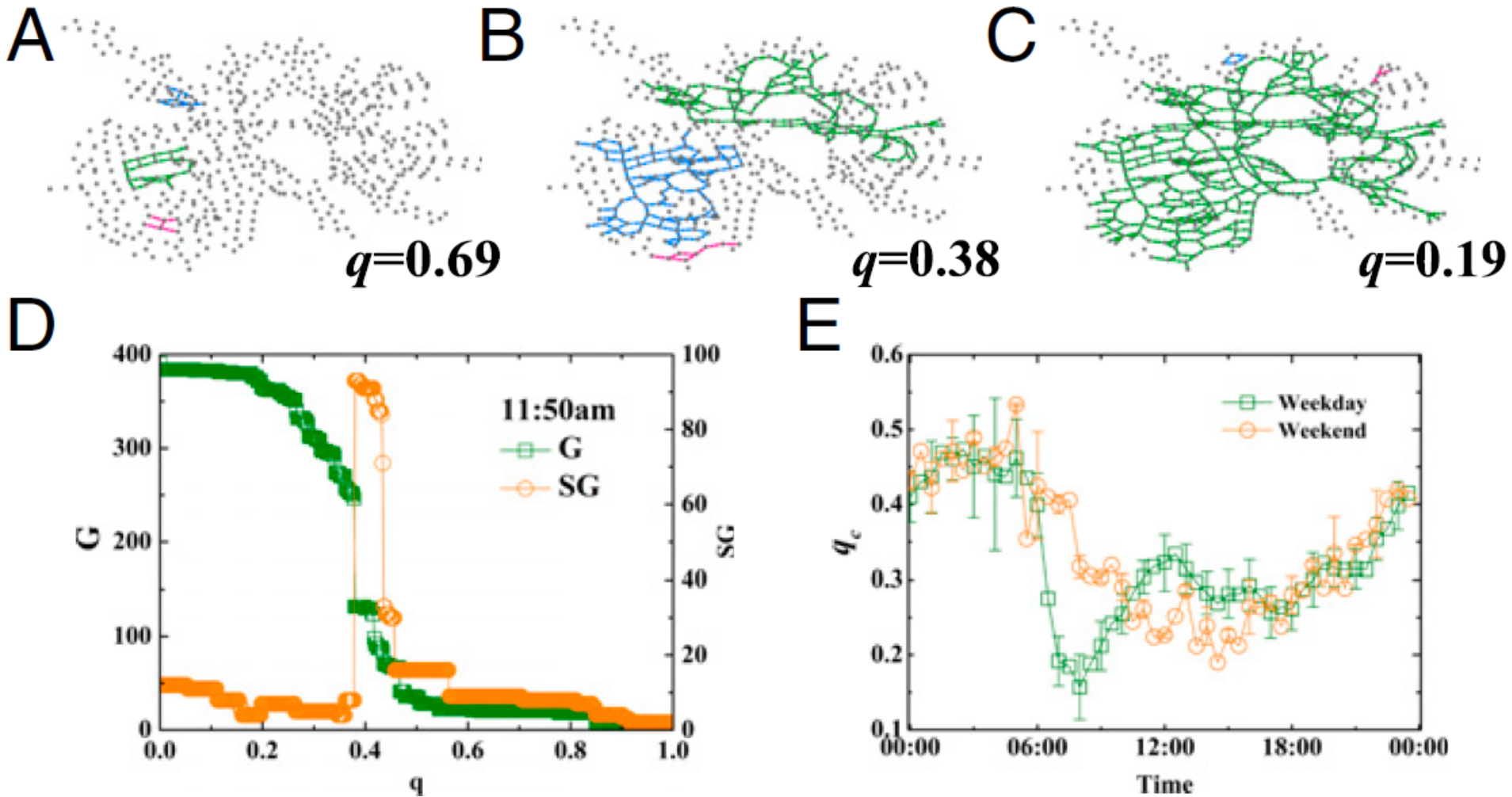
1. Mapping traffic in Beijing as a dynamic network
2. Percolation theory identify bottlenecks



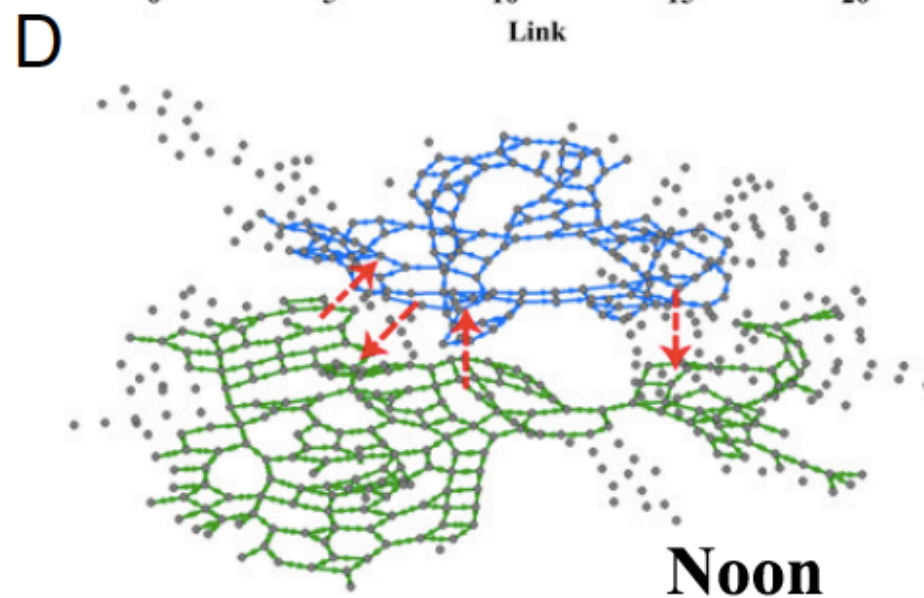
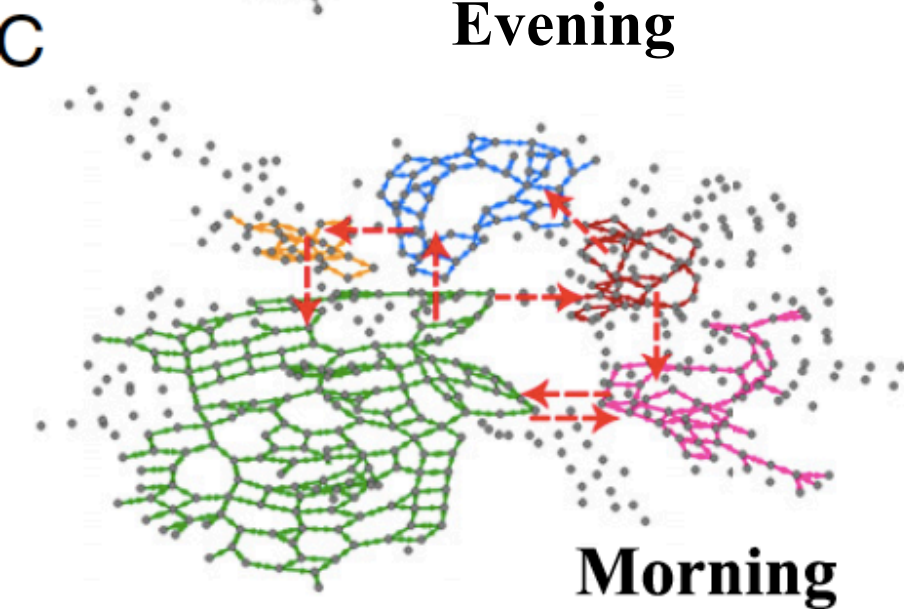
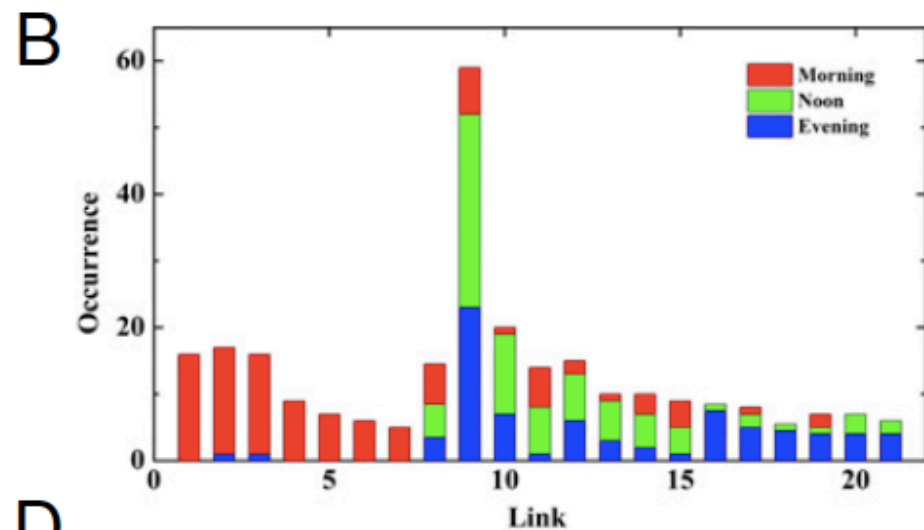
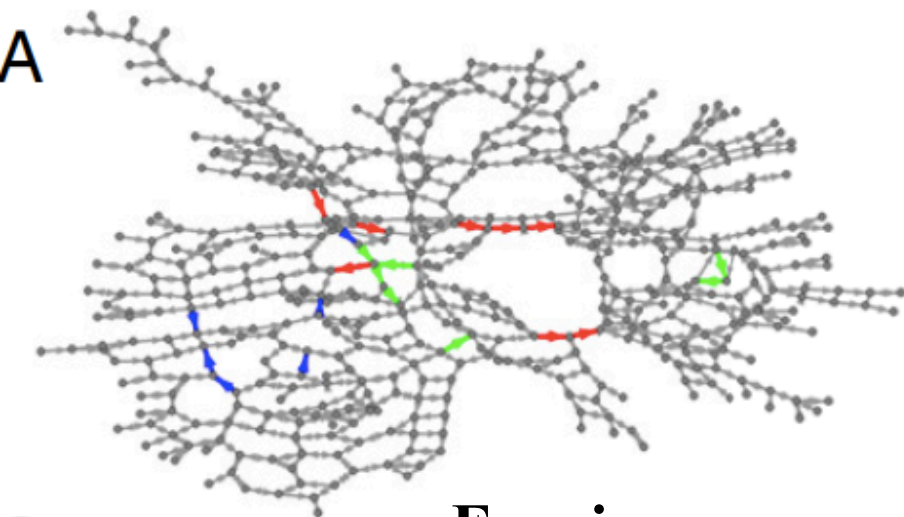
9:00am on 29/3/2013

