## Quantum Optics Group

Dipartimento di Fisica, Sapienza Università di Roma

## Lecture 3: Boson sampling



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http: $\backslash \backslash$ quantumoptics.phys.uniroma1.it
www.quantumab.It

## Quantum computation



- logical gate
- quantum algorithms


## Boson Sampling

## Quantum simulation



## Quantum walk



## How to achieve QUANTUM SUPREMACY ??

Proposed "quantum supremacy" for controlled quantum systems surpassing classical ones. Please sunnest alternatives.

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Proposed "quantum supremacy" for controlled quantum systems surpassing classical ones. Please suignest alternatives.

## BOSON SAMPLING

propagation on the chip with m modes


Input: $n$ bosons


## Output: n-photon state

Can a classical computer efficiently simulate the distribution of the output mode numbers?

## Answer: NO!!

## How to Achieve QUANTUM SUPREMACY ??

Proposed "quantum supremacy" for controlled quantum systems surpassing classical ones. Please sunnest alternatives.

## BOSON SAMPLING

The Extended Church-Turing (ECT) Thesis
Everything feasibly computable in the physical world is feasibly computable by a (probabilistic) Turing machine.

Can we experimentally disproof the ECT thesis?
$n$ bosons


## n-photon state

Can a classical computer efficiently simulate the distribution of the output mode numbers?

## Answer: NO!!

Arkhipov and Aaronson, The Computational Complexity of Linear Optics Proceedings of the Royal Society (2011)

## Boson Sampling

Sampling the output distribution (even approximately) of noninteracting bosons evolving through a linear network is hard to do with classical resources
n bosons m modes


Why? Transition amplitudes are related to the permanent of square matrices

$$
\langle T| U_{F}|S\rangle=\frac{\operatorname{Per}\left(U_{S, T}\right)}{\sqrt{s_{1}!\ldots s_{m}!t_{1}!\ldots t_{m}!}}
$$

classicaly hard

| input |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 1 | 1 | 0 | 1 |
| - | 0 | 0.212 | $-0.018+0.1652$ | -0.238-0.18 | $-0.429+0.32 i$ | $-0.715+0.2 t$ |
| 난 | 1 | -0.193-0.388i | $-0.045-0.3792$ | $0.19+0.3112$ | 0.328-0.2692 | $-0.594+0.03\}$ |
|  |  | $-0.723+0.3632$ | 0.087-0.09t | -0.076-0.155 | $0.206+0.443 z$ | -0.153-0.193 |
| $\bigcirc$ | 1 | $-0.092+0.0453$ | $-0.148-0.6452$ | $-0.588+0.1842$ | $-0.369-0.0862$ | $0.167+0.0258$ |
|  |  | -0.318-0.0097 | $-0.144-0.5048$ | 0.452-0.405ı | $0.037+0.3872$ | $0.071+0.0258$ |

S. Aaronson and A. Arkhipov, Proceedings of the 43rd Annual ACM Symposium on Theory of Computing, 333-342

## EVOLUTION OF BOSONIC STATES IN LINEAR OPTICS

How does the transition amplitude links with the permanent?

$$
\begin{aligned}
& U \longrightarrow \text { Unitary evolution of the system, } m \times m \text { matrix } \\
& |T\rangle=\left|t_{1}, \cdots t_{m}\right\rangle \\
& |S\rangle=\left|s_{1}, \cdots s_{m}\right\rangle \longrightarrow \text { Input } n \text {-photon state: } t_{1}+\ldots+t_{m}=n \\
& \text { Output } n \text {-photon state: } s_{1}+\ldots+s_{m}=n
\end{aligned}
$$

## EVOLUTION OF BOSONIC STATES IN LINEAR OPTICS

## How does the transition amplitude links with the permanent?

$U \leadsto$ Unitary evolution of the system, $m \times m$ matrix
$|T\rangle=\left|t_{1}, \cdots t_{m}\right\rangle \Longrightarrow$ Input $n$-photon state: $t_{1}+\ldots+t_{m}=n$
$|S\rangle=\left|s_{1}, \cdots s_{m}\right\rangle \Longrightarrow$ Output $n$-photon state: $s_{1}+\ldots+s_{m}=n$
Unitary evolution in the Hilbert space: $\mathcal{H}_{l}$ with dimension: $l=\binom{m+n-1}{n}$
$n \times n$ matrix, composed by repeating:

$$
\left.\begin{aligned}
& l=\text { all possible combinations of } \\
& n \text { photons in } m \text { modes }
\end{aligned} \right\rvert\, \begin{aligned}
& (-) s_{i} \text { times the } \\
& (-) t_{j} \text { times the }
\end{aligned}
$$

