Int. School of Physics “Enrico Fermi”- Course 201
Nuclear Physics with Stable and Radioactive Ion Beams
Varenna
July 14-19, 2017

Recent developments in shell model studies of atomic nuclei

Takaharu Otsuka

This work has been supported by MEXT and JICFuS as a priority issue (Elucidation of the fundamental laws and evolution of the universe) to be tackled by using Post ‘K’ Computer

From the shell model perspectives,
three (possible) pillars combined for future

computation

Monte Carlo Shell Model (MCSM)

(almost) unlimited dimensionality

massive parallel computers

Hamiltonian

pf

pfg9d5 (A3DA) (Ni)

8+8 on ^{56}Ni core (Zr)

8+8 on ^{80}Zr core (Sn)

8+10 on ^{132}Sn core (Sm)

…

island of stability

+

χEFT based (multi-)shell int.

many-body dynamics

Shell evolution (Type I & II)

Quantum Phase Transition

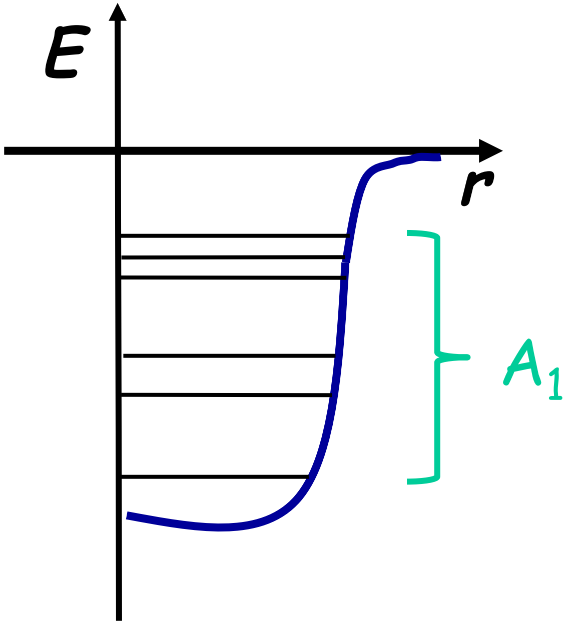
Shape coexistence

Quantum Self-organization

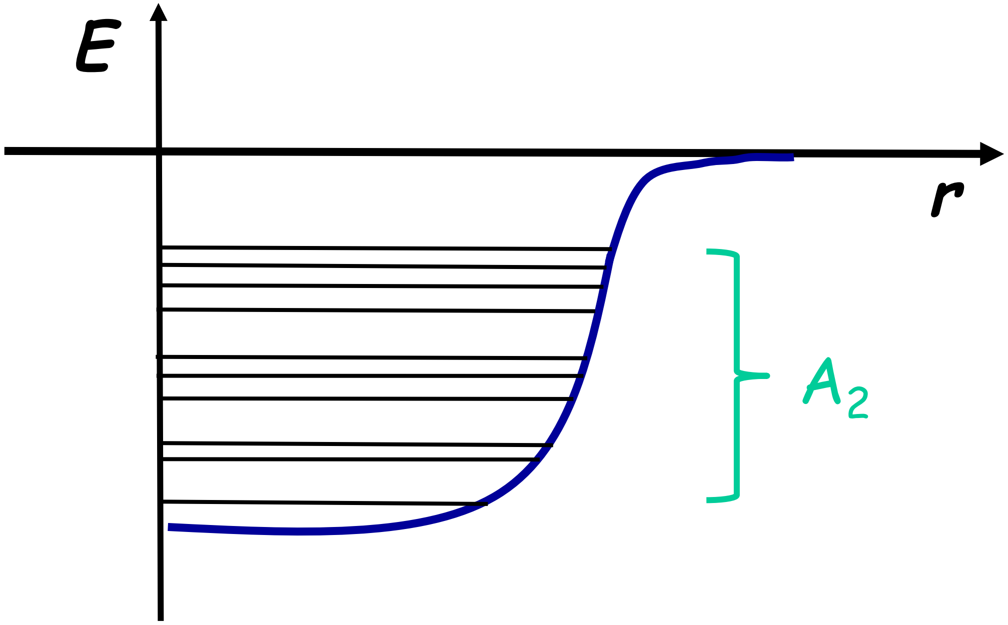
Single-particle states - starting point -

Mean potential becomes wider so as to cast *A* nucleons with the same separation energy, keeping its depth.

light nuclei



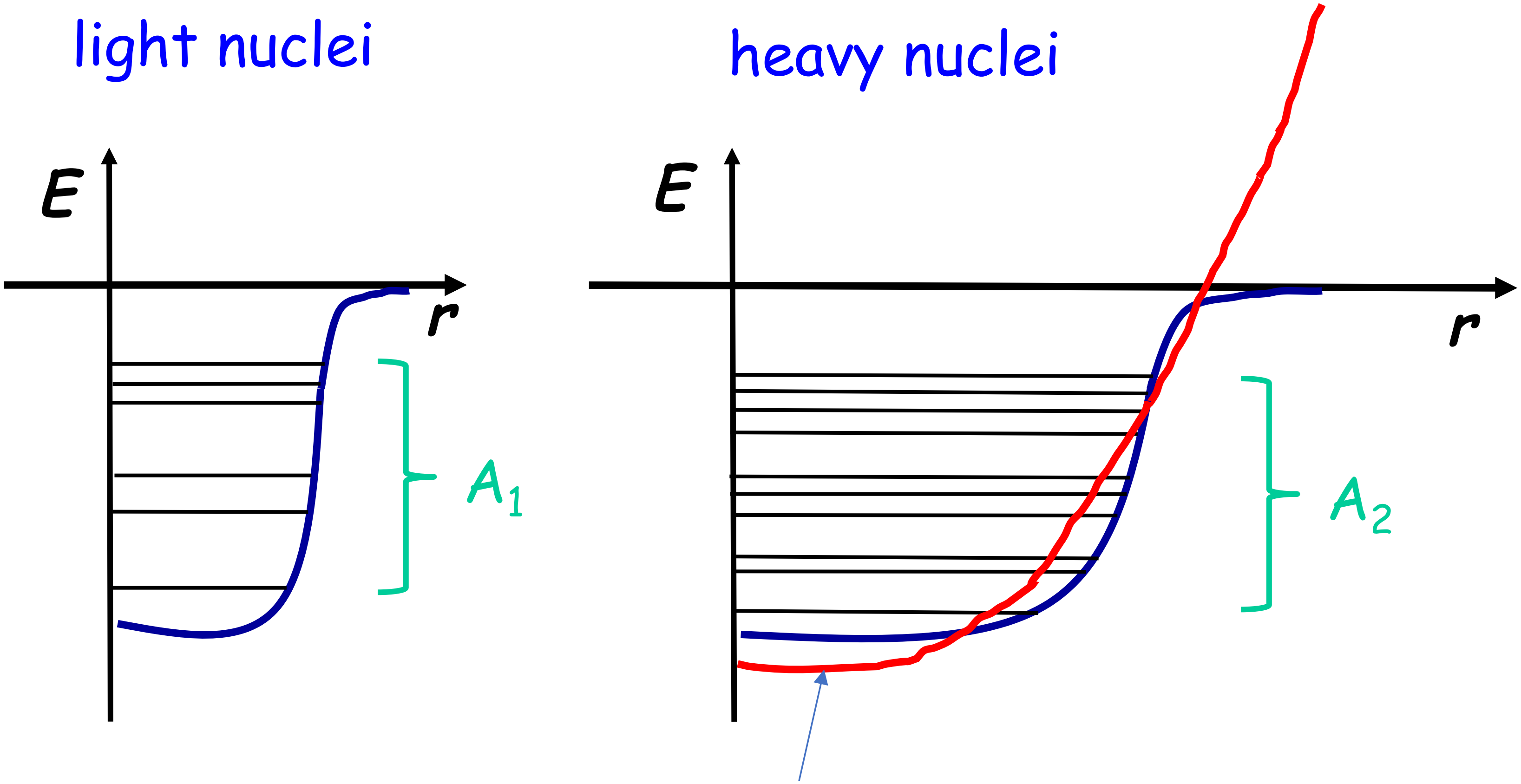
heavy nuclei



This potential can be approximated by a Harmonic Oscillator potential.

Single-particle states - starting point -

Mean potential becomes wider so as to cast *A* nucleons with the same separation energy, keeping its depth.



This potential can be approximated by a Harmonic Oscillator potential.