**Daqing Li, Bowen Fu,
Yungpeng Wang, Guangquan
Lu et al, PNAS (2015)**

PERCOLATION AND TRAFFIC

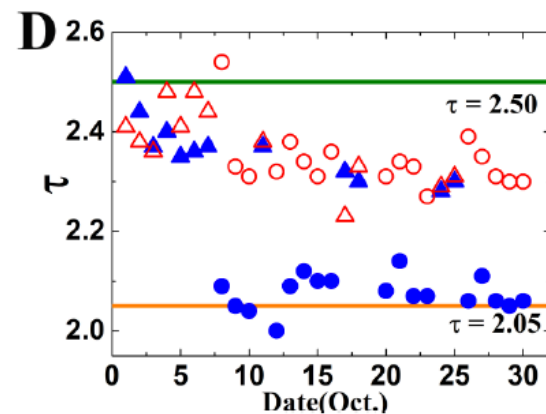
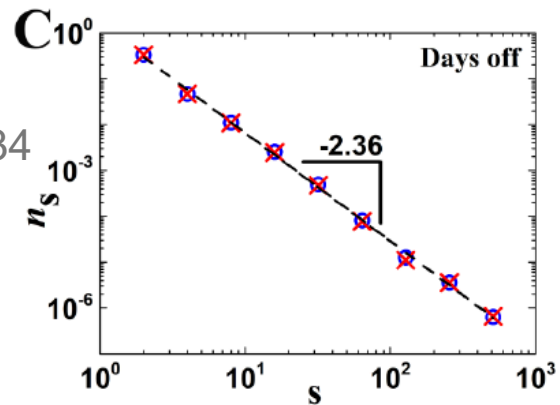
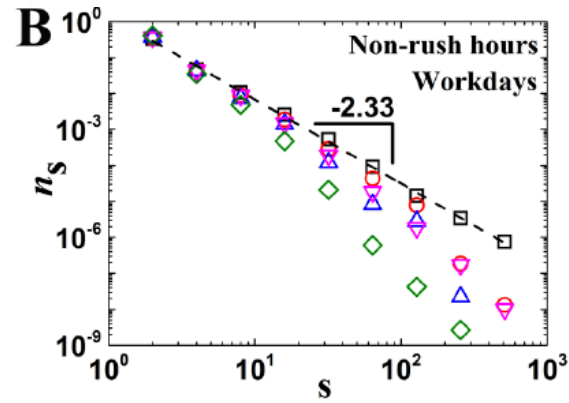
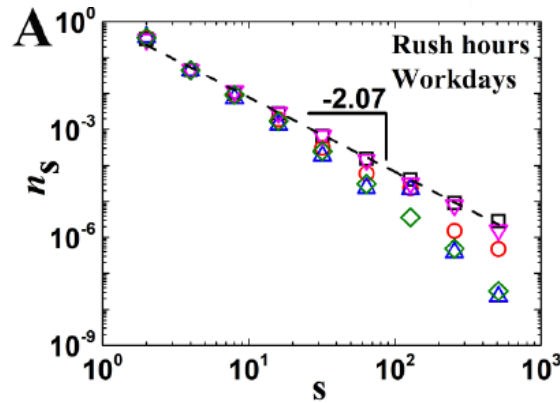