## An Example of Boson Sampling

Input:
$|1,1,1,0,0,0,0,0\rangle$
propagation on the chip with $\mathrm{m}=8$ modes
n=3
bosons

possible outcomes

$$
\begin{gathered}
l=\binom{m+n-1}{n} \\
l=120
\end{gathered}
$$

$$
\mathrm{P}(\mathrm{~s} 1, \ldots, \mathrm{sm})=\operatorname{per}(\mathrm{U})
$$

O(n!) elements
permanent of $n \times n$ matrix

## COMPUTATIONAL COMPLEXITY OF THE PERMANENT

How complex is the calculation of the permanent of a $n \times n$ matrix?

$$
\begin{aligned}
& \text { Permanent, bosons } \\
& \operatorname{per}(A)=\sum_{\sigma \in S_{n}} \prod_{i=1}^{n} a_{i, \sigma(i)}
\end{aligned}
$$

faster algorithms scale as $\mathrm{O}\left(\begin{array}{l} \\ 2^{n}\end{array}\right)$ can be computed in $\operatorname{poly}(\mathrm{n})$ time

No linear algebra rules
can be exploited to
simplify the calculation

Determinant, fermions
$\operatorname{det}(A)=\sum_{\sigma \in S / n} \operatorname{sgn}(\sigma) \prod_{i=1}^{n} a_{i, \sigma(i)}$

No linear transformations can map the permanent to a determinant


## APPROXIMATION OF THE PERMANENT COMPLEXITY CLASSES AND THE COLLAPSE OF THE POLYNOMIAL HIERARCHY

HARDNESS CONJECTURE (S. Aaronson and A. Arkhipov, Proceedings of ACM STOC 2011, 333-342)

If the Boson Sampling problem could be (even approximately) solved by a classical algorithm in poly(n) times

Collapse of the polynomial hierarchy!

Based on two conjectures on the computational complexity of the permanent of a Gaussian matrix

## \#P COMPUTATIONAL COMPLEXITY AND THE PERMANENT

\#P Class: includes the sets of counting problems associated to a decision problem in NP

Counts the number of solutions satisfying a certain constraint

Machine state and the tape symbol do not uniquely determine the next step of the machine

Decision problems solvable with a non-deterministic Turing machine

Turing machine where the transition rule is not single-valued

Permanent of a Matrix with $\mathbf{N}(0,1)$ gaussian entries: \#P-hard problem

Permanent of a Matrix with $\{0,1\}$ entries: \#P-complete problem (Valiant, 1979)

Every problem in \#P can be reduced to the permanent of a $\{0,1\}$ entries matrix in polynomial time

## Boson Sampling


«Small-scale quantum computers made from an array of interconnected waveguides on a glass chip can now perform a task that is considered hard to undertake on a large scale by classical means. »
T. Ralph, News \& Views, Nature Photonics 7, 514 (2013)

## Boson Sampling

## Photons naturally solve the BosonSampling problem

Experimental platform: photons in linear optical interferometers


Hard to implements with bulk optics

Require a technological step recently available due to integrated photonics

ค Universidade
Federal
Fluminen

UNIVERSITA DI ROMA photonics

## Integrated multimode interferometers with arbitrary designs for photonic boson sampling

Andrea Crespil ${ }^{3}$, Roberto Osellame ${ }^{12 *}$, Roberta Ramponi ${ }^{12}$, Daniel J. Brod, Ernesto F. Galvabo ${ }^{1 *}$ Nicolò Spagnolo ${ }^{4}$, Chiara Vitellits, Enrico Malorino ${ }^{4}$, Paolo Mataloni ${ }^{4}$ and Fabio Sciarrino ${ }^{4 *}$

The evolution of bosent undergoing arbitary linear uniturg iranstormations quikily becoment hard to predict uting classical Piotons propagating in a mullipert inferferveneler noturally solve this se-called hosios samplias problem, therely motival. Ing the development of tednoologles that enable preeise centrel of multiphoton interference in large interferometern ${ }^{2-c}$. Mere, me wes novel three-dimenaional manufacturing tectriques to