Eigenvalues of HO potential

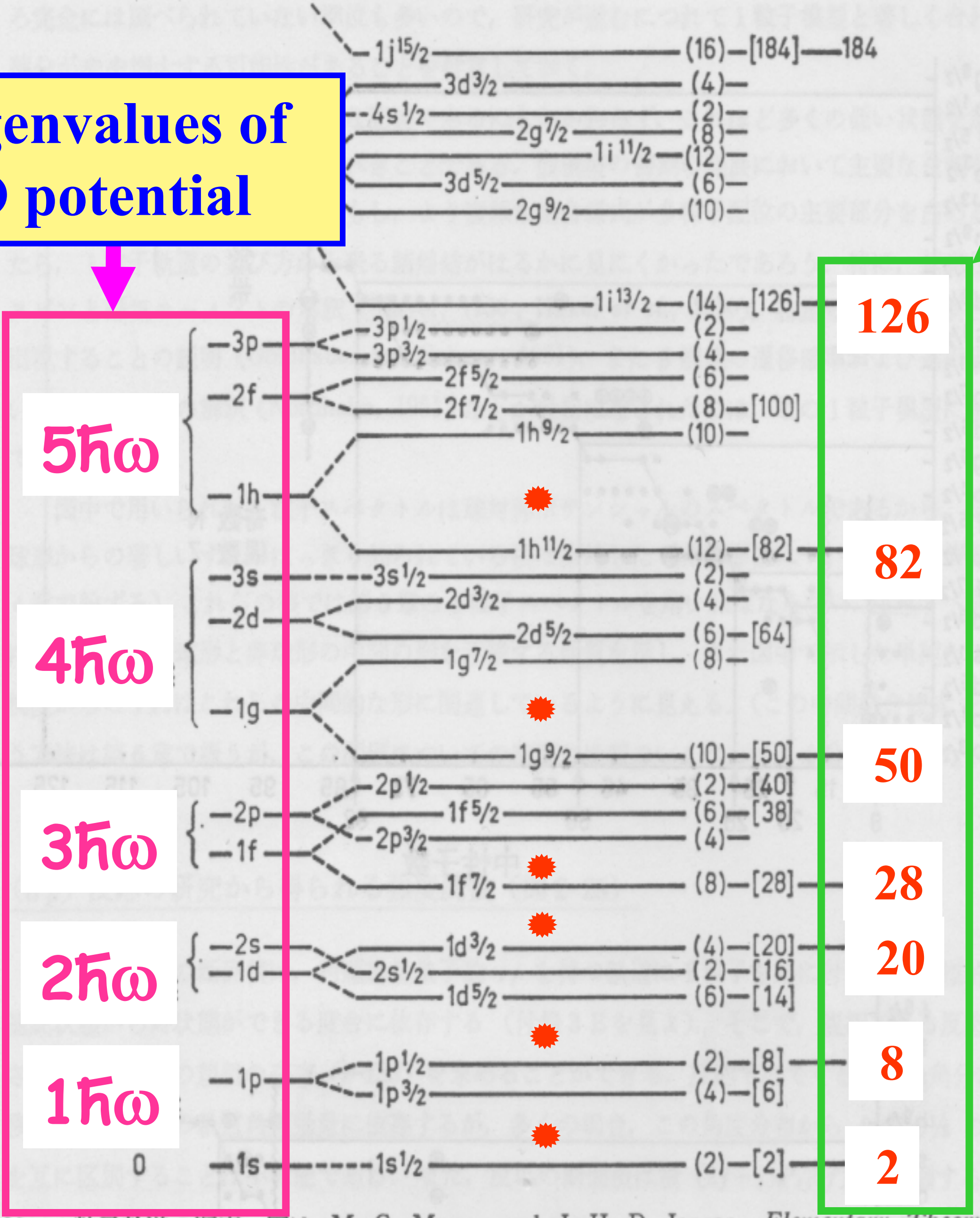
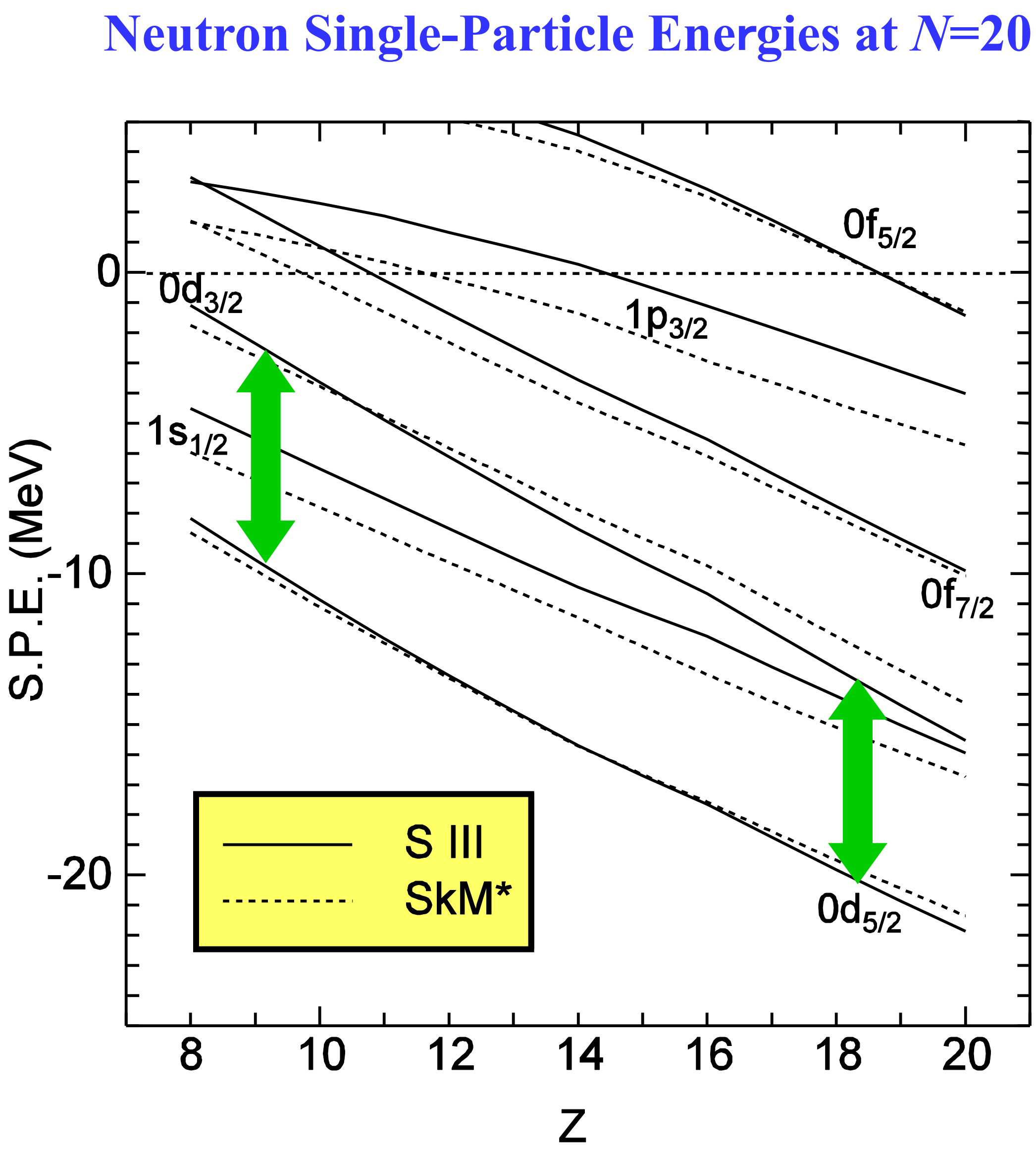


図2-23　1粒子軌道の順序．図は M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955 からとった．

Magic numbers by Mayer and Jensen (1949)

