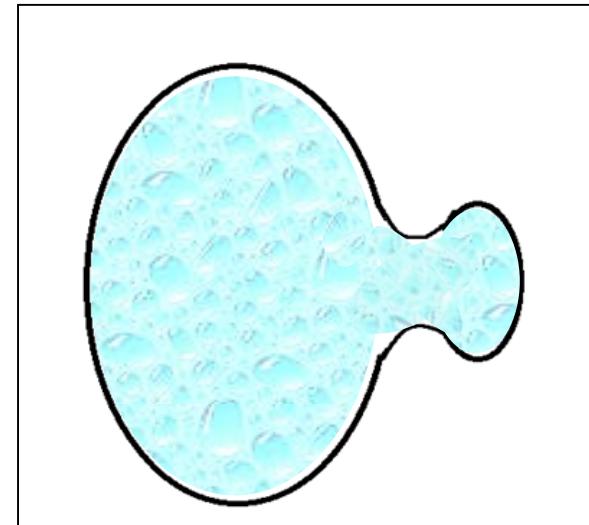
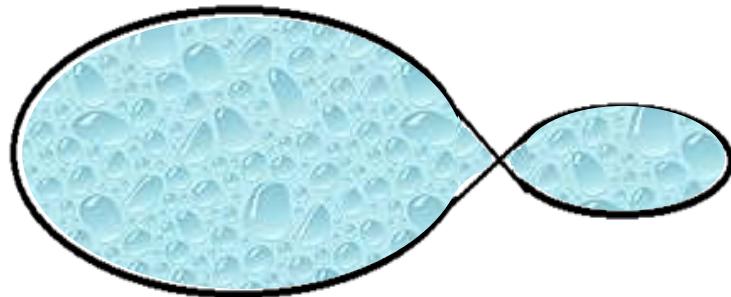


# Structure Function and Collective Effects in Exit Channel Particle Evaporation

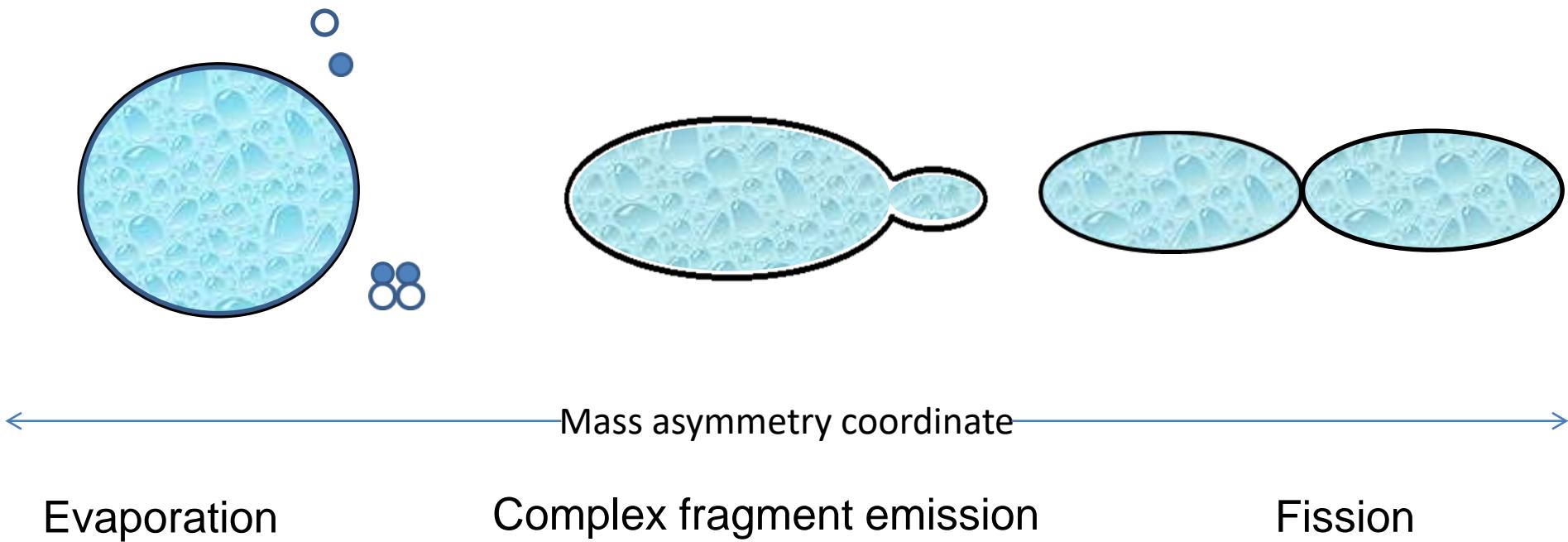
***Luciano G. Moretto***

*University of California Berkeley and LBNL*



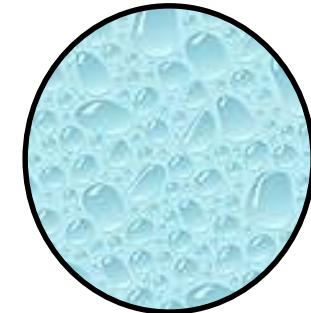
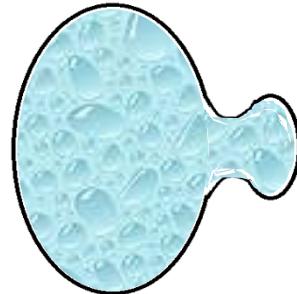
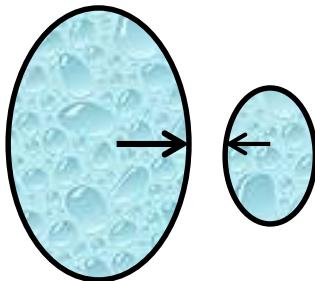
# Statistical Emission from Compound Nucleus

Evaporation and Fission are two extremes of a continuous process

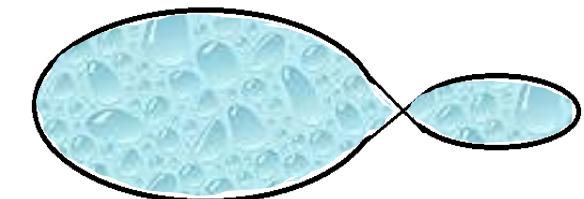
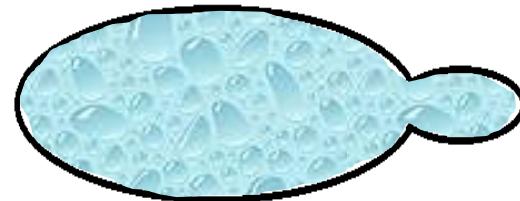


Complex interplay of collective and intrinsic degrees of freedom

# Collective and Intrinsic Degrees of Freedom in entrance channel (sub-barrier) fusion reactions.



Very different from exit channel



**non Hermiticity ?!**

# Conditional Saddle Points and Thermal Yields

$$P(z) \approx \exp - V(z)/T$$

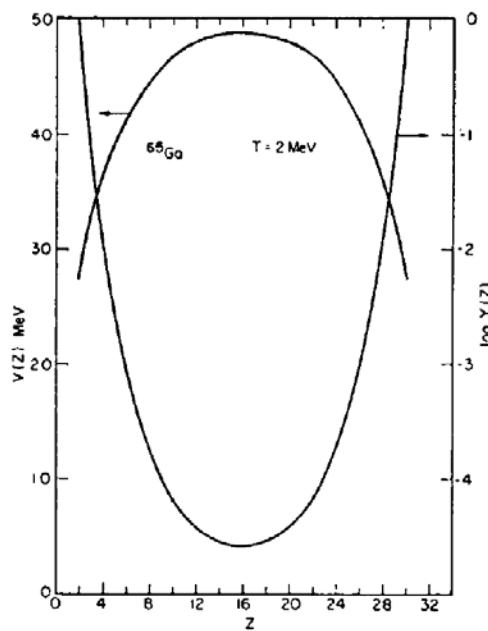
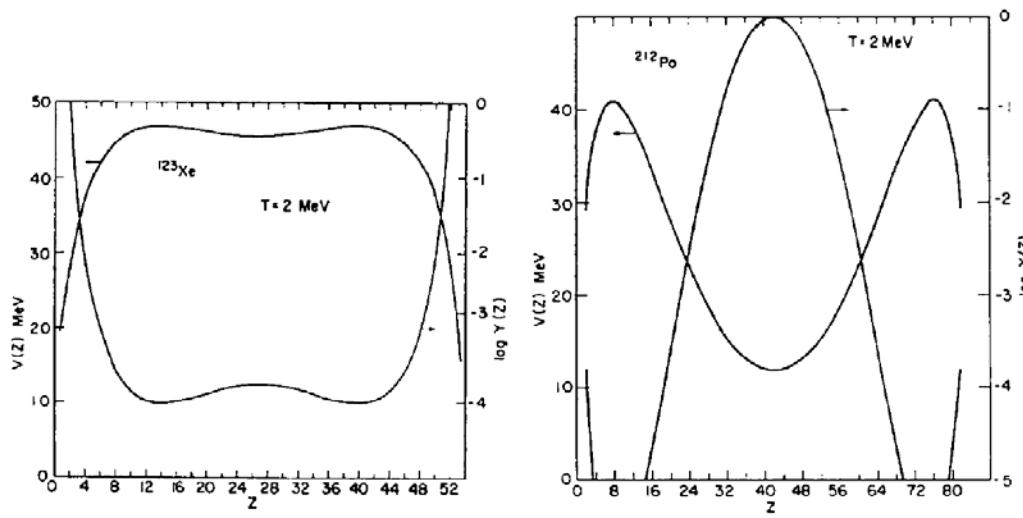
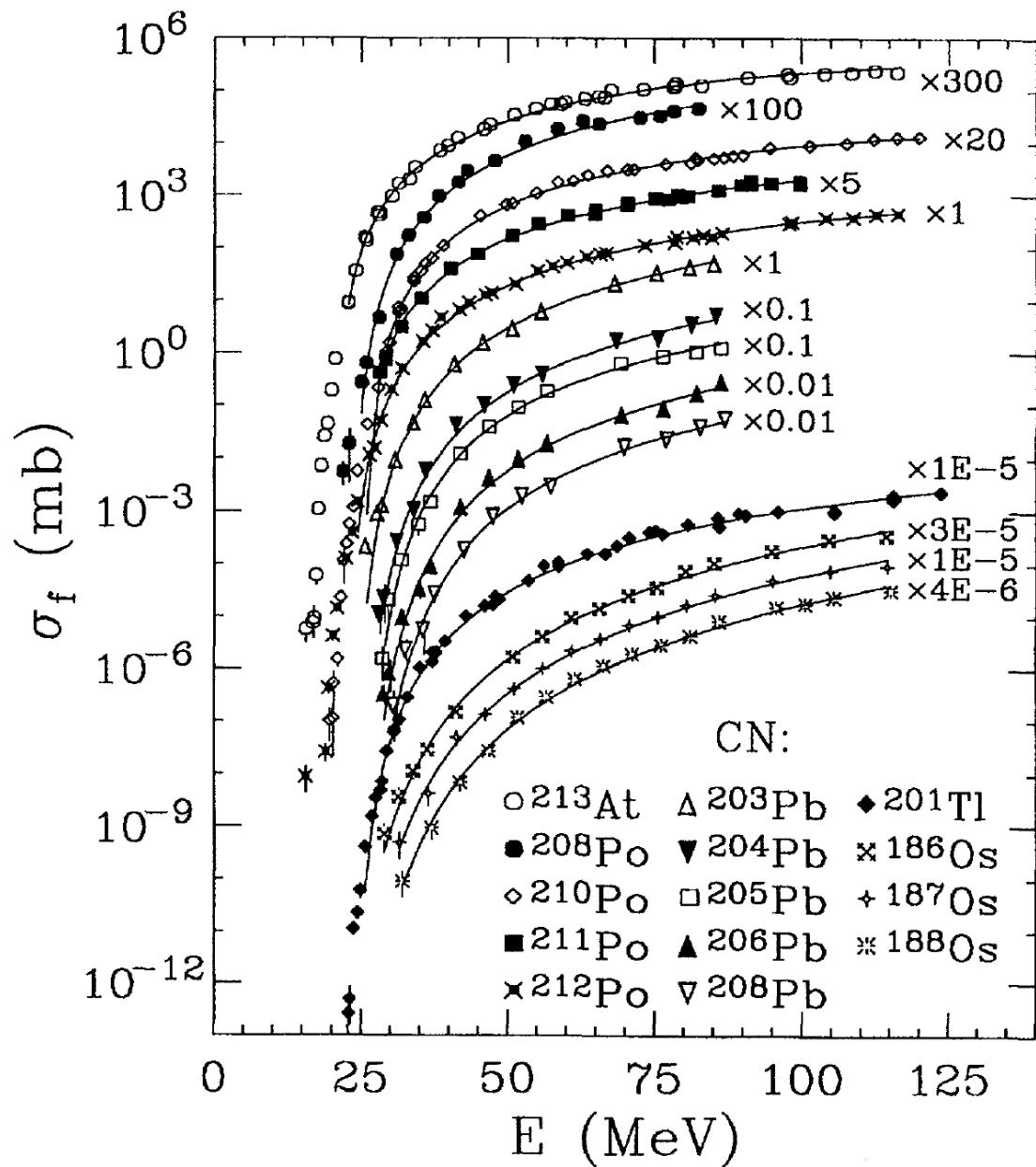


Fig. 1a.





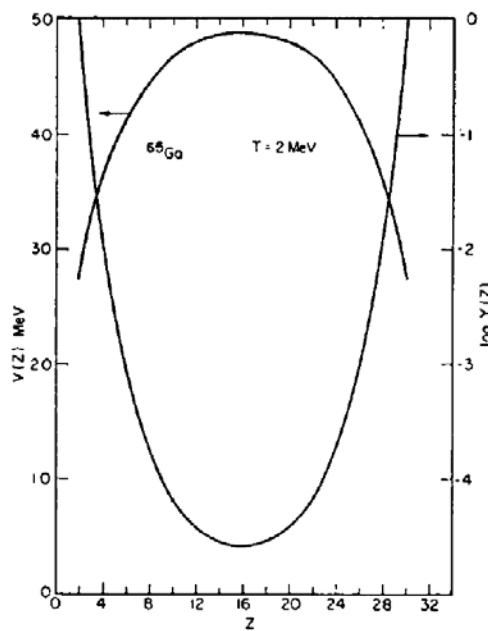
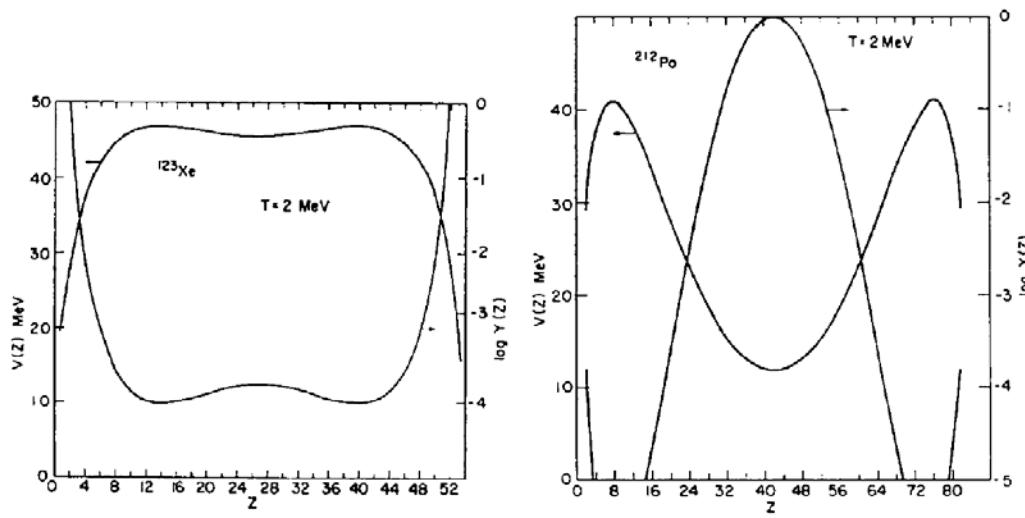
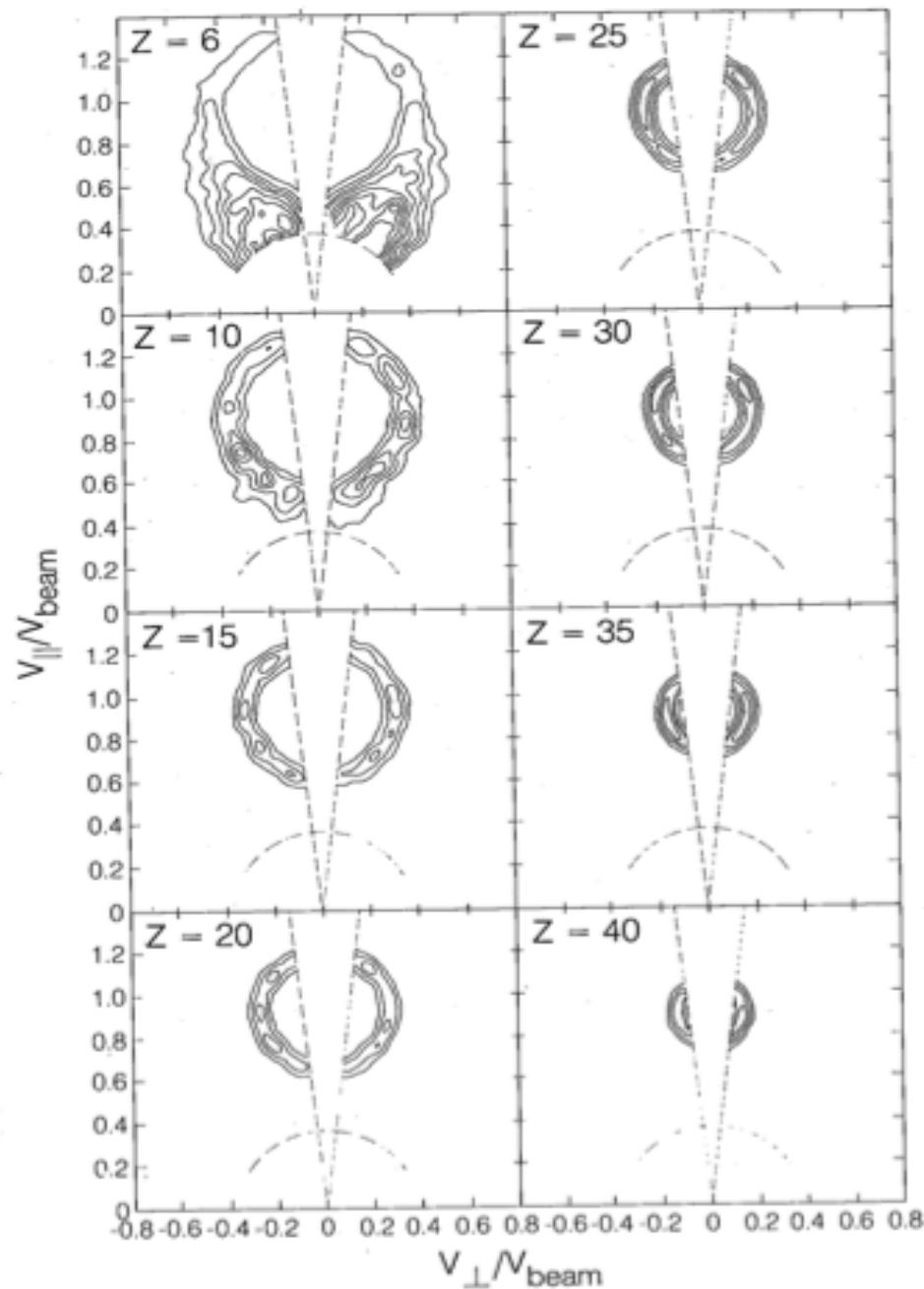