DIFFERENT HOURS DIFFERENT BOTTLENECKS



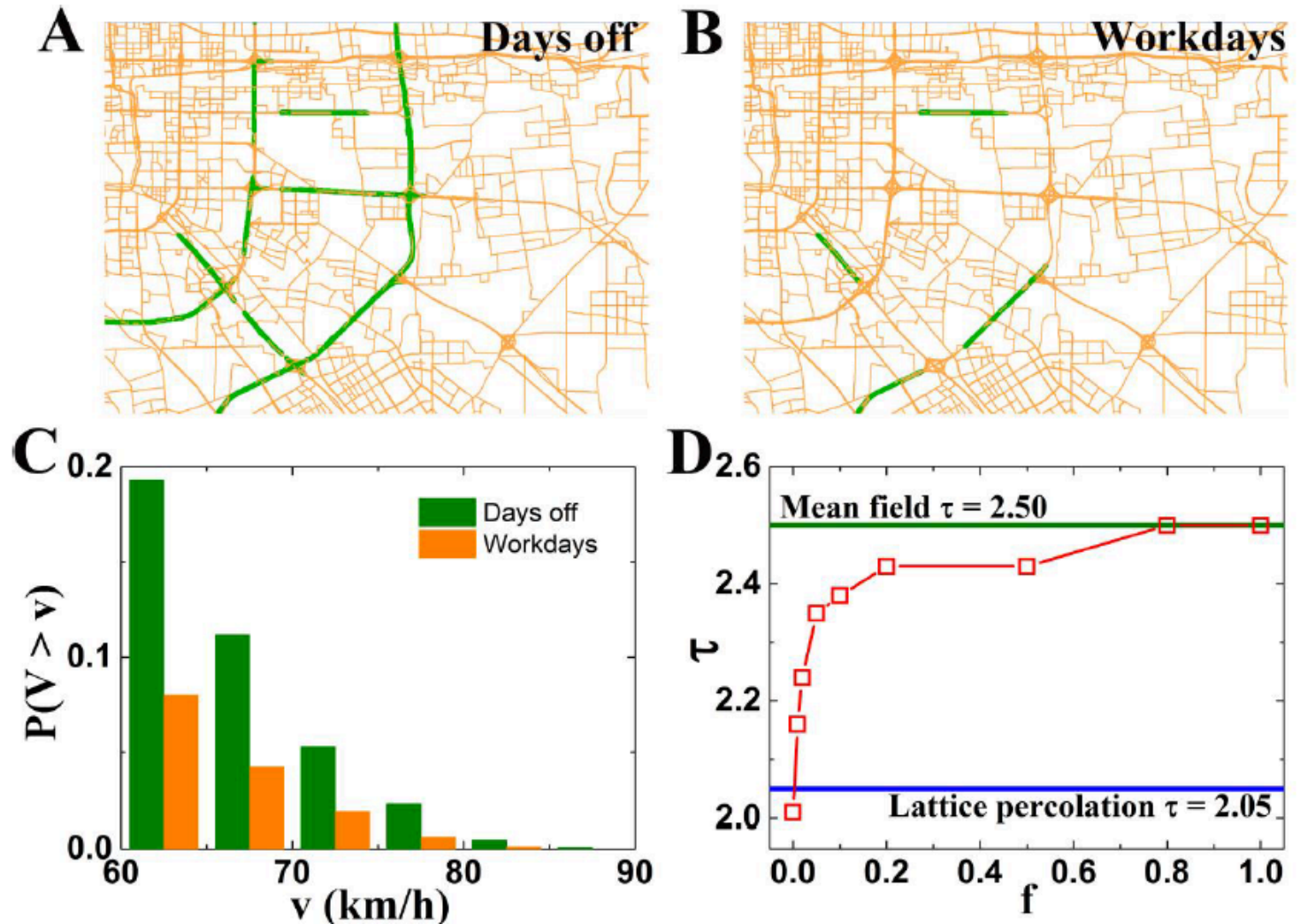
Percolation critical exponents

A 8:00AM, Oct. 1st **B** 8:00AM, Oct. 15th 2015

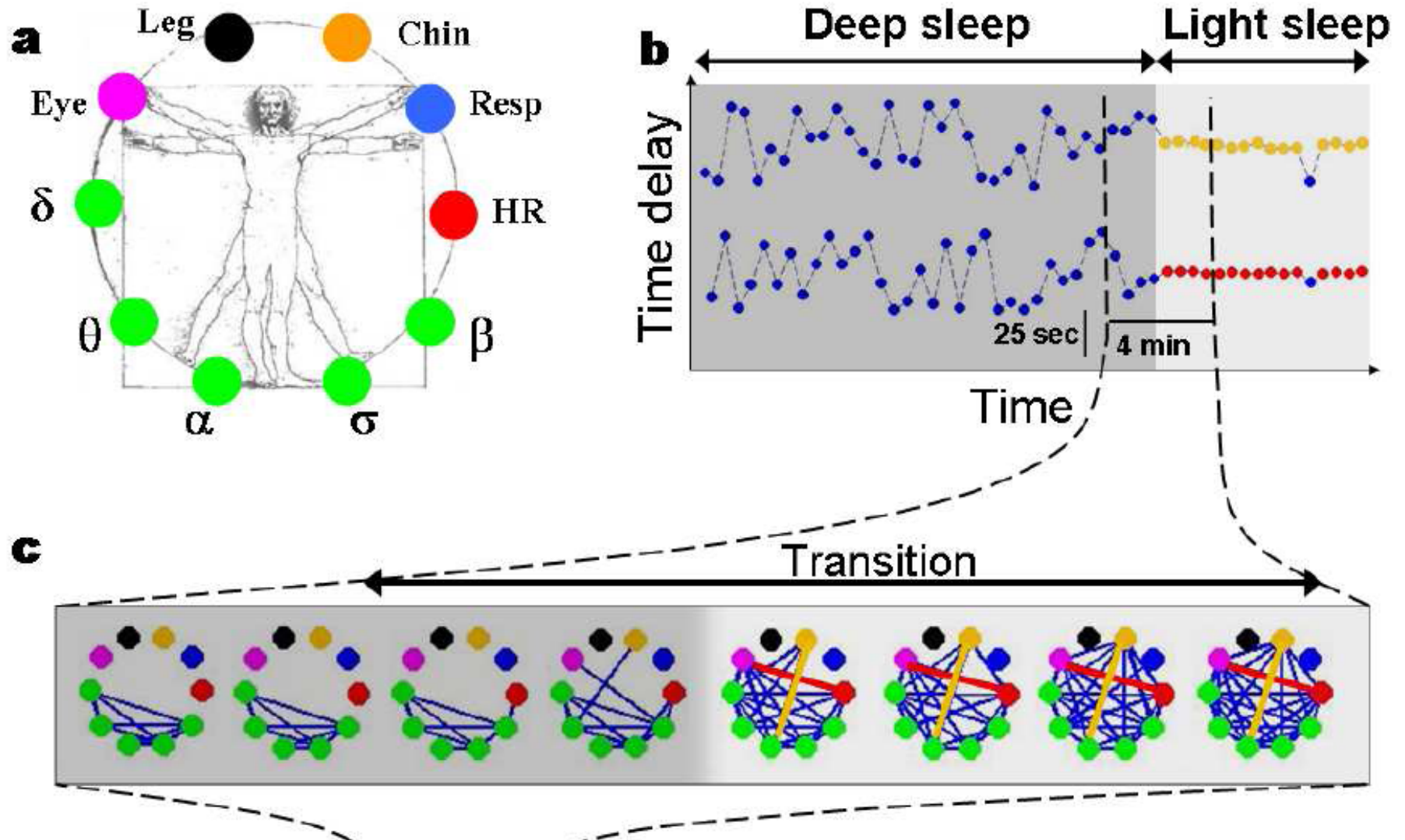


Daqing Li et al
arXiv:1709.03134

Percolation Model



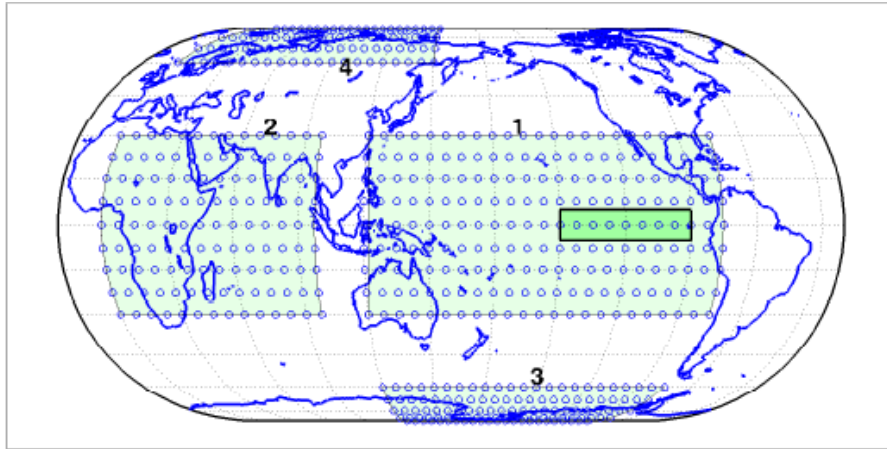
Physiological Networks



Bashan, Ivanov et al, Nature Communication [2012]