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| nature |
| :--- | :--- | :--- |
| Photonics |$\quad$ LETTERS

## Experimental boson sampling

Max Tillmann ${ }^{12 \star}$, Borivoje Dakić', René Heilmann³, Stefan Nolte3, Alexander Szameit ${ }^{3}$ and Philip Walther ${ }^{\text {l2 }}$

Universal quantum compoters' peomise a dramatic speed over classical computers, but their full-sia remains challenging?. However, intermediate quan tational models ${ }^{3-3}$ have been proposed that are a but can solve problems that are believed to
hard. Aarensen and Arkhiport hawe shown that hard. Aarenson and Arkhipor have shown that hard problem of sampling the bosonic sutput Remarkably, this computation does not require m
hased intaractions ${ }^{\geqslant p}$ or adantive fead.forward


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Boson Sampling on a Photonic Chip




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Massachusetts Institute of Technology

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efficent compotaten or timulation If Be ula be flongly ceritadided by phovical aculet lor dinikal compoten sech a bex a bowes katwed by ver puilet inem

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ww.sciencemag.org

# THE SOLUTION: INTEGRATED PHOTONICS 



Laser writing technology: unique capability to transmit any polarization state

- Femtosecond pulse tightly focused in a glass
- Waveguides writing by translation of the sample


## Boson Sampling: chip

Requirement for Boson Sampling design arbitrary interferometers

Requires independent control of phases and beam-splitter operation



TRANSMISSION CONTROL



Reck, et al., PRL 73, 58 (1994)

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Reck, et al., PRL 73, 58 (1994)

## Characterization of the unitary

Geometry for arbitrary


| $i$ | $t_{i}$ | $\alpha_{i}[\mathrm{rad}]$ | $\beta_{i}[\mathrm{rad}]$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.19 | 0 | 0 |
| 2 | 0.40 | 0.64 | 0 |
| 3 | 0.48 | 0 | 1.37 |
| 4 | 0.44 | 0 | 1.10 |
| 5 | 0.55 | 2.21 | 0 |
| 6 | 0.54 | 0 | 1.02 |
| 7 | 0.51 | 2.93 | 0 |
| 8 | 0.76 | 1.08 | 0 |
| 9 | 0.99 | 2.58 | 0 |
| 10 | 0.95 | 0 | 0 |

Randomly sampled matrix

$$
U^{t}=\left(\begin{array}{ccccc}
0.212 & -0.018+0.165 i & -0.238-0.18 i & -0.429+0.32 i & -0.715+0.2 i \\
-0.193-0.388 i & -0.045-0.379 i & 0.19+0.311 i & 0.328-0.269 i & -0.594+0.03 i \\
-0.723+0.363 i & 0.087-0.09 i & -0.076-0.155 i & 0.206+0.443 i & -0.153-0.193 i \\
-0.092+0.045 i & -0.148-0.645 i & -0.588+0.184 i & -0.369-0.086 i & 0.167+0.025 i \\
0.318-0.009 i & -0.144-0.594 i & 0.452-0.405 i & 0.037+0.387 i & 0.071+0.025 i
\end{array}\right)
$$

Reconstructed matrix

$$
U^{r}=\left(\begin{array}{ccccc}
0.37 & 0.007+0.151 i & -0.164-0.31 i & -0.442+0.138 i & -0.702+0.099 i \\
-0.109-0.465 i & -0.013-0.585 i & 0.121+0.381 i & 0.076-0.134 i & -0.474-0.147 i \\
-0.677+0.18 i & 0.134-0.027 i & -0.283-0.133 i & 0.036+0.498 i & -0.206-0.319 i \\
-0.039+0.24 i & -0.08-0.572 i & -0.496-0.046 i & -0.475-0.22 i & 0.265+0.125 i \\
0.262+0.133 i & 0.09-0.524 i & 0.479-0.377 i & 0.055+0.486 i & 0.143+0.007 i
\end{array}\right)
$$

Gate fidelity: $F=\left|\operatorname{Tr}\left(U^{t} U^{r \dagger}\right)\right| / 5=0.95$.
A. Crespi, R. Osellame, R. Ramponi, D. J. Brod, E. F. Galvao, N. Spagnolo, C. Vitelli, E. Maiorino, P. Mataloni, F. Sciarrino, Integrated multimode interferometers with arbitrary designs for photonic boson sampling, Nature Photonics 7, 545 (2013).