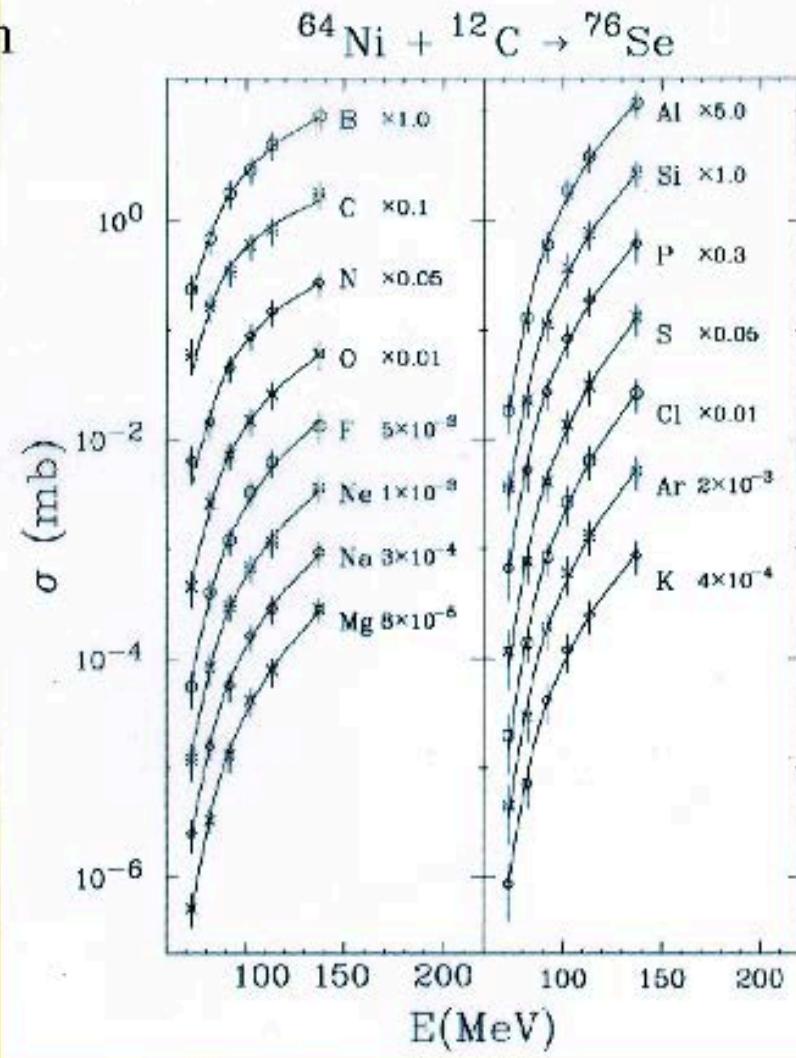
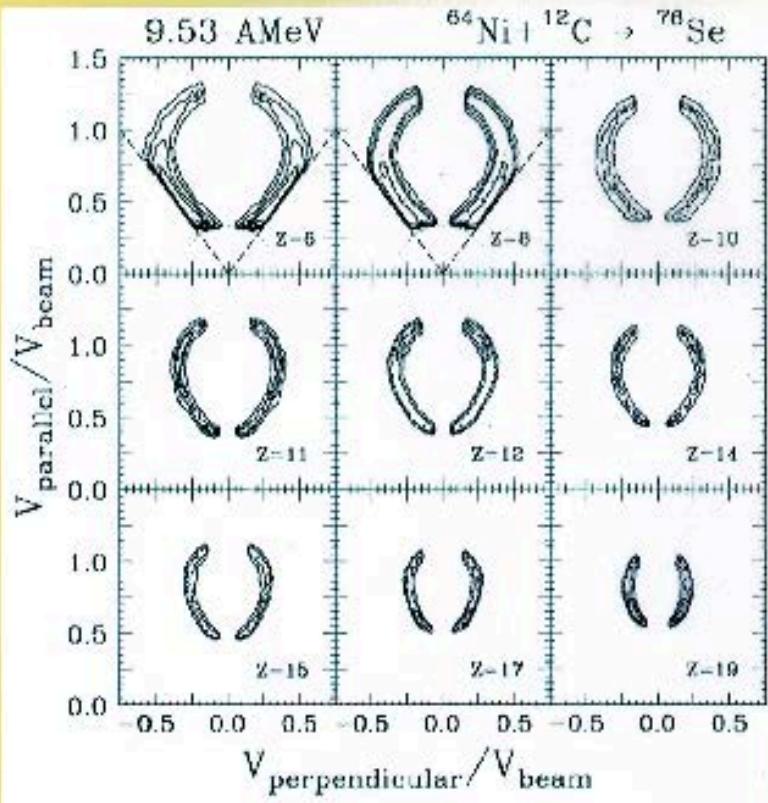
Fig. 1a.

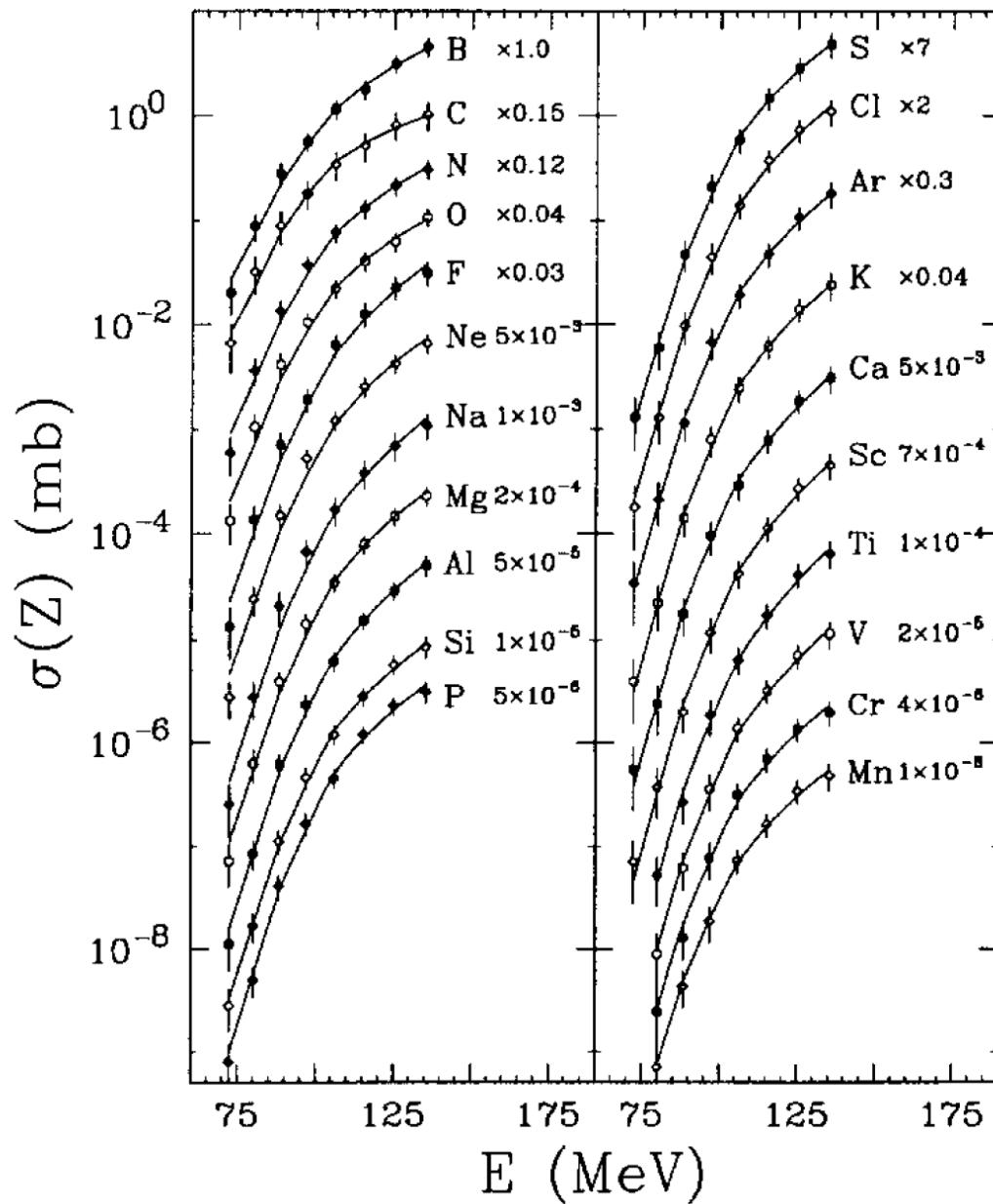
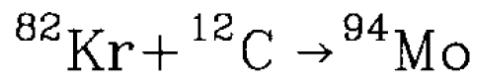


$E/A = 18 \text{ MeV}$   $^{139}\text{La} + ^{12}\text{C}$



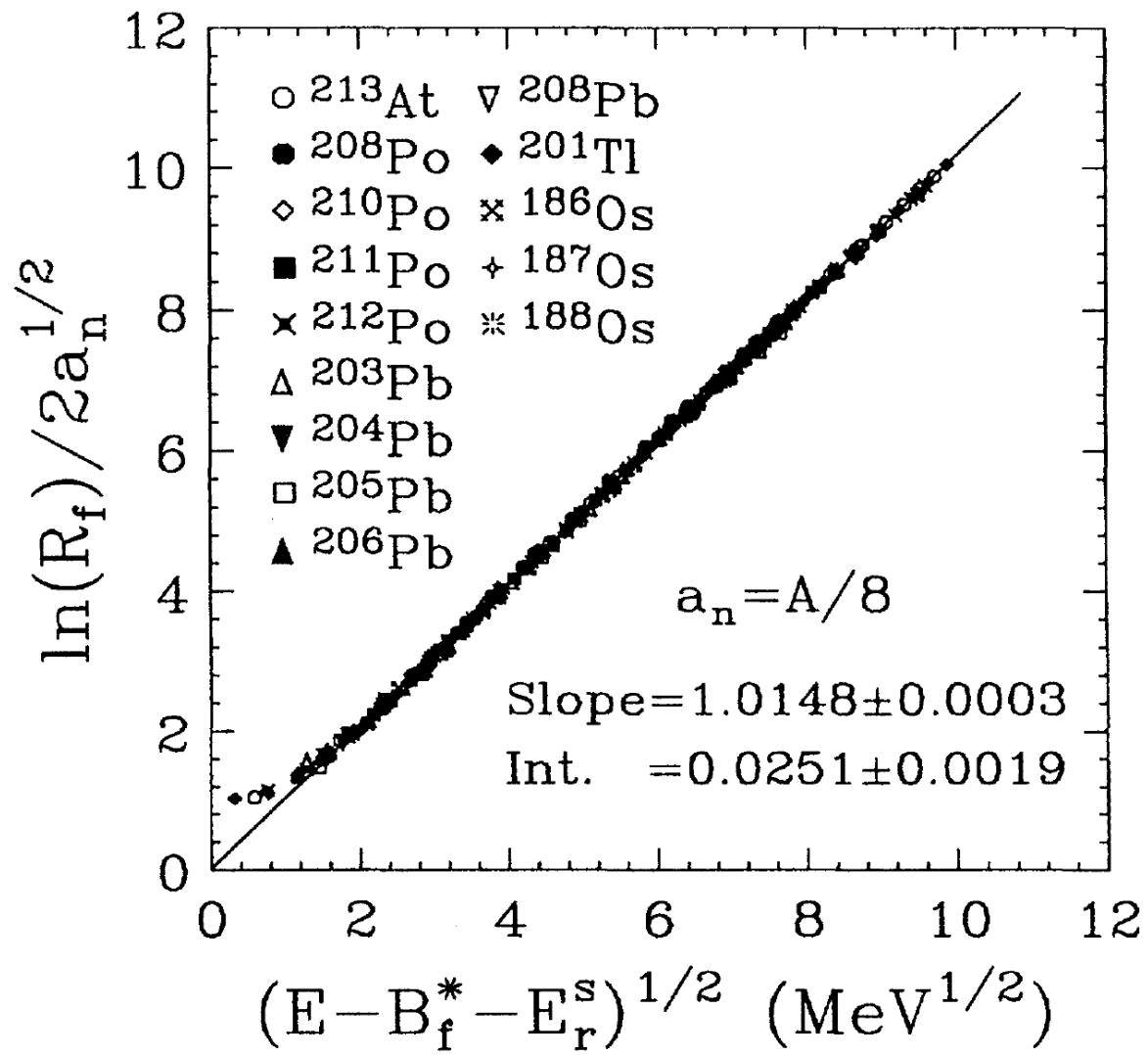
## Complex fragment emission at the 88-inch cyclotron