Structure and Function

Climate networks are very sensitive to El Nino

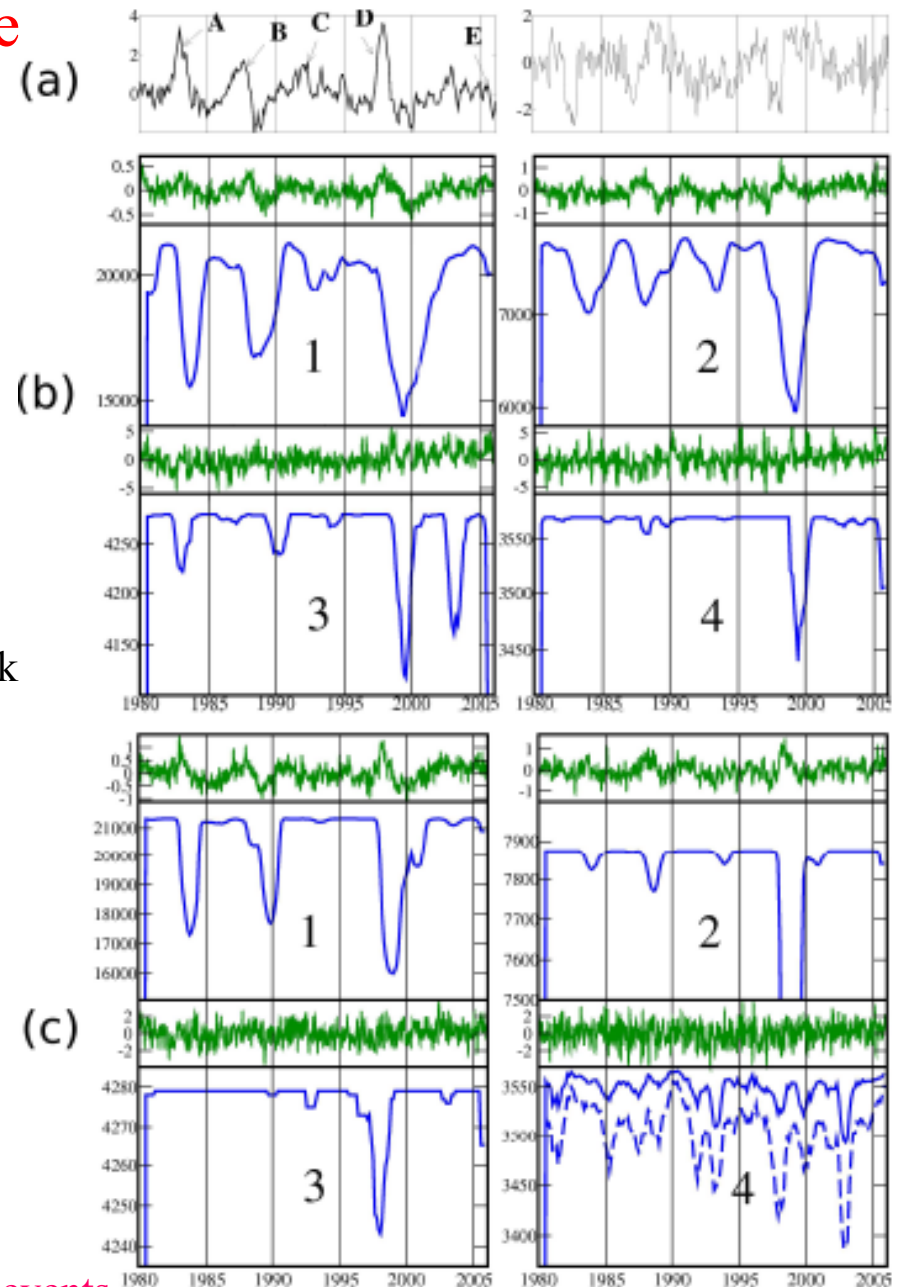


Sea surface temperature network

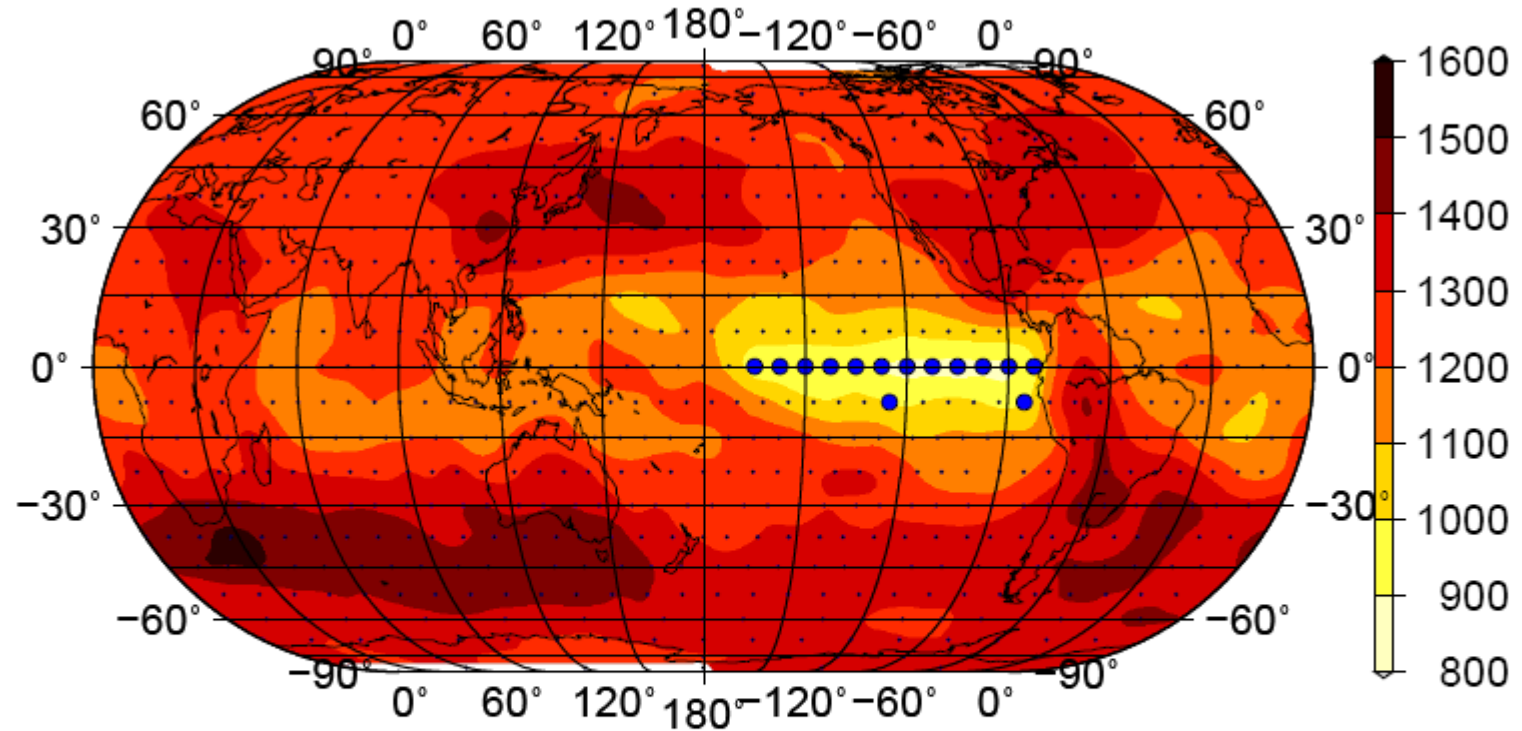
5km height temperature network

Yamasaki, Gozolchiani, SH (PRL 2008, 2011)

Challenge: Porecasting El-Nino and other extreme events
Ludescher et al PNAS (2014, 2015)



EL-NINO BECOMES **AUTONOMOUS**: ONLY INFLUENCE-NOT INFLUENCED

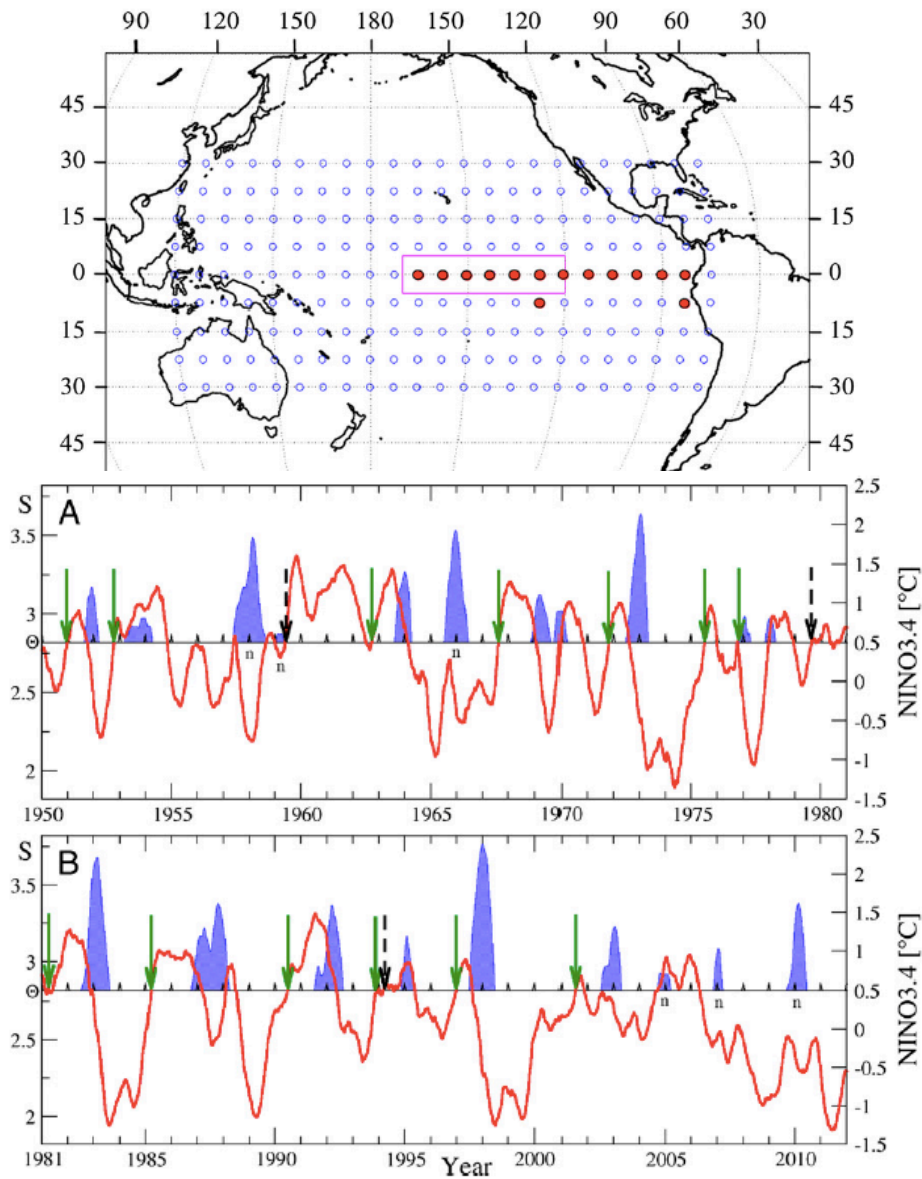


$$\langle I_l^y \rangle_{y \in El-Nino}$$

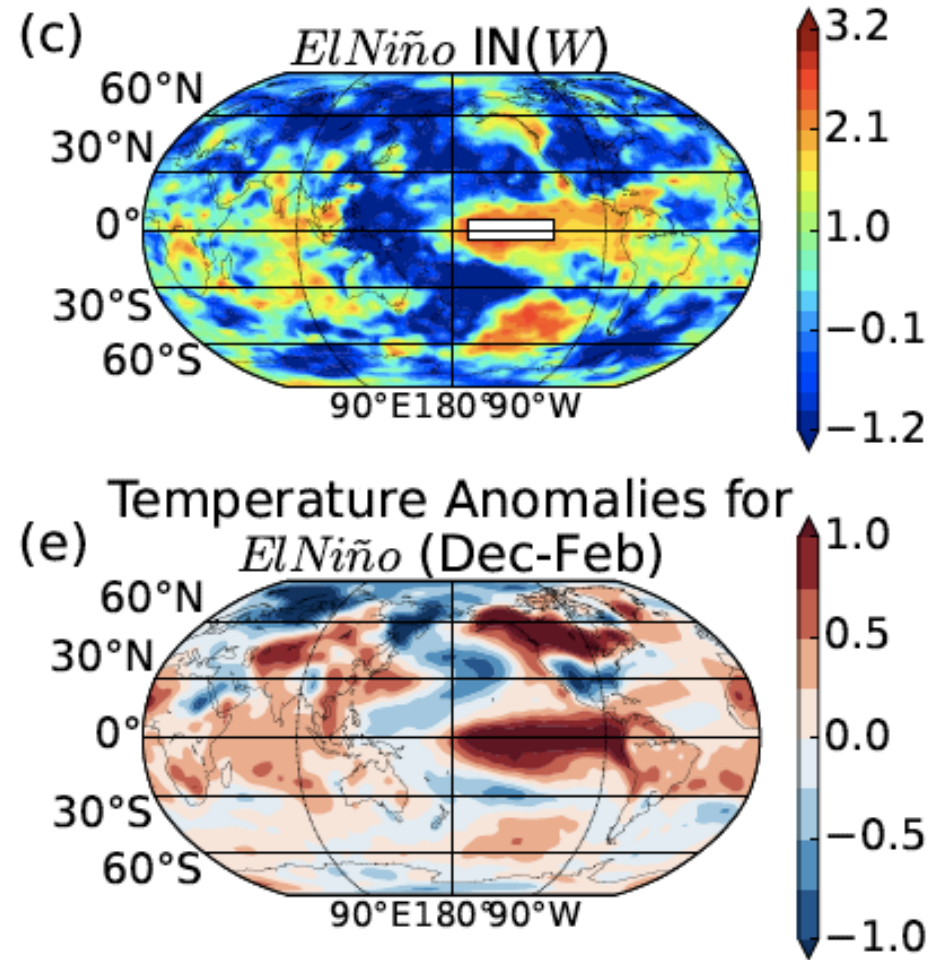
Gozolchiani et al PRL (2011)

VERY EARLY PREDICTION!!! Ludescher et al, PNAS (2014)