## Boson Sampling: apparatus


A. Crespi, R. Osellame, R. Ramponi, D. J. Brod, E. F. Galvao, N. Spagnolo, C. Vitelli, E. Maiorino, P. Mataloni, F. Sciarrino, Integrated multimode interferometers with arbitrary designs for photonic boson sampling, Nature Photonics 7, 545 (2013).

## Experimental Boson Sampling

Single-photon probabilities $S_{\text {exp }, r}^{1}=0.990 \pm 0.005$


Two-photon probabilities $S_{\text {exp }, r}^{2}=0.977 \pm 0.027$

$$
S_{e x p, r p}^{3}=0.983 \pm 0.045
$$

Three-photon probabilities

Good agreement between experimental data and the probabilities expected from the permanent formula:

$$
\langle T| U_{F}|S\rangle=\frac{\operatorname{per}\left(U_{S, T}\right)}{\sqrt{s_{1}!. . s_{m}!t_{1}!. t_{m}!}}
$$

## Boson Sampling in a 13-mode device

Input: $(6,7,8)$


Output: 286 different possible no-bunching configurations

91 different fabrication phases
N. Spagnolo, C. Vitelli, M. Bentivegna, D. J. Brod, A. Crespi, F. Flamini, S. Giacomini, G. Milani, R. Ramponi, P. Mataloni, R. Osellame, E. F. Galvao, and F. Sciarrino, Nature Photonics 8, 614 (2014) Similar experiment in Bristol: J. Carolan, et al., Nature Photonics 8, 619 (2014)

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## Validation of the Boson Sampling output

Boson Sampling: hard problem with classical computer
but may be very hard also to validate/certify!

We need to develop different methodologies to validate/ certify the output



Validation against the uniform

N. Spagnolo, C. Vitelli, M. Bentivegna, D. J. Brod, A. Crespi, F. Flamini, S. Giacomini, G. Milani, R. Ramponi, P. Mataloni, R. Osellame, E. F. Galvao, and F. Sciarrino, Nature Photonics 8, 614 (2014) Similar experiment in Bristol: J. Carolan, et al., Nature Photonics 8, 619 (2014)

## Validation of Boson Sampling....

Boson Sampling: hard problem with classical computer
but may be very hard also to validate/certify!!
Two sources of hard computability: permanent calculation and the exponential growth of the Hilbert space

## Validation of Boson Sampling....

## Boson Sampling: hard problem with classical computer

but may be very hard also to validate/certify!!
Two sources of hard computability: permanent calculation and the exponential growth of the Hilbert space

Can we discriminate the Boson Sampling distribution from the Uniform Distribution efficiently, hence without requiring an exponential number of measurements?

## Boson-Sampling in the light of sample complexity

C. Gogolin, M. Kliesch, L. Aolita, and J. Eisert

Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany
C. Gogolin, M. Kliesch, L. Aolita, J. Eisert, arXiv:1306.3995 (2013)

## Distinguishing Boson Sampling from Uni

Can we discriminate the Boson Sampling distribution from the Uniform Distribution efficiently (hence without requiring an exponential number of measurements) ?
Black box settings:

no information on the system including the unitary
Symmetric algorithms:
 decision depends only on the outcome frequencies and not on the labels of the collected samples

the two distribution cannot be distinguished efficiently

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Black box settings:

no information on the system including the unitary

Symmetric algorithms:
 decision depends only on the outcome frequencies and not on the labels of the collected samples

the two distribution cannot be distinguished efficiently


The unitary $U$ and the input state are known problem parameters

The adoption only of symmetric algorithms is too restrictive

Can we efficiently distinguishing the BosonSampling distribution from a Uniform distribution by exploiting information on the unitary?
C. Gogolin, M. Kliesch, L. Aolita, J. Eisert, arXiv:1306.3995 (2013)

## Distinguishing Boson Sampling from Un

Can we efficiently distinguishing the BosonSampling distribution from a Uniform distribution by exploiting information on the unitary?