Realization in Hartree-Fock energies by Skyrme model



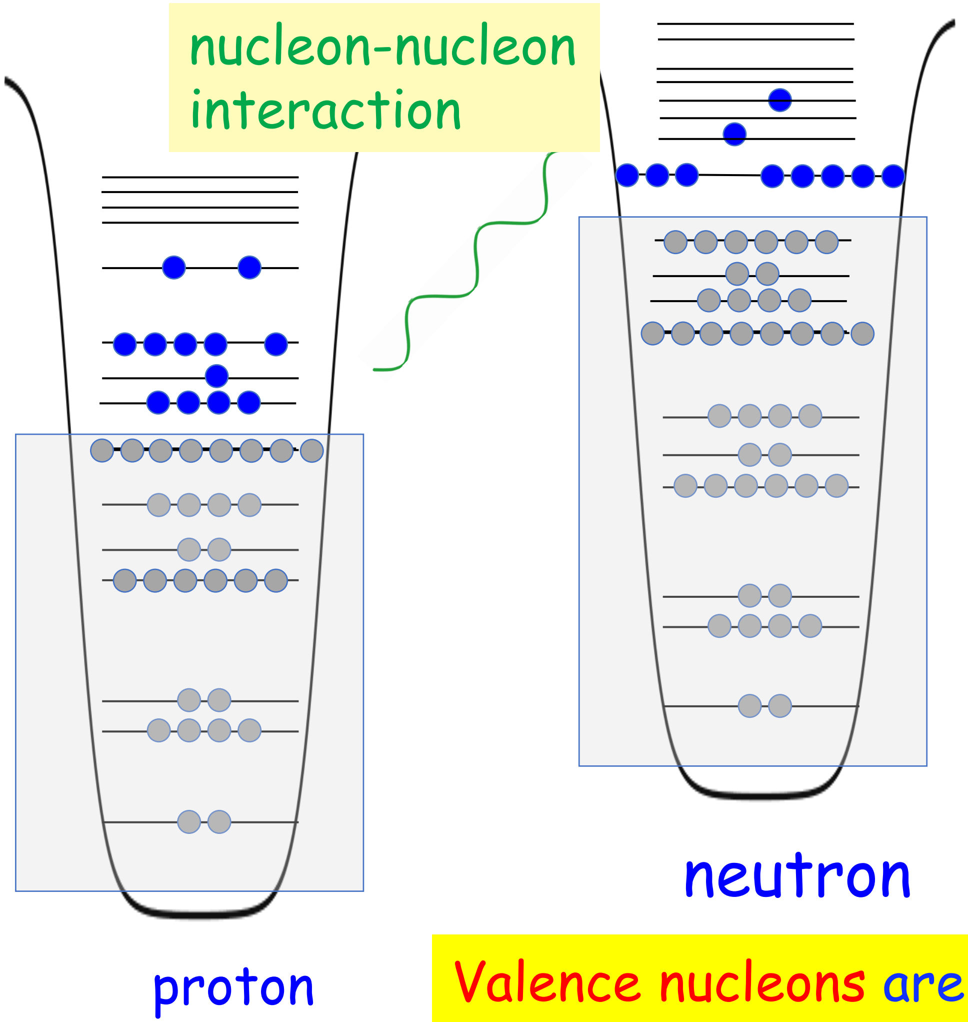
The shell structure remain rather unchanged

-- orbitals shifting together

-- change of potential depth

~ Woods-Saxon.

shell structure and nucleon-nucleon interaction



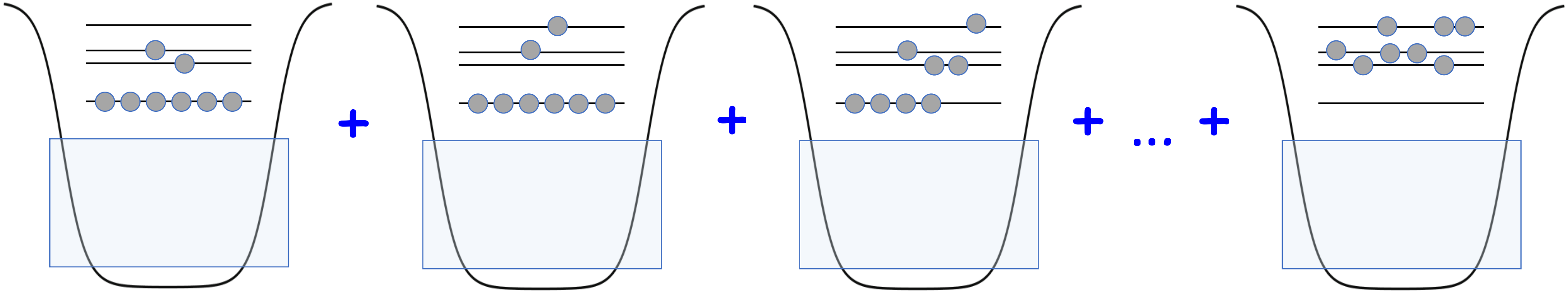
Protons and neutrons are orbiting in the mean potential like a "vase"
→ single-particle energies

Lower orbits form the inert core (or closed shell) (*shaded parts in the figure*)

Upper orbits are only partially occupied (valence orbits and nucleons).

Valence nucleons are the major source of nuclear dynamics at low excitation energy, because the inert core is *frozen (implicitly taken into account in terms of effective interaction and operators).*

Possible configurations : dimension of the shell-model calculation



How can we find the solution of this problem ?

Hamiltonian in shell model calculations

ε_i : Single Particle Energy (SPE)

<j1, j2, J, T | V | j3, j4, J, T >

: Two-Body Matrix Element (TBME)

$$H = \Sigma_i \boxed{\epsilon_i} n_i + \Sigma_{i,j,k,l} \boxed{v_{ij,kl}} a_i^\dagger \, a_j^\dagger \, a_l \, a_k$$

Step 1: Calculate matrix elements

$$\boxed{< \phi_1 | H | \phi_1 >, \quad < \phi_1 | H | \phi_2 >, \quad < \phi_1 | H | \phi_3 >, \quad ....}$$

where ϕ_1 , ϕ_2 , ϕ_3 are Slater determinants

In the second quantization,

closed shell

$$\phi_1 = a_\alpha^+ \; a_\beta^+ \; a_\gamma^+ \; ..... \boxed{| 0 >}$$

$$\phi_2 = a_{\alpha'}^+ \; a_{\beta'}^+ \; a_{\gamma'}^+ \; ..... \boxed{| 0 >}$$

$$\phi_3 = ...$$

$$H = \Sigma_i \, \epsilon_i \, n_i + \Sigma_{i,j,k,l} \, v_{ij,kl} \, a_i^\dagger \, a_j^\dagger \, a_l \, a_k$$

Step 2 : Obtain the matrix of Hamiltonian, **H**

$$\mathbf{H} = \begin{bmatrix} <\phi_1 |\mathbf{H}| \phi_1> & <\phi_1 |\mathbf{H}| \phi_2> & <\phi_1 |\mathbf{H}| \phi_3> & \cdots\cdot \\ <\phi_2 |\mathbf{H}| \phi_1> & <\phi_2 |\mathbf{H}| \phi_2> & <\phi_2 |\mathbf{H}| \phi_3> & \cdots\cdot \\ <\phi_3 |\mathbf{H}| \phi_1> & <\phi_3 |\mathbf{H}| \phi_2> & <\phi_3 |\mathbf{H}| \phi_3> & \cdots\cdot \\ <\phi_4 |\mathbf{H}| \phi_1> & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

Step 3: Solve the eigenvalue problem : $H \Psi = E \Psi$

$$\begin{pmatrix} < \phi_1 |H| \phi_1 > & < \phi_1 |H| \phi_2 > & \cdot \;\; \cdot \;\; \cdot \\ < \phi_2 |H| \phi_1 > & < \phi_2 |H| \phi_2 > & \cdot \;\; \cdot \;\; \cdot \\ < \phi_3 |H| \phi_1 > & \cdot & \cdot \;\; \cdot \;\; \cdot \\ < \phi_4 |H| \phi_1 > & \cdot & \cdot \;\; \cdot \;\; \cdot \\ \cdot & \cdot & \cdot \;\; \cdot \;\; \cdot \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ \cdot \\ \cdot \end{pmatrix} = E \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ \cdot \\ \cdot \end{pmatrix}$$

$$\Psi = c_1 \phi_1 + c_2 \phi_2 + c_3 \phi_3 + \ldots..$$

c_i **probability amplitudes**

With Slater determinants ϕ_1, ϕ_2, ϕ_3 ,...,

the eigen wave function is expanded as

$$\Psi = c_1 \phi_1 + c_2 \phi_2 + c_3 \phi_3 + ...$$

c_i **probability amplitudes**

With this, we can calculate various physical quantities by $< \Psi' | T | \Psi >$.