Forecasting the onset and global influence



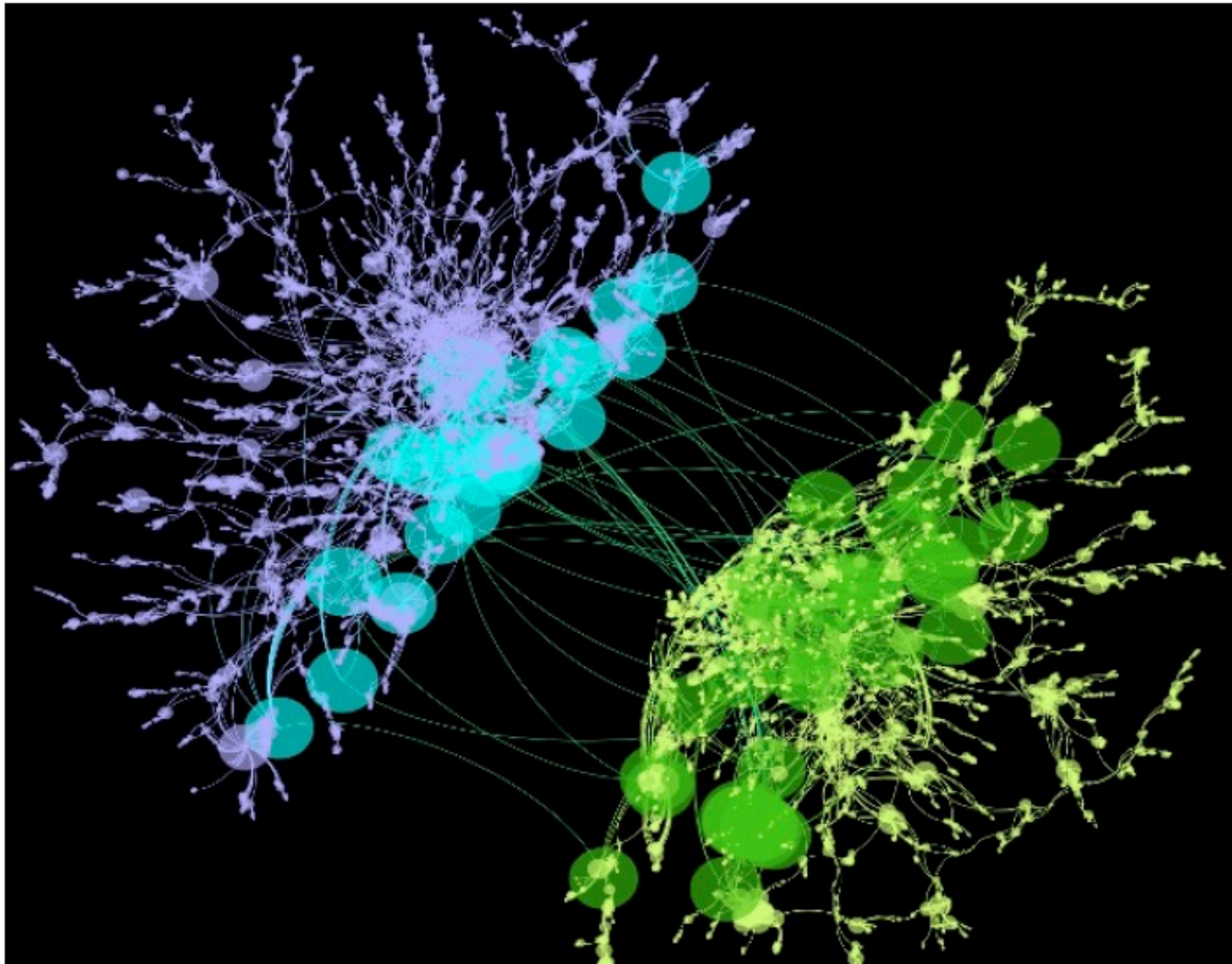
Ludescher et al, PNAS (2013,2014)



Jing-fang Fan et al, PNAS (2017)

Resilience of Networks with Community Structure: External Field Interlinks - Flights between Continents-Back to Physics

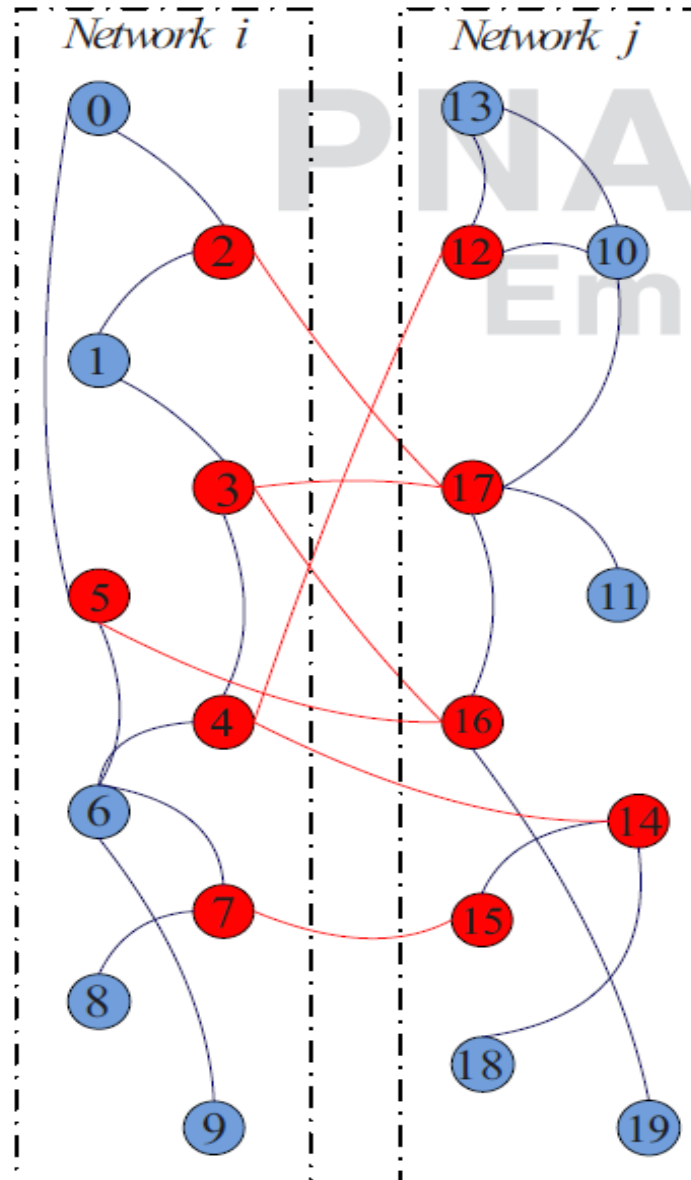
A



Network Model

A fraction r of nodes are interconnected nodes

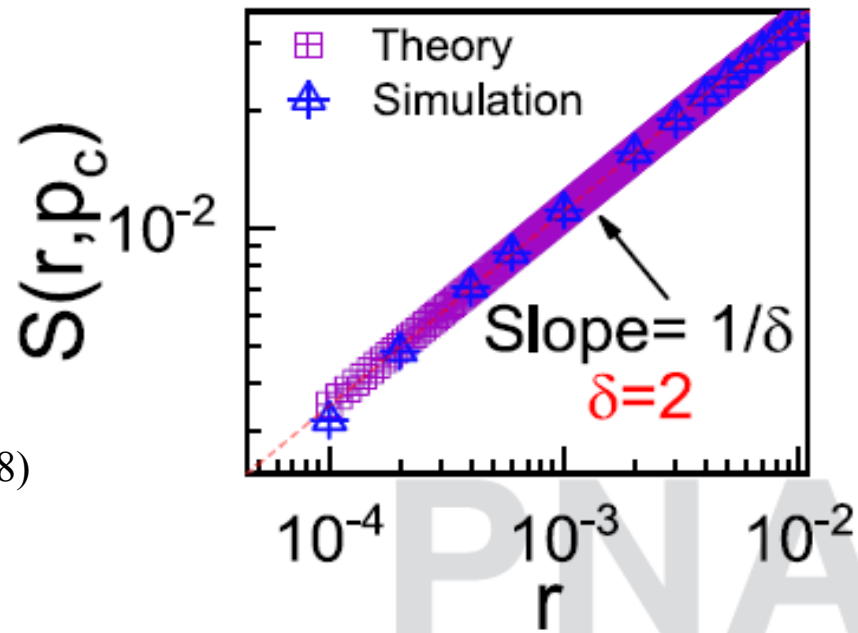
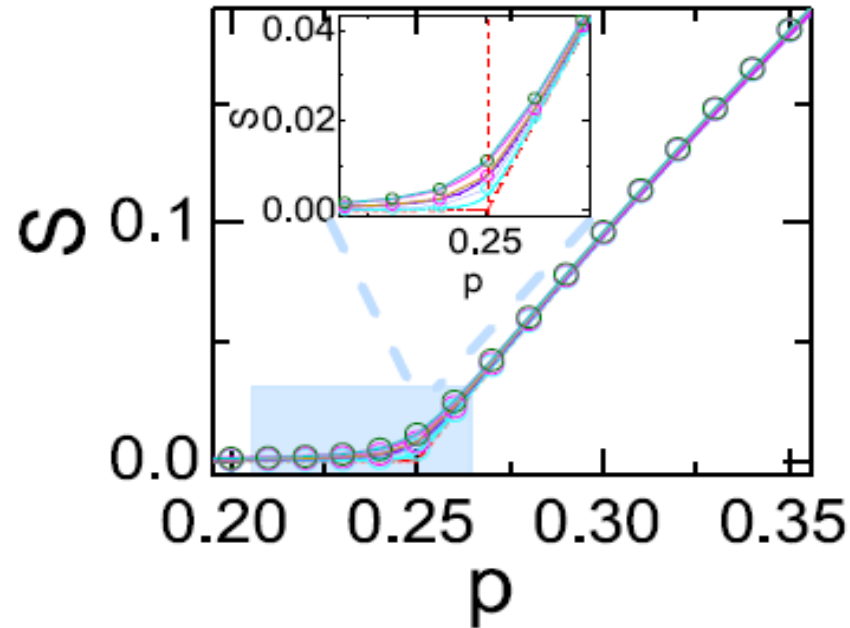
$$r=0.5$$