Validation stage
The algorithm: for each outcome $T=\left\{t_{1}, t_{2}, \ldots, t_{n}\right\}$, input $S=\left\{s_{1}, s_{2}, \ldots, s_{n}\right\}$

$$
\underset{\substack{\text { Define } \\
A_{i, j}=U_{s_{i}, t_{j}} .}}{\text { Calculate }} \underbrace{\substack{\text { computationally efficient }}}_{\substack{P=\prod_{i=1}^{n} \sum_{j=1}^{n}\left|A_{i, j}\right|^{2} .}} \begin{aligned}
& \text { If } P>\left(\frac{n}{m}\right)^{n} \begin{array}{c}
\text { BosonSampling } \\
\text { Else }
\end{array} \\
& \text { UniformSampler }
\end{aligned}
$$

## Experimental Results - 1


${ }^{0.06} \quad$ b) 9-mode interferometer

d) 5 -mode interferometer

e) 7-mode interferometer

f) 9-mode interferometer


The BosonSampling distribution can be efficiently discriminated from the Uniform
N. Spagnolo, C. Vitelli, M. Bentivegna, D. J. Brod, A. Crespi, F. Flamini, S. Giacomini, G. Milani, R. Ramponi, P. Mataloni, R. Osellame, E. F. Galvao, and F. Sciarrino, Nature Photonics 8, 614 (2014) Similar experiment in Bristol: J. Carolan, et al., Nature Photonics 8, 619 (2014)

## Experimental Results - 2

5-mode interferometer


Uniform Sampler



$>95 \%$ success probability achieved with sample size of $\mathrm{N} \sim 100$ data

N. Spagnolo, C. Vitelli, M. Bentivegna, D. J. Brod, A. Crespi, F. Flamini, S. Giacomini, G. Milani, R. Ramponi, P. Mataloni, R. Osellame, E. F. Galvao, and F. Sciarrino, arXiv:1311.1622 (2013)

## Validation of Boson Sampling in a 13-mode device

Input: (6,7,8)


91 different fabrication phases
Output: 286 different possible no-
Validation against the uniform distribution
 bunching configurations


## Validation of Boson Sampling

## What about other distributions?

- Uniform (simplest and most general case of non-informed-about-U distributions) can be ruled out efficiently
- What about validating against"smarter cheaters"?
- Fermion Sampler
- Classical Mockud Sampler

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nature
photonics
```


## Experimental validation of photonic boson sampling

Nicolò Spagnolo', Chiara Vitelli12, Marco Bentivegna¹, Daniel J. Brod³, Andrea Crespi44, Fulvio Flamini', Sandro Giacomini', Giorgio Milani', Roberta Ramponi44, Paolo Mataloni', Roberto Osellame ${ }^{4,5 \star}$, Emesto F. Galvão ${ }^{3 \star}$ and Fabio Sciarrino ${ }^{1 \star}$

## Distinguishing Boson Sampling from alternative distributions

Further Step: can we discriminate the BosonSampling distribution from Alternative distribution?

Example: Indistinguishable Bosons vs Distinguishable Bosons
Strategy: Compare outcome-by-outcome the probabilities between the two cases and assign the event (up to a threshold) to the cases with higher probability

Requires calculating permanents
Does not require calculating the whole distribution

b) 9-mode interferometer

N. Spagnolo, C. Vitelli, M. Bentivegna, D. J. Brod, A. Crespi, F. Flamini, S. Giacomini, G. Milani, R. Ramponi, P. Mataloni, R. Osellame, E. F. Galvao, and F. Sciarrino, arXiv:1311.1622 (2013)

Nature Photonics (in press)

First experimental results with integrated photonics :
$\left[\begin{array}{l}\text { nature } \\ \text { photonics }\end{array}\right.$

Integrated multimode interferometers with arbitrary designs for photonic boson sampling
Andrea Crespi ${ }^{12}$, Roberto Osellame ${ }^{12 *}$, Roberta Rampon ${ }^{22}$, Daniel J. Brod3, Ernesto F. Galvaio ${ }^{3 *}$, Nicolo Spagnolot, Chiara Vitelliks, Enrico Maiorino ${ }^{4}$, Paolo Matalonit and Fabio Selarrino ${ }^{4 *}$


## The Extended Church-Turing (ECT) Thesis

Everything feasibly computable in the physical world is feasibly computable by a (probabilistic) Turing machine.

## Can we experimentally

 disproof the ECT thesis?
## GOAL: to achieve Boson Sampling with $n=10-20$ photons and $m=100-200$ modes

## Open questions:

Challenges

- How to increase the complexity of Boson sampling?
- Does it exist simpler experimental schemes achieving a similar goal?
- How to certify the well-functioning of boson-sampling experiment?

How realistic noise and imperfections affect the hardness claim?

- Single photon sources
- Manipulation on a chip
- Single photon detectors


## Scattershot Boson Sampling

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FSL H5, mavee [9030
PHYSICAL REVIEW LETTERS м
```