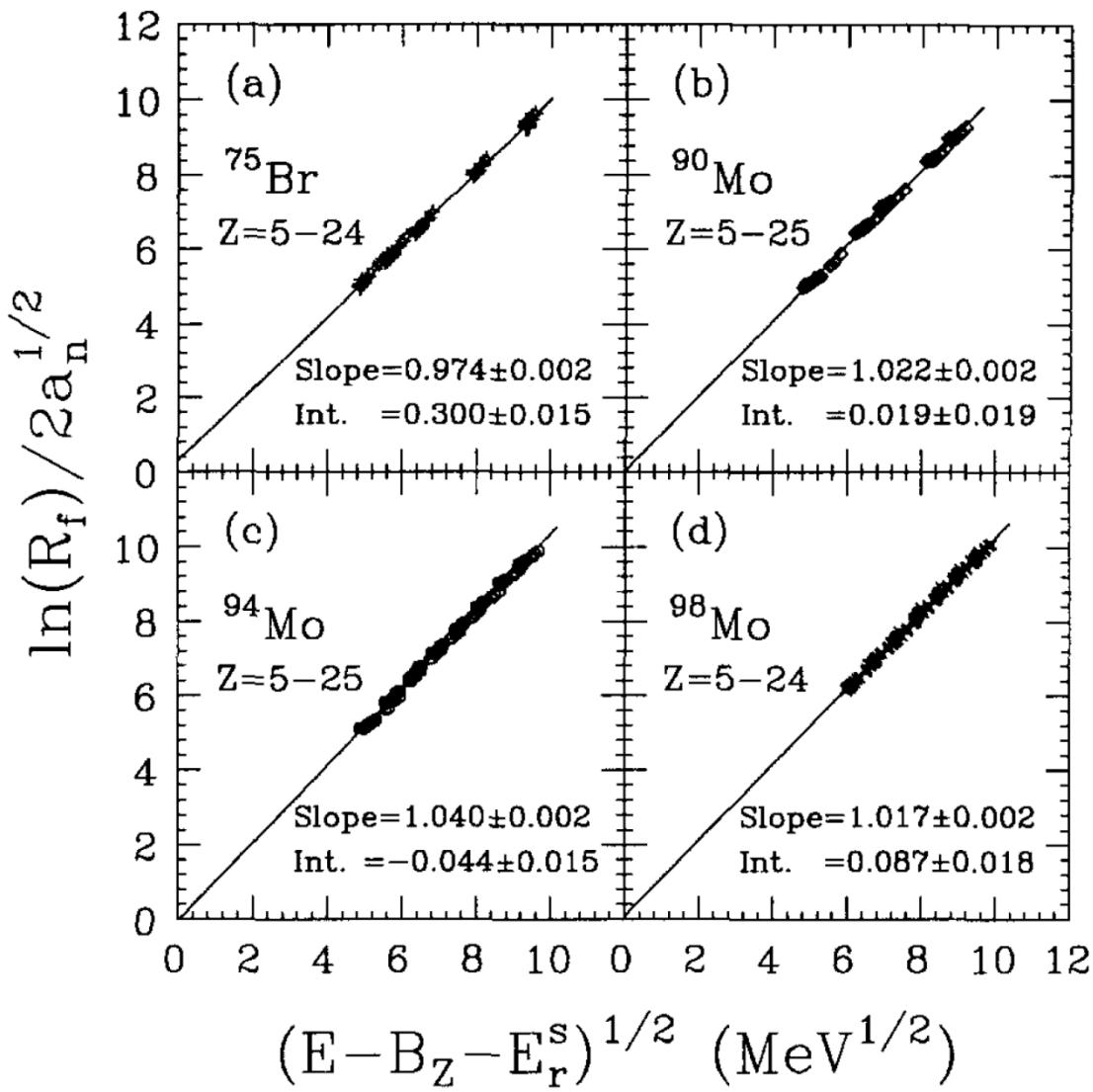


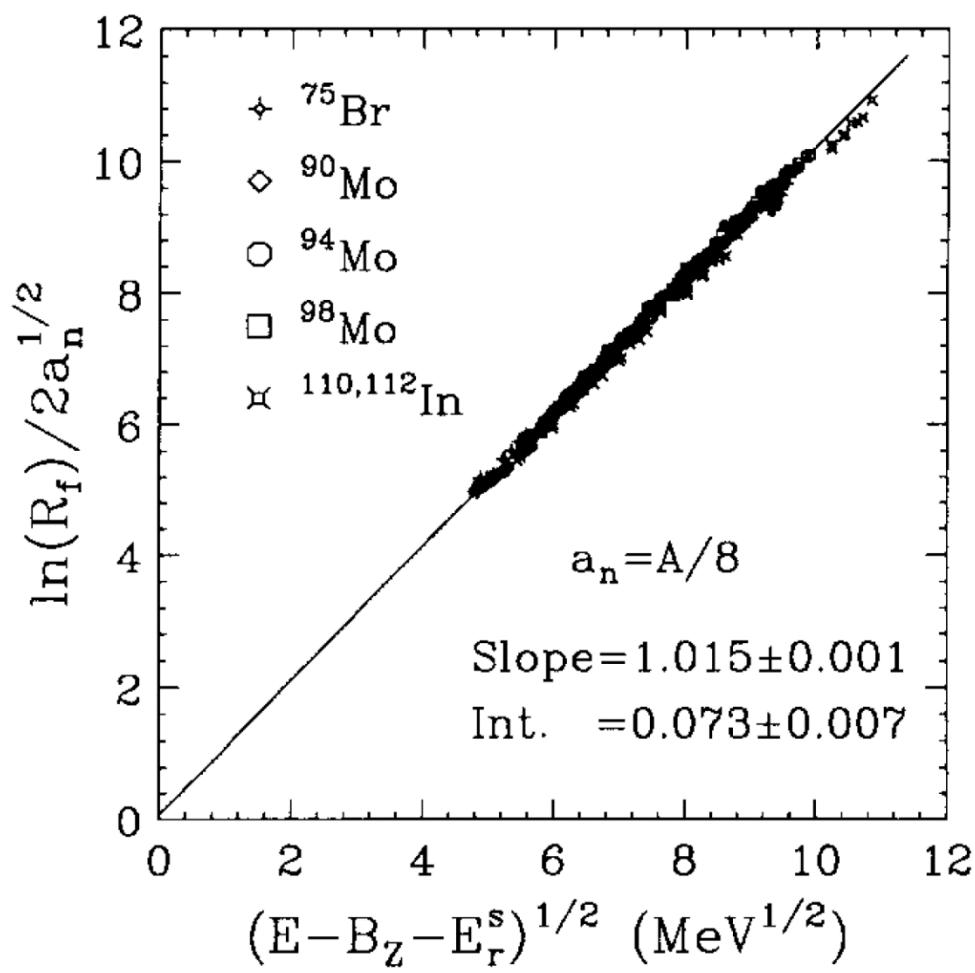


# Universal Transition State Scaling Function

$$\begin{aligned} & \frac{1}{2\sqrt{a_n}} \ln \left[ \frac{\sigma_f}{\sigma_0} \Gamma_T \frac{2\pi\rho_n(E - E_r^{gs})}{T_s} \right] \\ &= \frac{\ln R_f}{2\sqrt{a_n}} = \sqrt{\frac{a_f}{a_n} (E - B_f^* - E_r^s)}. \end{aligned}$$







# Saddle Point and Normal Modes

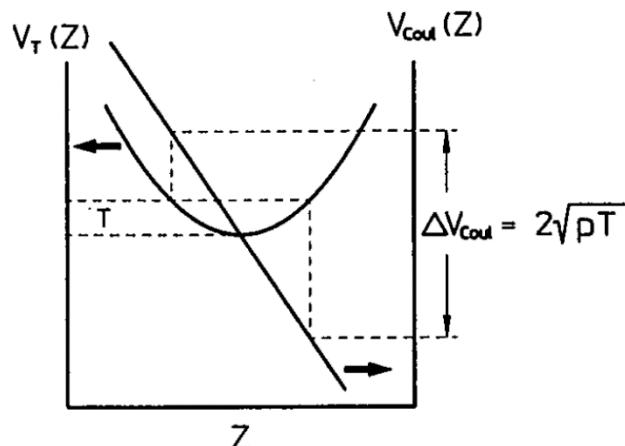
i) decay mode:



ii) non-amplifying mode:



iii) amplifying mode:



**Figure 2.** Top: Normal modes at the saddle point. Bottom: Total potential energy  $V_T$  and Coulomb energy  $V_{\text{Coul}}$  as a function of the deformation coordinate  $Z$ .

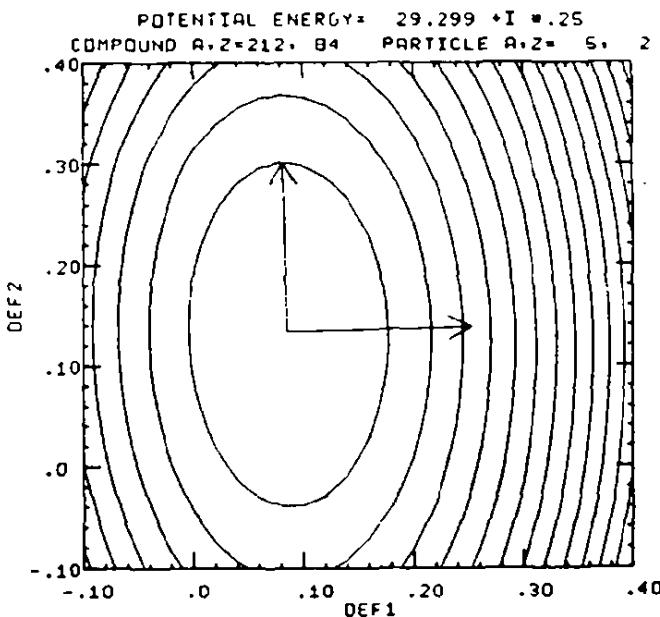


Fig. 2a.

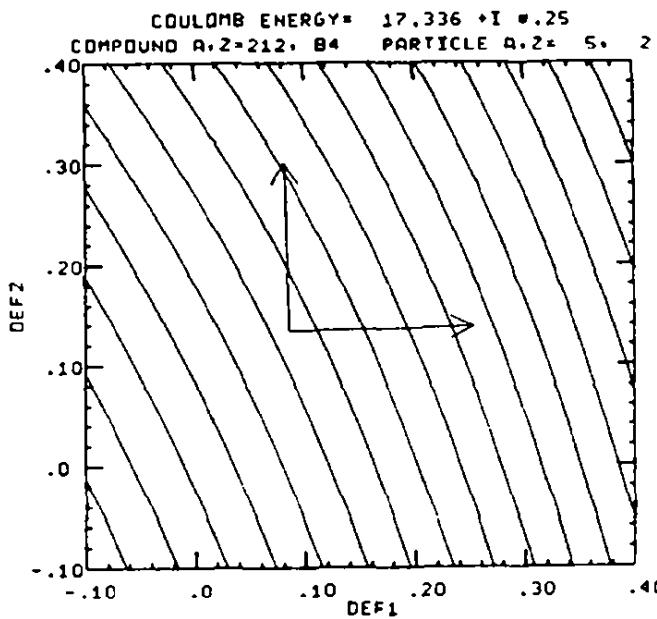
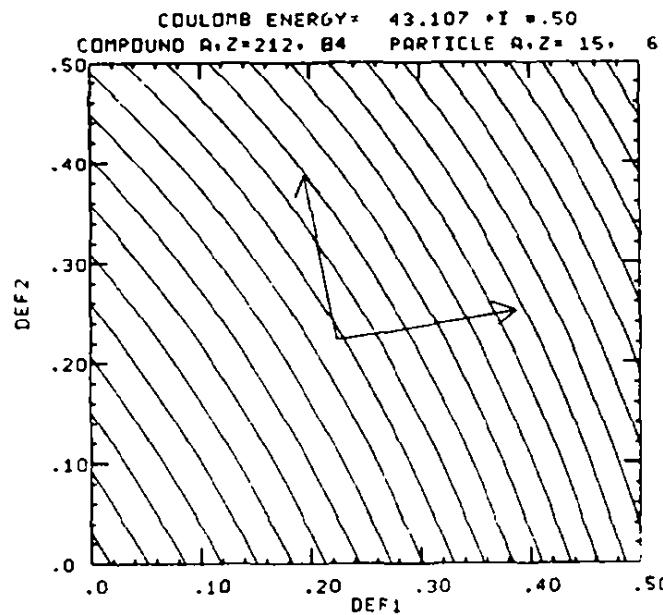
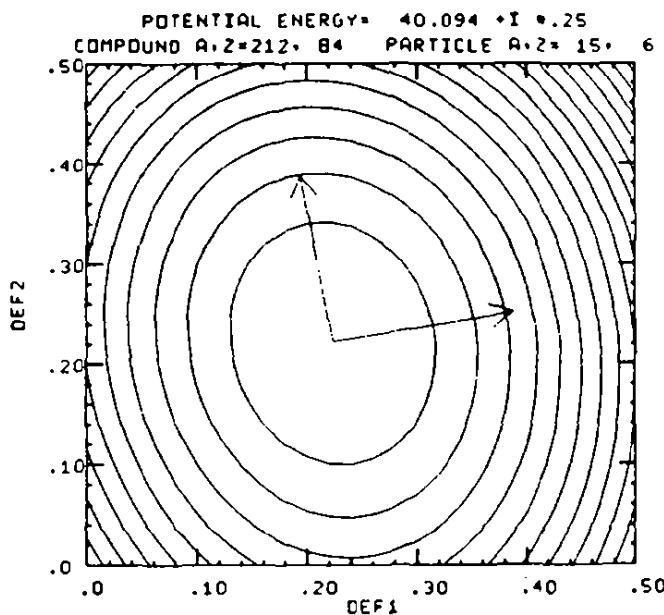
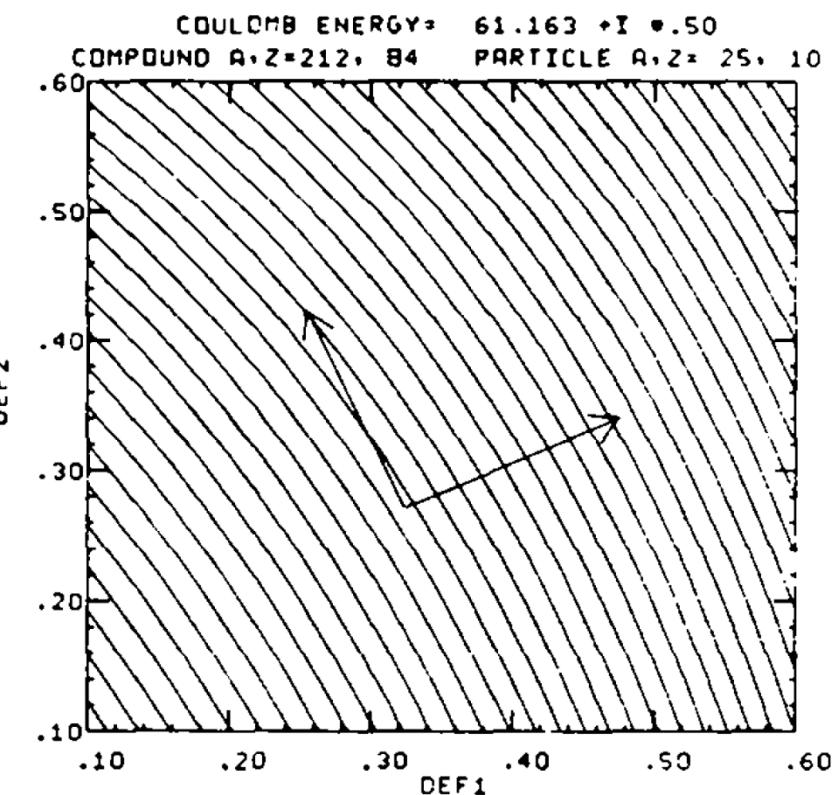
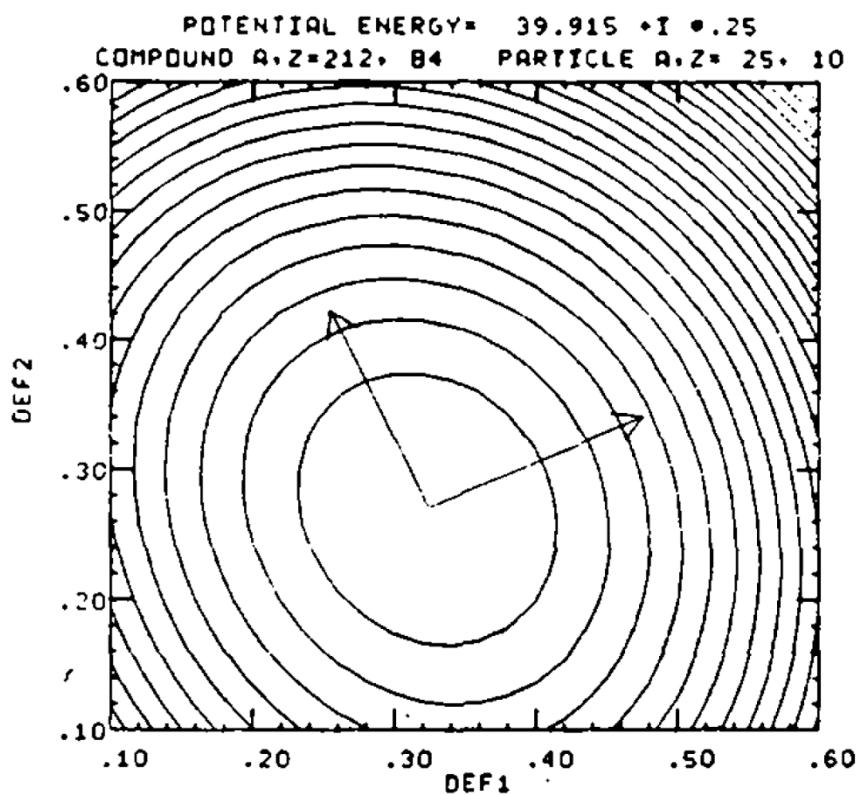
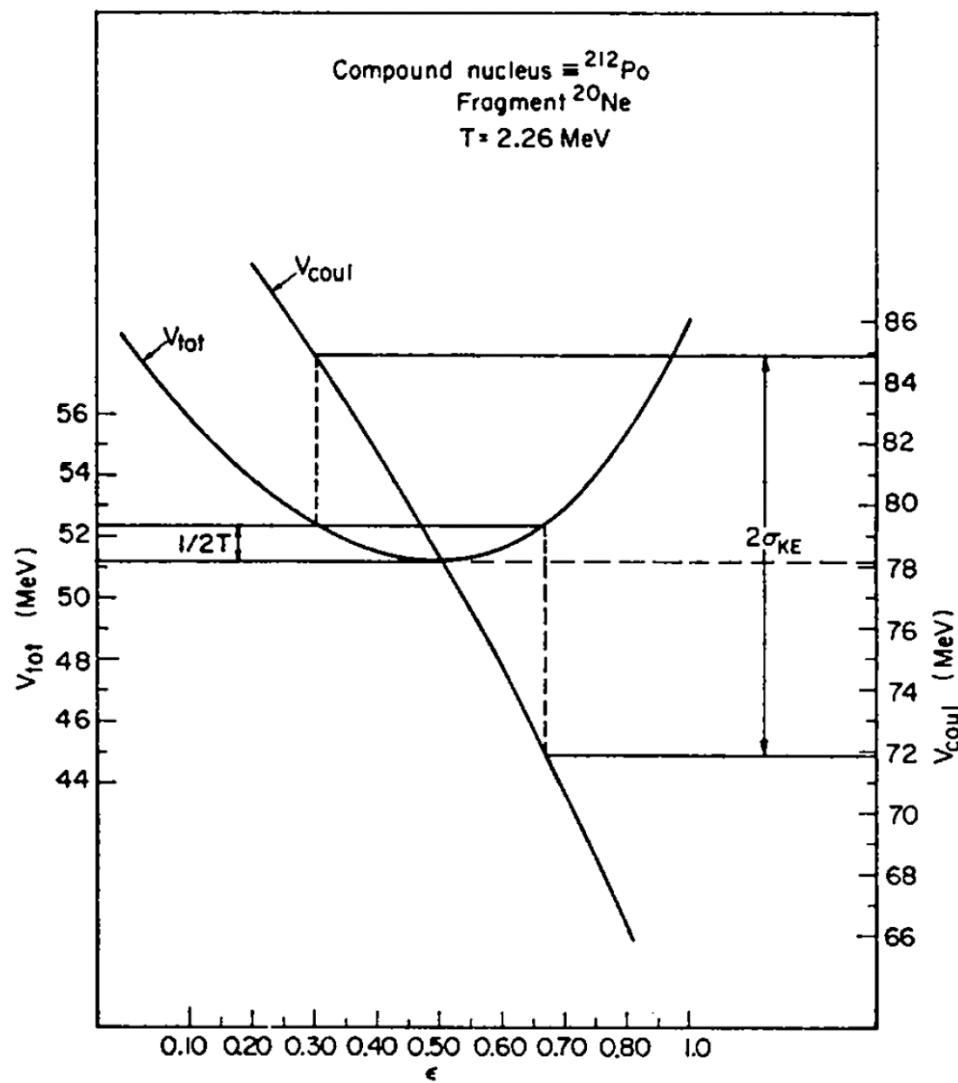


Fig. 2b.







