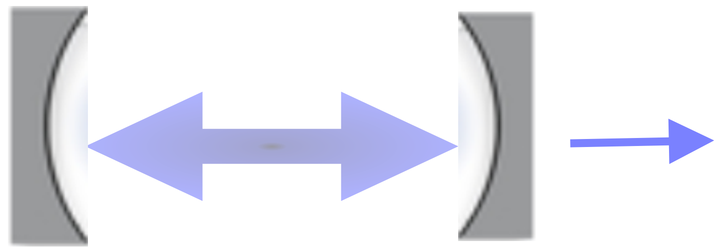
Ion chains in a cavity: a novel paradigma of static friction

Cavity Quantum Electrodynamics



Single-mode cavity:
damped quantum harmonic oscillator

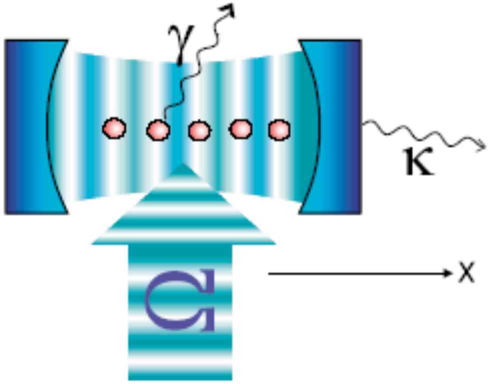
Energy $\hbar\omega\hat{a}^\dagger\hat{a}$

Electric field $E_0\cos(kx)(\hat{a}^\dagger+\hat{a})$

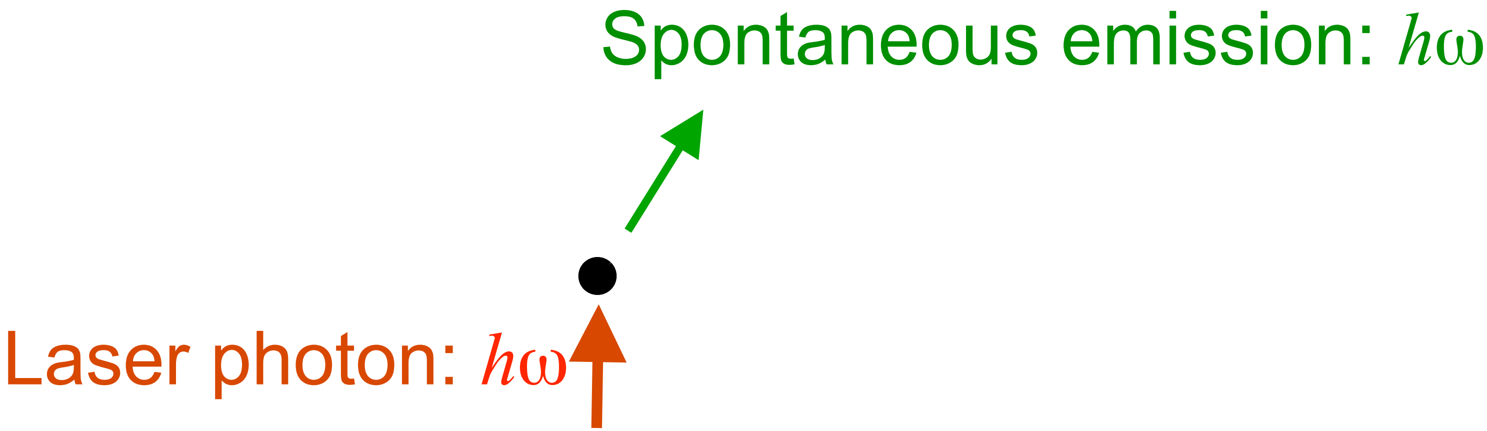
$$\partial_t\varrho=\frac{1}{\mathrm{i}}[\omega\hat{a}^\dagger\hat{a},\varrho]+\kappa(2\hat{a}\varrho\hat{a}^\dagger-\hat{a}^\dagger\hat{a}\varrho-\varrho\hat{a}^\dagger\hat{a})$$

Quantum structures in cavity QED

Originate from the mechanical effects of light in a high-finesse cavity

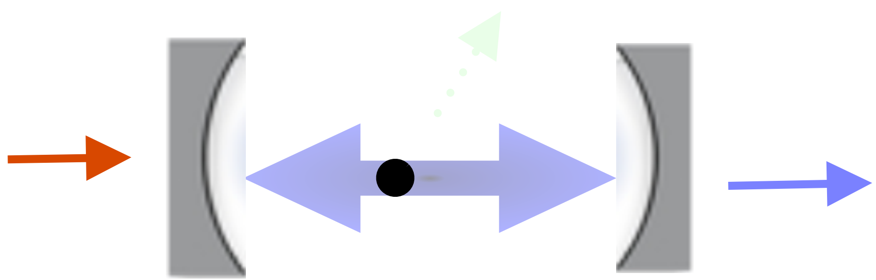


Mechanical effects of light



$\omega < \omega$: energy is transferred from the atom center of mass into the electromagnetic field.

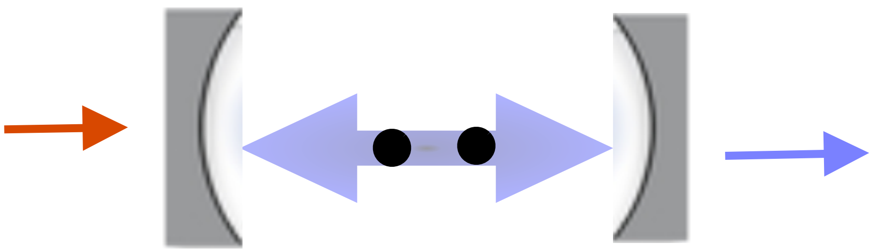
Mechanical effects of light in a cavity



atom coherently scatter into the cavity field
The phase of the emitted light depends on the atom position in the cavity mode

ω < ω: (cavity) cooling

Photon-mediated interactions

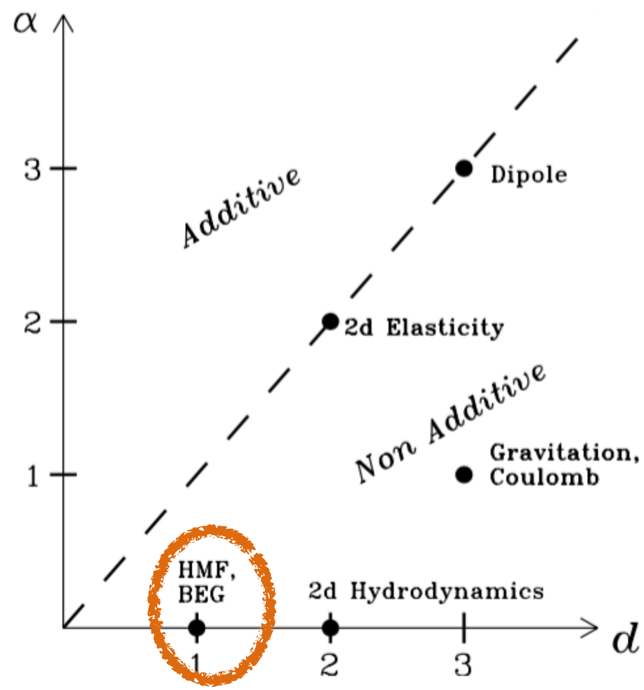


The phase of the emitted light depends on the atomic positions in the cavity

The cavity field mediates an effective interaction

Long-range interactions

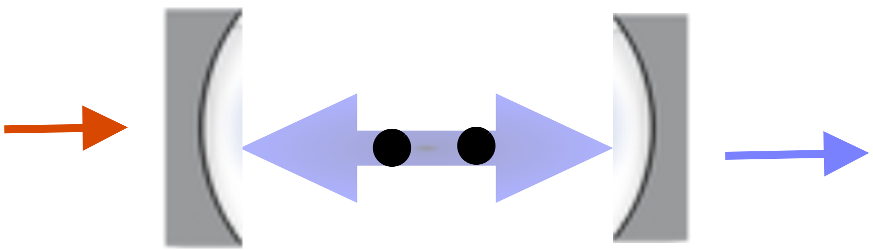
Potential scales with $1/r^{a}$
with exponent a < dimension d



Cavity Quantum Electrodynamics
Trapped ions (effective models)

Photon-mediated interactions depend on the pump intensity

Correlations can form when the field is sufficiently strong

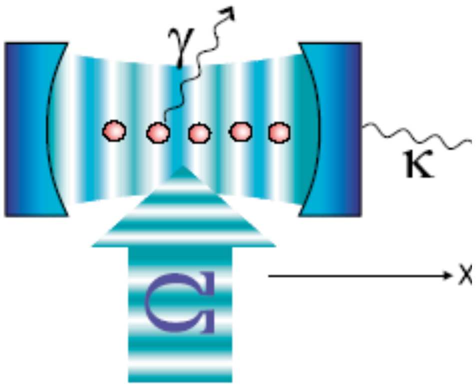


Interplay between pump and losses

Dynamics and phase transitions
are intrinsically out-of-equilibrium

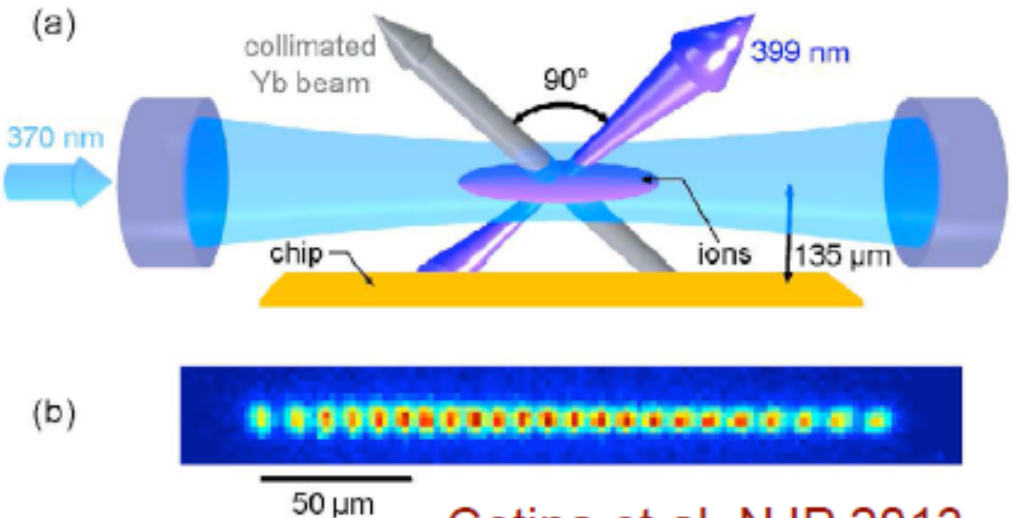
Theoretical model

- Atoms driven far-off resonance: coherent scattering into the cavity mode - classical dipoles
- Atoms move (quantum motion): dynamical refractive index



$$\hat{\mathcal{H}}_{\text{eff}} = \sum_{j=1}^{N} \frac{\hat{p}_j^2}{2m_j} - \hbar\left[\Delta_c - \sum_{j=1}^{N} U_j \cos^2(k\hat{x}_j)\right]\hat{a}^\dagger\hat{a} + \hbar\sum_{j=1}^{N} S_j \cos(k\hat{x}_j)(\hat{a} + \hat{a}^\dagger).$$

Ion crystal in a high-Q cavity

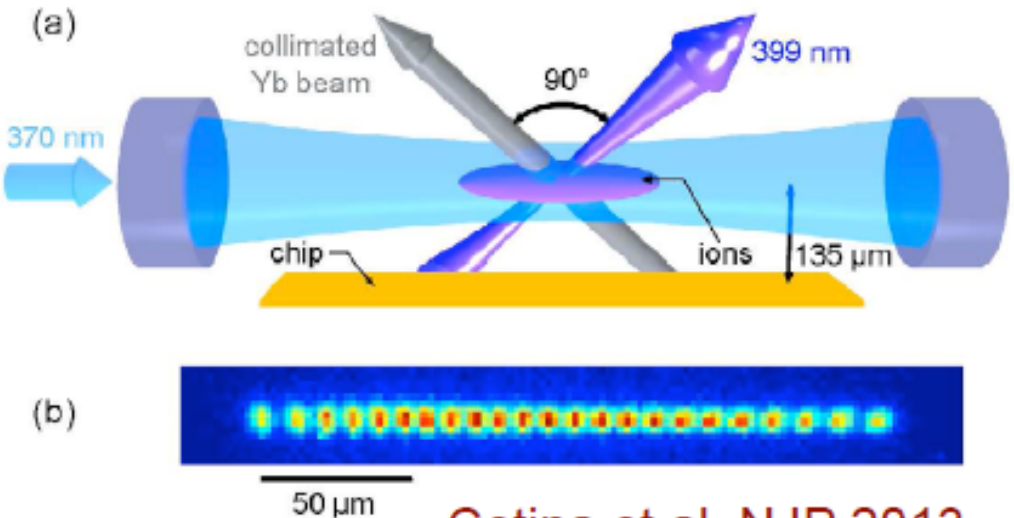


Cetina et al, NJP 2013

- Ion chain forms an elastic crystal
- Optical lattice of the cavity mode forms a substrate potential