## Boses Sampling from a Ganvia Stuw











A. P. Lund, A. Laing, S. Rahimi-Keshari, T. Rudolph,
J. L. O'Brien, T. C. Ralph, Phys. Rev. Lett. 113, 100502 (2014)
$m$ heralded single photon sources


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Twenty Reasons to Delicer Opsask Acted A lore * Scattershot BosonSampling: A new approach to scalable BosonSampling experiments

Scott Aaronson's blog, acknowledged to S. Kolthammer, http://www.scottaaronson.com/blog/?p=1579

$$
\begin{aligned}
& \text { Generalization of Boson } \\
& \text { Sampling problem with } \\
& \text { computational complexity }
\end{aligned}
$$

Corresponds to sampling both from the input and the output modes

Potential huge increase of the brightness of the quantum hardware

## Scattershot Boson Sampling

$p=$ probability of generating a photon pair in a single source
(typical values $p=0.01-0.015$ )

## Conventional Boson Sampling, n photons

$p^{n}$ probability of generating the $n$-photon input

## Scattershot Boson Sampling, n-photon term

$p^{n}(1-p)^{m-n}$ probability of generating one of the $n$-photon input configurations
$\binom{m}{n}$ number of possible output configurations
Total generation rate: $\quad \sim p^{n}(1-p)^{m-n}\binom{m}{n}$


Hardness of Scattershot Boson Sampling: as hard as the conventional Boson Sampling
Corresponds to sampling both from the input and the output
A. P. Lund, A. Laing, S. Rahimi-Keshari, T. Rudolph, J. L. O’Brien, T. C. Ralph, Phys. Rev. Lett. 113, 100502 (2014)

## Scalability of down conversion


"Conventional" Boson Sampling $n$ bosons evolving in $m$ modes, random Unitary, fixed input state
Sampling from the output distribution

Photonic implementations

## Probabilistic process

based on parametric down-conversion sources
Probability of producing $n$ photons drops exponentially in $n$
Low process efficiency, gain must be kept low to prevent multi-photon events

## Is it possible to devise a generalized Boson Sampling problem?

Same computational complexity of conventional Boson Sampling


## BOSON SAMPLING

## SCATTERSHOT BOSON SAMPLING



## Experimental scattershot boson sampling

## Photon source

## Input state preparation

## Chip and detection



6 different parametric down-conversion sources


3 physical crystals and separation by polarization

Interference between photons generated from two subsequent laser pulses

M. Bentivegna, N. Spagnolo, C. Vitelli, F. Flamini, N. Viggianiello, L. Latmiral, P. Mataloni, D. J. Brod, E. F. Galvao, A. Crespi, R. Ramponi, R. Osellame, and F. Sciarrino,
"Experimental scattershot boson sampling", Science Advances 1, e1400255 (2015).

## Experimental setup: preparation

## Photon source



1) Photon I (input 6) [fixed]

Three photon events: 2) Photon III (input 8) [fixed]
3) Random input heralded by $T_{i}$


Input state preparation


Input randomness further enhanced by sequential switching of photon VII

M. Bentivegna, N. Spagnolo, C. Vitelli, F. Flamini, N. Viggianiello, L. Latmiral, P. Mataloni, D. J. Brod, E. F. Galvao, A. Crespi, R. Ramponi, R. Osellame, and F. Sciarrino,
"Experimental scattershot boson sampling", Science Advances 1, e1400255 (2015).

## Experimental setup: chip and detection

## Photon <br> source

## Input state preparation



Evolution through $\mathrm{m}=9$ and $\mathrm{m}=13$ interferometers with random (but known) structure

Coincidence detection for:
Three-photon events with one heralding trigger
Two-photon events with two heralding triggers

M. Bentivegna, N. Spagnolo, C. Vitelli, F. Flamini, N. Viggianiello, L. Latmiral, P. Mataloni, D. J. Brod, E. F. Galvao, A. Crespi, R. Ramponi, R. Osellame, and F. Sciarrino,
"Experimental scattershot boson sampling", Science Advances 1, e1400255 (2015).