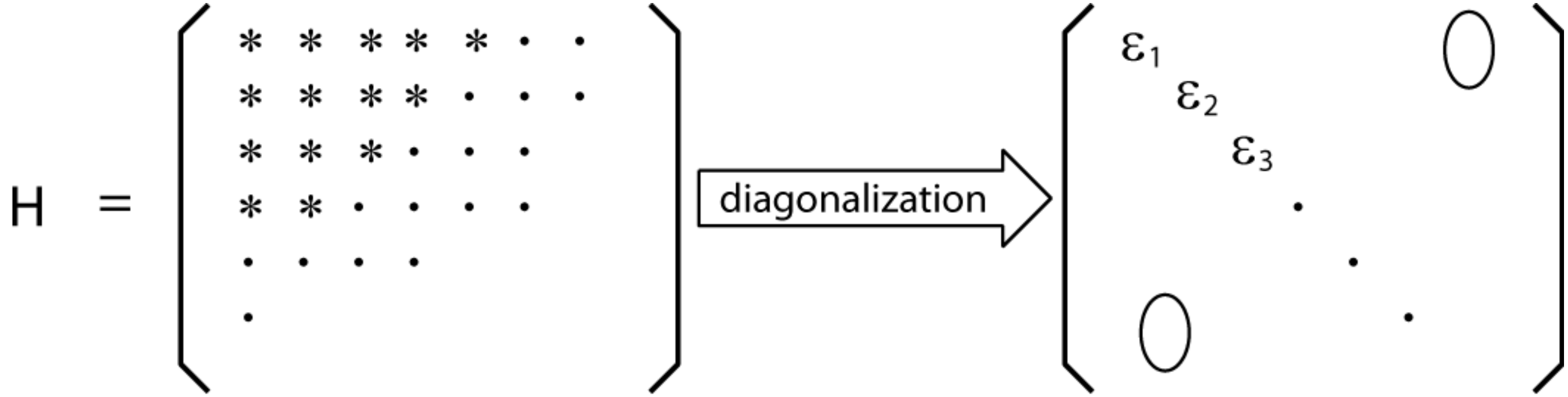
For instance, E2, M1, ... matrix elements

(transitions and moments)

spectroscopic factors (SF) and two-nucleon amplitudes (TNA) for transfer reactions

Two types of shell-model calculations

dim. <~ 10^{10}



Conventional Shell Model

all Slater determinants

Direct diagonalization

For even bigger problem,

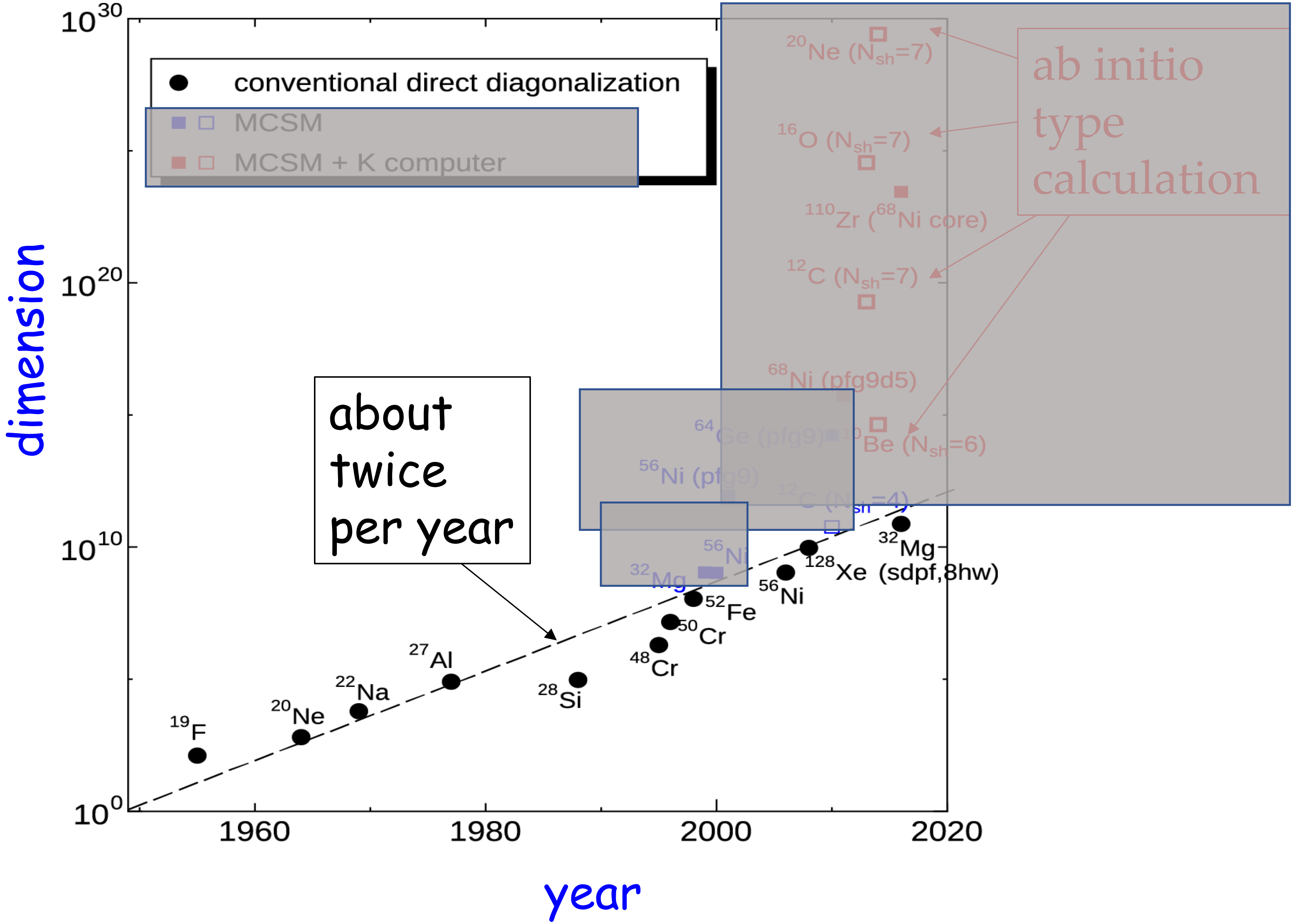
~100 dim.

Monte Carlo Shell Model

bases important for a specific eigenstate

Selected
important basis vectors

Dimension of the shell-model calculation



about twice per year

ab initio type calculation

Monte Carlo Shell Model

Auxiliary-Field Monte Carlo (AFMC) method

general method for quantum many-body problems

For nuclear physics, **Shell Model Monte Carlo (SMMC)** calculation has been introduced by **Koonin** et al. Good for finite temperature.

- **minus-sign problem**
- **only ground state, not for excited states in principle.**

Quantum Monte Carlo Diagonalization (QMCD) method

No sign problem. Symmetries can be restored.
Excited states can be obtained.

→ **Monte Carlo Shell Model (MCSM)**

Background of Monte Carlo Shell Model (I)

Two-body interaction can be rewritten as $V = (1/2)\, \Sigma_\alpha\ \mathbf{v}_\alpha\, \mathbf{O}_\alpha{}^2$

α : index
$\mathbf{O}_\alpha$: one-body operators (rearranged by diagonalization)

Hubberd-Stratonovichi transformation

True eigenstate : $\psi = \Sigma_\sigma\, e^{-\beta h(\sigma)}\ e^{-\beta h(\sigma')}\ \ldots\ \psi_0$

imaginary time (β) evolution by one-body field $h(\sigma)$

One-body operator is introduced as $h(\sigma) = \Sigma_\alpha\ \mathbf{s}_\alpha\, \sigma_\alpha\, \mathbf{v}_\alpha\, \mathbf{O}_\alpha$

σ_α : random number (Gaussian) σ : set of σ_α's
s_α : phase (1 for $\mathbf{v}_\alpha < 0$, i for $\mathbf{v}_\alpha > 0$)

Background of Monte Carlo Shell Model (II)

True eigenstate : $\psi = \Sigma_{\sigma,\sigma', \dots} e^{-h(\sigma)} e^{-h(\sigma')} \dots \psi_0$

Use $\phi(\sigma,\sigma',\dots) = e^{-\beta h(\sigma)} e^{-\beta h(\sigma')} \dots \psi_0$
as a basis for shell-model diagonalization

$\phi(\sigma,\sigma',\dots)$ are selected and refined :

(i) Random sampling -> only those lowering energy are kept
(ii) Polished by varying **$\sigma,\sigma',\dots$** gradually (random noise reduced)
(iii) Symmetry restoration (Angular momentum, parity)

Slater determinants (or Cooper-pair type wave functions) are used

Usually, 20~50 bases are kept (many more thrown away)

Advanced Monte Carlo Shell Model (currently used)

N_B : number of basis vectors (dimension)

N_p : number of (active) particles

N_{sp} : number of single-particle states

$$|\Psi(D)\rangle = \sum_{n=1}^{N_B} c_i P^{J,\Pi} |\phi(D^{(n)})\rangle$$

amplitude

Projection op.

$$|\phi(D^{(n)})\rangle = \prod_{\alpha=1}^{N_p} \underbrace{\left(\sum_{i=1}^{N_{sp}} a_i^\dagger D_{i\alpha}^{(n)} \right)}_{\text{Stochastically “deformed” single-particle state}} |-\rangle$$

n-th basis vector (Slater determinant)

$$E(D) = \langle \Psi(D) | H | \Psi(D) \rangle$$

Minimize $E(D)$ as a function of D utilizing qMC and conjugate gradient methods

Step 1 : stochastic generation of candidates of the n-th MCSM basis vector

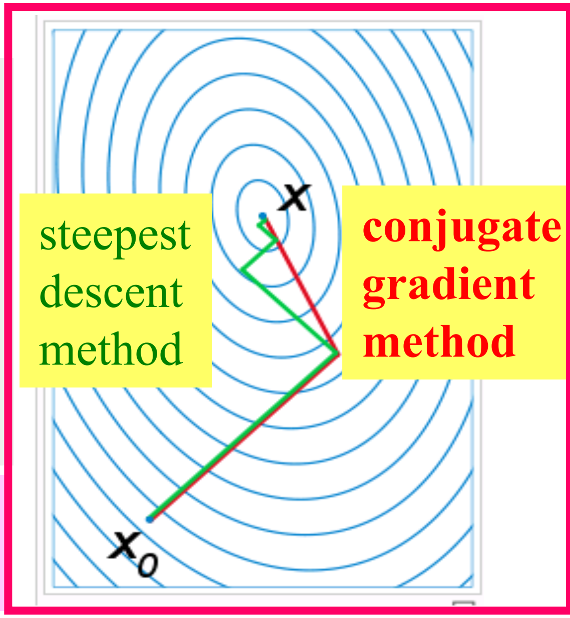
$\cancel{|\phi(\sigma)\rangle = \prod e^{\Delta\beta \cdot h(\sigma)}} \cdot |\phi^{(0)}\rangle$ *only theoretical background*

Shift randomly matrix elements of the matrix D.