Mismatch between the ordering of the ions due to the Coulomb force and the periodicity of the cavity-light field

The ingredients

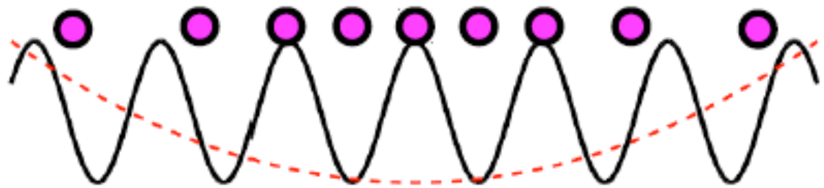


Cetina et al, NJP 2013

Ion chain in a lattice: Frenkel-Kontorova model & friction

Ion chain in a cavity: long-range interactions lead to a novel paradigma of friction

Ion crystal in a lattice



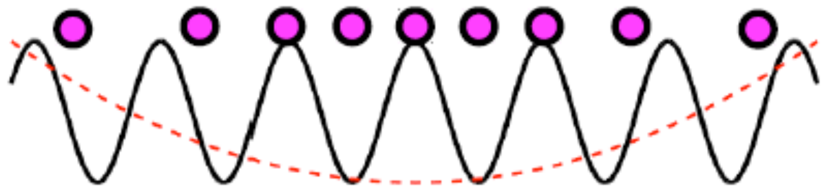
The ions

$$H_{ions} = \sum_{j=1}^{N}\left[\frac{p_j^2}{2m} + \frac{1}{2}m\omega^2 x_j^2 + \sum_{k=j+1}^{N}\frac{q^2}{4\pi\epsilon_0}\frac{1}{|x_j - x_k|}\right]$$

The substrate potential

$$H_{lattice} = V\sum_j \cos^2(kx_j)$$

Ion crystal in a lattice



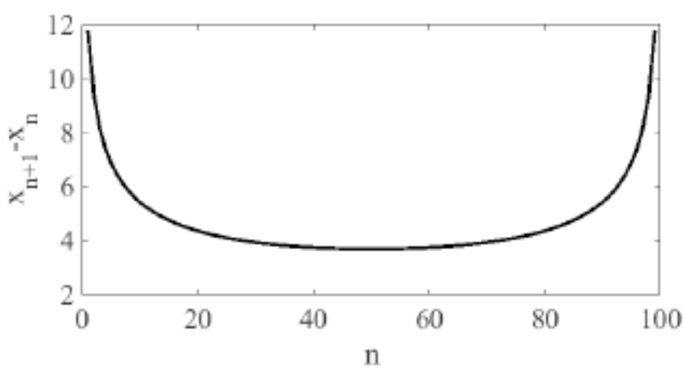
The ions

$$H_{ions} = \sum_{j=1}^{N} \left[\frac{p_j^2}{2m} + \frac{1}{2} m\omega^2 x_j^2 + \sum_{k=j+1}^{N} \frac{q^2}{4\pi\epsilon_0} \frac{1}{|x_j - x_k|} \right]$$

The substrate potential

$$H_{lattice} = V \sum \cos^2(kx_j)$$

The trapping potential ensures that the ions have a non-uniform ion density

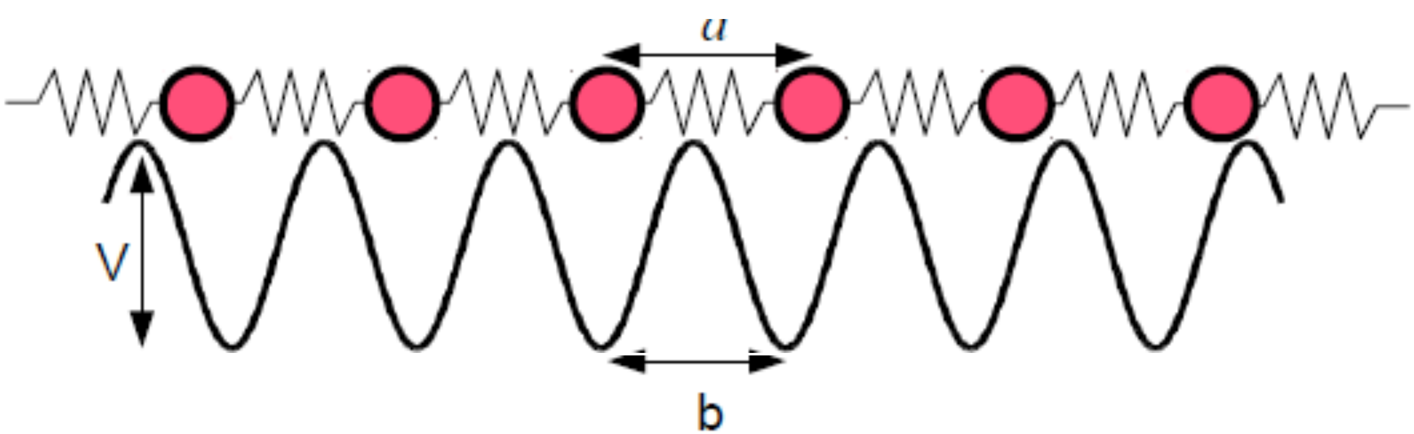


Inherent mismatch between wavelength and particle spacing

Can simulate stick-slip motion and friction – **Frenkel-Kontorova model**!

Microscopic models for friction

From sliding to stick-slip motion between two crystal surfaces: Features reproduced in 1D by the Frenkel-Kontorova model

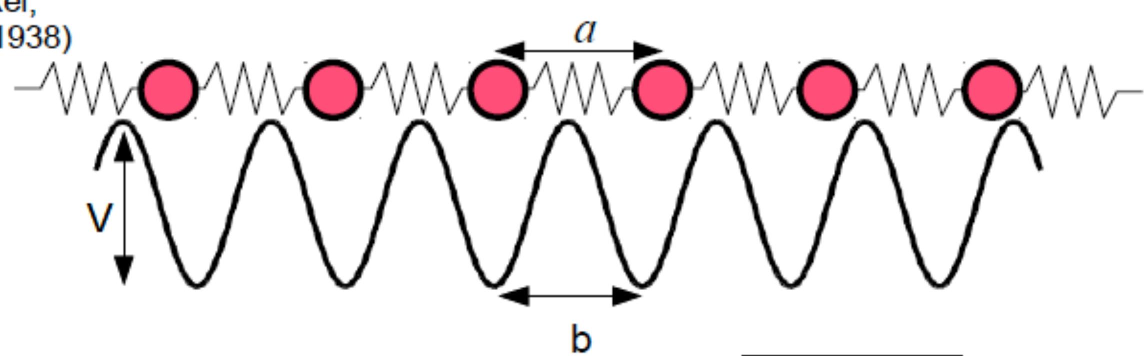


$$H = \sum_n \left[\frac{1}{2}(x_{n+1} - x_n - a)^2 - \frac{V}{(2\pi)^2} \cos\left(\frac{2\pi}{b} x_n \right) \right]$$

T. Kontorova & J. Frenkel,
Zh. Eksp. Teor. Fiz. 8 (1938)

The Frenkel-Kontorova model

T. Kontorova & J. Frenkel,
Zh. Eksp. Teor. Fiz. **8** (1938)



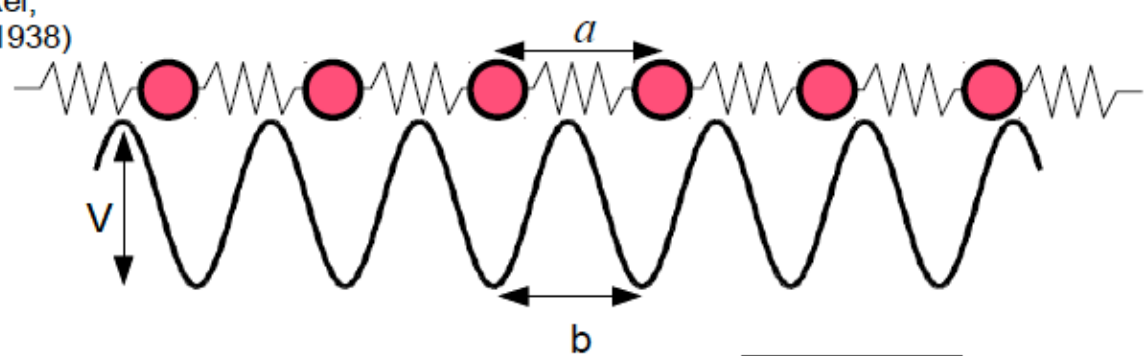
$$H = \sum_n \left[\frac{1}{2}(x_{n+1} - x_n - a)^2 - \frac{V}{(2\pi)^2} \cos\left(\frac{2\pi}{b} x_n \right) \right]$$

Impose a mismatch

$$\boxed{\frac{a}{b} = \frac{\sqrt{5}+1}{2}}$$

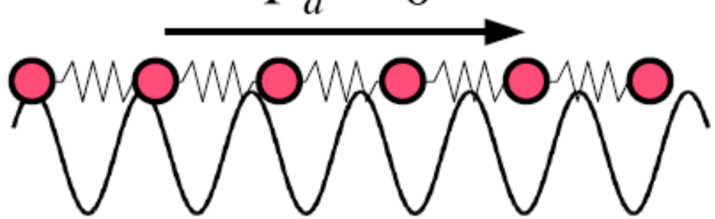
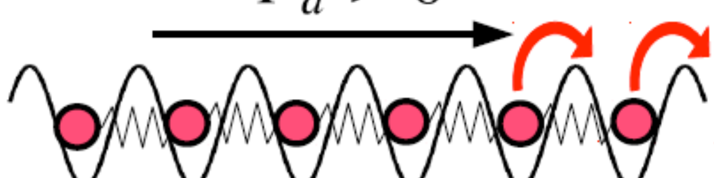
The Frenkel-Kontorova model

T. Kontorova & J. Frenkel,
Zh. Eksp. Teor. Fiz. **8** (1938)



$$H = \sum_n \left[\frac{1}{2}(x_{n+1} - x_n - a)^2 - \frac{V}{(2\pi)^2} \cos\left(\frac{2\pi}{b} x_n \right) \right]$$

Impose a mismatch $\rightarrow$ $\boxed{\dfrac{a}{b} = \dfrac{\sqrt{5}+1}{2}}$

$V < V_C$ Sliding phase	$F_d = 0$ 	**Frictionless**
$V > V_C$ Pinned phase	$F_d > 0$ 	**Emergence of static friction**

Frenkel-Kontorova features

Depinning Force

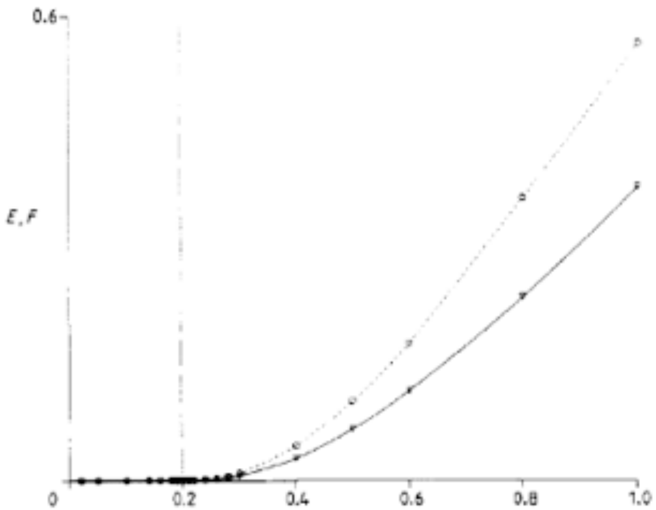


Figure 5. Variation of the Peierls–Nabarro barrier E_{PN} (broken curve) and of the depinning force F_c (full curve) as a function of λ.