Let us plot the ridge point potential energy for the sphere-spheroid model as a function of the spheroid deformation (fig. 3). On the same graph let us plot the Coulomb interaction energy of the two touching fragments, also as a function of the spheroid deformation. In second order in the deformation parameter  $z = \beta - \beta_{\text{eq}}$ , the potential energy has the form:

$$V_T = V_R + kz^2, \quad (6)$$

while the Coulomb interaction energy in first order has the form:

$$V_C = E_0 - cz. \quad (7)$$

The fluctuation in potential energy associated with the deformation mode in equilibrium with a thermostat with temperature  $T$  is of the order  $\frac{1}{2}T$ . The corresponding fluctuation in Coulomb energy is:

$$\sigma = \sqrt{\frac{c^2 T}{2k}} = \sqrt{\frac{1}{2} p T}, \quad (8)$$

where the parameter  $p = c^2/k$  is dependent only upon the potential energy of the ridge point mode in question. The width  $\sigma$  for sufficiently large values of the parameter  $p$  may become the dominant contribution to the spread in kinetic energy.

### 5.1. ONE DECAY MODE AND ONE AMPLIFYING MODE

The decay width takes the following form:

$$\Gamma^{(4)} dy d\varepsilon dz dp_z = \frac{dy(2\pi T m_y)^{\frac{1}{2}} \rho_R(E - B_R)}{h 2\pi \rho(E)} \exp \left\{ -\frac{1}{T} \left( \varepsilon + \frac{p_z^2}{2m_z} + V(z) \right) \right\} \frac{dz dp_z d\varepsilon}{h}. \quad (9)$$

In this expression  $z, p_z, m_z$ , and  $V(z)$  are the coordinate, conjugate momentum, inertia and potential energy of the amplifying mode;  $\varepsilon$  is the kinetic energy of the decay mode.

Since the kinetic energy associated with the amplifying mode is not expected to contribute to the final kinetic energy, one can integrate directly over  $p_z$ . Furthermore, one can express  $V(z)$  in the quadratic approximation:

$$V(z) = B_R + kz^2. \quad (10)$$

One then obtains:

$$\Gamma^{(3)} dy d\varepsilon dz = \frac{dy(2\pi T m_y)^{\frac{1}{2}}}{h} \frac{\rho_R(E - B_R)}{2\pi \rho(E)} \frac{(2\pi m_z T)^{\frac{1}{2}}}{h} \exp \left[ -\frac{1}{T} (\varepsilon + kz^2) \right] d\varepsilon dz. \quad (11)$$

Let us now assume that the kinetic energy at infinity is given by:

$$E_k = E_C + \varepsilon \approx E_0 - cz + \varepsilon, \quad (12)$$

where  $E_0$  is the Coulomb interaction energy at the ridge point and  $cz$  is its first order dependence upon the deformation parameter  $z$ . Then the kinetic energy distribution at infinity is:

$$P(E_k) dE_k \propto dE_k \int_0^{E_k} \exp \left( -\frac{1}{T} \left\{ \varepsilon + \frac{k}{c^2} (E_k - E_0 - \varepsilon)^2 \right\} \right) d\varepsilon, \quad (13)$$

where all the irrelevant multiplicative factors have been dropped. Letting  $c^2/k = p$  and  $E_k - E_0 = x$  one obtains:

$$P(x) dx \propto \exp \left( -\frac{x}{T} \right) \left\{ \operatorname{erf} \left( \frac{2E_0 + p}{2\sqrt{pT}} \right) - \operatorname{erf} \left( \frac{p - 2x}{2\sqrt{pT}} \right) \right\} dx. \quad (14)$$

Even for small charged particles, and rather large temperatures, the argument of the first error function is quite large. Consequently,

$$\operatorname{erf} \left( \frac{2E_0 + p}{2\sqrt{pT}} \right) \approx 1,$$

$$P(x) dx \propto \exp \left( -\frac{x}{T} \right) \operatorname{erfc} \left( \frac{p - 2x}{2\sqrt{pT}} \right) dx. \quad (15)$$

## 5.2. ONE DECAY MODE, ONE AMPLIFYING MODE AND ONE NON-AMPLIFYING MODE

$$P(x)dx \propto \left\{ (2x - p) \exp\left(-\frac{x}{T}\right) \left[ \operatorname{erf}\left(\frac{2E_0 + p}{2\sqrt{pT}}\right) - \operatorname{erf}\left(\frac{p - 2x}{2\sqrt{pT}}\right) \right] + \frac{2\sqrt{pT}}{\sqrt{\pi}} \left[ \exp\left(-\frac{p^2 + 4x^2}{4pT}\right) - \exp\left(-\frac{(2E_0 + p)^2 + 4px}{4pT}\right) \right] \right\} dx.$$

Again, if  $E_0 \gg \sqrt{pT}$ , the above expression can be simplified as follows:

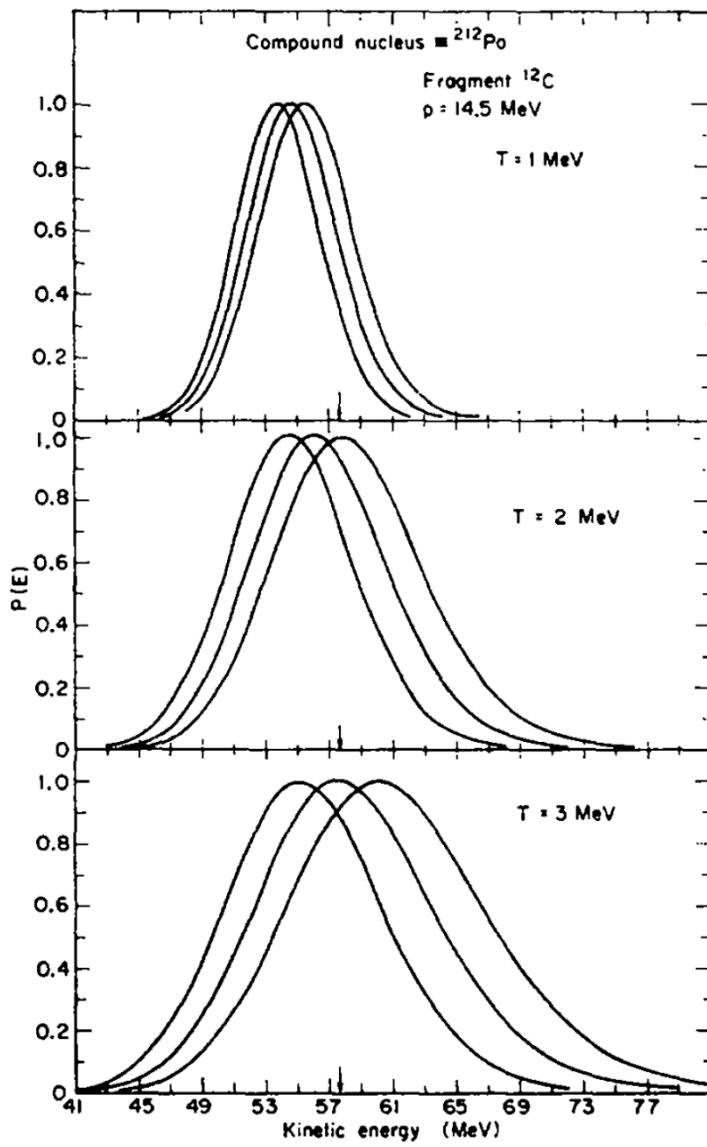
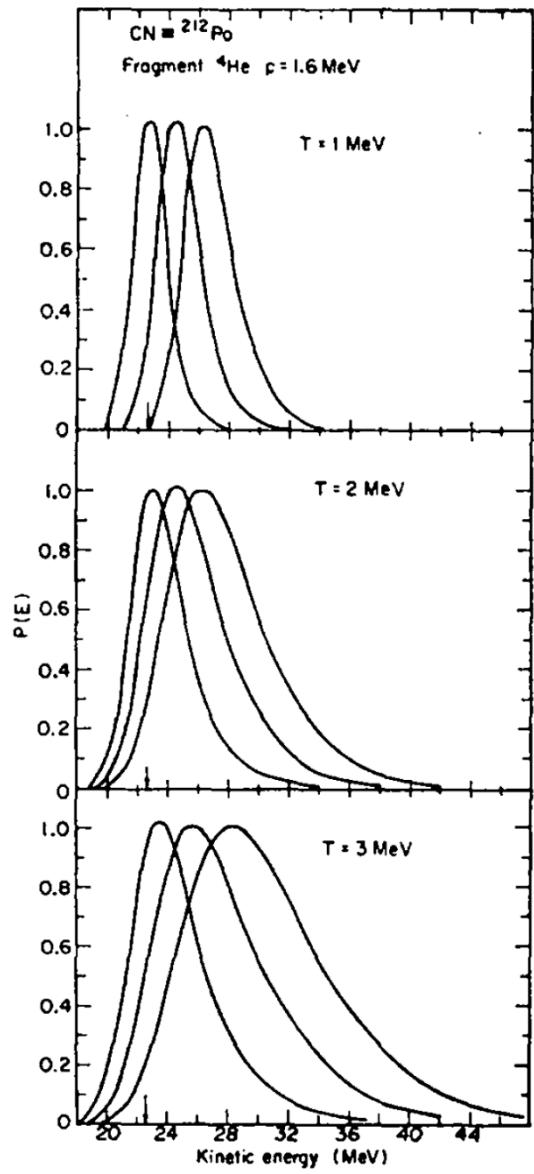
$$P(x)dx \propto \left\{ (2x - p) \exp\left(-\frac{x}{T}\right) \operatorname{erfc}\left(\frac{p - 2x}{2\sqrt{pT}}\right) + 2\sqrt{\frac{pT}{\pi}} \exp\left(-\frac{p^2 + 4x^2}{4pT}\right) \right\} dx.$$

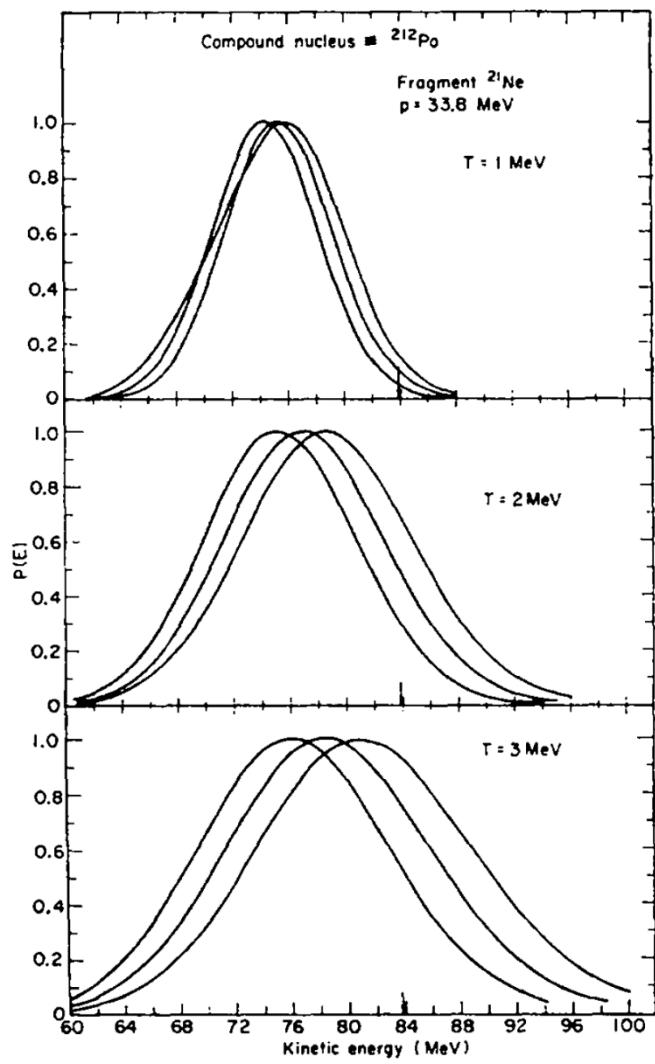
# ONE DECAY MODE, ONE AMPLIFYING MODE AND TWO NON-AMPLIFYING MODES

$$P(x)dx \propto \left\{ \left( \frac{1}{4}p^2 + \frac{1}{2}pT + x^2 - px \right) \exp \left( -\frac{x}{T} \right) \left[ \operatorname{erf} \left( \frac{2E_0 + p}{2\sqrt{pT}} \right) - \operatorname{erf} \left( \frac{p - 2x}{2\sqrt{pT}} \right) \right] \right. \\ \left. + \frac{\sqrt{pT}}{2\sqrt{\pi}} \left[ (2x - p) \exp \left( -\frac{p^2 + 4x^2}{4pT} \right) - (2E_0 + p) \exp \left( -\frac{(2E_0 + p)^2 + 4px}{4pT} \right) \right] \right\} dx.$$

Again, if  $E_0 \gg \sqrt{pT}$  the above expression becomes:

$$P(x)dx \propto \left\{ \left( \frac{1}{4}p^2 + \frac{1}{2}pT + x^2 - px \right) \exp \left( -\frac{x}{T} \right) \operatorname{erfc} \left( \frac{p - 2x}{2\sqrt{pT}} \right) \right. \\ \left. + \frac{\sqrt{pT}}{2\sqrt{\pi}} (2x - p) \exp \left( -\frac{p^2 + 4x^2}{4pT} \right) \right\} dx.$$





## 6.1. LIMITING EXPRESSIONS FOR $p = 0$

First, let us consider the limit to which the three expressions, eqs. (15), (20), and (23), tend when the amplification parameter  $p$  tends to zero. This occurs when the charge of the emitted particle goes to zero. Under these conditions  $x \equiv E_k$ . By noticing that in this limit the function erfc tends to a non-zero constant [ $\lim_{y \rightarrow \infty} \text{erfc}(-y) = 2$ ] one obtains:

$$P(E_k) dE_k \propto \begin{cases} \exp(-E_k/T) dE_k & (24a) \\ E_k \exp(-E_k/T) dE_k & (24b) \\ E_k^2 \exp(-E_k/T) dE_k . & (24c) \end{cases}$$

It appears that one can write a general expression as:

$$P(E_k) dE_k \propto E_k^n \exp(-E_k/T) dE_k , \quad (25)$$

## LIMITING EXPRESSIONS FOR LARGE AMPLIFICATION PARAMETERS AND FOR MORE THAN ONE AMPLIFYING MODE

$$P(x)dx \propto \exp\left[-\frac{x^2}{(p_1 + p_2)T + 2(n+1)T^2}\right] dx,$$

where  $x = E_k - E_0 - (n+1)T$ .

# The angular distributions

$$\frac{d\sigma}{d\Omega} = \int_0^{I_{\max}} dI \sigma_I \int_{-I}^{+I} dK \frac{\Gamma_f^I(K)}{\Gamma_T^I} W_K^I(\theta),$$

where

$$\Gamma_f^I(K) = \frac{\Gamma_f^0}{2I+1} \exp \left[ -\frac{\hbar^2 I^2}{2T} \left( \frac{1}{\mathcal{J}_\perp} - \frac{1}{\mathcal{J}_c} \right) \right] \exp \left( -\frac{K^2}{2K_0^2} \right)$$

$$W_K^I(\theta) \propto \frac{2I+1}{\sqrt{\sin^2 \theta - K^2/I^2}}$$

$K_0^2$  is the standard deviation of the statistical distribution of  $K$ -values and is given by:

$$K_0^2 = \mathcal{J}_{\text{eff}} T/\hbar^2. \quad (39)$$

The quantity  $\mathcal{J}_{\text{eff}}$  is related to the principal moments of inertia,  $\mathcal{J}_{\parallel}$  and  $\mathcal{J}_{\perp}$ , of the system at the ridge point by the relation:

$$\frac{1}{\mathcal{J}_{\text{eff}}} = \frac{1}{\mathcal{J}_{\parallel}} - \frac{1}{\mathcal{J}_{\perp}}. \quad (40)$$

If one assumes that  $\Gamma_T \approx \Gamma_n$ , the integration over  $K$  of eq. (36) gives:

$$W(\theta) \propto \int_0^{I_{\max}} \frac{2I dI \exp\left(-\frac{I^2 \sin^2 \theta}{4K_0^2}\right) I_0\left(\frac{I^2 \sin^2 \theta}{4K_0^2}\right)}{\exp(-\beta I^2)}. \quad (41)$$

In this expression  $I_0$  is the modified Bessel function of order 0 and

$$\beta = \frac{\hbar^2}{2T} \left( \frac{1}{\mathcal{J}_n} - \frac{1}{\mathcal{J}_{\perp}} \right), \quad (42)$$

$\mathcal{J}_n$  being the moment of inertia of the residual nucleus after neutron emission. If  $\beta I^2 \ll 1$  then  $\exp(-\beta I^2) \approx 1$  and the integral becomes of the form:

$$W(\theta) \propto \exp\left(-\frac{I_{\max}^2 \sin^2 \theta}{4K_0^2}\right) \left[ I_0\left(\frac{I_{\max}^2 \sin^2 \theta}{4K_0^2}\right) + I_1\left(\frac{I_{\max}^2 \sin^2 \theta}{4K_0^2}\right) \right].$$