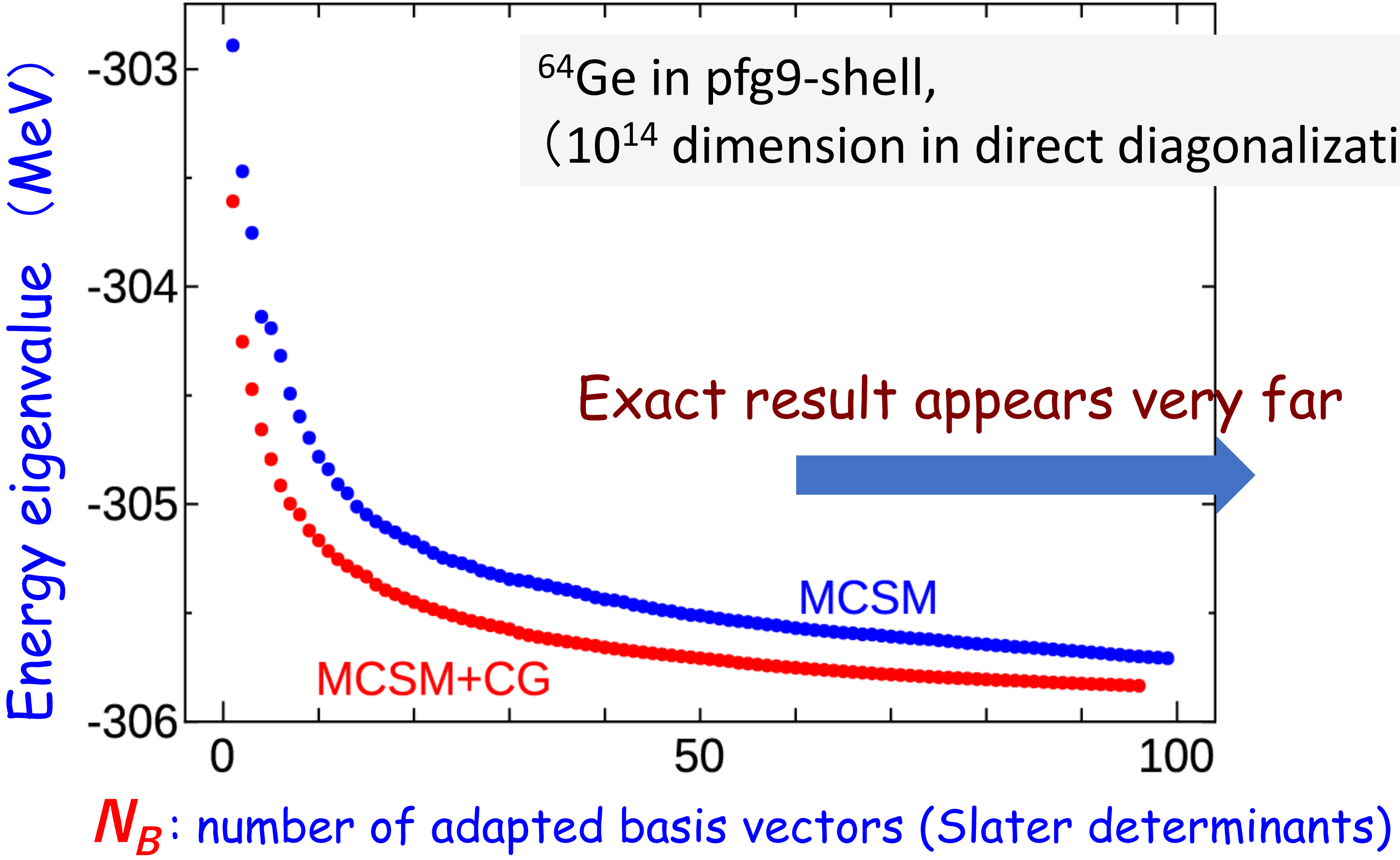
(The very initial one can be a Hartree-Fock state.)

Select the one producing the lowest *E(D)* (rate < 0.1 %)

Step 2 : polish *D* by means of the conjugate gradient (CG) method “variationally”.



Example of MCSM calculation



^{64}Ge in pfg9-shell,
(10^{14} dimension in direct diagonalization)

Exact result appears very far

Numerous MC trials and CG optimization for each basis vector

Advanced Monte Carlo Shell Model (currently used)

N_B : number of basis vectors (dimension)

$$|\Psi(D)\rangle = \sum_{n=1}^{N_B} c_i P^{J,\Pi} \left|\phi(D^{(n)})\right\rangle$$

amplitude

Projection op.

N_p : number of (active) particles

N_{sp} : number of single-particle states

$$\left|\phi(D^{(n)})\right\rangle = \prod_{\alpha=1}^{N_p} \underbrace{\left(\sum_{i=1}^{N_{sp}} a_i^\dagger D_{i\alpha}^{(n)}\right)}_{\text{Deformed single-particle state}} |-\rangle$$

n-th basis vector
(Slater determinant)

$$E(D) = \langle \Psi(D) | H | \Psi(D) \rangle$$

Minimize *E(D)* as a function of *D* utilizing qMC and conjugate gradient methods

Step 1 : stochastic generation of candidates of the n-th MCSM basis vector

$$\cancel{|\phi(\sigma)\rangle = \prod e^{\Delta\beta \cdot h(\sigma)} \cdot} |\phi^{(0)}\rangle$$

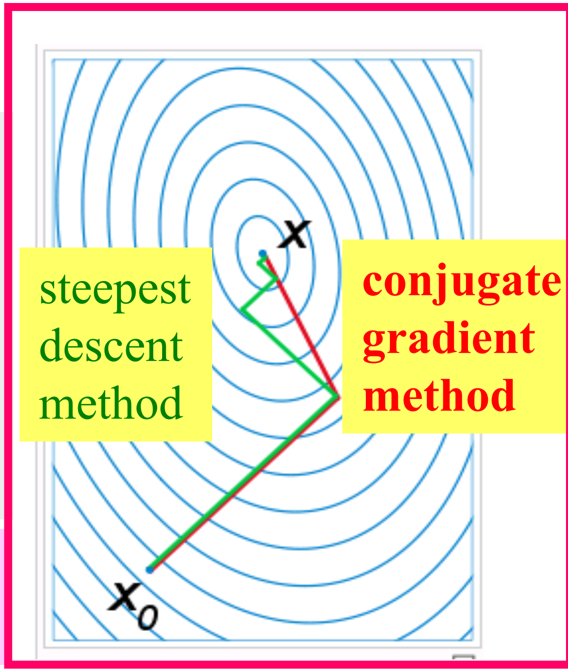
only theoretical background

Shift randomly matrix elements of the matrix D.

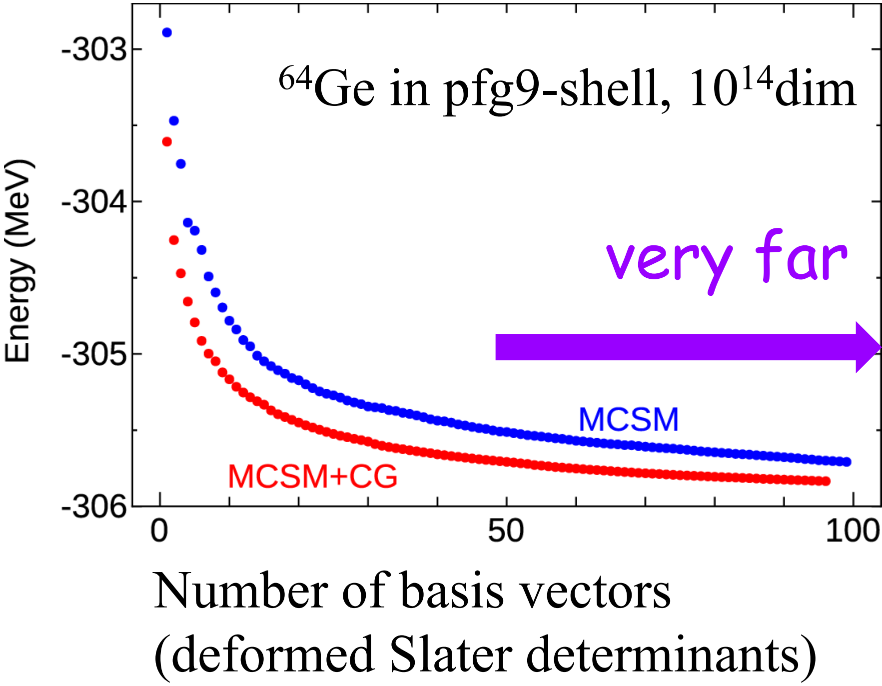
(The very initial one can be a Hartree-Fock state.)