## Experimental setup: generation

## Photon source

## Input state preparation

## Chip and detection



6 different parametric down-conversion sources


3 physical crystals and separation by polarization

Interference between photons generated from two subsequent laser pulses

## Experimental setup: preparation

## Photon source



1) Photon I (input 6) [fixed]

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Input randomness further enhanced by sequential switching of photon VII

## Experimental setup: chip and detection

## Photon <br> source <br> Input state preparation

Chip and detection


Evolution through $\mathrm{m}=9$ and $\mathrm{m}=13$ interferometers with random (but known) structure

Coincidence detection for:
Three-photon events with one heralding trigger
Two-photon events with two heralding triggers

## Scattershot - sampling with random input

Three-photon events


Two-photon events

Number of input/output configurations
$\mathrm{m}=9$ interferometer 2288 combinations
$\mathrm{m}=13$ interferometer 1680 combinations

Few events per input/ output configurations



Data with three-photon and two-photon input collected simultaneously

## Validation of Scattershot Boson Sampling

Validation against the uniform distribution with the Aaronson-Arkhipov test


Validation against distinguishable photons with shot-by-shot likelihood ratio test


Data can be validated against a first set of alternative hypothesis

## Witness of genuine multi-photon interference

Is it possible to derive a witness able to identify true-many photon interference occurring in Boson Sampling ?

## Quantum certification of Boson Sampling..

Is it possible to derive a witness able to identify true-many photon interference occurring in Boson Sampling ?
Boson Sampling with random unitaries: output states hard to predict..

$$
\text { Fourier matrices with m modes: } U_{1, q}^{F o u}=\frac{1}{\sqrt{m}} e^{i \frac{2 \pi l q}{m}}
$$

Quantum suppression law (forbidden output state) generalization of the Hong-Ou-Mandel effect to multiport device

1) Efficient approach for increasing $\boldsymbol{n}$ and $\boldsymbol{m}$

- many output states are suppressed
- computationally easy to predict which states are suppressed

2) Stringent approach

- genuine n-photon quantum interference

Theoretical proposal - Stringent and efficient assessment of Boson-Sampling devices
M. Tichy, K. Mayer, A. Buchleitner, K. Mølmer, PRL 113, 020502 (2014)

## Implementation of Fast Fourier Transform with 3D-integrated photonics



Crespi, Osellame, Ramponi, Bentivegna, Flamini, Spagnolo, Viggianiello, Innocenti, Mataloni, and Sciarrino Quantum suppression law in a 3-D photonic chip implementing the Fast Fourier Transform, Nature Communications 7, 10469 (2016).

## Implementation of Fast Fourier Transform with 3D-integrated photonics




## Implementation of Fast Fourier Transform with 3D-integrated photonics



| 8 modes |
| :--- |
| Fast Fourier |
| Transform |


phase

Scalable approach for the implementation of fast Fourier transform using 3-D photonic integrated interferometers fabricated via femtosecond laser writing technique.

Crespi, Osellame, Ramponi, Bentivegna, Flamini, Spagnolo, Viggianiello, Innocenti, Mataloni, and Sciarrino Quantum suppression law in a 3-D photonic chip implementing the Fast Fourier Transform, Nature Communications 7, 10469 (2016).


## Implementation of Fast Fourier Transform with 3D-integrated photonics


directional coupler

## Injection of cyclic input states

For $\mathrm{n}=2$ and $\mathrm{m}=8$ there are 4 possible (collision-free) cyclic inputs:
(1,0,0,0,1,0,0,0), (0,1,0,0,0,1,0,0), ( $0,0,1,0,0,0,1,0$ ), ( $0,0,0,1,0,0,0,1$ )

directional coupler
phase

## Quantum suppression law

Suppression of all output noncyclic output states!

Crespi, Osellame, Ramponi, Bentivegna, Flamini, Spagnolo, Viggianiello, Innocenti, Mataloni, and Sciarrino Quantum suppression law in a 3-D photonic chip implementing the Fast Fourier Transform, Nature Communications 7, 10469 (2016).