This expression has two interesting limits: as  $g = I_{\max}^2/4K_0^2$  tends to infinity (either because  $K_0^2$  tends to zero or because  $I_{\max}$  becomes very large) one can use the asymptotic expression for the Bessel functions:

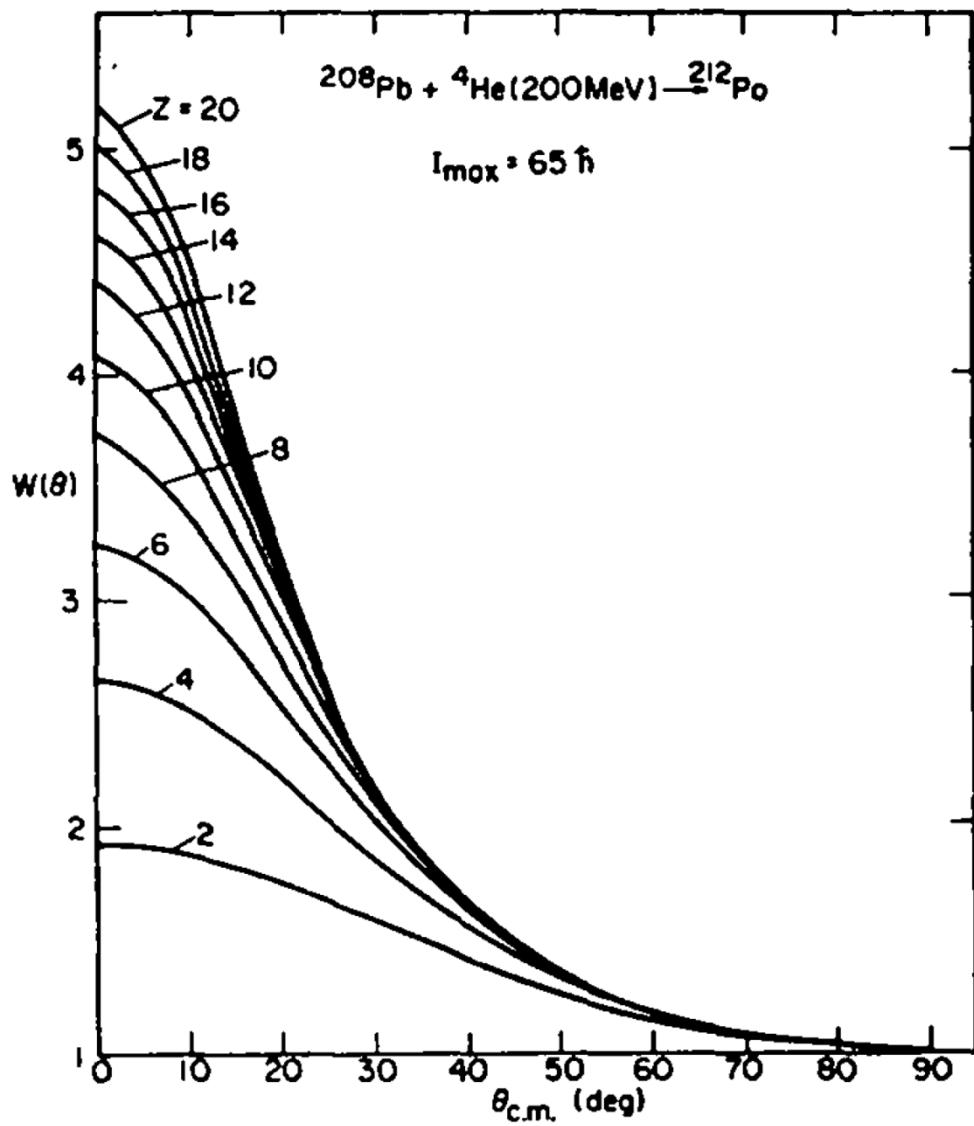
$$I_v(z) = \frac{e^z}{\sqrt{2\pi z}} \left( 1 - \frac{4v^2 - 1}{8z} + \dots \right). \quad (47)$$

Then if one keeps only the lowest term in the  $z^{-\frac{1}{2}}$  expansion one obtains:

$$\lim_{\theta \rightarrow \infty} W(\theta) \propto \frac{1}{\sin \theta}. \quad (48)$$

On the other hand, as  $g \rightarrow 0$  (either because  $I_{\max} \approx 0$  or  $K_0^2 \rightarrow \infty$ ) one obtains:

$$\lim_{g \rightarrow 0} W(\theta) = \text{constant}. \quad (49)$$



# Neutron Emission

The ridge point configuration for the neutron emission is represented by a neutron just outside the nucleus. The principal moments of inertia can be approximately expressed as follows:

$$\mathcal{I}_{||} \approx \mathcal{I}, \quad \mathcal{I}_{\perp} \approx \mathcal{I} + \mu R^2, \quad (50)$$

where  $\mathcal{I}$  is the moment of inertia of the residual nucleus,  $\mu$  is the reduced mass of the neutron nucleus system and  $R$  is the distance between neutron and nucleus when they are in contact. In many cases  $\mathcal{I} \gg \mu R^2$ . Thus the quantity  $z$  takes the approximate form:

$$\begin{aligned} z &= \frac{\hbar^2 I_{\max}^2 \sin^2 \theta}{4T} \left( \frac{1}{\mathcal{I}_{||}} - \frac{1}{\mathcal{I}_{\perp}} \right) \approx \frac{\hbar^2 I_{\max}^2 \sin^2 \theta}{4T \mathcal{I}} \left( 1 - \frac{1}{1 + \mu R^2 / \mathcal{I}} \right) \\ &\approx \frac{\hbar^2 I_{\max}^2 \sin^2 \theta}{4T \mathcal{I}} \frac{\mu R^2}{\mathcal{I}} = \frac{\bar{E}_R}{T} \frac{\mu R^2}{\mathcal{I}} \sin^2 \theta, \end{aligned} \quad (51)$$

where  $\bar{E}_R$  is the mean rotational energy of the residual nucleus. Similarly,

$$\mathcal{I}_c \approx \mathcal{I}_{||} \approx \mathcal{I}, \quad \frac{1}{2} \beta I^2 = \frac{z}{\sin^2 \theta} = \frac{\bar{E}_R}{T} \frac{\mu R^2}{\mathcal{I}} = \alpha. \quad (52)$$

Expanding eq. (46) to first order in  $z$  we obtain:

$$\begin{aligned} W(\theta) \approx 1 - \frac{1}{2}z + \frac{1}{2}\beta I_{\max}^2 \left(1 - \frac{2}{3}z\right) &\approx 1 + \frac{\bar{E}_R}{T} \frac{\mu R^2}{\mathcal{J}} - \frac{1}{2} \frac{\bar{E}_R}{T} \frac{\mu R^2}{\mathcal{J}} \sin^2 \theta \\ &= 1 + \alpha - \frac{1}{2}\alpha \sin^2 \theta. \quad (53) \end{aligned}$$

The normalized angular distribution in first order takes the form:

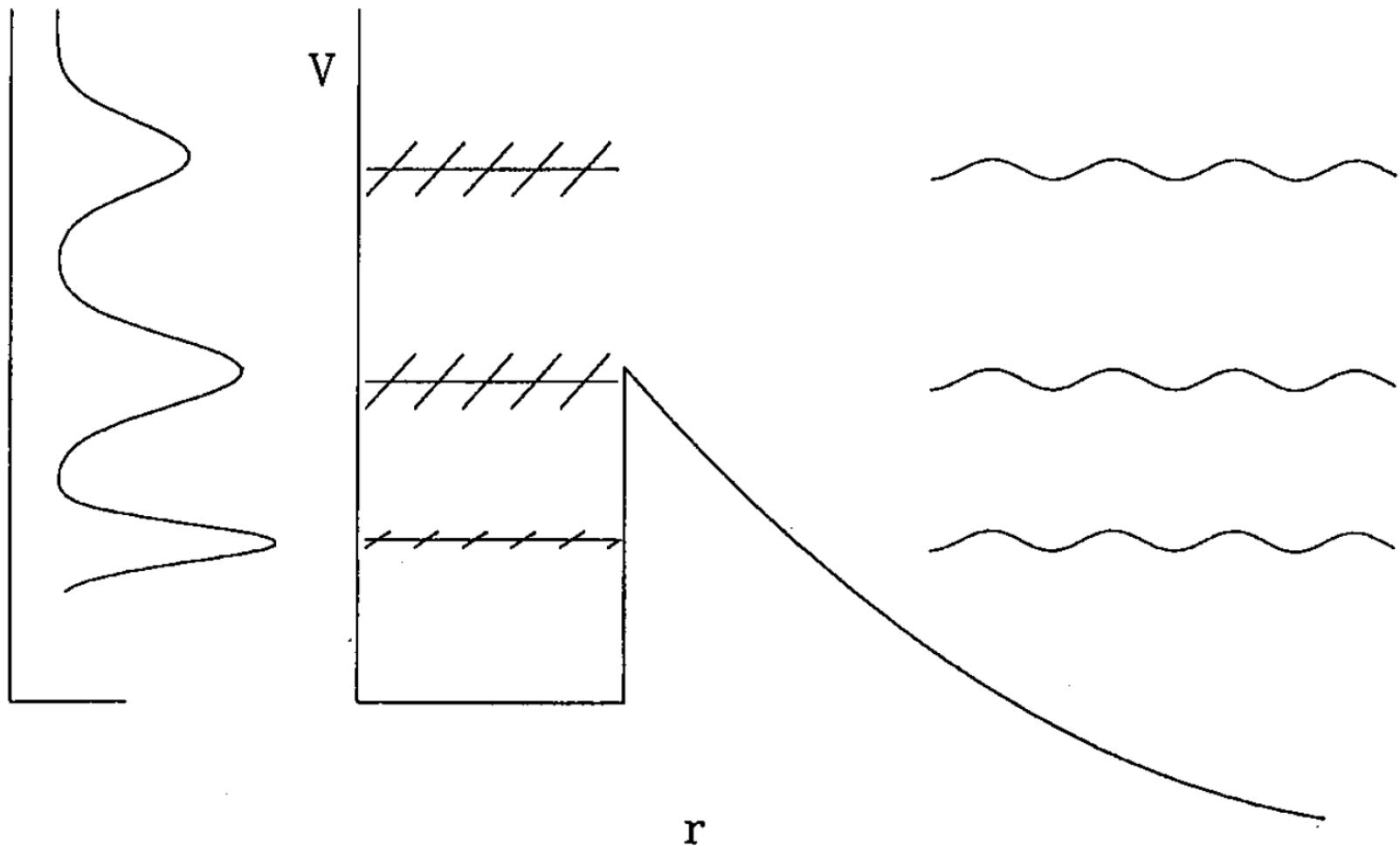
$$\begin{aligned} \frac{W(\theta)}{W(90)} &= \frac{1 + \alpha - \frac{1}{2}\alpha \sin^2 \theta}{1 + \frac{1}{2}\alpha} \approx (1 - \frac{1}{2}\alpha)(1 + \alpha - \frac{1}{2}\alpha \sin^2 \theta) \\ &= 1 + \frac{1}{2}\alpha \cos^2 \theta = 1 + \frac{1}{2} \frac{\bar{E}_R}{T} \frac{\mu R^2}{\mathcal{J}} \cos^2 \theta. \end{aligned}$$

The very same normalized distribution has been obtained by Ericson from a more conventional evaporation theory <sup>17</sup>).

## Particle Structure Functions

The existence of complex particles in a nucleus may be compared to that of a solute molecule in a solvent. The effect of the solvent on a solute molecule varies from a modest modification of its properties to full dissociation. Particles ( $\alpha$ ,  $d$ ,  $t$ , etc.) inside a nucleus can be seen as solutes in the nuclear solvent. Their existence as independent particles in the nuclear medium relates directly to the question of the extent to which the total nuclear wave function is factorizable into the product of the wave function of the particle and that of the residual nucleus.

*Particle structure function and sub-barrier fusion in hot nuclei*



**Figure 1.** Schematic drawing of the states of a particle in a potential well.

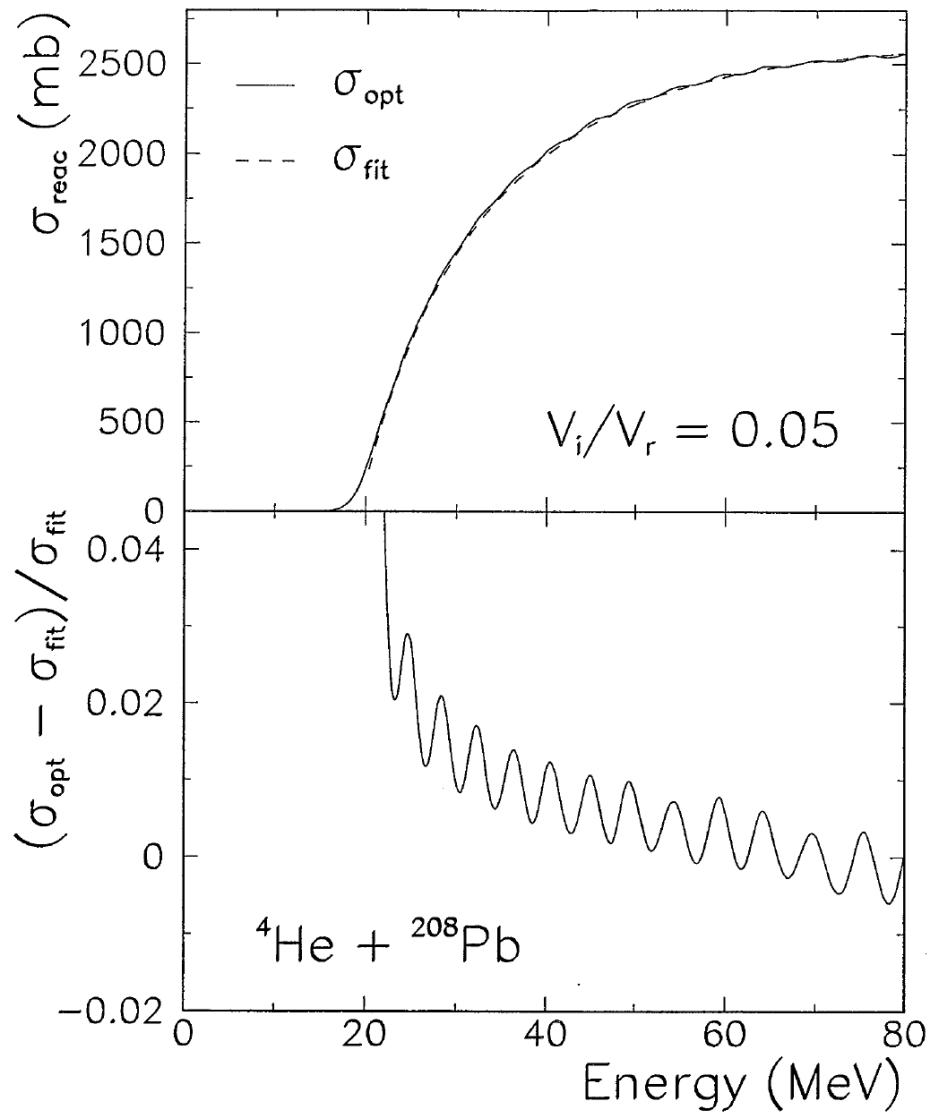
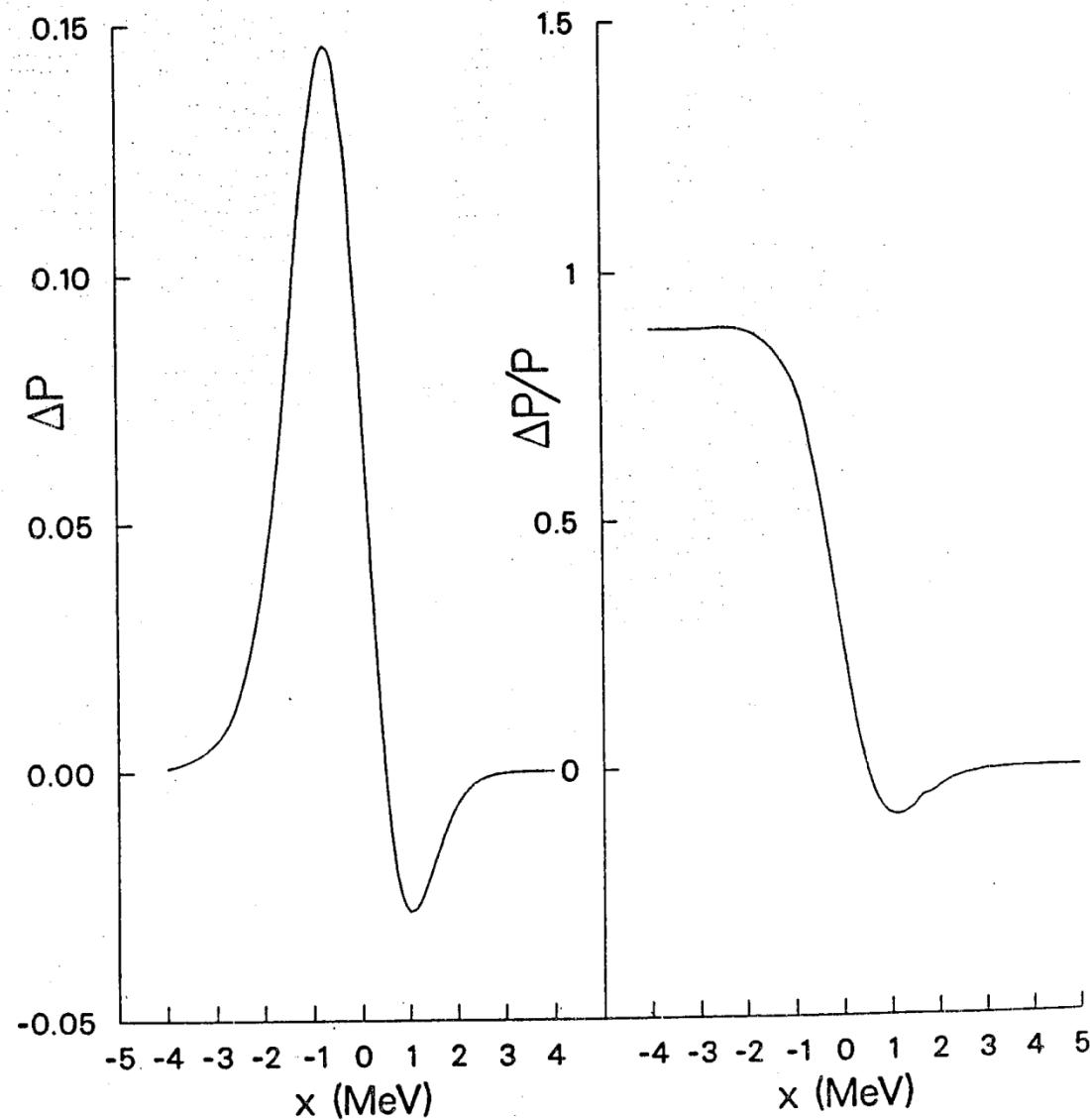