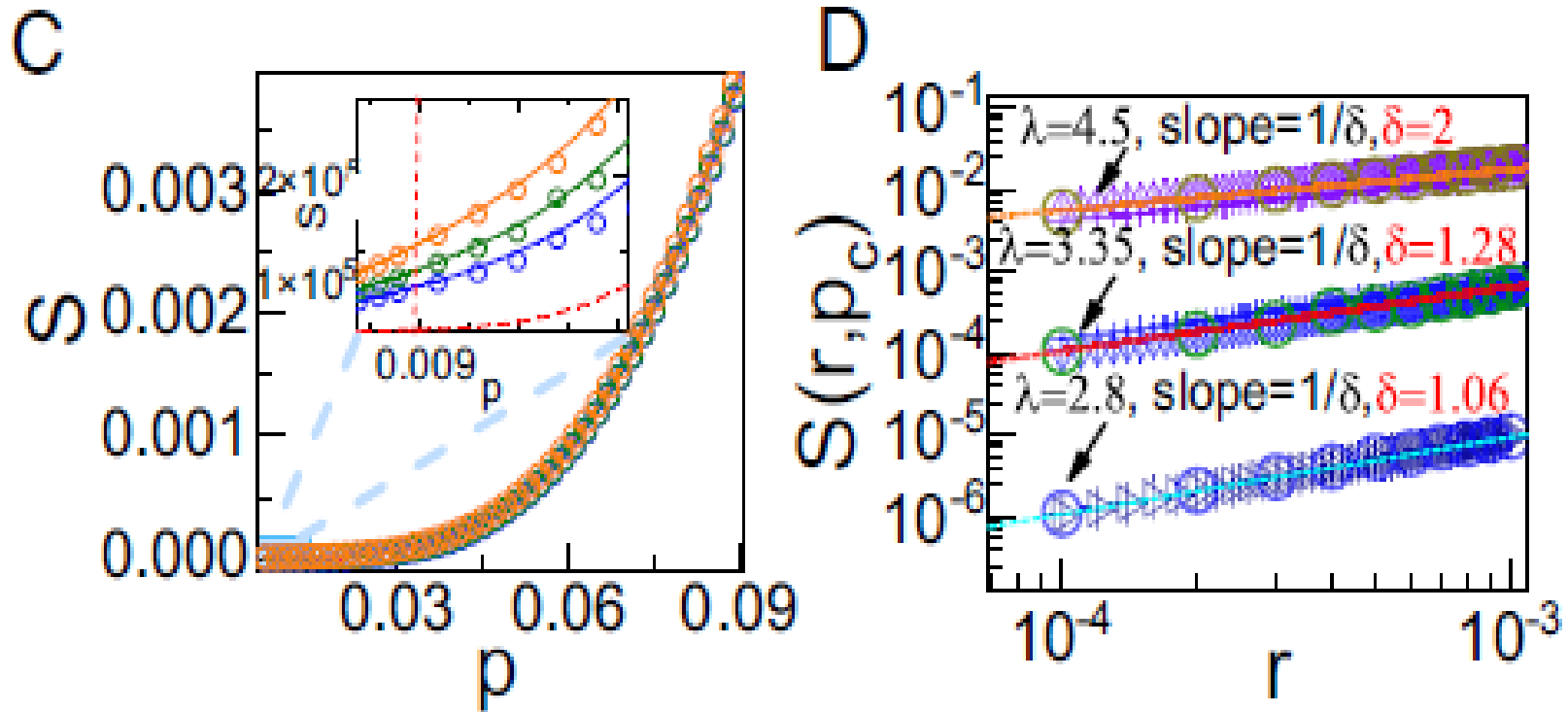
Scaling with field r

The field r removes the transition
Like magnetic field in spin system

$$S(p_c, r) \sim r^{1/\delta}$$

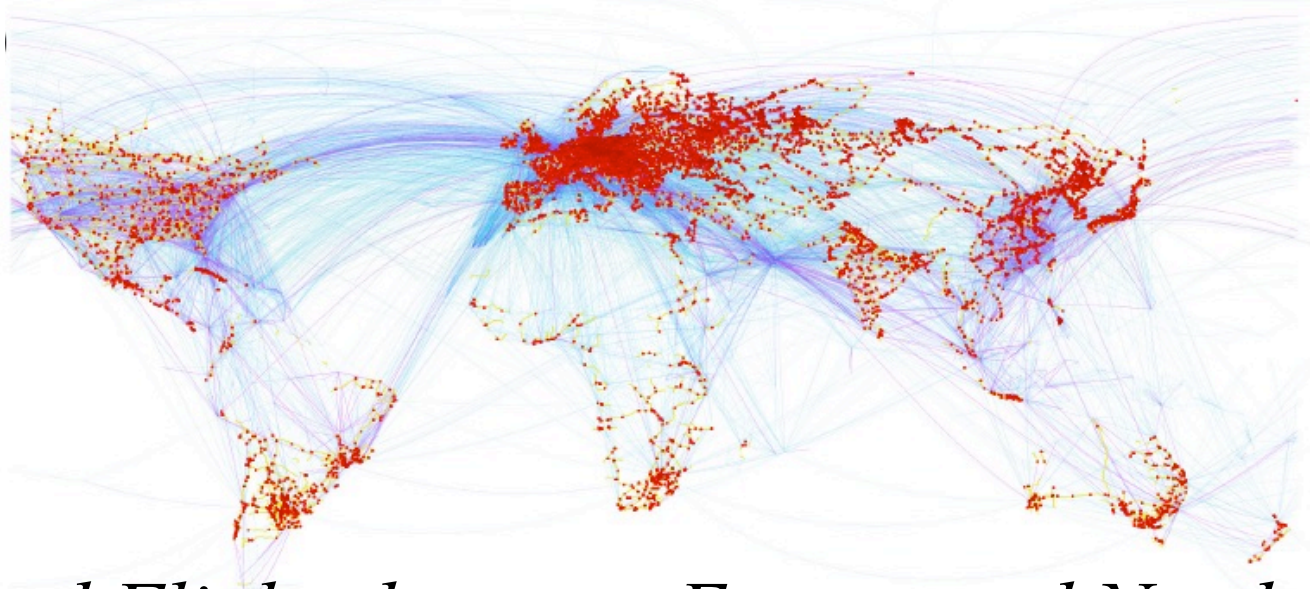


Scaling with field r in scale free modules

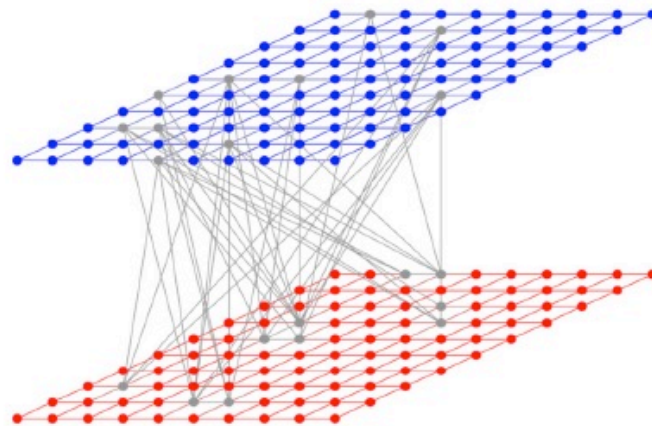


G. Dong et al PNAS (2018)
arXiv:1805.01032

Scaling with field r in spatially embedded modules