Phonon Gap

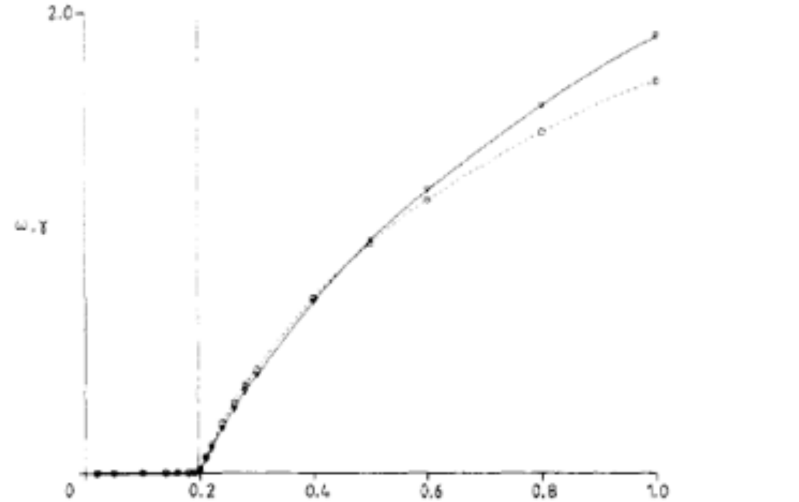


Figure 4. Variation of the gap in the phonon spectrum ω_0 (broken curve) and Lyapun exponent γ of the ground state (full curve) as a function of λ.

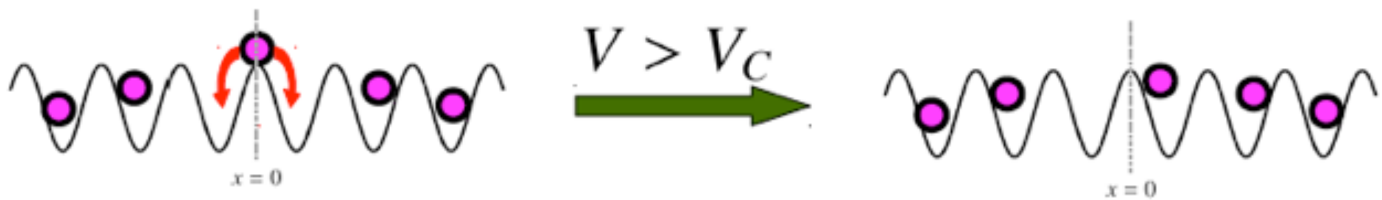
In the sliding phase there is a zero phonon mode

The sliding-to-pinned transition is known as the **Aubry Transition**

M. Peyrard & S Aubry
J. Phys. C: Solid State Phys. **16**, 1593 (1983)

Frenkel-Kontorova with ions

NO Aubry transition – Translational invariance is broken
replaced by **symmetry breaking structural transition**
Mimics at finite size the onset of friction expected at ideal Aubry transition



Depinning force will be finite in the sliding phase – Instead use **restoring force**

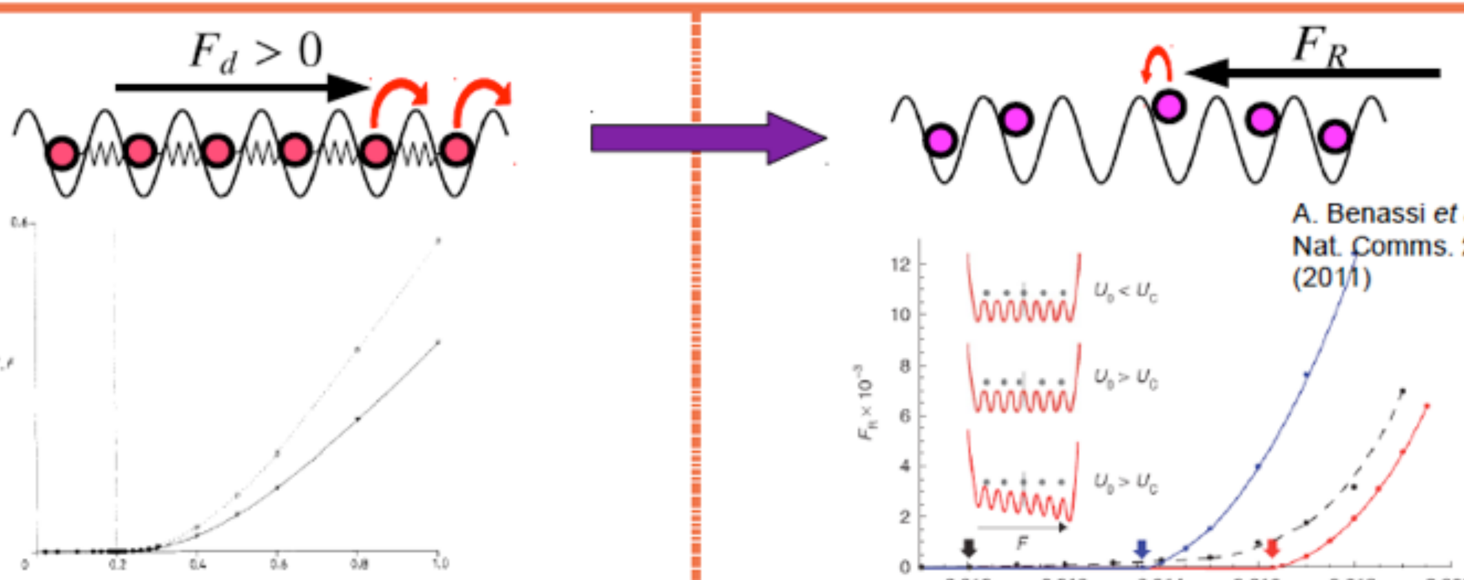
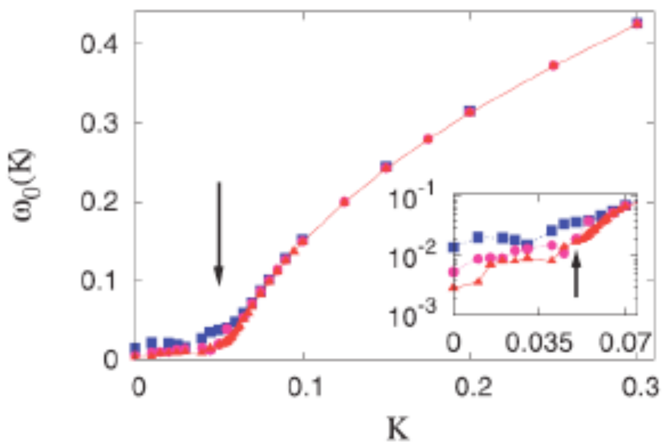


Figure 5. Variation of the Peierls–Nabarro barrier E_{PN} (broken curve) and of the depinning force F_c (full curve) as a function of λ.

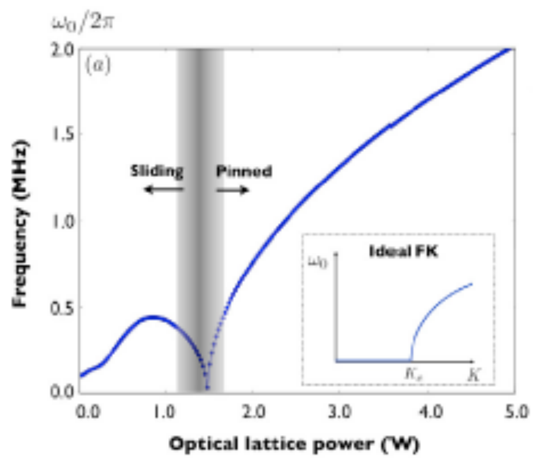
A. Benassi *et al.* Nat. Comms. **2**, 236 (2011)

Frenkel-Kontorova with ions

NO zero phonon mode – **Sliding phase:** phonon gap may **increase or decrease**
Pinned phase: phonon gap will only **increase monotonically**



I. García-Mata *et al.*
Eur. Phys. J. D **41**, 325–330 (2007)



T. Pruttivarasin *et al.*
New J. Phys. **13,** 075012 (2011)

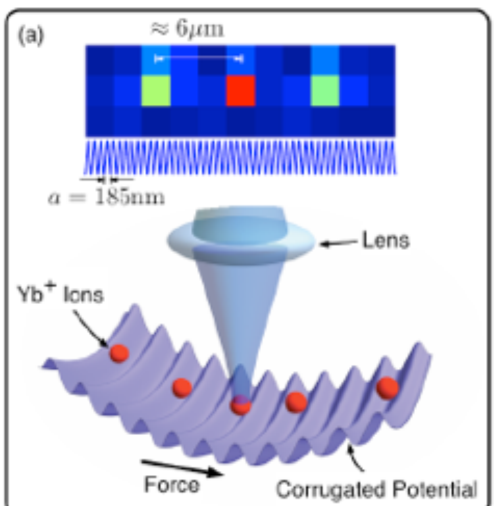
Experimental realizations

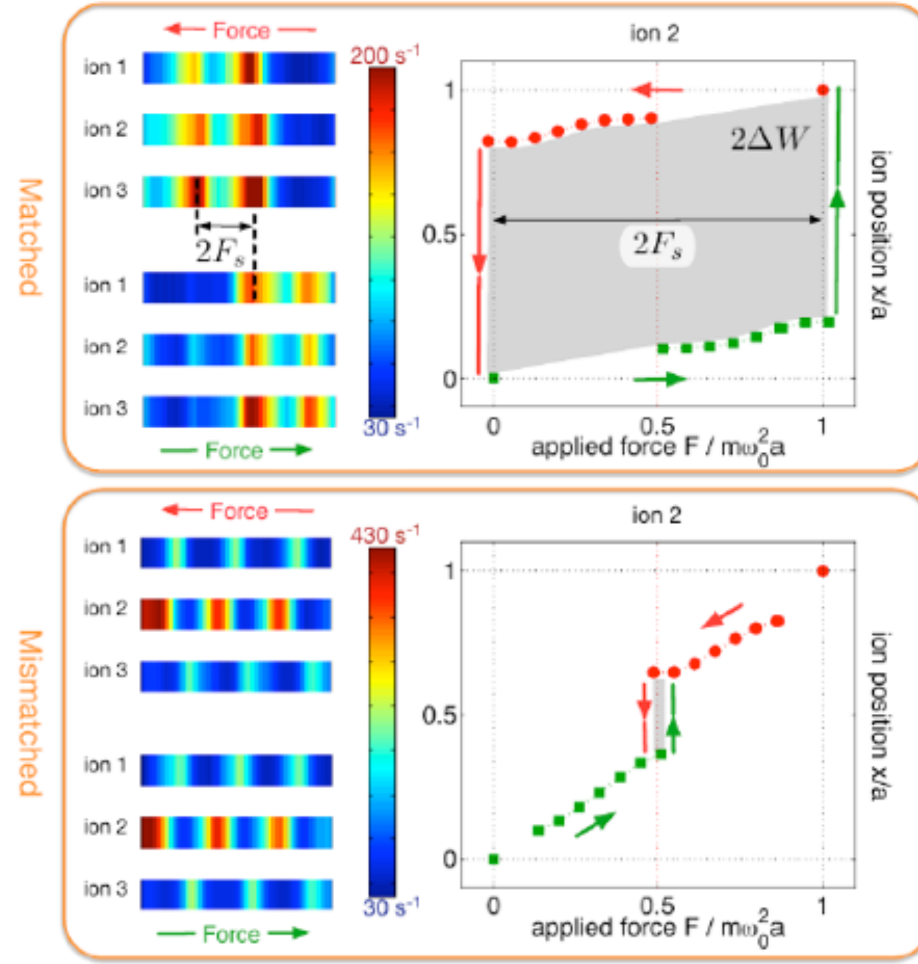
Blatt – Innsbruck
Drewsen – Aarhus
Schaetz – Freiburg
Haeffner – Berkley

Vuletic - MIT

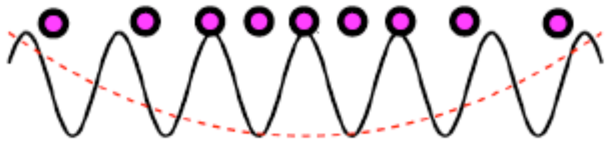
Science **348**, 1115
Tuning friction atom-by-atom in an ion-crystal simulator

Nature Physics **11**, 915
Velocity tuning of friction with two trapped atoms

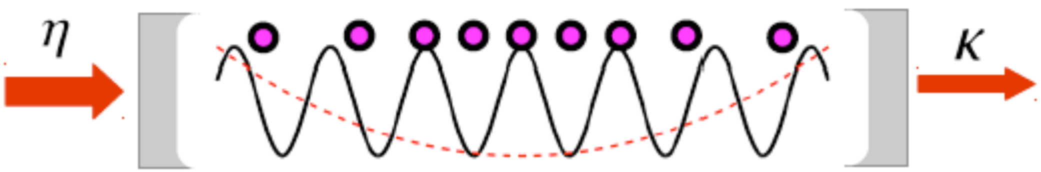




Ion crystal in a cavity

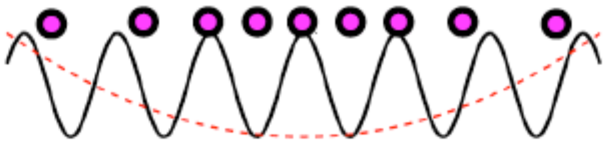


$$H_{lattice} = V \sum_j \cos^2(kx_j)$$

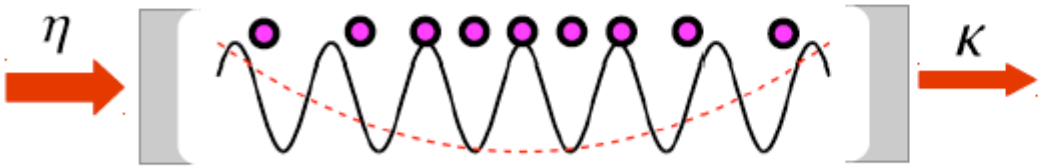


$$H_{lattice} = U_0 \hat{n} \sum_j \cos^2(kx_j)$$

Ion crystal in a cavity



$$H_{lattice} = V \sum_j \cos^2(kx_j)$$



$$H_{lattice} = U \hat{n} \sum_j \cos^2(kx_j)$$

Number of photons dynamically fluctuating variable!!