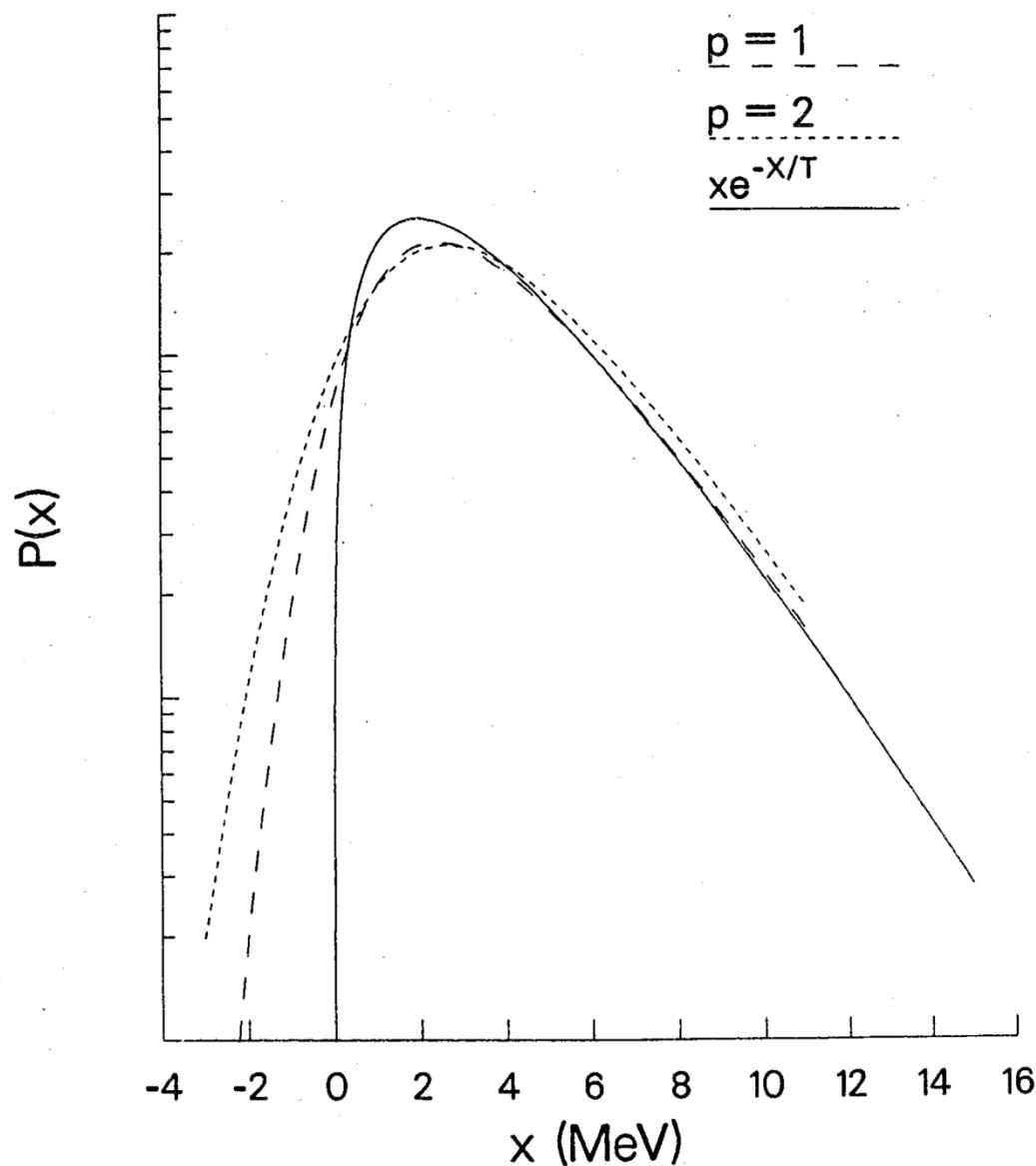
Figure 5.2: Top panel: The reaction cross section as a function of energy calculated for  ${}^4\text{He} + {}^{208}\text{Pb}$  using an optical model (solid line). In order to enhance the optical resonance, the imaginary part of the optical potential has been reduced in the calculation from 16% to 5% of the real potential. The dashed line is a smooth fit to reproduce the general trends of the cross section. Bottom panel: Fractional deviation of the optical model calculation from the smooth fit.

$$\begin{aligned}
P(\epsilon) \propto & e^{-x/T} \left\{ \operatorname{erf}\left((2V_{\text{Coul}}^0 + p)/2\sqrt{pT}\right) - \operatorname{erf}\left((p - 2x)/2\sqrt{pT}\right) \right. \\
& + \frac{1}{2} e^{-(p-2x)^2/4pT} \left[ e^{(p-2x-\gamma pT)^2/4pT} \left( 1 + \operatorname{erf}\left((p - 2x - \gamma pT)/2\sqrt{pT}\right) \right) \right. \\
& - e^{(p-2x+\gamma pT)^2/4pT} \left( \operatorname{erf}\left((2V_{\text{Coul}}^0 + p + \gamma pT)/2\sqrt{pT}\right) \right. \\
& \left. \left. - \operatorname{erf}\left((p - 2x + \gamma pT)/2\sqrt{pT}\right) \right) \right] \right\}, \quad (1)
\end{aligned}$$