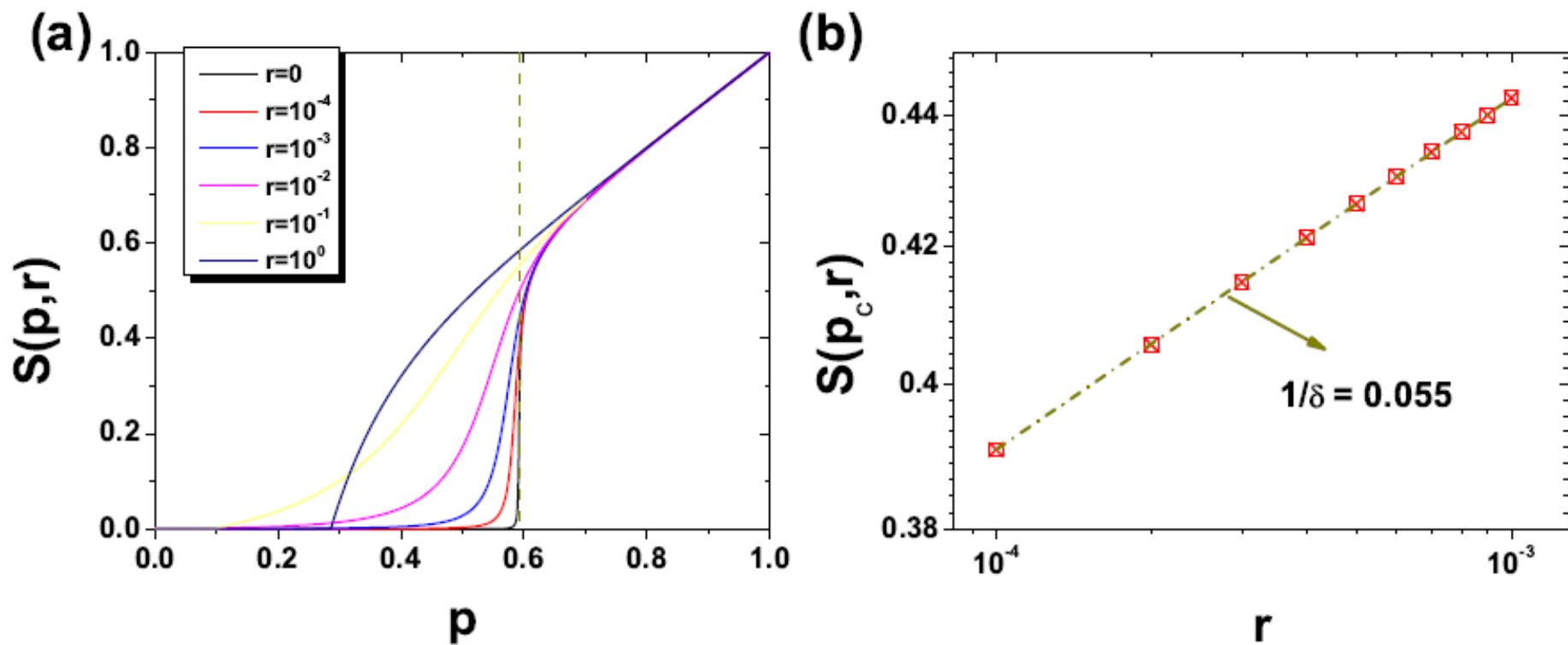


Trains and Flights between Europe and North America



Fan et al arXiv:1806.00756

Scaling with field r in spatially embedded modules

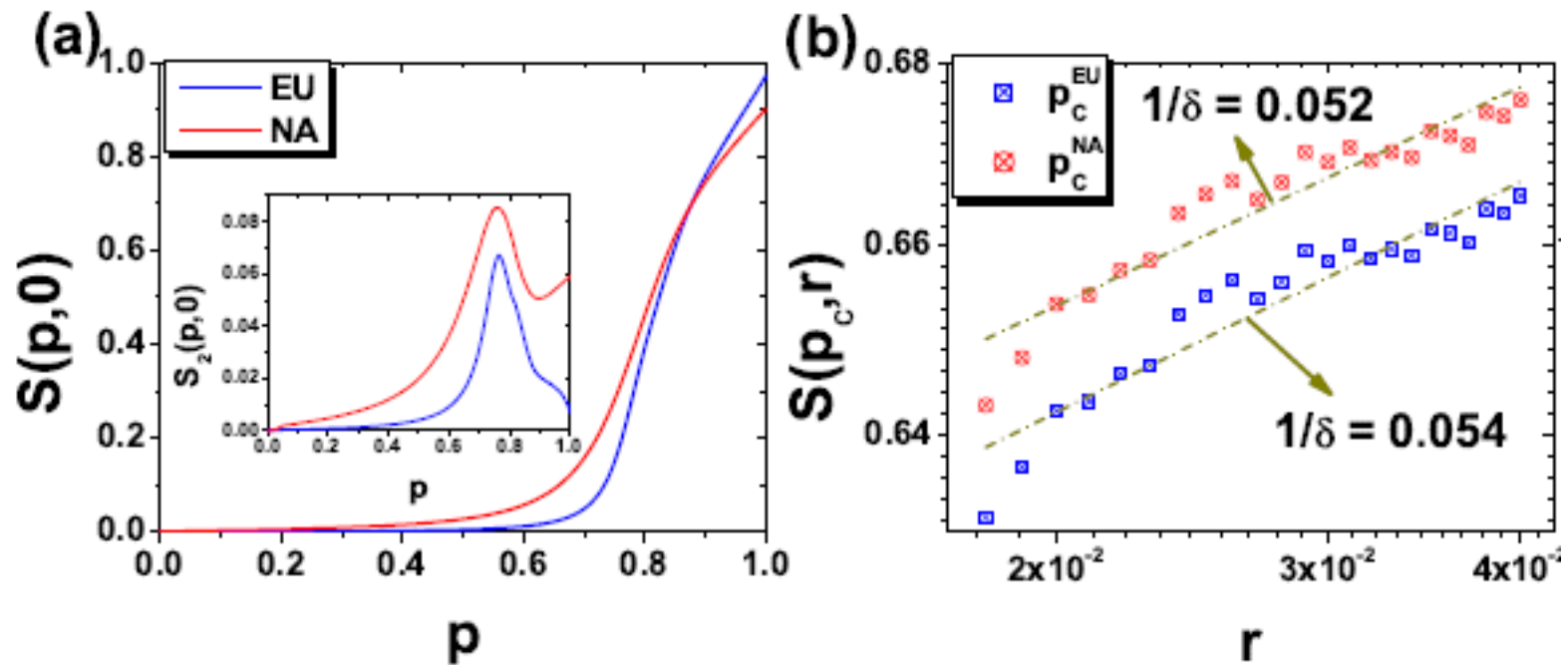


$$S(p_c, r) \sim r^{1/\delta},$$

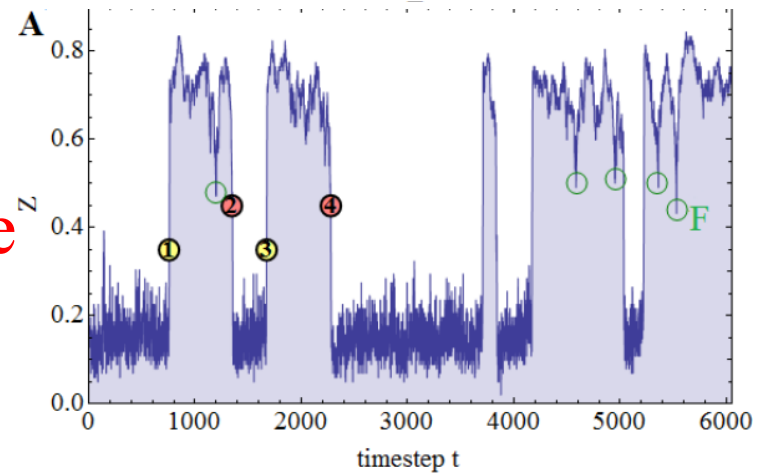
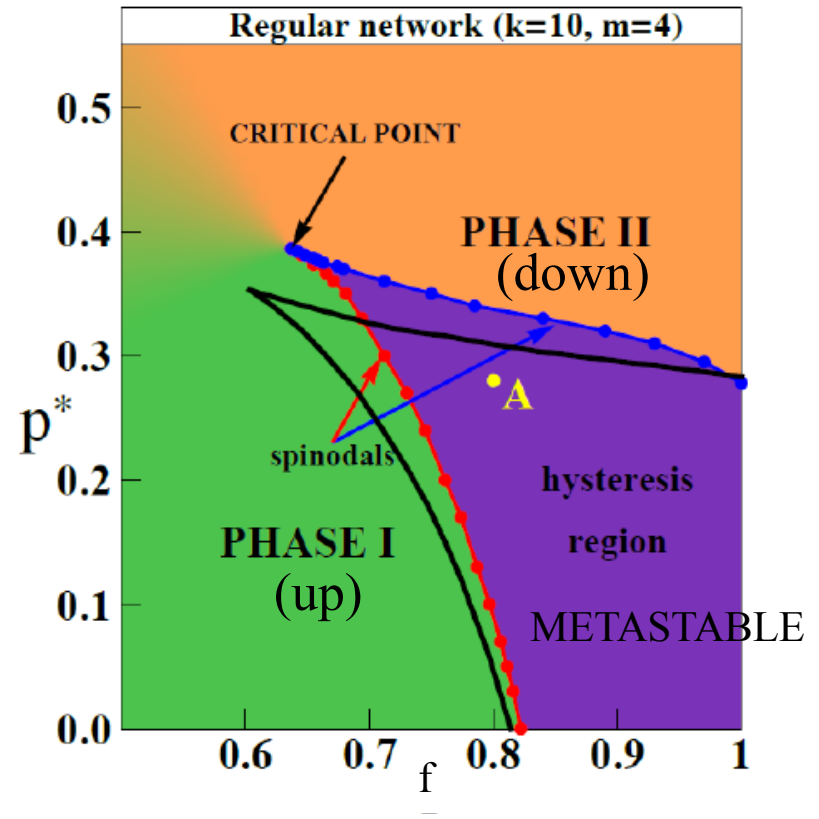
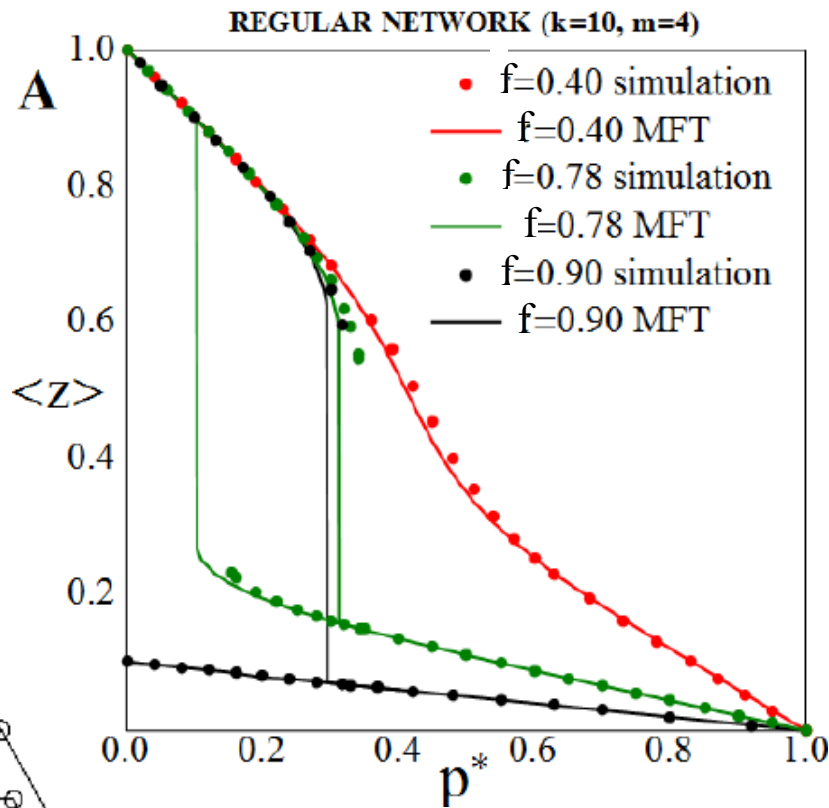
Fan et al
arXiv preprint arXiv:1806.00756