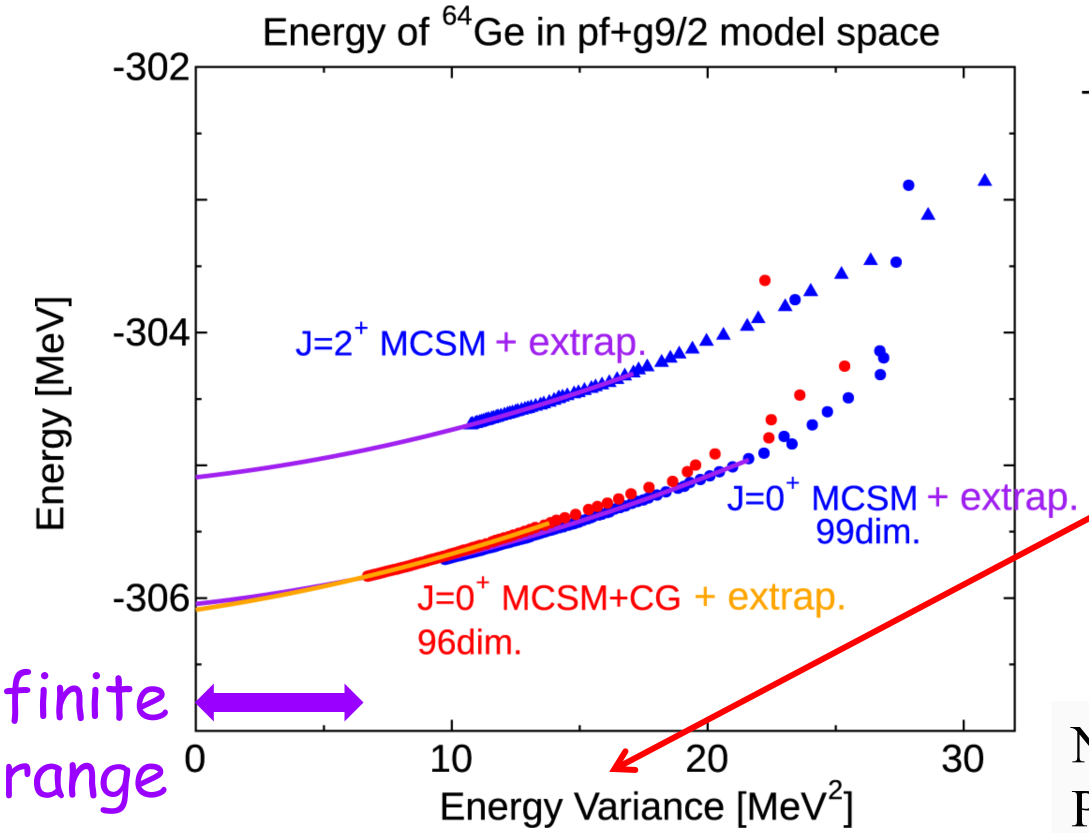
Select the one producing the lowest *E(D)* (rate < 0.1 %)

Step 2 : polish *D* by means of the conjugate gradient (CG) method “variationally”.



Extrapolation by Energy Variance





Variance : $\langle \Delta H^2 \rangle = \langle H^2 \rangle - \langle H \rangle^2$

$$\langle H \rangle = E_0 + a\langle \Delta H^2 \rangle + b\langle \Delta H^2 \rangle^2 + \ldots$$

N. Shimizu, et al.,
Phys. Rev. C **82,** 061305(R) (2010).

MCSM (Monte Carlo Shell Model -*Advanced version*-)

1. Selection of important many-body basis vectors
   by quantum Monte-Carlo + diagonalization methods
   basis vectors : about 100 selected Slater determinants
   composed of "deformed" single-particle states
2. Variational refinement of basis vectors
   conjugate gradient method
3. Variance extrapolation method -> exact eigenvalues

+ innovations in algorithm and code (=> now moving to GPU)