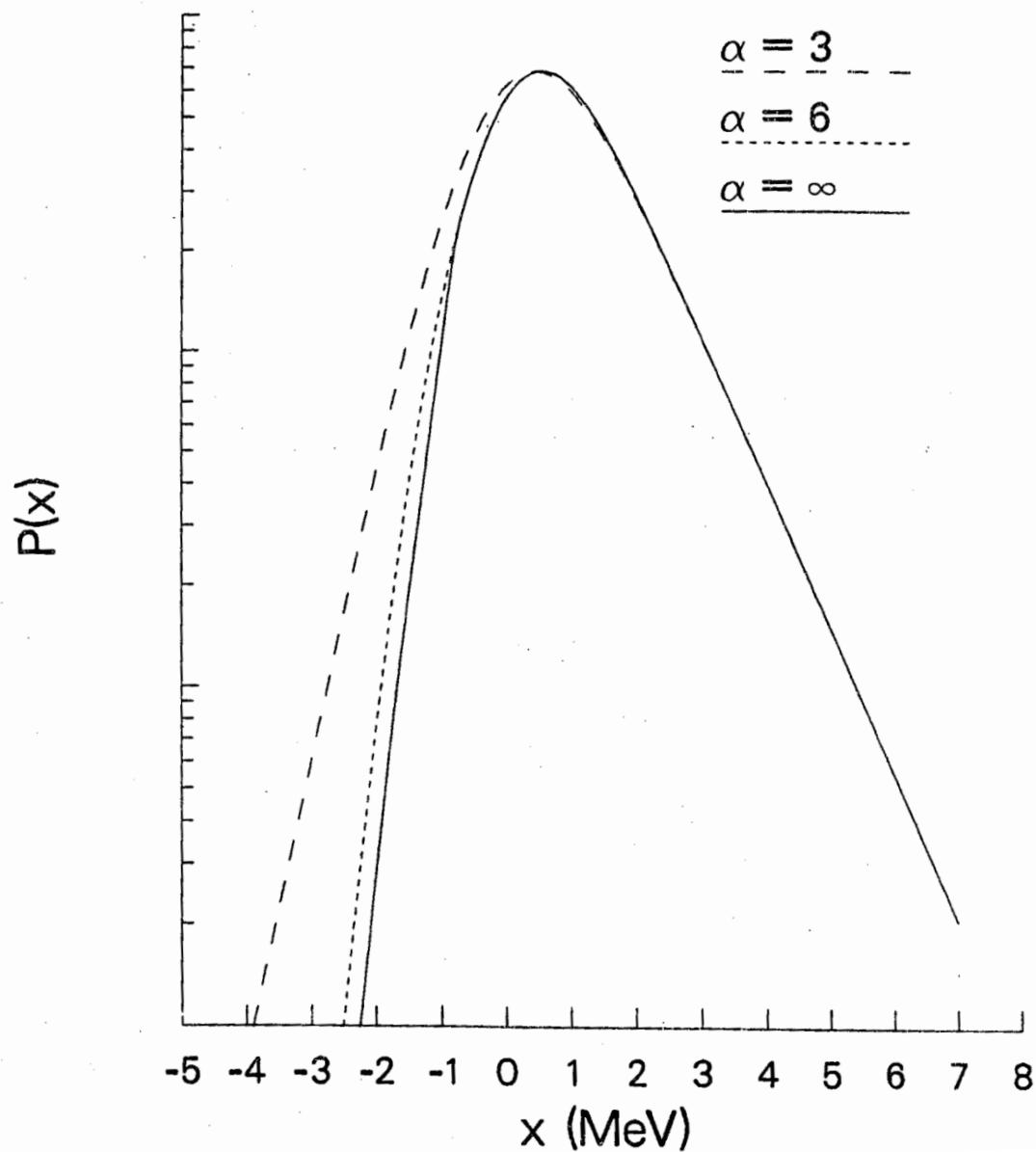
$\alpha=3$  ;  $p=1$  ;  $T=1$  MeV



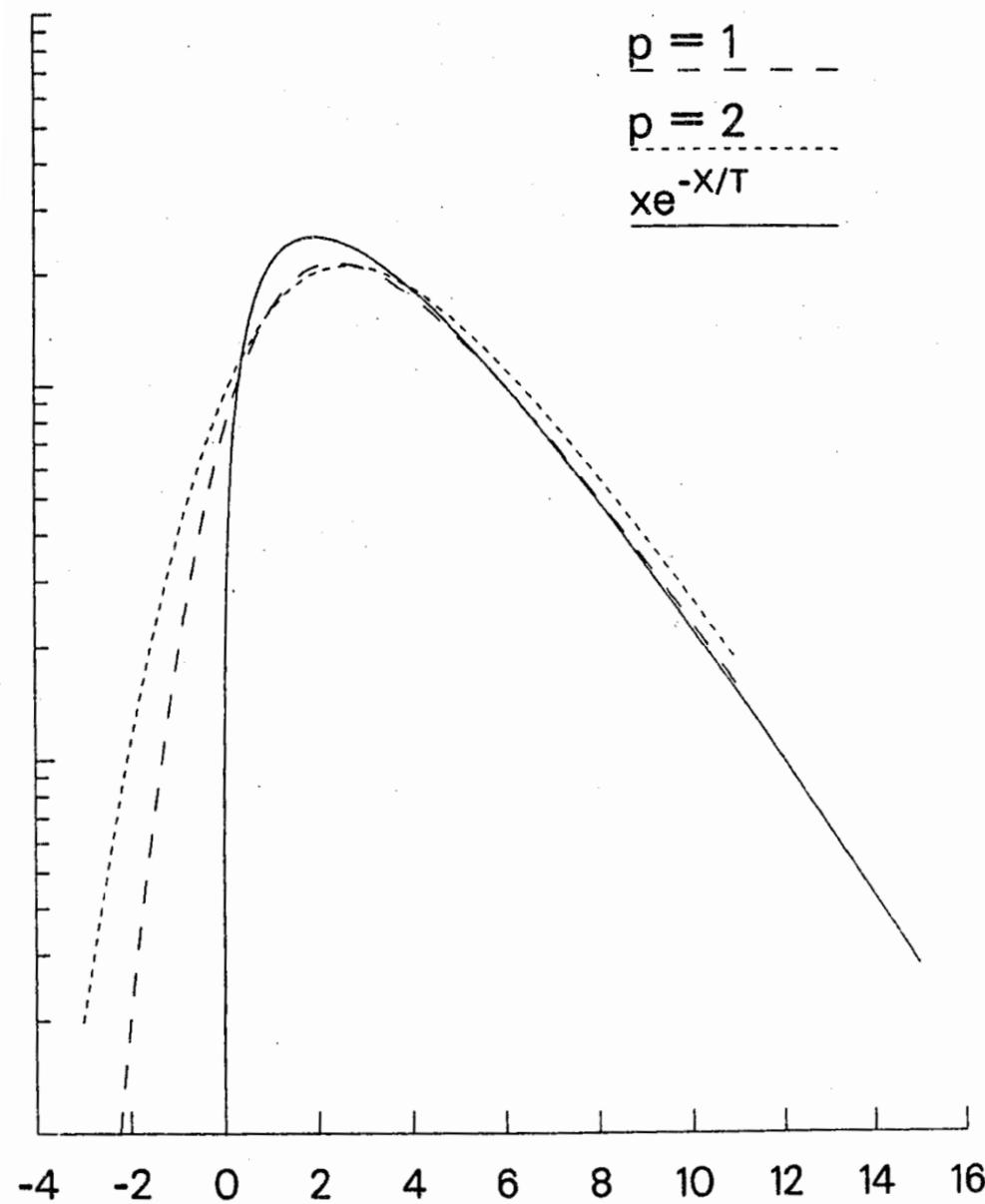
$T = 2 \text{ MeV}$



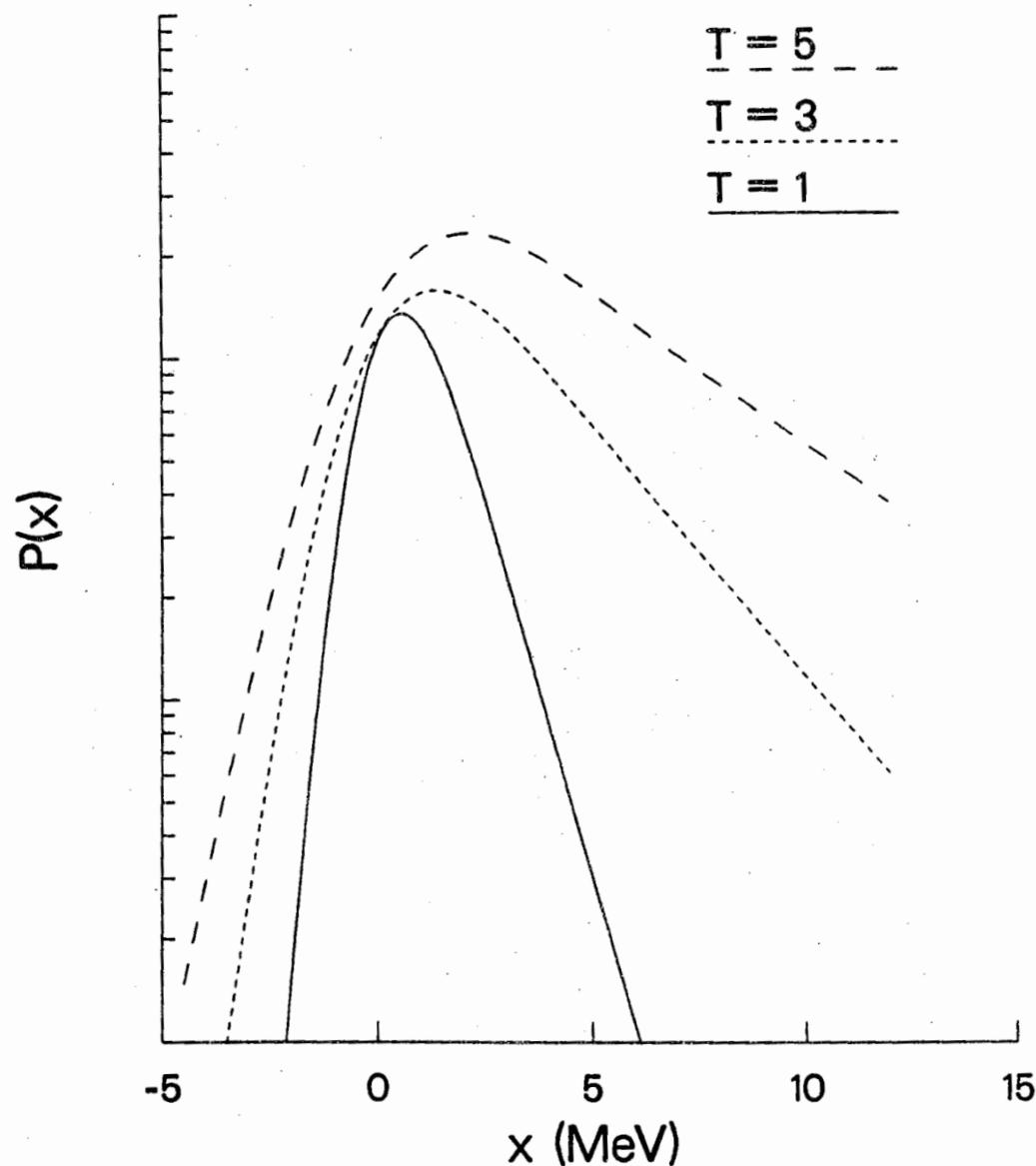
$p = 1; T = 1 \text{ MeV}$



$T = 2 \text{ MeV}$



$$p = 1; \alpha = 6$$



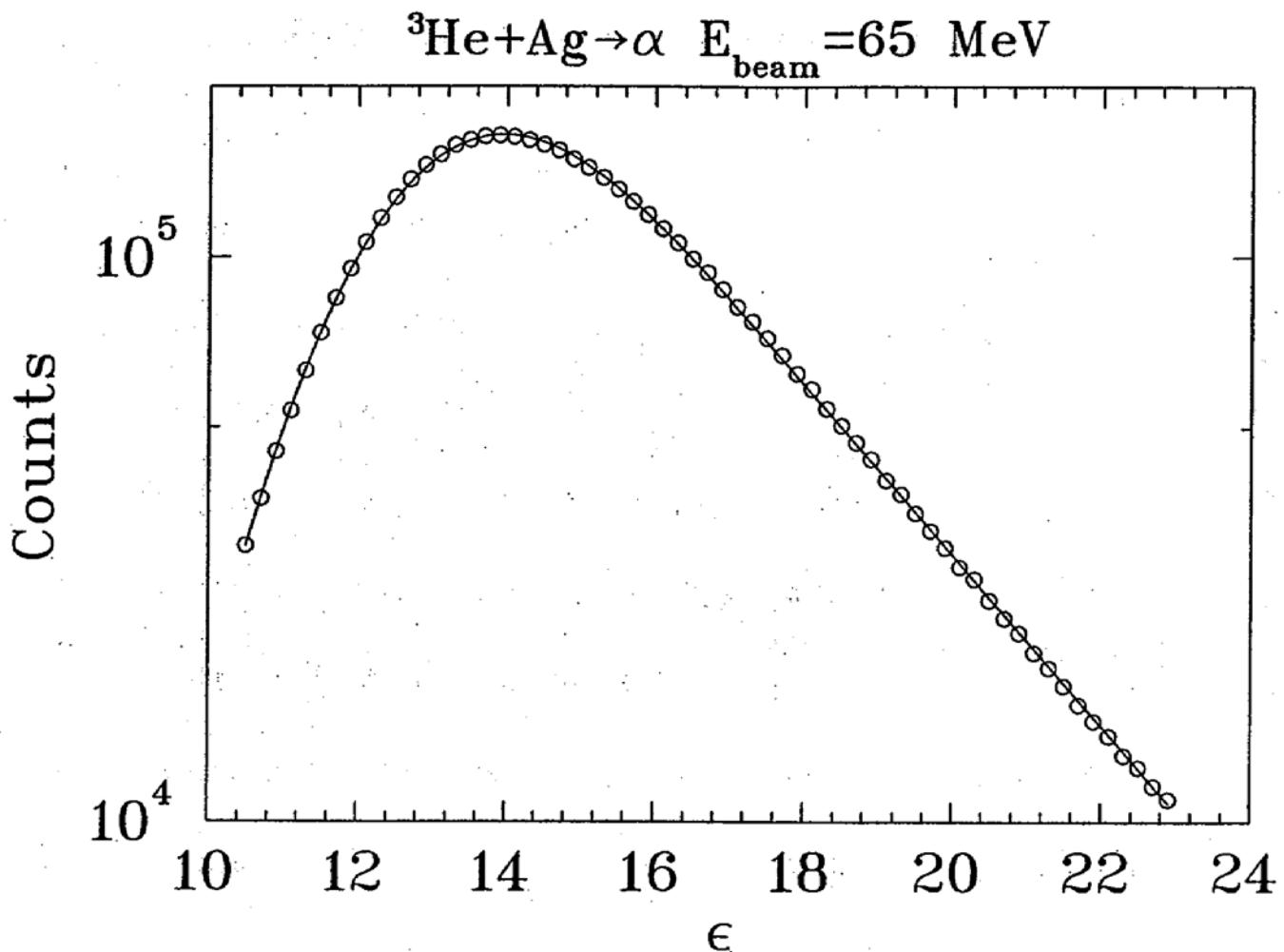


Figure 5.1: The quality of fitting experimental data with Equation 5.1. The circles represent the data, and the solid line is the fit.

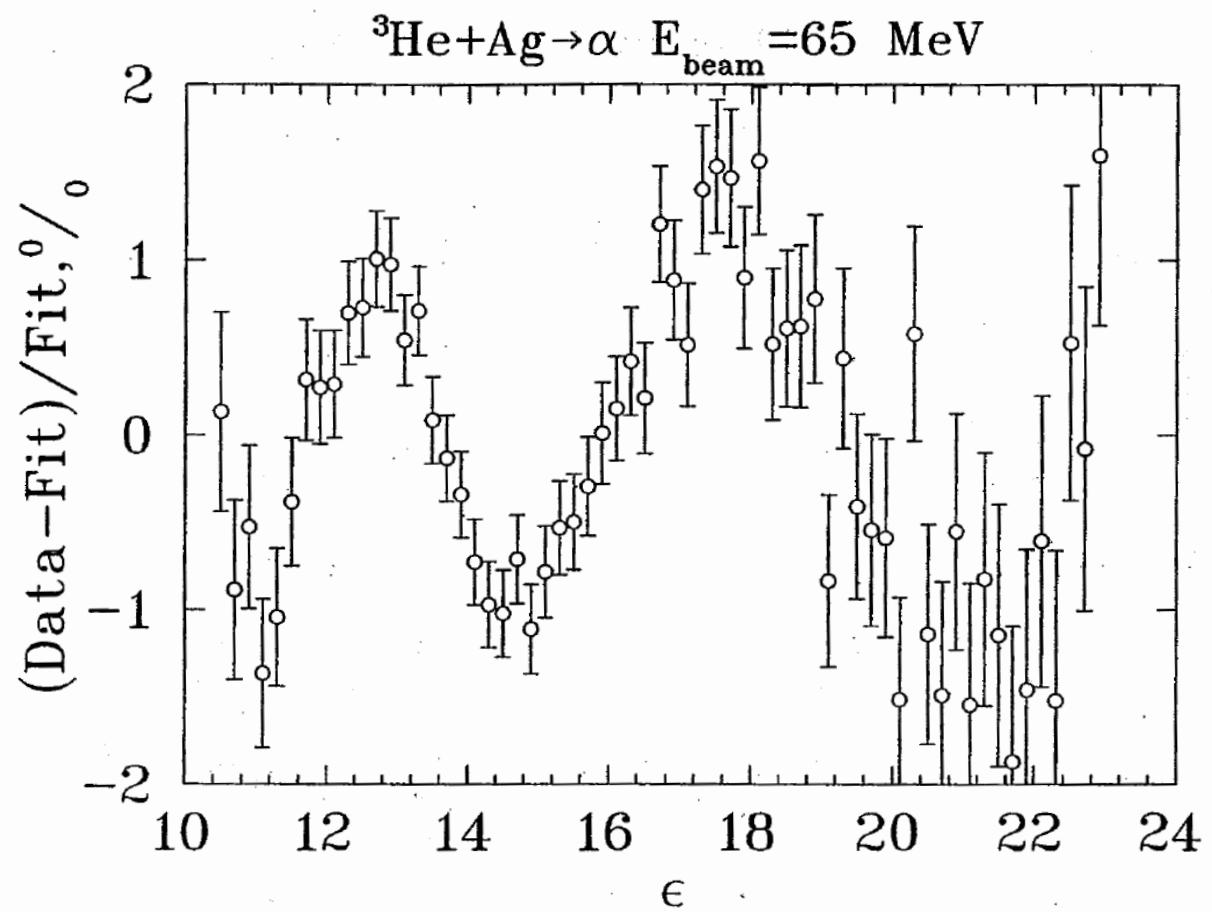


Figure 5.2: An example of oscillations observed in fitting data with Equation 5.1.

$$P(\epsilon) = A \exp\left(-\frac{\epsilon - B}{T}\right) \operatorname{erfc}\left(\frac{p - 2(\epsilon - B)}{2\sqrt{pT}}\right),$$

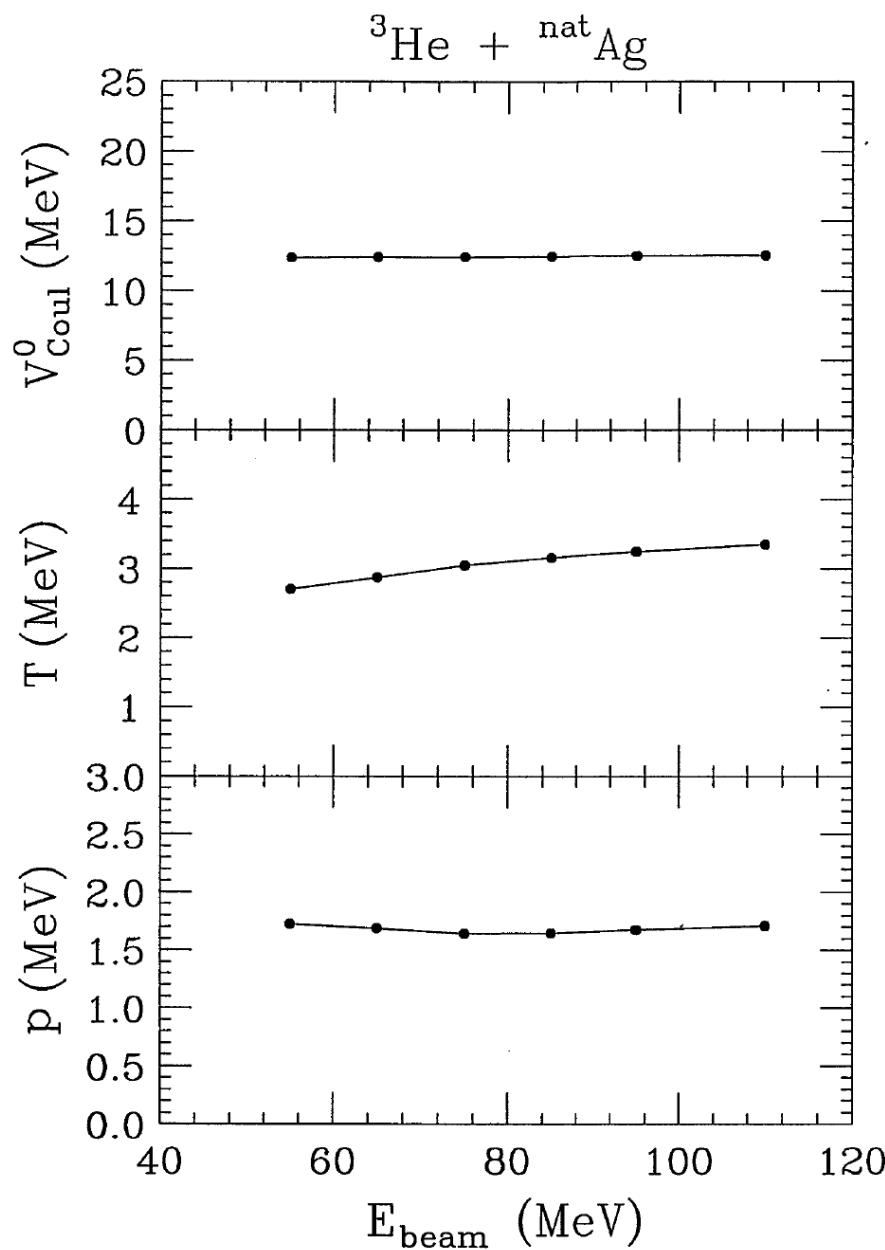


Figure 7.4: The values for the Coulomb barrier  $V_{\text{Coul}}^0$ , the temperature  $T$  of the residual nucleus, and the amplification parameter  $p$ , extracted from the fits shown in the lower panels of sextants in Fig. 7.2 for nuclei formed in the  ${}^3\text{He} + {}^{\text{nat}}\text{Ag}$  reactions, are plotted against bombarding energy. The error bars are smaller than the size of symbols.

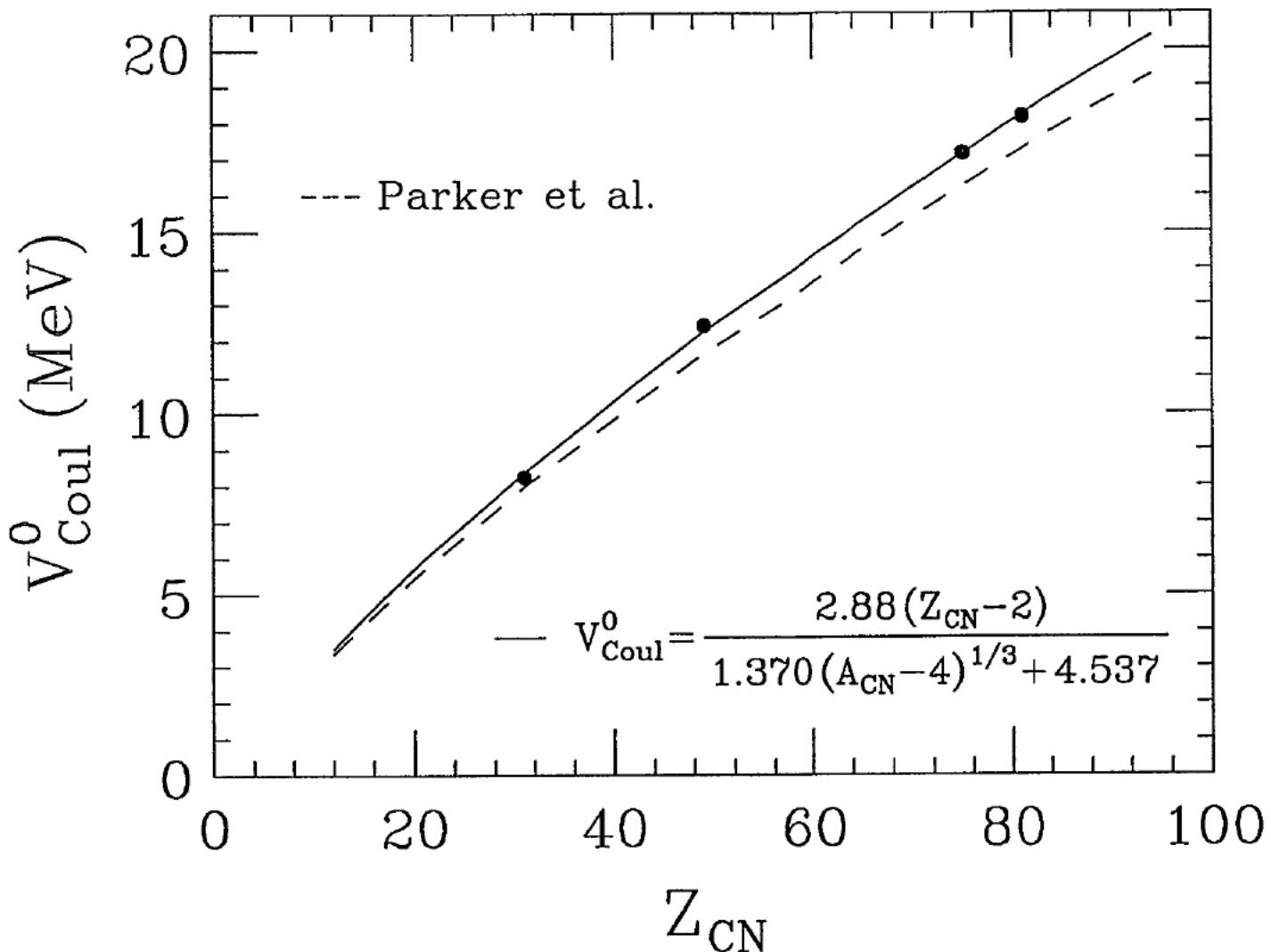


Figure 7.8: The extracted values of the Coulomb barriers  $V_{\text{Coul}}^0$  for alpha evaporation are plotted versus the atomic number  $Z_{\text{CN}}$  of the nucleus from which alpha particle is emitted. The solid line is the systematic of  $V_{\text{Coul}}^0$  established with the data points shown. The dashed line is the systematic of Coulomb barriers for alpha evaporation given by Parker *et al.* [Park 91].

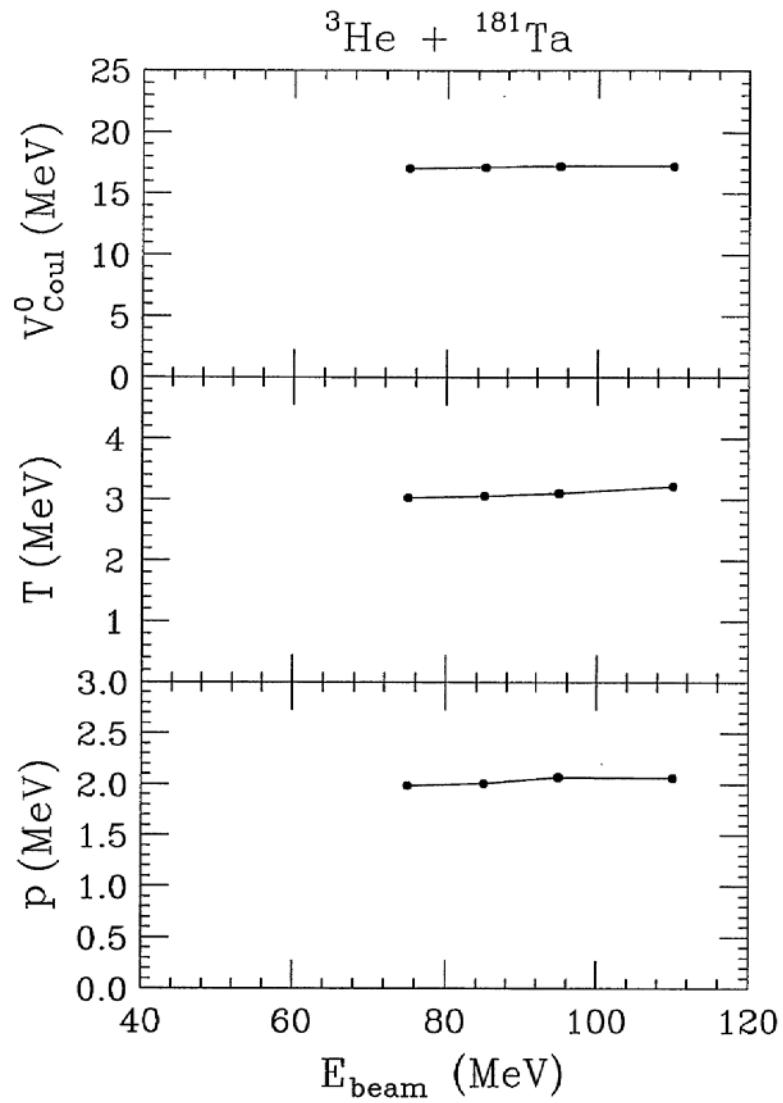


Figure 7.6: The values for the Coulomb barrier  $V_{\text{Coul}}^0$ , the temperature  $T$  of the residual nucleus, and the amplification parameter  $p$ , extracted from the fits using Eq. 7.3 to the measured alpha spectra for nuclei formed in the  ${}^3\text{He} + {}^{181}\text{Ta}$  reactions, are plotted against bombarding energy. The error bars are smaller than the size of symbols.