## Quantum certification of Boson Sampling

## n=2 photons over 8 modes Fast Fourier Transform



16 suppressed states over 28 output states

(c)

$$
\mathrm{Re}_{8} \text { (aft }
$$

$$
\left.\mathrm{Re}_{\mathrm{e}} \text { (OFT} \mathrm{T}_{\text {mes }}\right)
$$

(d)

$$
\operatorname{lm}\left(Q F T_{\text {meol }}\right)
$$

$$
\operatorname{Im}\left(\text { QFT }_{\text {tun }}\right)
$$

Quantum suppression of a large number of output states with 4- and 8-mode optical circuits: the experimental results demonstrate genuine quantum interference between the injected photons

| Paper | Group | Contents | Validation |
| :---: | :---: | :---: | :---: |
| Science 339, 794 (2013) | Brisbane, Boston | $\mathrm{n}=2,3$ photons, $\mathrm{m}=6$ modes - fiber | No |
| Science 339, 798 (2013) | Oxford | $\mathrm{n}=3$ photons, $\mathrm{m}=6$ modes $+n=4$ photons with (lower complexity) bunched input | No |
| Nat. Photon. 7, 540 (2013) | Vienna, Jena | $n=3$ photons, $m=5$ modes | No |
| Nat. Photon. 7, 548 (2013) | Roma, Milano, Niteroi | $n=3$ photons, $m=5$ modes Haar-Random unitary | No |
| PRL 111, 130503 (2013) | Roma, Milano, Niteroi | Bosonic Birthday paradox, and verification of full-bunching law | No |
| Nat. Photon. 8, 615 (2014) | Roma, Milano, Niteroi | $n=3$ photons, $m=5,7,9,13$ modes validation tests | Uniform distribution, distinguishable particles |
| Nat. Photon. 8, 621 (2014) | Bristol | $n=3+n=4,5$ photons (subtracting bunching), $\mathrm{m}=21$ qwalk <br> $\mathrm{n}=3$ photons in $\mathrm{m}=9$ Haar Unitary | Uniform distribution, distinguishable particles |
| Phys. Rev. X 5, 041015 (2015) | Vienna, Jena | investigation on complexity with partial photon distinguishability, $\mathrm{n}=3$ photons, $\mathrm{m}=5$ modes | No |
| Science Advances 1, e1400255 (2015) | Roma, Milano, Niteroi | $n=3$ photons, $m=9,13$ modes scattershot of 8 input states | Uniform distribution, distinguishable particles |
| Science 349, 711 (2015) | Bristol | implementation of $6 \times 6$ fully reconfigurable circuit, Haar-random. $n=3$ : zero-transmission in Fourier matrix $\mathrm{n}=6$ with bunched input ( 2 modes) | Distinguishable particles |
| Nature Communications 7, 10469 (2016). | Roma, Milano | $n=2$ photons, $m=4,8$ modes suppression law in Fourier matrix with scalable 3D architecture | Distinguishable particles, mean-field state |

## hybrid integrated quantum photonics



Near-optimal single-photon sources in the solid-state

$$
\text { arXiv: } 1510.06499
$$

N. Somaschi ${ }^{1, *}$, V. Giesz ${ }^{1, *}$, L. De Santis ${ }^{1,2 *}$, J. C. Loredo ${ }^{3}$, M. P. Almeida ${ }^{3}$, G. Hornecker ${ }^{45}$, S. L. Portalupi ${ }^{1}$, T. Grange ${ }^{4.5}$, C. Anton ${ }^{1}$, J. Demory ${ }^{1}$, C. Gomez ${ }^{1}$, I. Sagnes ${ }^{1}$, N. D. Lanzillotti-Kimura ${ }^{1}$, A. Lemaitre $^{1}$, A. Auffeves ${ }^{4,5}$, A. G. White ${ }^{3}$, L. Lanco $^{1,6}$ and P. Senellart ${ }^{1,7, *}$

## Integrated devices

## Polarization independent

Beam Splitter


Phys. Rev. Lett. 105, 200503 (2010)

## Polarization dependent CNOT



Nat. Comm.
2, 566 (2011) and Birthday Paradox

Boson Sampling On chip

111, 130503 . (2013)

Nat. Phot. 7, 545 (2013)

Science Advances 1, e1400255 (2015).

Validation

Nat. Phot. $\overline{8}, 614$ (2014)

## Quantum simulation

## Ordered systems



Pbys. Rev. Lett.
108, 010502 (2012)

Disordered Systems Phase Control


Nat. Pbot. 7, 322
(2013)

3D devices
Integrated tritter

3d interferometry
Sc. Reports 2, 862 (2012)
(2013)

Nat. Com. 4, 1606


Integrated waveplates
Nat. Com. 5, 2549 (2014)