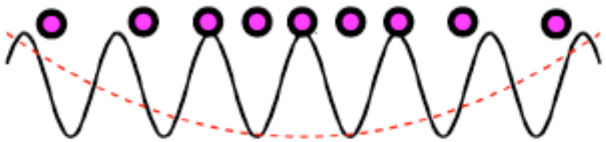
At steady state:

$$\bar{n} = \langle \hat{n} \rangle = \frac{|\eta|^2}{\kappa^2 + (\Delta_{eff}\{\bar{x}_j\})^2}$$

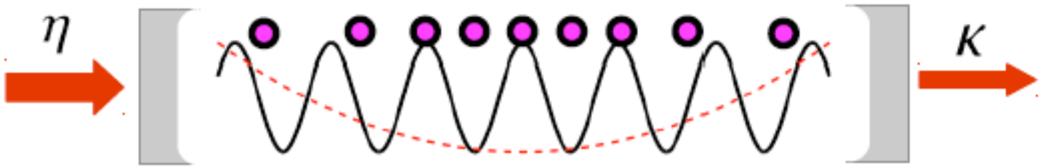
$$\Delta_{eff}\{\bar{x}_j\} = \Delta_c - \kappa C \frac{\sum_j \cos^2(k\bar{x}_j)}{N}$$

Number of photons depends nonlinearly on the ions positions

Ion crystal in a cavity



$$H_{lattice} = V \sum_j \cos^2(kx_j)$$



$$H_{lattice} = U_0 \hat{n} \sum_j \cos^2(kx_j)$$

Number of photons dynamically fluctuating variable!!

At steady state:

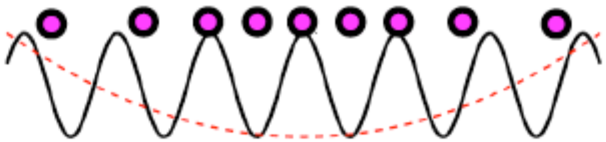
$$\bar{n} = < \hat{n} > = \frac{|\eta|^2}{\kappa^2 + (\Delta_{eff}\{\bar{x}_j\})^2}$$

$$\Delta_{eff}\{\bar{x}_j\} = \Delta_c - \kappa C \boxed{\frac{\sum_j \cos^2(k\bar{x}_j)}{N}}$$

Number of photons depends nonlinearly on the ions positions

$B_N(\{x_j\})$ **Bunching Parameter**

Ion crystal in a cavity



$$H_{lattice} = V \sum_j \cos^2(kx_j)$$

$$H_{lattice} = U_0 \hat{n} \sum_j \cos^2(kx_j)$$

$$\Delta_{eff}(\{x_j\}) = \Delta_c - \kappa C B_N(\{x_j\})$$

Effective cavity potential

$$V_{cav} = -\frac{\hbar|\eta|^2}{\kappa} \arctan\left(\frac{\Delta_{eff}(\{x_j\})}{\kappa}\right)$$

Substrate depth dependent on:

Pump strength	η
Cooperativity	$C = NU_0/\kappa$

Nonlinearity depends on Cooperativity

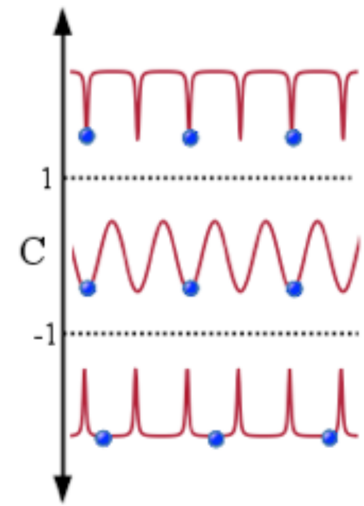
A deformable substrate

$$V_{cav} = -\frac{\hbar|\eta|^2}{\kappa}\arctan\left(\frac{\Delta_{eff}(\{x_j\})}{\kappa}\right)$$

$|C| \ll 1$ the cavity potential is

$$V_{cav} \propto |\eta|^2 C \sum_j \cos^2(kx_j)$$

$|C| \geq 1$ then the potential gives rise to an effective **long-range force** between the ions



Sign of C changes the effect of the long range interaction!

Cavity field acts as a **deformable potential** due to scattering of photons

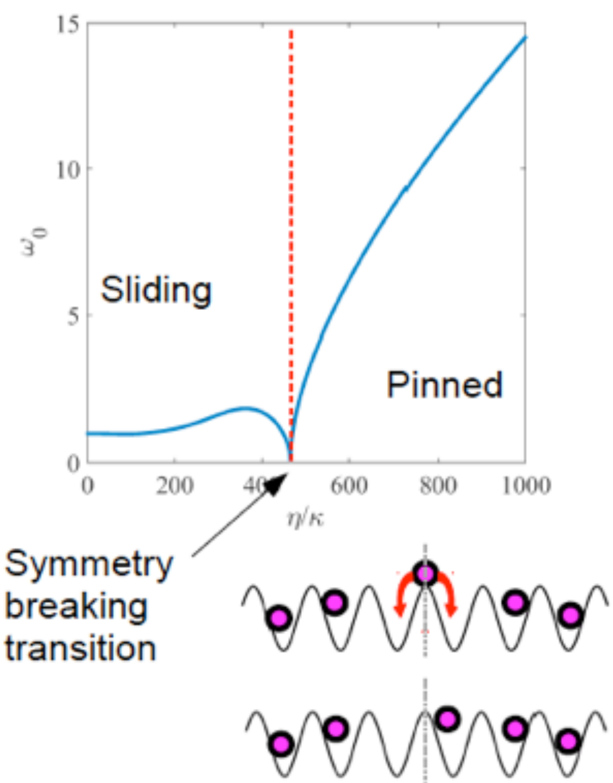
There is a **multi-body long range interaction** mediated by the cavity!

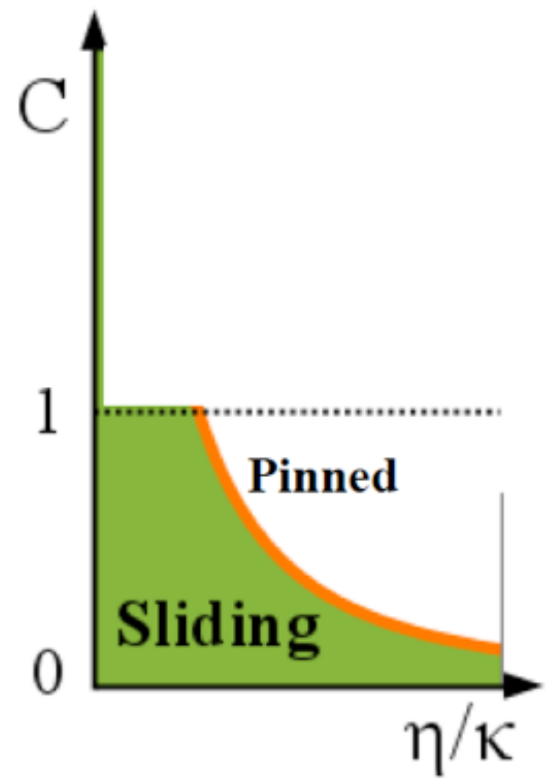
Phase diagrams: preliminary

$$|C| \ll 1$$

Vanishing phonon gap

Sliding to pinned transition!





Phase diagrams: preliminary

Cannot use winding number to discern phases – instead use **bunching parameter**

$$w = \lim_{n\to\infty} \frac{x_n - x_0}{n}$$

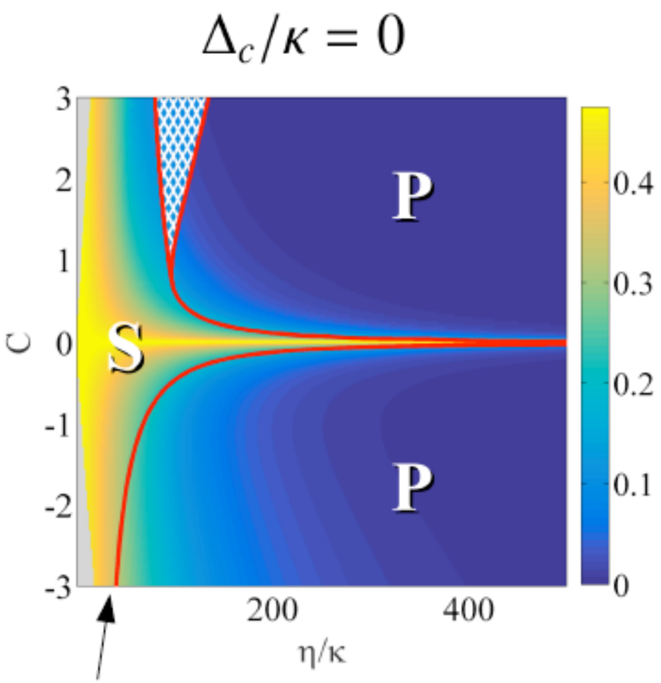
$$\boxed{B_N = \frac{\sum_j \cos^2(kx_j)}{N}}$$

Ions commensurate with cavity field $\begin{cases} B_N = 1 & \text{Intensity maxima} \\ B_N = 0 & \text{Intensity minima} \end{cases}$

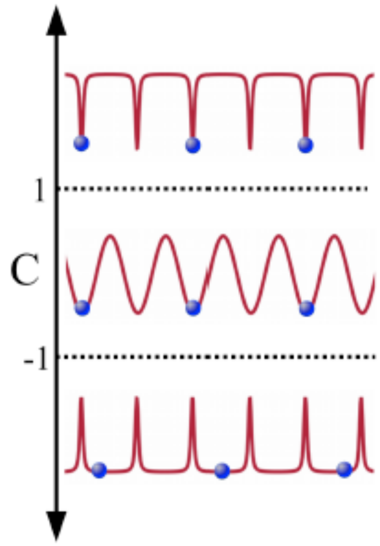
Ions incommensurate $0 < B_N < 1$

Phase diagrams

$$B_N = \frac{\sum_j \cos^2(kx_j)}{N}$$

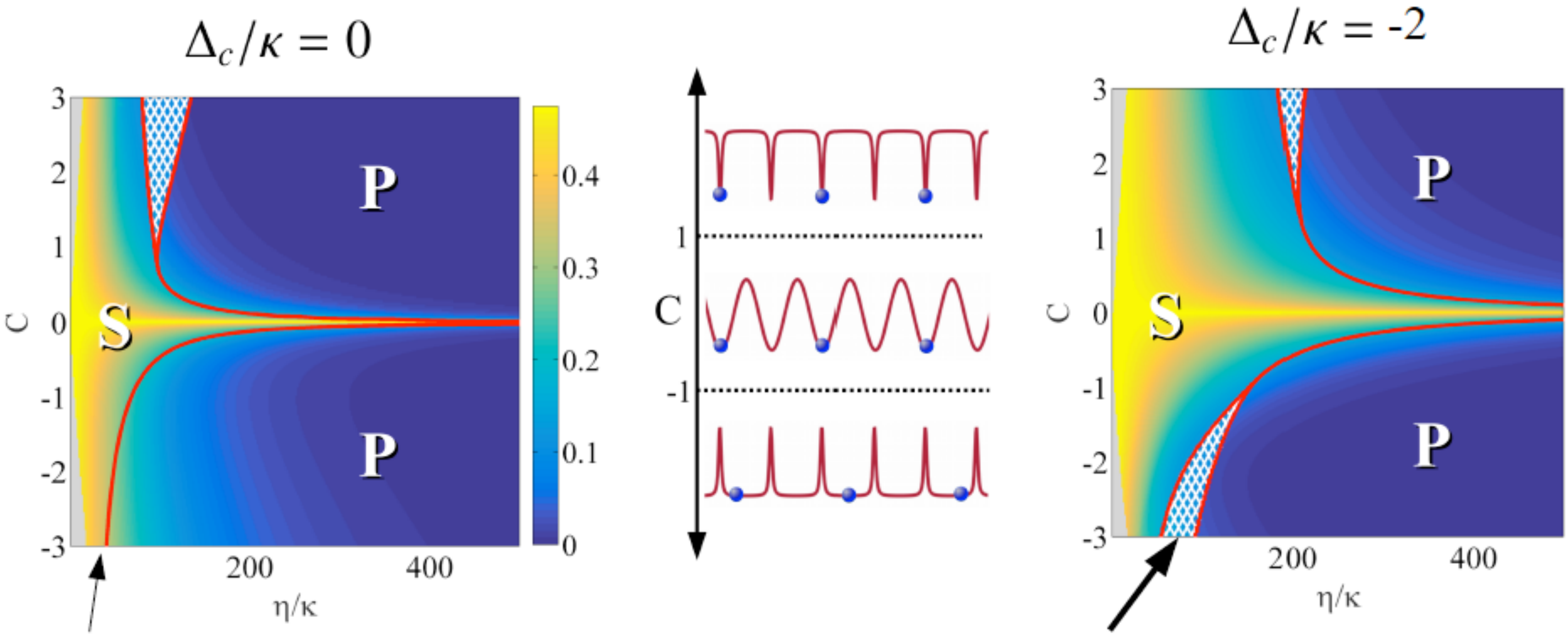


Symmetry breaking transition



Phase diagrams

$$B_N = \frac{\sum_j \cos^2(kx_j)}{N}$$



Symmetry breaking transition

Optomechanical resonance

$$\Delta_{eff}(\{x_j\}) = \Delta_c - \kappa C B_N(\{x_j\})$$

Beyond mean-field: Crystal vibrations

$$\partial_t \varrho = \frac{1}{\mathrm{i}\hbar}[\delta H, \rho] + \mathcal{L}[\varrho]$$