# Collective Scaling (Weight) Function and Associated Orthogonal Polynomials

We write down the experimental

spectrum  $F(\epsilon)$  as a linear combination of orthogonal polynomials  $P_n(\epsilon)$

$$F(\epsilon) = \sum c_n S(\epsilon) P_n(\epsilon), \quad (7.7)$$

where  $S(\epsilon)$  is a suitably chosen weight function that generates the polynomials  $P_n(\epsilon)$ ;  $c_n$  is the coefficient which can be considered as the amplitude of a spectral mode corresponding to the  $n^{\text{th}}$  order polynomial  $P_n(\epsilon)$ . The orthogonality condition is

$$\int_a^b S^2(\epsilon) P_n(\epsilon) P_m(\epsilon) d\epsilon = \begin{cases} 0, & n \neq m, \\ \dots & n = m \end{cases} \quad (7.8)$$

amplitudes  $c_n$  can be obtained from the dot product of the experimental spectrum with the  $n^{\text{th}}$  polynomial

$$c_n = \int_a^b F(\epsilon) S(\epsilon) P_n(\epsilon) d\epsilon, \quad (7.9)$$

and the corresponding strength  $s_n$  can be defined as

$$s_n = c_n^2 / \int_a^b F^2(\epsilon) d\epsilon. \quad (7.10)$$

By definition we have  $\sum s_n = 1$ .

# Collective Scaling (Weight) Function and Associated Orthogonal Polynomials

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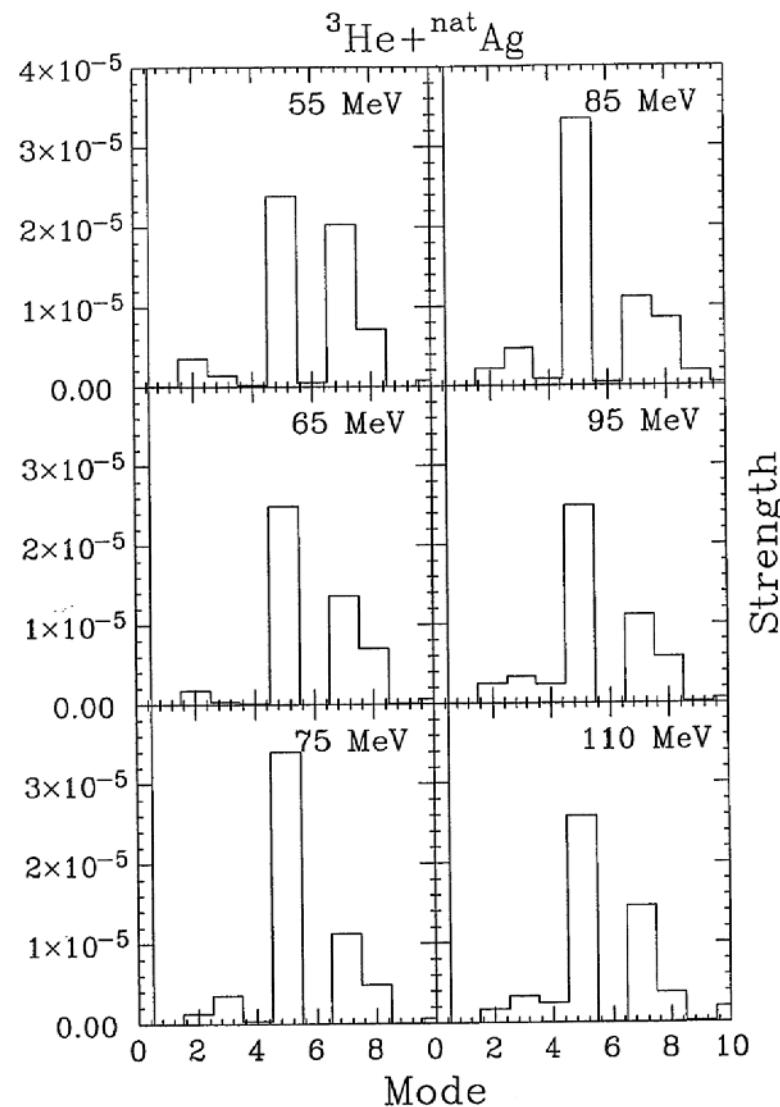


Figure 7.12: The strength  $s_n$  of the  $n^{\text{th}}$  order as defined in Eq. 7.10 plotted against the order  $n$ , for the  ${}^3\text{He} + {}^{\text{nat}}\text{Ag}$  reactions at 55, 65, 75, 85, 95, and 110 MeV bombarding energies.

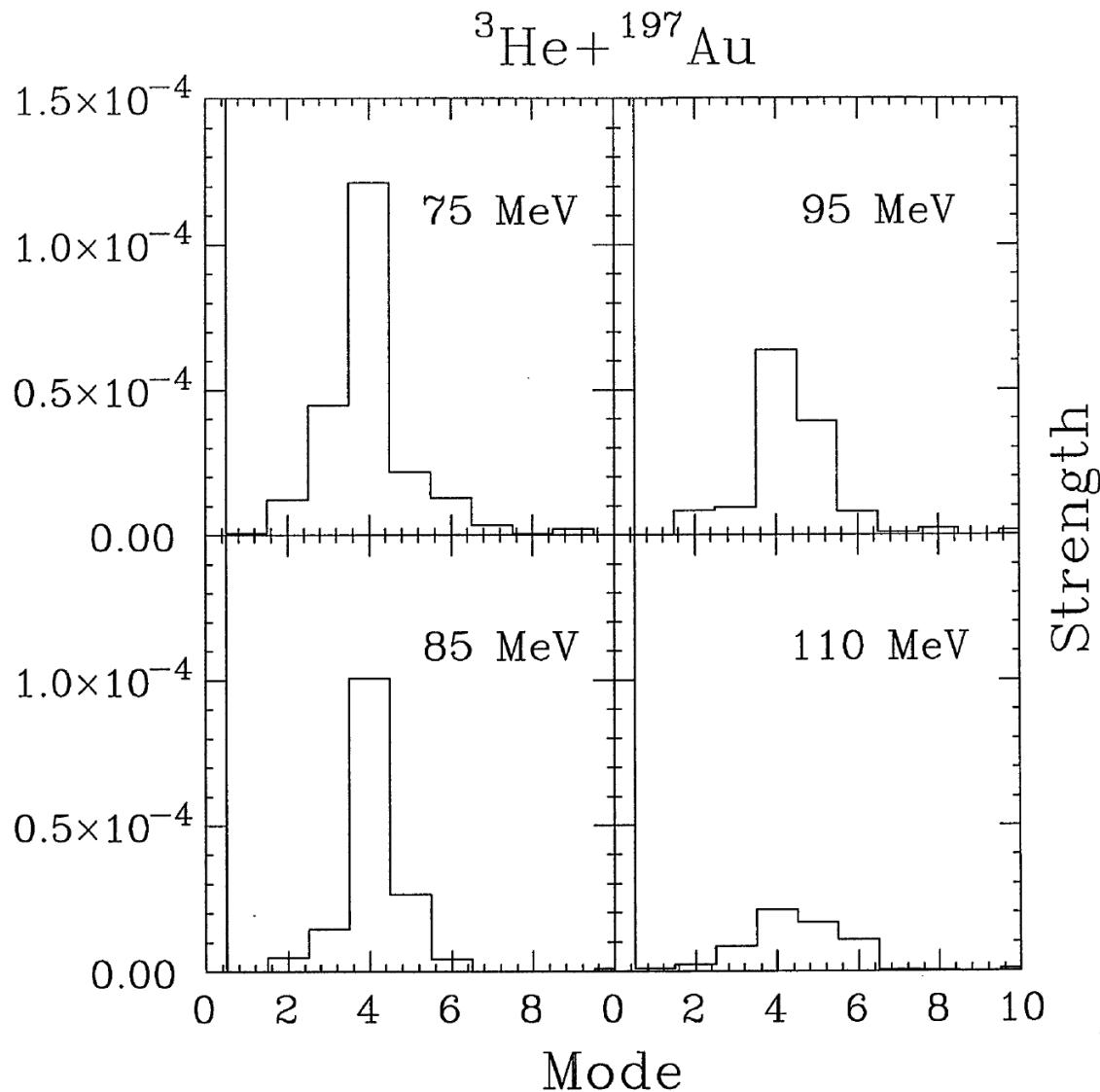


Figure 7.14: The strength  $s_n$  of the  $n^{\text{th}}$  order as defined in Eq. 7.10 plotted against the order  $n$ , for the  ${}^3\text{He} + {}^{197}\text{Au}$  reactions at 75, 85, 95, and 110 MeV bombarding energies.

# A trivial example...

Let us assume that the actual spectrum is smeared over small changes of T and B

$$\begin{aligned} P(\epsilon, B, T) &= P(\epsilon, \bar{B}, \bar{T}) \\ &+ \left. \frac{\partial P}{\partial B} \right|_{\bar{B}, \bar{T}} (B - \bar{B}) + \frac{1}{2} \left. \frac{\partial^2 P}{\partial B^2} \right|_{\bar{B}, \bar{T}} (B - \bar{B})^2 \\ &+ \left. \frac{\partial P}{\partial T} \right|_{\bar{B}, \bar{T}} (T - \bar{T}) + \frac{1}{2} \left. \frac{\partial^2 P}{\partial T^2} \right|_{\bar{B}, \bar{T}} (T - \bar{T})^2 \\ &+ \left. \frac{\partial^2 P}{\partial B \partial T} \right|_{\bar{B}, \bar{T}} (B - \bar{B})(T - \bar{T}) + \dots \end{aligned}$$

$$\overline{P}(\epsilon, B, T) = P(\epsilon, \overline{B}, \overline{T}) + \frac{1}{2} \left. \frac{\partial^2 P}{\partial B^2} \right|_{\overline{B}, \overline{T}} \sigma_B^2 + \frac{1}{2} \left. \frac{\partial^2 P}{\partial T^2} \right|_{\overline{B}, \overline{T}} \sigma_T^2 + \left. \frac{\partial^2 P}{\partial B \partial T} \right|_{\overline{B}, \overline{T}} \text{Cov}(B, T) + \dots \quad (5.8)$$

where

$$\begin{aligned} \sigma_B^2 &= \int_B \int_T w(B, T)(B - \overline{B})^2 dB dT \\ \sigma_T^2 &= \int_B \int_T w(B, T)(T - \overline{T})^2 dB dT \\ \text{Cov}(B, T) &= \int_B \int_T w(B, T)(B - \overline{B})(T - \overline{T}) dB dT \end{aligned} \quad (5.9)$$

$$\begin{aligned}
\overline{P}(\epsilon, B, T) &= A \exp\left(-\frac{\epsilon - \overline{B}}{\overline{T}}\right) \operatorname{erfc}\left(\frac{p - 2(\epsilon - \overline{B})}{2\sqrt{p\overline{T}}}\right) \\
&\quad \left[ 1 + \frac{\sigma_B^2}{2\overline{T}^2} + \frac{\sigma_B^2(\epsilon - \overline{B})}{2\overline{T}^3} \left( \frac{\epsilon - \overline{B}}{\overline{T}} - 2 \right) + \frac{\operatorname{Cov}(B, T)}{\overline{T}^2} \left( \frac{\epsilon - \overline{B}}{\overline{T}} - 1 \right) \right] \\
&+ \frac{A}{\sqrt{\pi}(pT)^{3/2}} \exp\left(-\frac{\epsilon - \overline{B}}{\overline{T}}\right) \exp\left(-\left(\frac{p - 2(\epsilon - \overline{B})}{2\sqrt{p\overline{T}}}\right)^2\right) \\
&\quad \left[ -\sigma_B^2(p - 2(\epsilon - \overline{B})) + \frac{\sigma_T^2}{16\overline{T}^2} ((p - 2(\epsilon - \overline{B}))^2 - 6p\overline{T})(p - 2(\epsilon - \overline{B})) \right. \\
&\quad \left. + \frac{p\overline{T} - (\epsilon - \overline{B})p - 2(\epsilon - \overline{B})^2}{T} \operatorname{Cov}(B, T) \right]
\end{aligned} \tag{5.11}$$

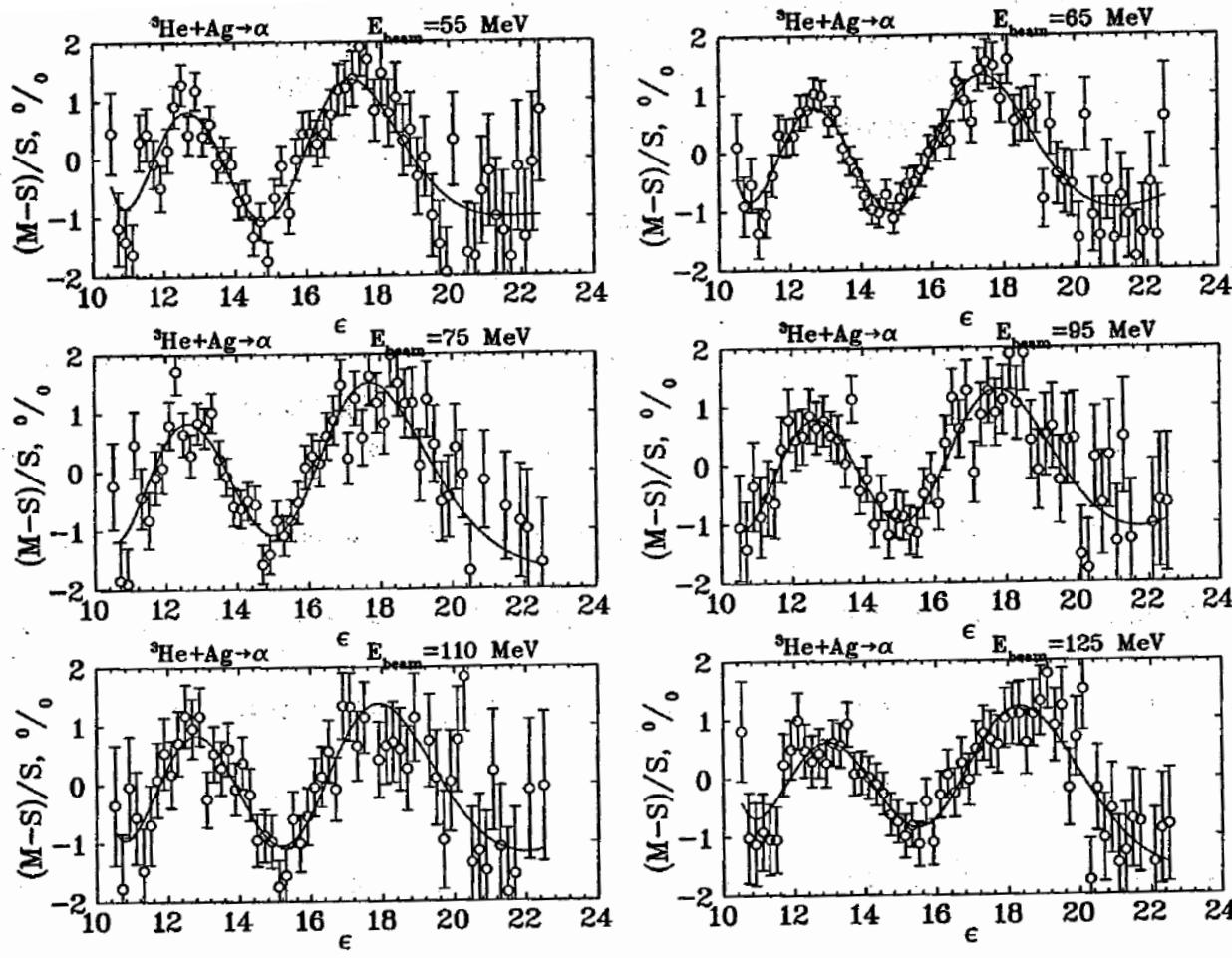


Figure 5.5: Examples of fitting the data with the moment expansion methodology at various excitation energies. The oscillations are completely described. The circles represent the relative residuals of the single-chance fit to the data, while the solid lines stand for the relative difference between the multiple-chance and single-chance fits.

# Conclusions

- 1) The introduction of amplifying and non amplifying collective modes at the conditional saddles explains the kinetic energy distributions with an accuracy of about 1%.
- 2) Residuals accumulate in a small number of higher modes, whose interpretation is yet to be found.