K computer (in Kobe) 10 peta flops machine

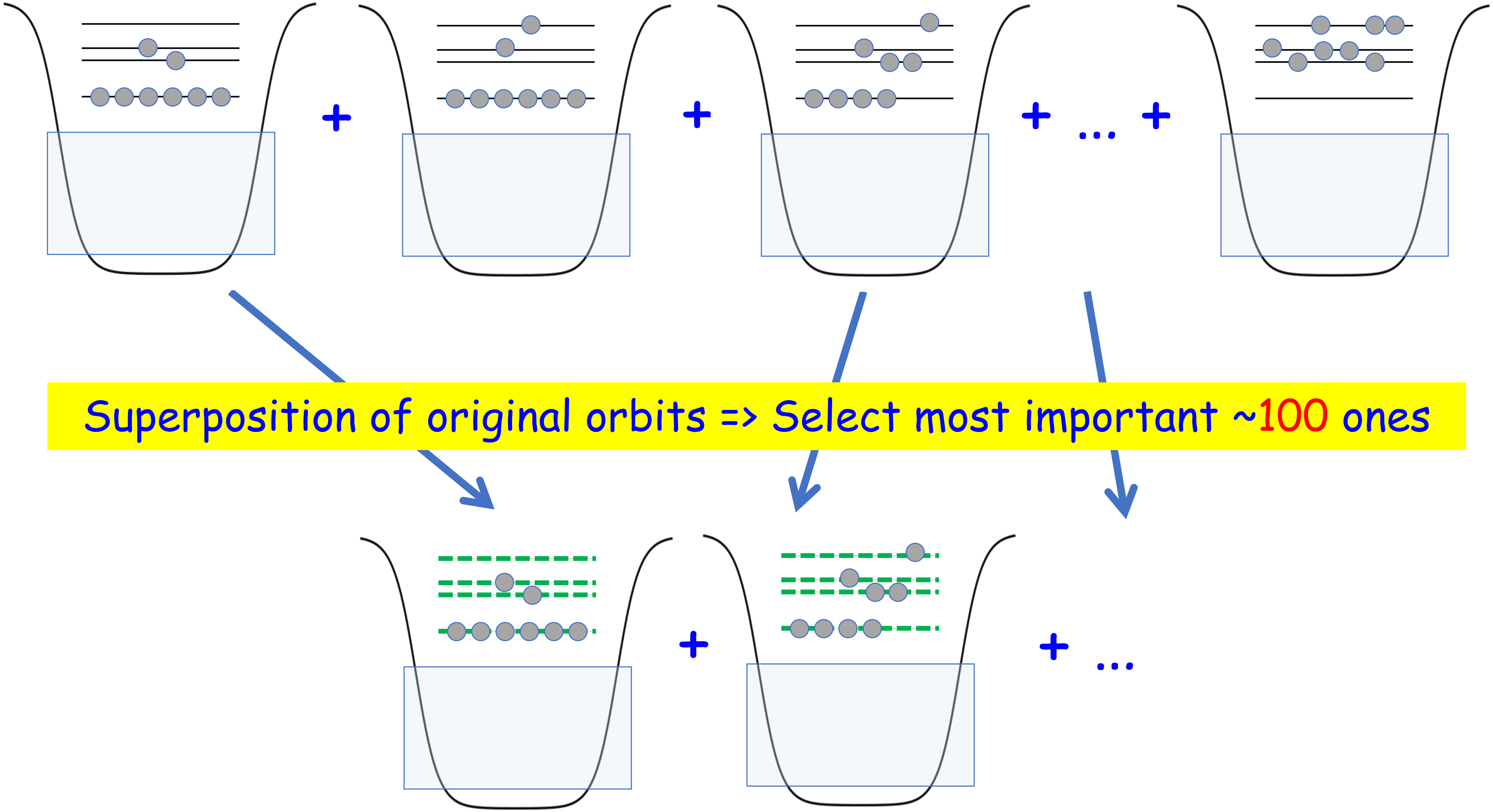
⇒ *Projection of basis vectors*

Rotation with three Euler angles

with about 50,000 mesh points

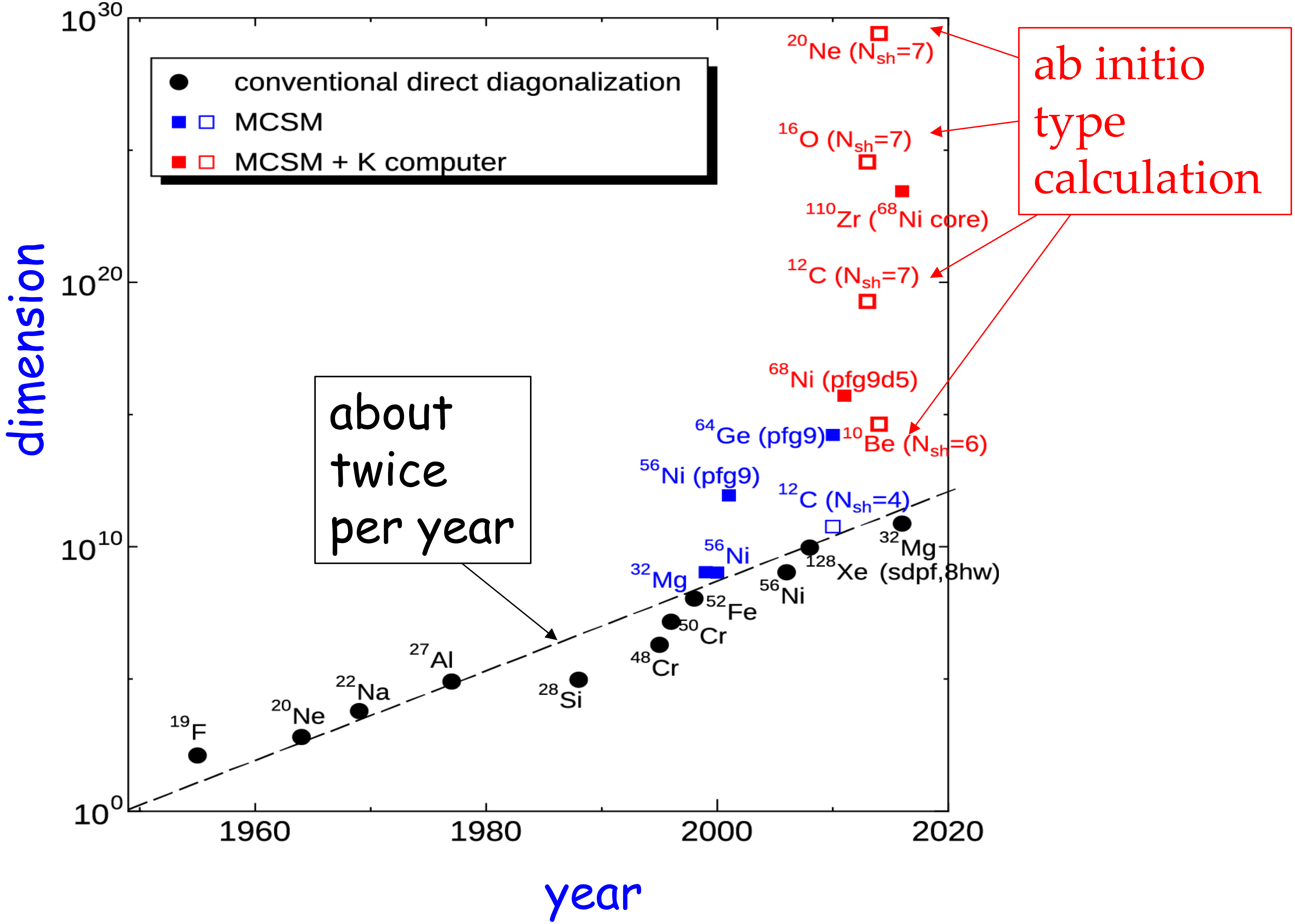
Example : 8^+ ^{68}Ni 7680 core x 14 h

Possible configurations: 10^{23} ways at maximum for Zr isotopes to be discussed



Superposition of original orbits => Select most important ~100 ones

Dimension of the shell-model many-body Hilbert space



about twice per year

ab initio type calculation

MCSM basis vectors on Potential Energy Surface

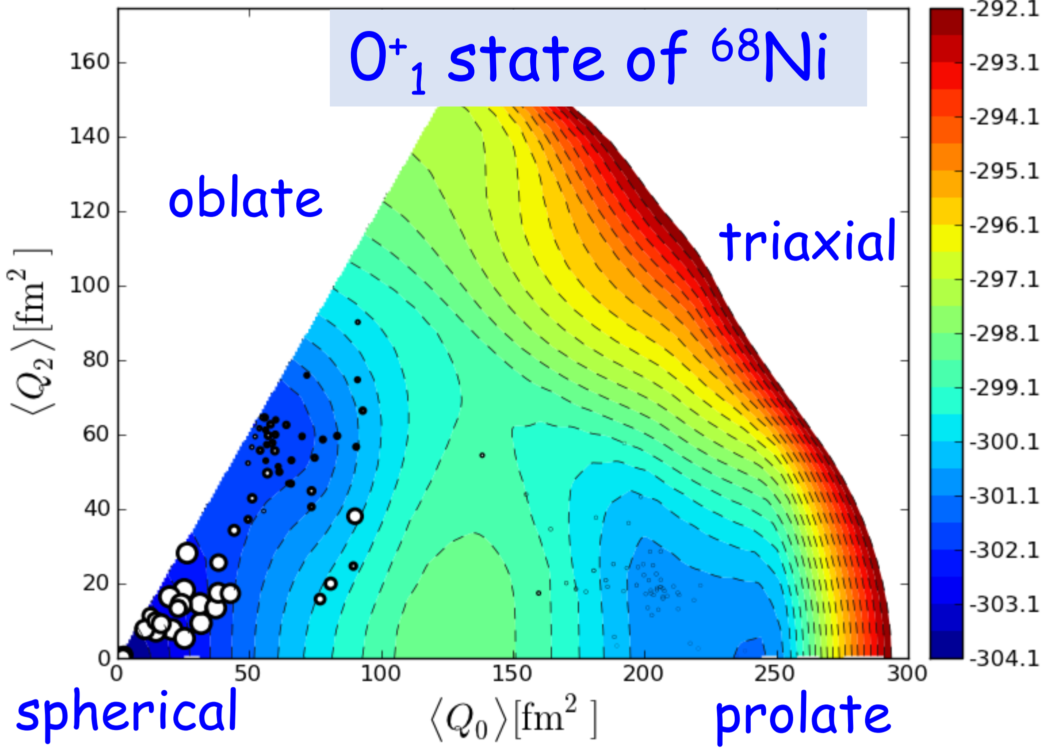
eigenstate $$\Psi = \sum_i c_i P[J^\pi] \boxed{\Phi_i}$$

amplitude

projection onto J^π

Slater determinant of deformed s. p. states → intrinsic shape

- PES is calculated by CHF for the shell-model Hamiltonian
- Location of circle : quadrupole deformation of unprojected MCSM basis vectors
- Area of circle : overlap probability between each projected basis and eigen wave function



Called ***T-plot*** in reference to

Y. Tsunoda, *et al.*

PRC 89, 031301 (R) (2014)