Dynamics of crystal vibrations and field fluctuations

$$\mathcal{L}[\varrho] = \kappa(2\delta a\,\varrho\,\delta a^\dagger - \delta a^\dagger \delta a\,\varrho - \varrho\,\delta a^\dagger \delta a)$$

cavity losses

Beyond mean-field: Crystal vibrations

$$\partial_t \varrho = \frac{1}{\mathrm{i}\hbar}[\delta H, \rho] + \mathcal{L}[\varrho]$$

$$\mathcal{L}[\varrho] = \kappa(2\delta a\, \varrho\, \delta a^\dagger - \delta a^\dagger \delta a\, \varrho - \varrho\, \delta a^\dagger \delta a)$$

optomechanical coupling between crystal vibrations and field fluctuations

$$\delta H_0 = \sum_\alpha \hbar\omega_\alpha b_\alpha^\dagger b_\alpha - \hbar\Delta_{\mathrm{eff}}\delta a^\dagger \delta a\,,$$

$$\delta H_{\mathrm{opto}} = -\hbar \sum_\alpha (\chi_\alpha^* \delta a + \chi_\alpha \delta a^\dagger)(b_\alpha + b_\alpha^\dagger)$$

coupling strength

$$\chi_\alpha = \sqrt{\frac{\omega_R}{\omega_\alpha}}\bar{a} U_0 \sum_j \sin(2k\bar{x}_j) S_j^\alpha$$

Beyond mean-field: Crystal vibrations

$$\partial_t \varrho = \frac{1}{\mathrm{i}\hbar}[\delta H, \rho] + \mathcal{L}\left[\varrho\right]$$

$$\mathcal{L}\left[\varrho\right] = \kappa(2\delta a\, \varrho\, \delta a^\dagger - \delta a^\dagger \delta a\, \varrho - \varrho\, \delta a^\dagger \delta a)$$

optomechanical coupling between crystal vibrations and field fluctuations

$$\delta H_0 = \sum_\alpha \hbar\omega_\alpha b_\alpha^\dagger b_\alpha - \hbar\Delta_{\mathrm{eff}} \delta a^\dagger \delta a\,,$$

$$\delta H_{\mathrm{opto}} = -\hbar \sum_\alpha (\chi_\alpha^* \delta a + \chi_\alpha \delta a^\dagger)(b_\alpha + b_\alpha^\dagger)$$

Fluctuations & Stability

Cavity $$\delta\dot{a} = (i\Delta_{eff} - \kappa)\delta a - i\bar{a}\sum_n c_n(b_n + b_n^\dagger) + \sqrt{2\kappa}a_{in}$$

Phonons $$\dot{b}_n = -(i\omega_n + \Gamma_n)b_n - i\bar{a}c_n(\delta a + \delta a^\dagger) + \sqrt{2\Gamma_n}b_{in,n}$$

Coupling to a noise source

Coupling between cavity and motional modes

Input noise terms

$$\frac{d\vec{X}}{dt} = M\vec{X} + \vec{X}_{in}(t)$$

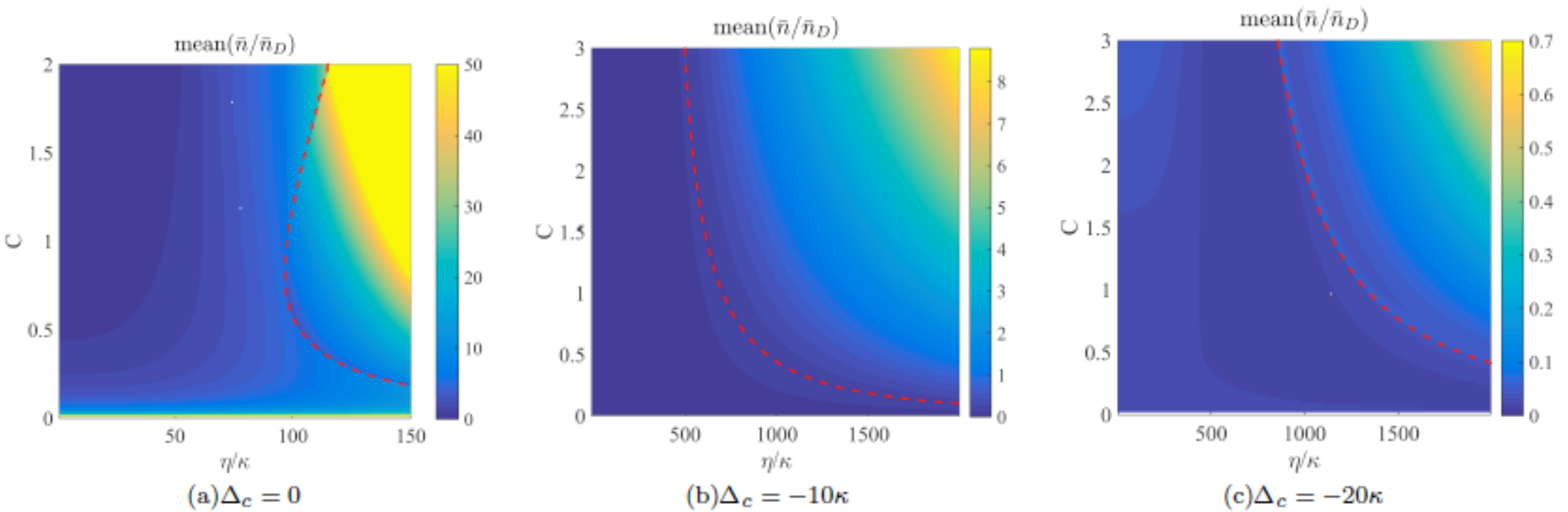
Time evolution of the eigenvectors of M depend on the exponential of the associated eigenvalues λ

$\mathrm{Re}(\lambda) > 0 \longrightarrow$ System is unstable

$\mathrm{Re}(\lambda) < 0 \longrightarrow$ System is stable

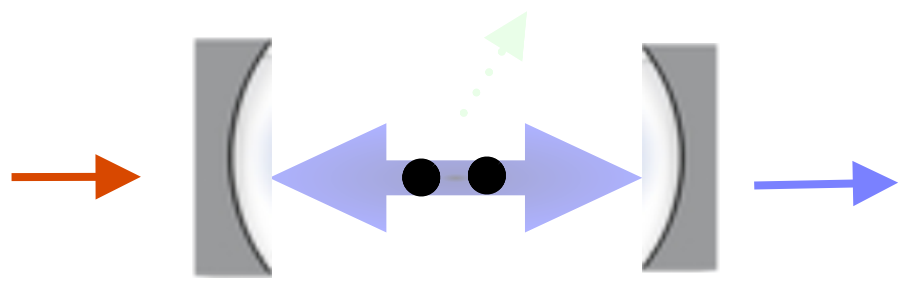
Crystal's steady state

mean occupation number / mode



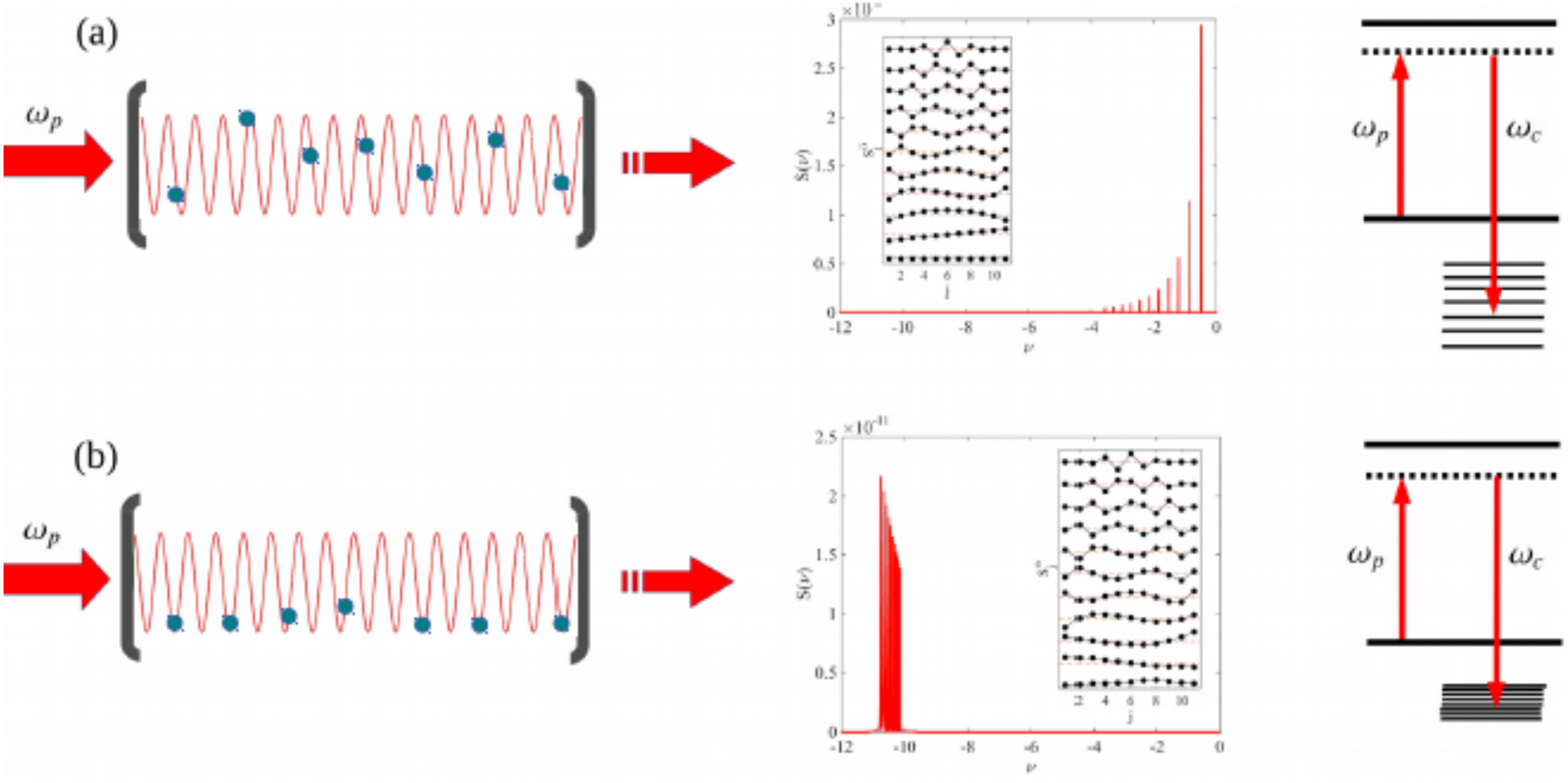
(C<0 is an unstable region)