Scaling with field r in real spatially embedded modules

Trains and Flights between Europe and North America



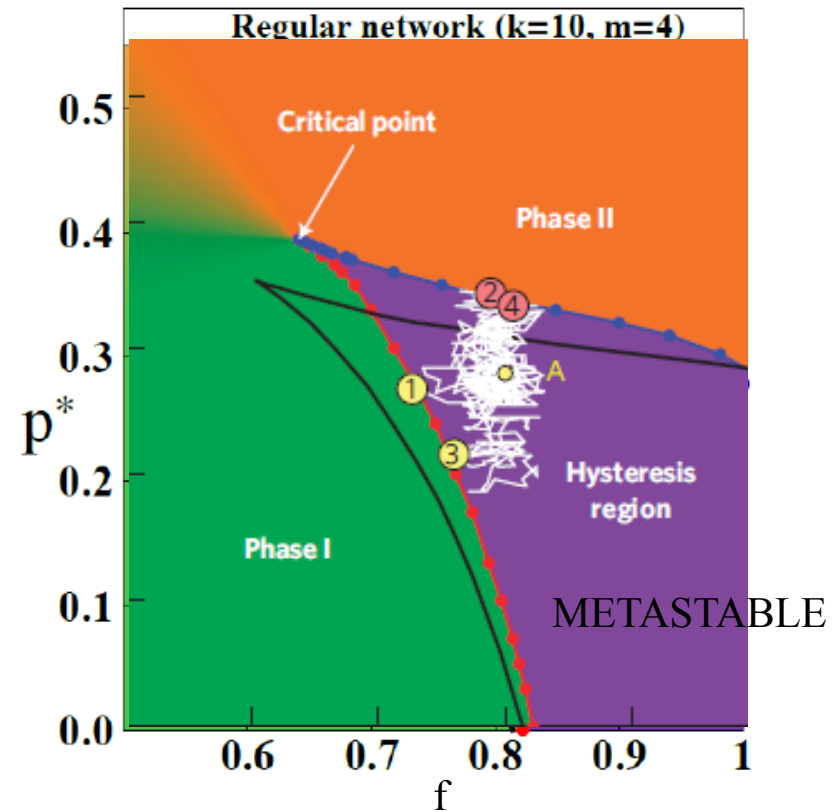
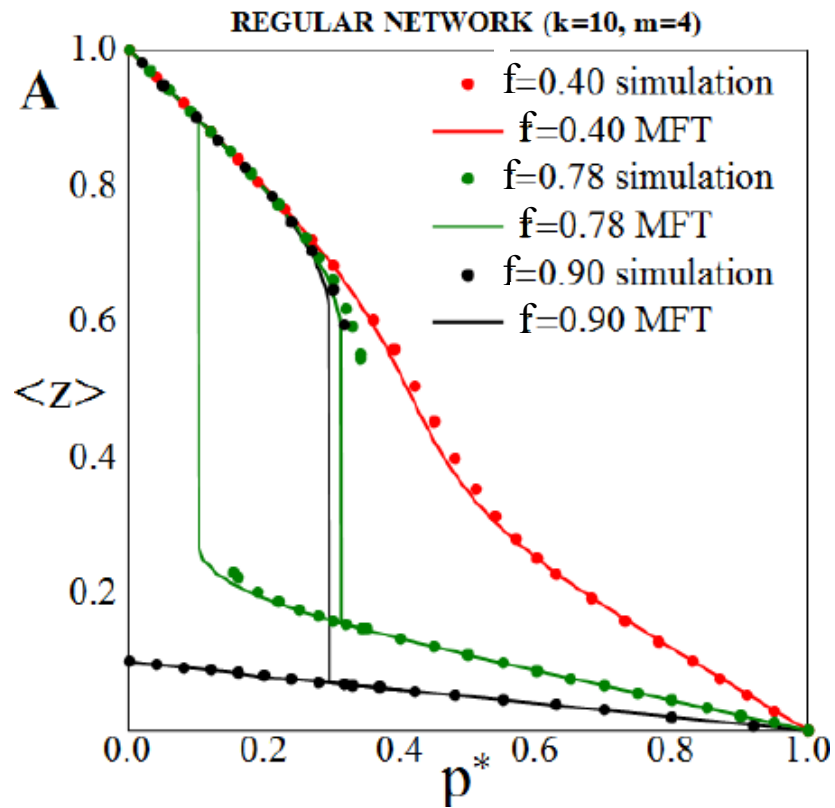
Introducing Recovery-Single Networks



Spontaneous Recovery and Failure

Majdanzik et al Nature Phys. 10, 3438 (2014)

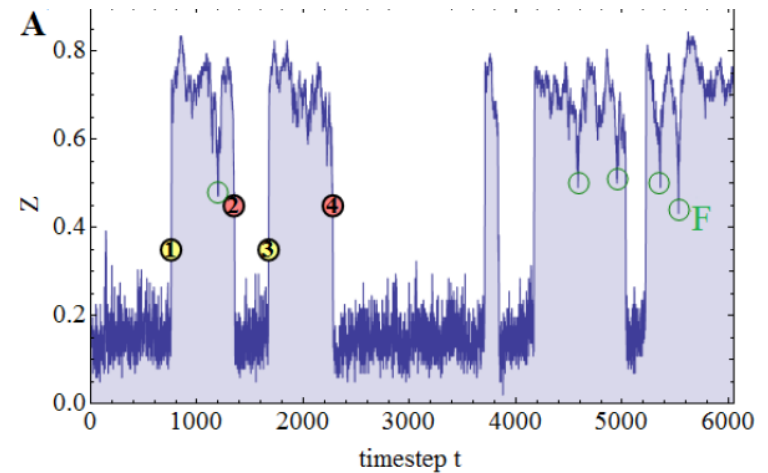
Introducing Recovery-Single Networks



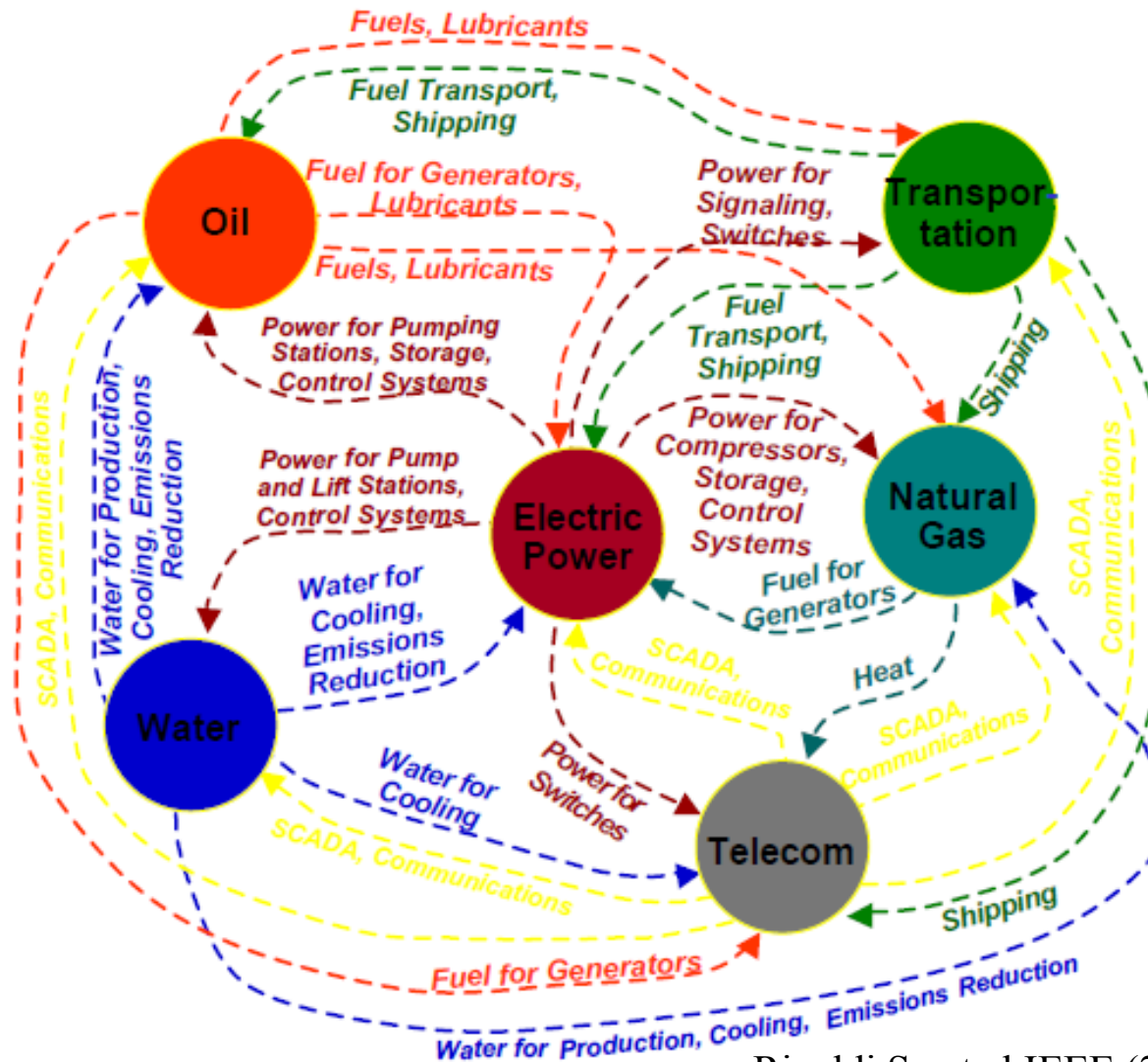
Diffusion of a SYSTEM in phase space

Spontaneous Recovery and Failure

Majdanzik et al Nature Phys. 10, 3438 (2014)



How interdependent are infrastructures?



Rinaldi S, et al IEEE (2001)