The cavity can cool the crystal vibrations



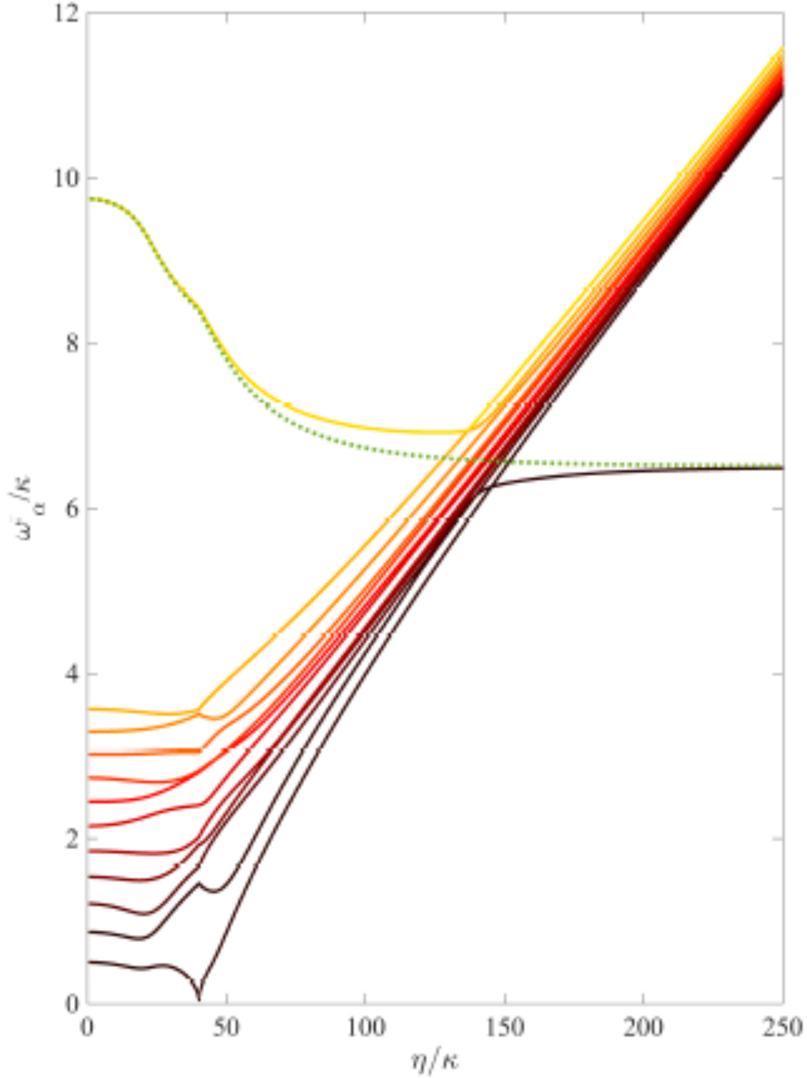
ω < ω: (cavity) cooling of the vibrations

Sideband cooling to the zero-point motion



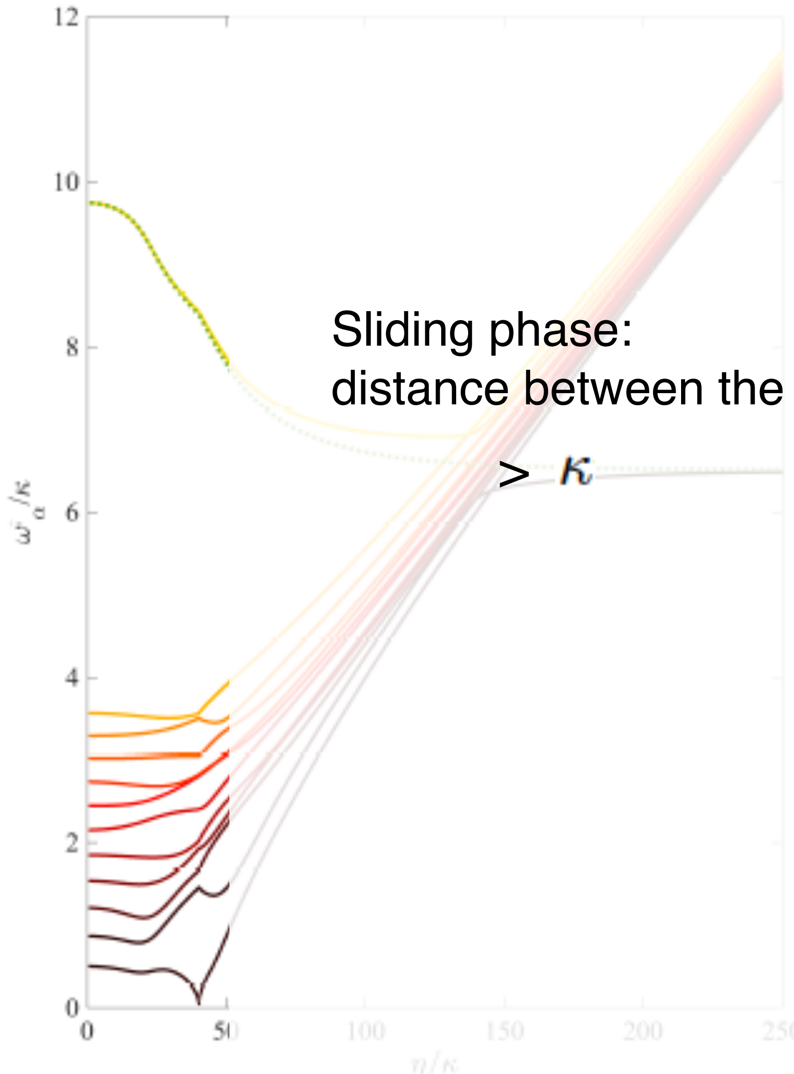
Spectrum

mode
spectrum (11 ions)



Resonances

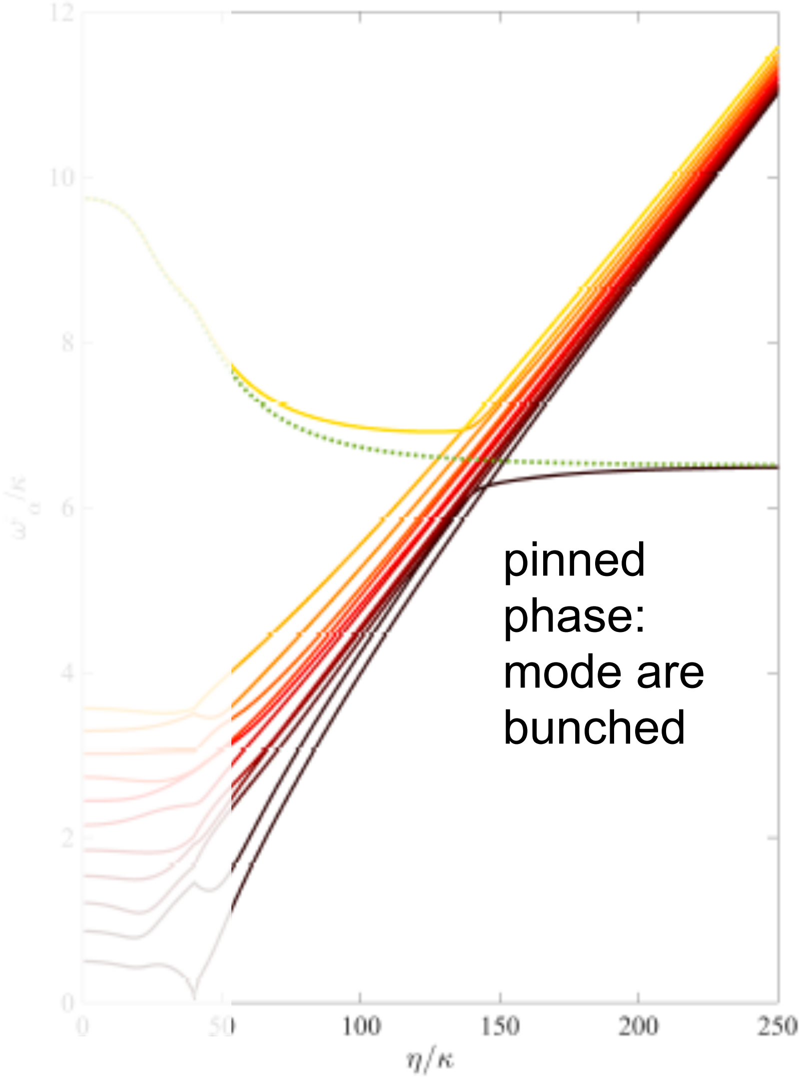
mode
spectrum (11 ions)



Sliding phase:
distance between the modes

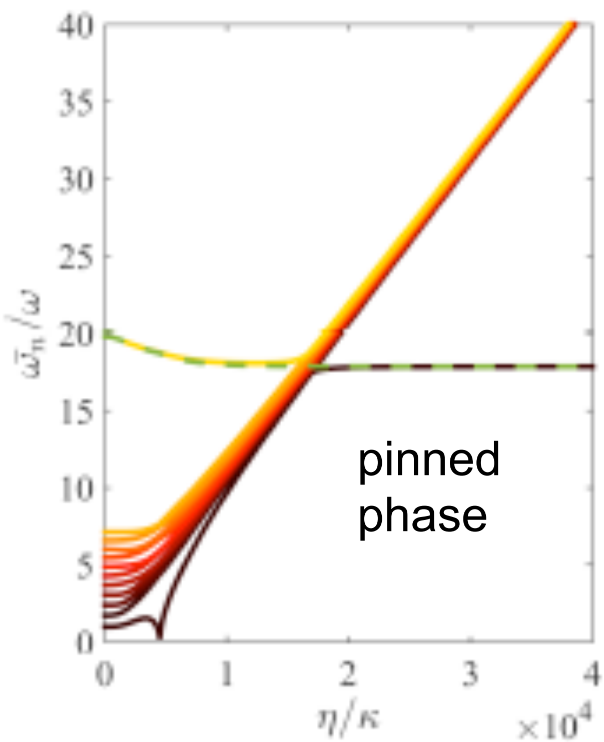
$> \kappa$

Resonances


pinned
phase:
mode are
bunched
0
2
4
6
8
10
12
0
50
100
150
200
250
η/κ

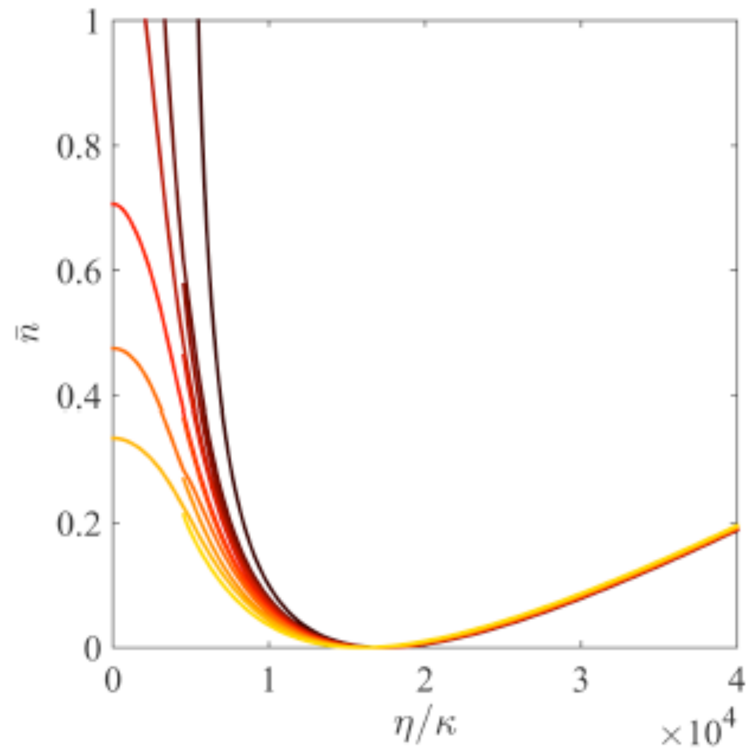
Cooling to the zero-point motion

mode spectrum (11 ions)



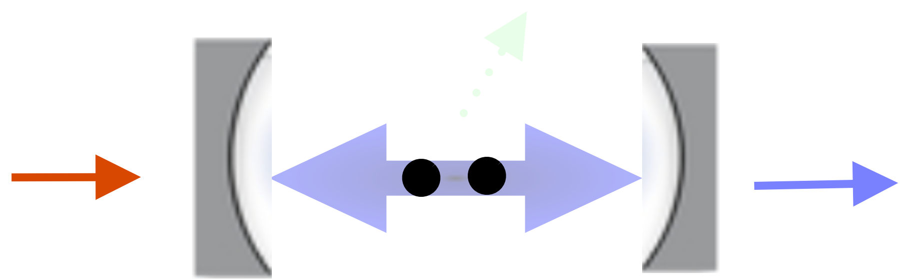
$\Delta_c = -100\kappa$

simultaneous ground-state cooling



cooling times: about 1-10 milliseconds

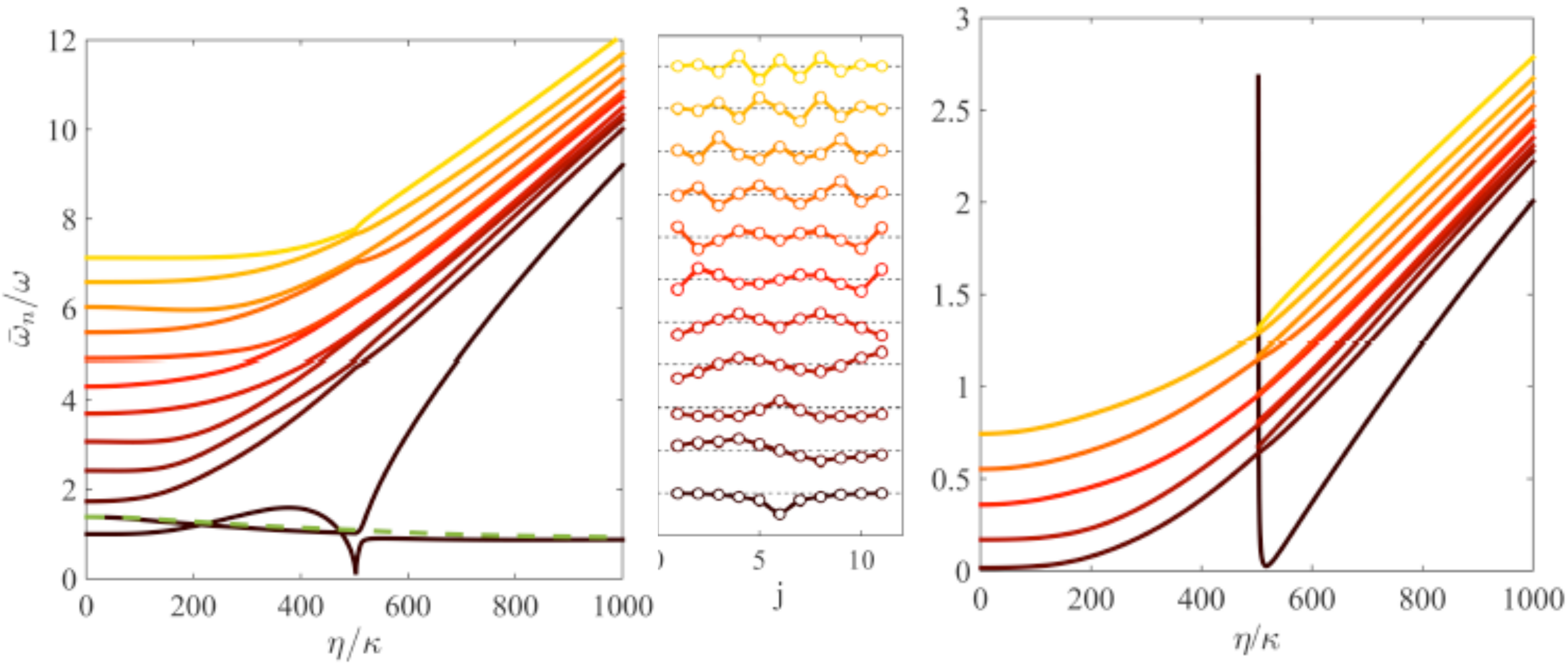
Quantum reservoir engineering of kinks



$\omega < \omega$: (cavity) cooling of the vibrations

Kink manipulation

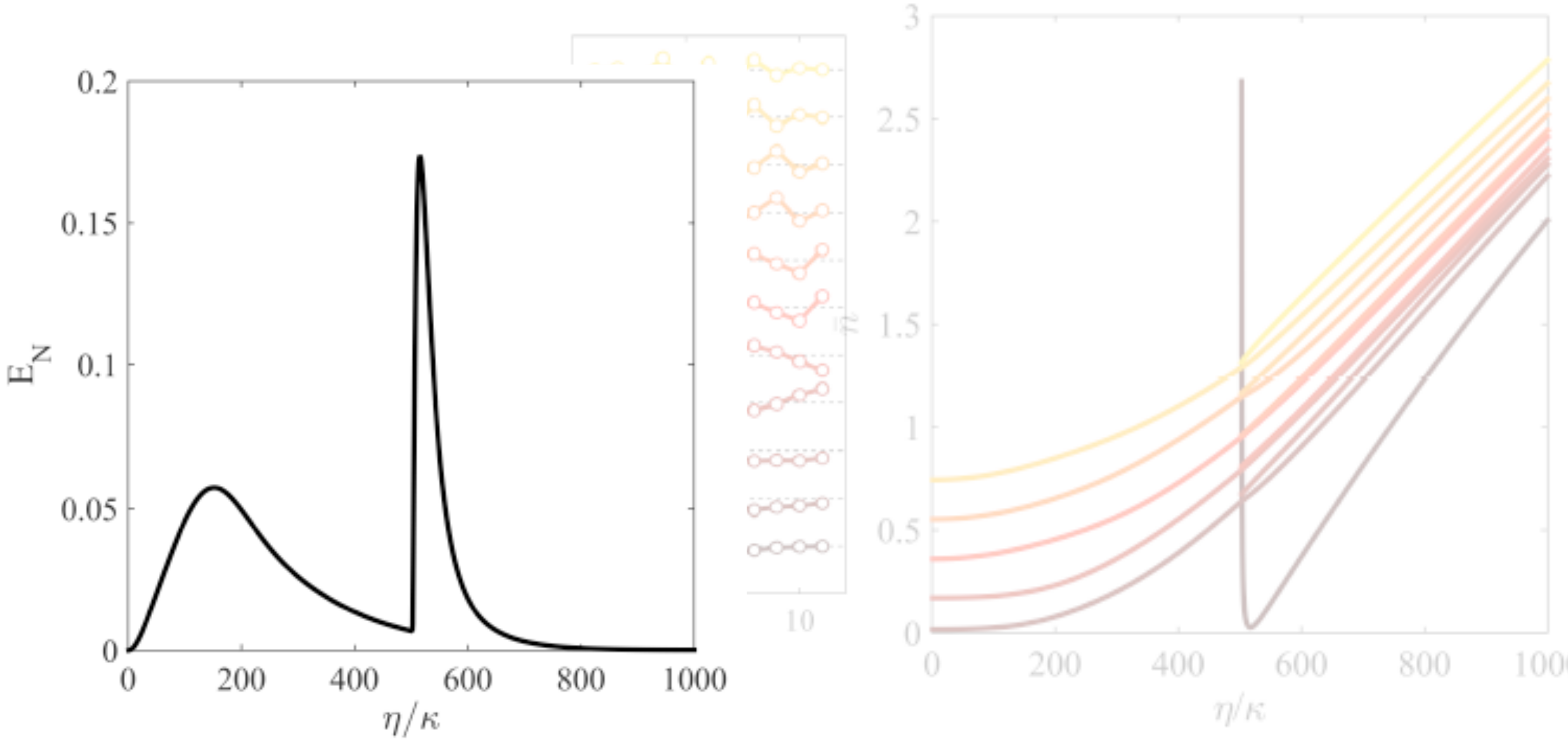
The cavity can cool selectively a localized mode (kink)



$C = 0.5$ and $\Delta_c = -5\kappa$

Kink manipulation

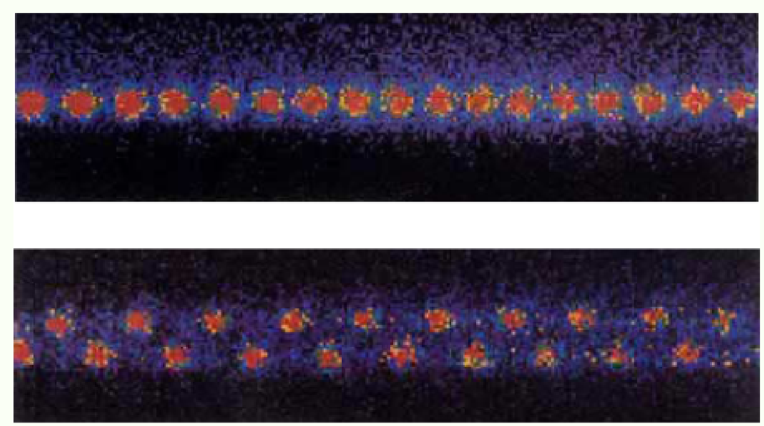
Cavity and kink are entangled
(quantum reservoir gives a stationary nonclassical state)



Visible in the spectra of light at the cavity output

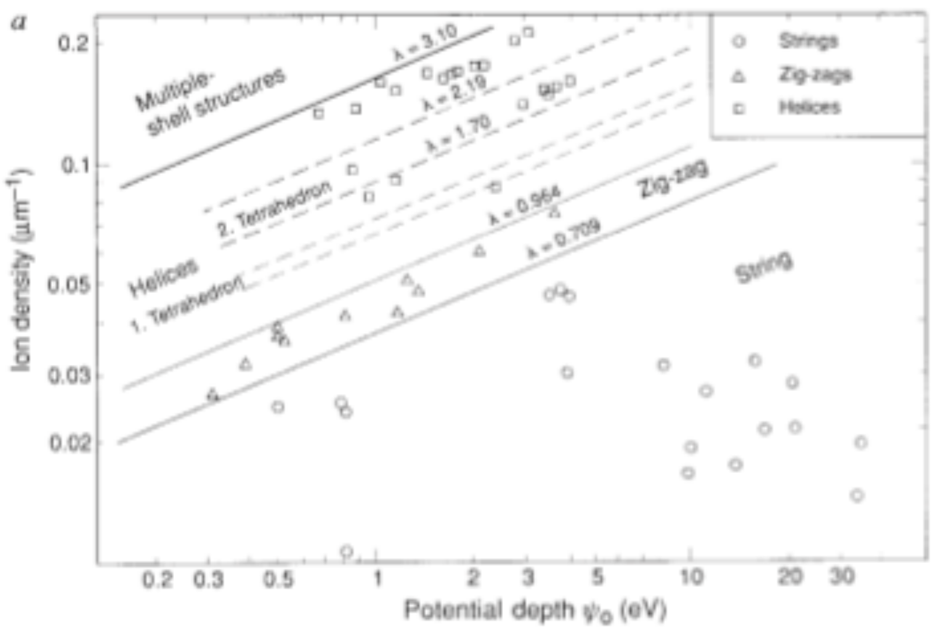
Outlook

- Entangle several kinks at steady state
- Explore the interplay between cavity and Coulomb interaction at the linear-zigzag instability (zero-phonon mode and soft mode)



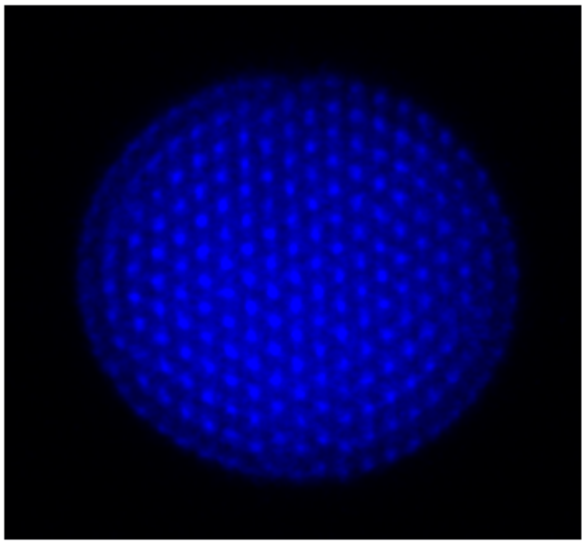
[Birkl et al., Nature 357, 310 (1992)]

Back to the structural diagram…

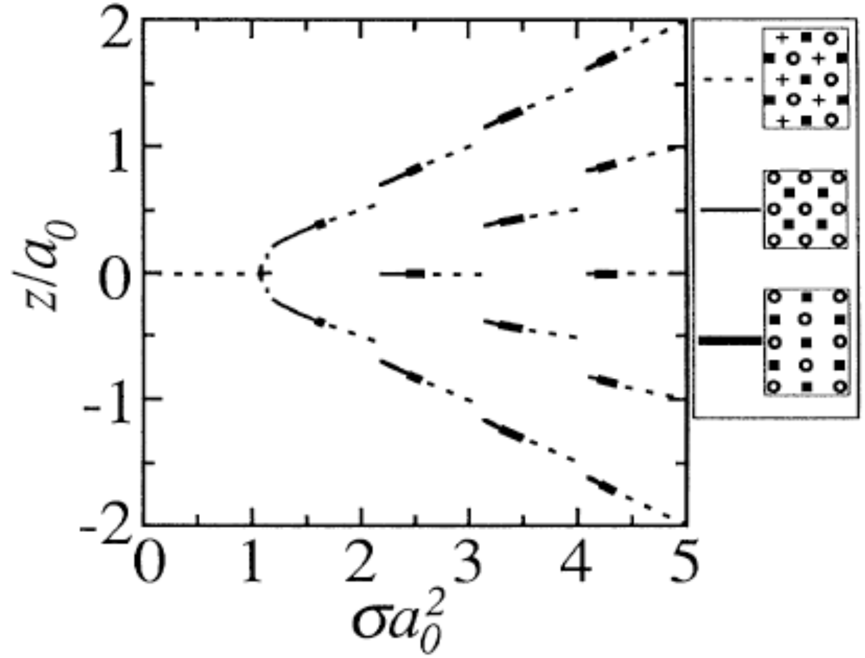


Topological Phase Transitions in Ion Crystals

Planar instability



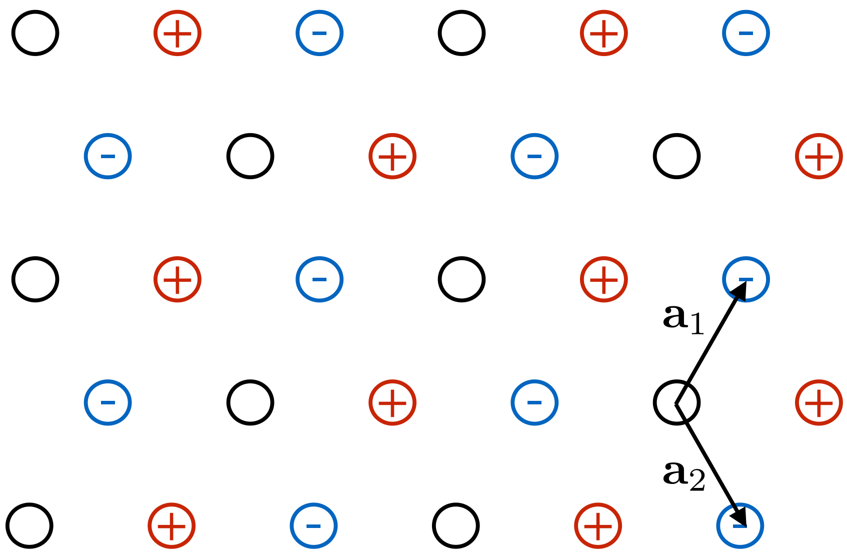
(M. Drewsen & coworkers, Aahrus)

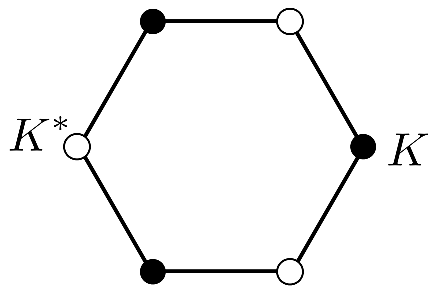


D.H.E. Dubin, PRL 1993

Continuous transition from a single to three planes

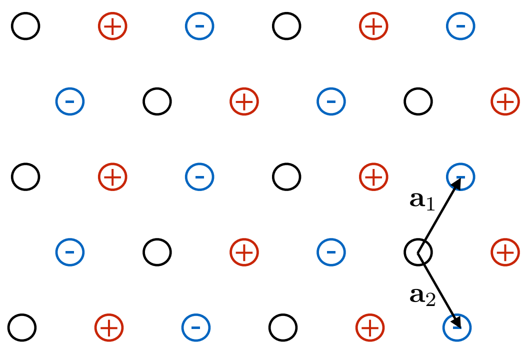
Order parameter

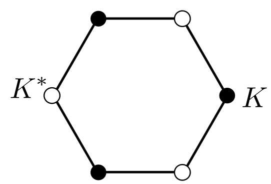




order parameter $\quad z_i = \mathrm{Re}\left[\psi e^{i\mathbf{K}\cdot\mathbf{r}_i}\right]$

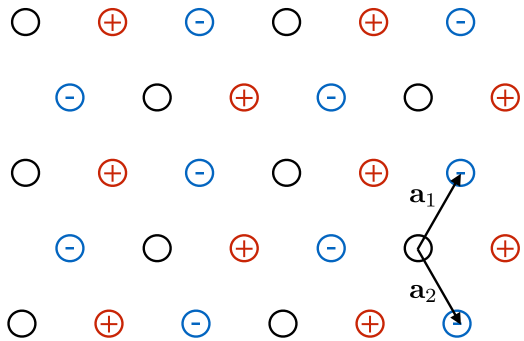
Symmetries and Model

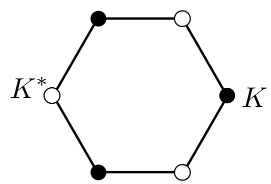




$$R_z : \psi \;\to\; -\psi\,,$$
$$T_{\mathbf{a}_1} : \psi \;\to\; \psi e^{2\pi i/3}\,,$$
$$R_x : \psi \;\to\; \psi^*\,,$$

Symmetries and Model





$$
\begin{aligned}
R_z : \psi &\rightarrow -\psi\,, \\
T_{\mathbf{a}_1} : \psi &\rightarrow \psi e^{2\pi i/3}\,, \\
R_x : \psi &\rightarrow \psi^*\,,
\end{aligned}
$$

Ginzburg-Landau free energy

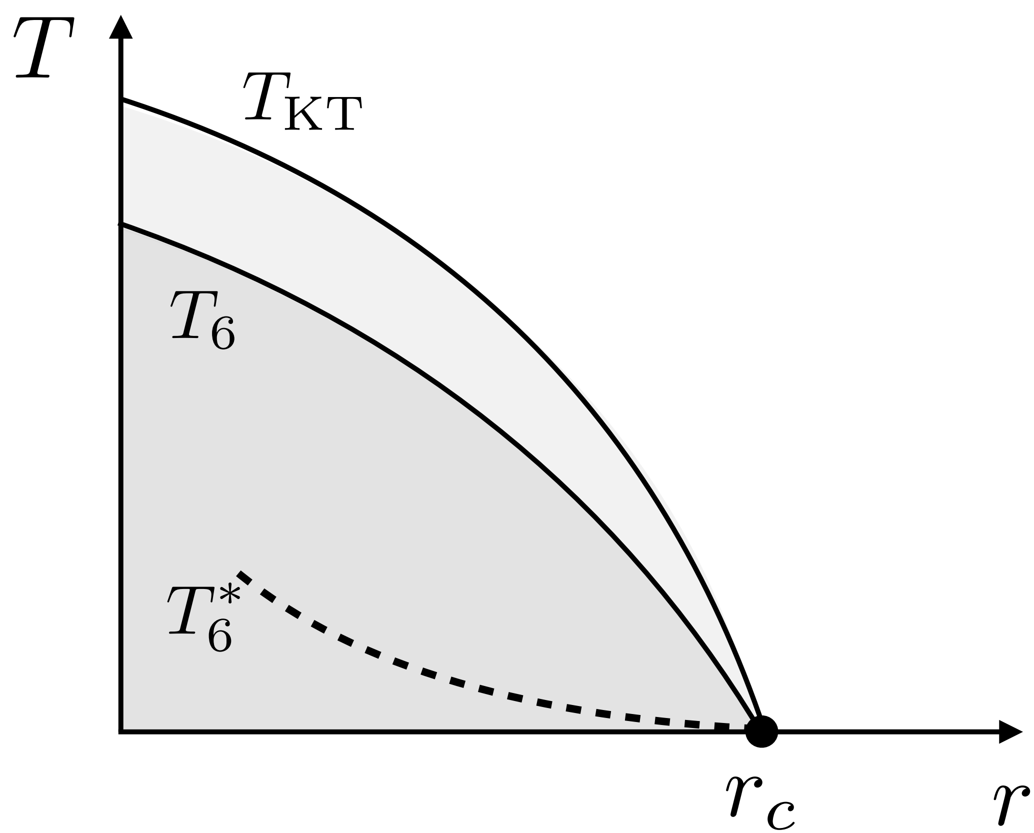
$$\frac{f_{\mathrm{GL}}}{\mathcal{K}} = \frac{\gamma}{2}|\nabla\psi|^2 + r|\psi|^2 + u|\psi|^4 + v|\psi|^6 + \frac{w}{2}\left[\psi^6 + (\psi^*)^6\right]$$

6-state clock model

Phase diagram

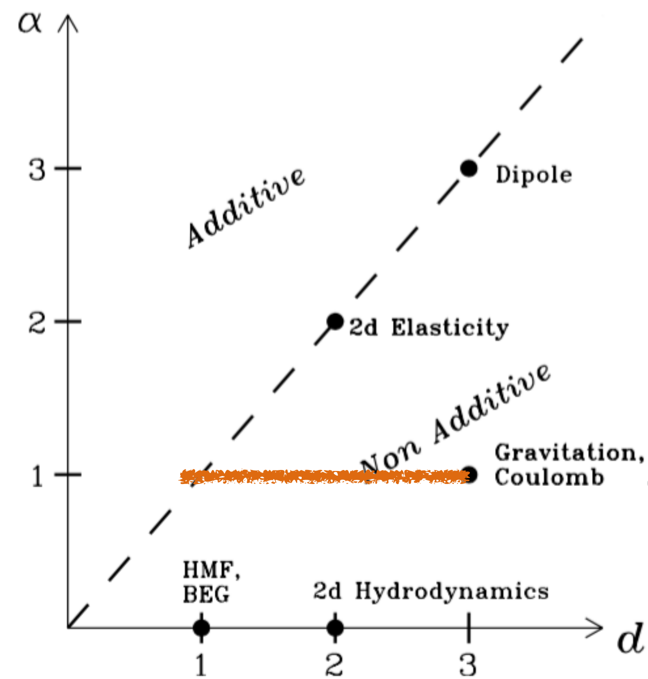
6-state clock model

$$\frac{f_{\mathrm{GL}}}{\mathcal{K}} = \frac{\gamma}{2}|\nabla\psi|^2 + r|\psi|^2 + u|\psi|^4 + v|\psi|^6 + \frac{w}{2}\left[\psi^6 + (\psi^*)^6\right]$$



Long-range interactions

Potential scales with $1/r^{a}$
with exponent a < dimension d



one-component plasmas

Negative Poisson's Ratios for Extreme States of Matter

Ray H. Baughman,[1*] Socrates O. Dantas,[2] Sven Stafström,[3] Anvar A. Zakhidov,[1] Travis B. Mitchell,[4] Daniel H. E. Dubin[5]

Negative Poisson's ratios are predicted for body-centered-cubic phases that likely exist in white dwarf cores and neutron star outer crusts, as well as those found for vacuumlike ion crystals, plasma dust crystals, and colloidal crystals (including certain virus crystals). The existence of this counterintuitive property, which means that a material laterally expands when stretched, is experimentally demonstrated for very low density crystals of trapped ions. At very high densities, the large predicted negative and positive Poisson's ratios might be important for understanding the asteroseismology of neutron stars and white dwarfs and the effect of stellar stresses on nuclear reaction rates. Giant Poisson's ratios are both predicted and observed for highly strained coulombic photonic crystals, suggesting possible applications of large, tunable Poisson's ratios for photonic crystal devices.

Thanks to

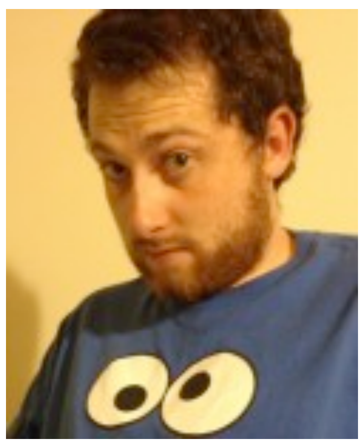
S. Fishman

E. Shimshoni

D. Podolsky

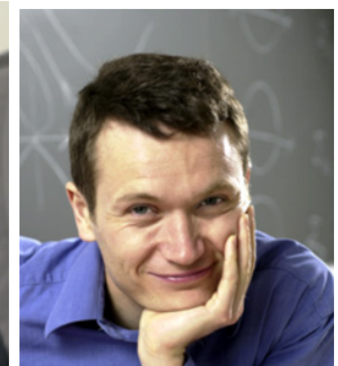
C. Cormick

H.Landa



T. Fogarty

V. Stojanovic



E. Demler

Thanks to....

UNIVERSITÄT
DES
SAARLANDES

Some literature

C. Cormick and G. Morigi,Phys. Rev. Lett. 109, 053003 (2012)

C. Cormick and G. Morigi, Phys. Rev. A 87, 013829 (2013)

T. Fogarty, et al, Phys. Rev. Lett. 115, 233602 (2015).

D. Podolsky, et al, to appear in PRX (2016), preprint arXiv:1511.08814

T. Fogarty, et al, preprint arXiv:1604.07548