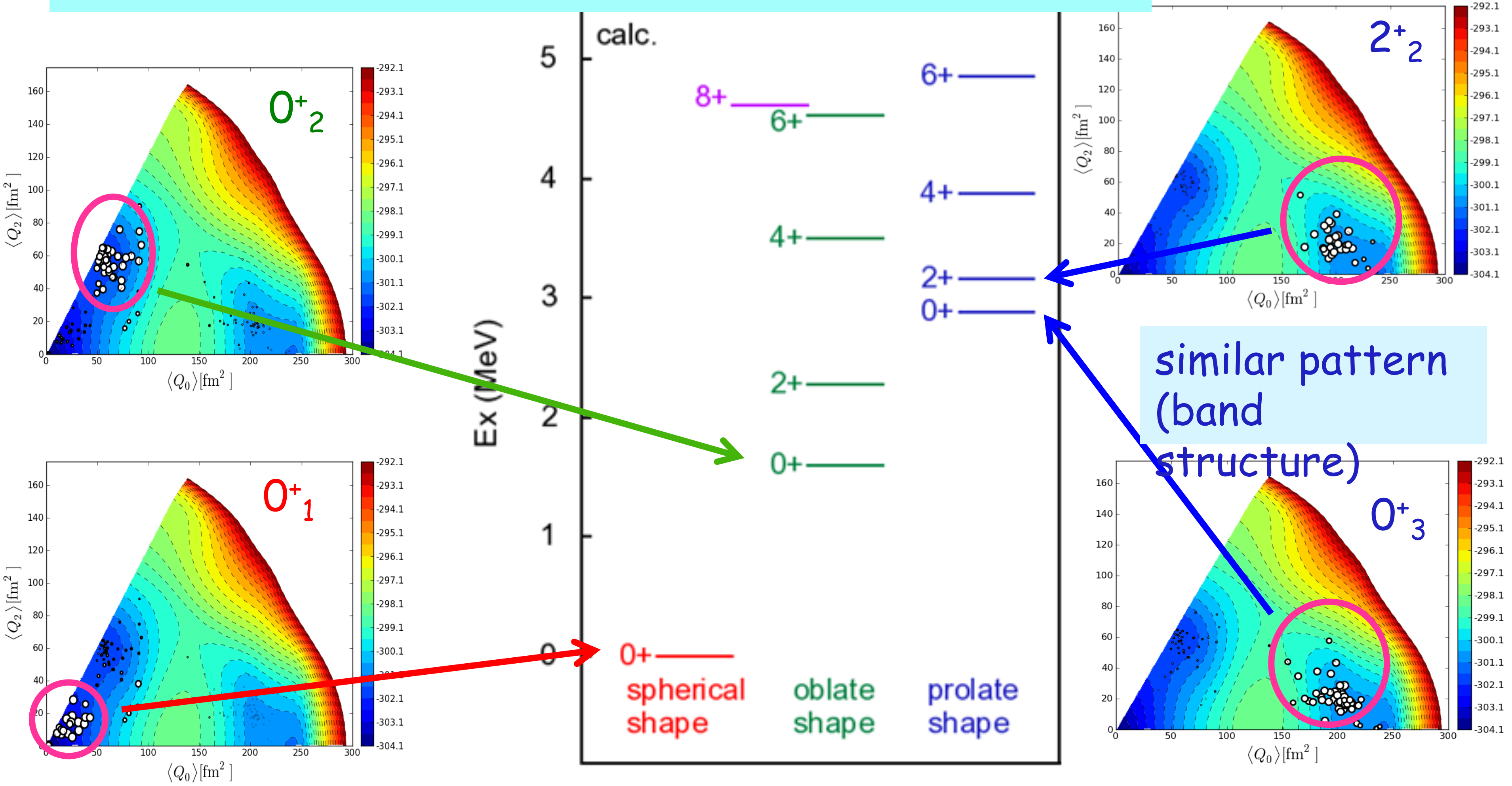
General properties of T-plot :

Certain number of large circles in a small region of PES

⇔ pairing correlations

Spreading beyond this can be due to shape fluctuation

Example : shape assignment to various 0^+ states of ^{68}Ni



similar pattern (band structure)

Three pillars combined for future

computation

Monte Carlo
Shell Model
(MCSM)

(almost)
unlimited
dimensionality

massive
parallel
computers

Hamiltonian

pf
pfg9d5 (A3DA) (Ni)
8+8 on ^{56}Ni core (Zr)
8+8 on ^{80}Zr core (Sn)
8+10 on ^{132}Sn core
(Sm)

…

island of stability
+
χEFT based
multi-shell int.

many-body dynamics

Shell evolution
(Type I & II)

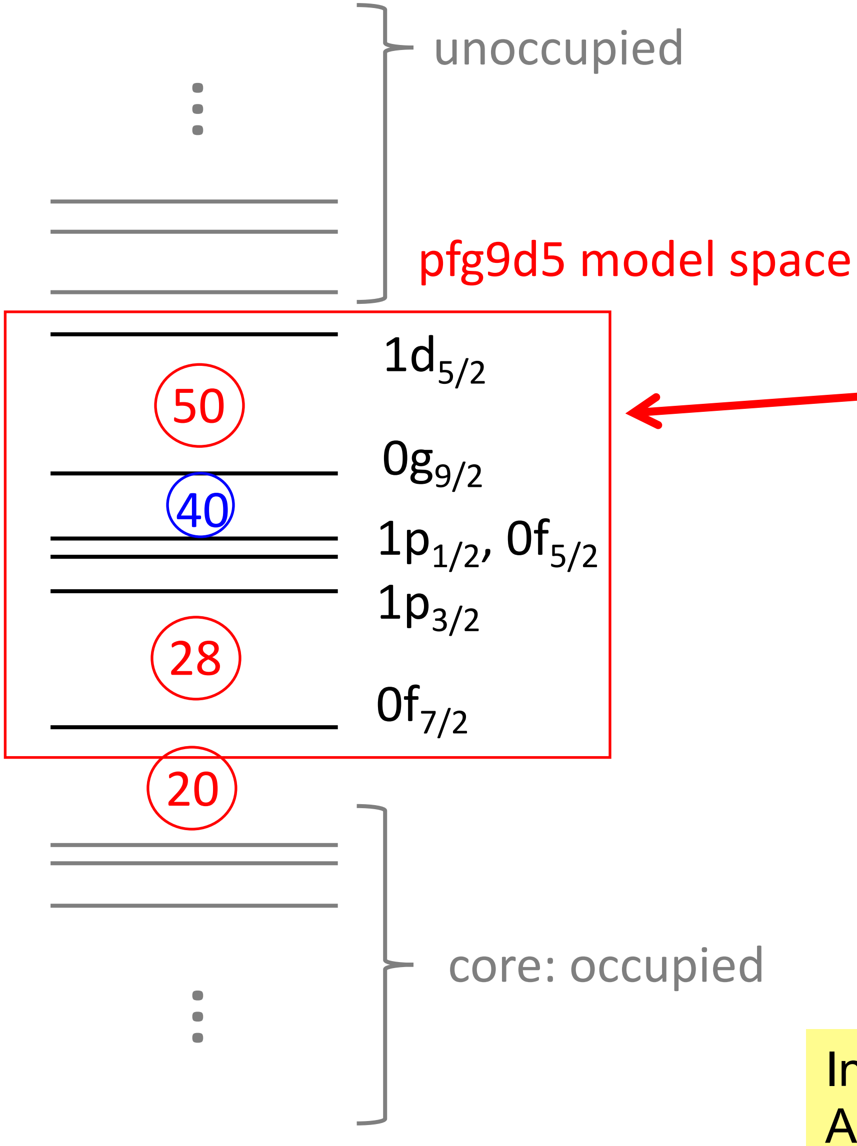
Quantum Phase
Transition

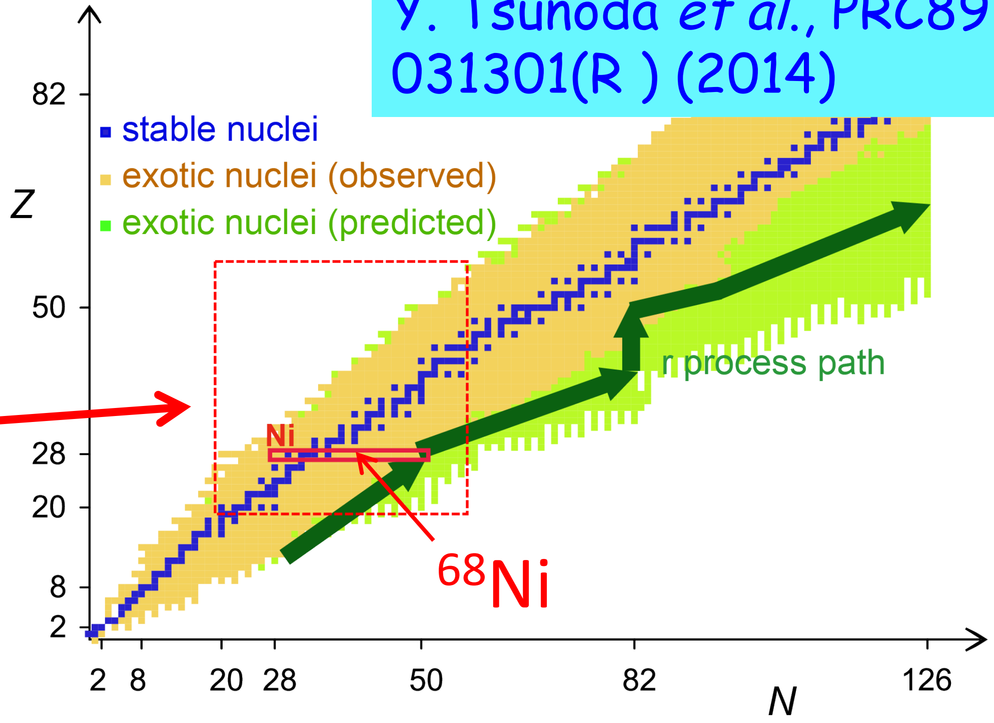
Shape coexistence

Quantum
Self-organization

Monte Carlo Shell Model (MCSM) calculation on Ni isotopes

Y. Tsunoda *et al.*, PRC89, 031301(R) (2014)





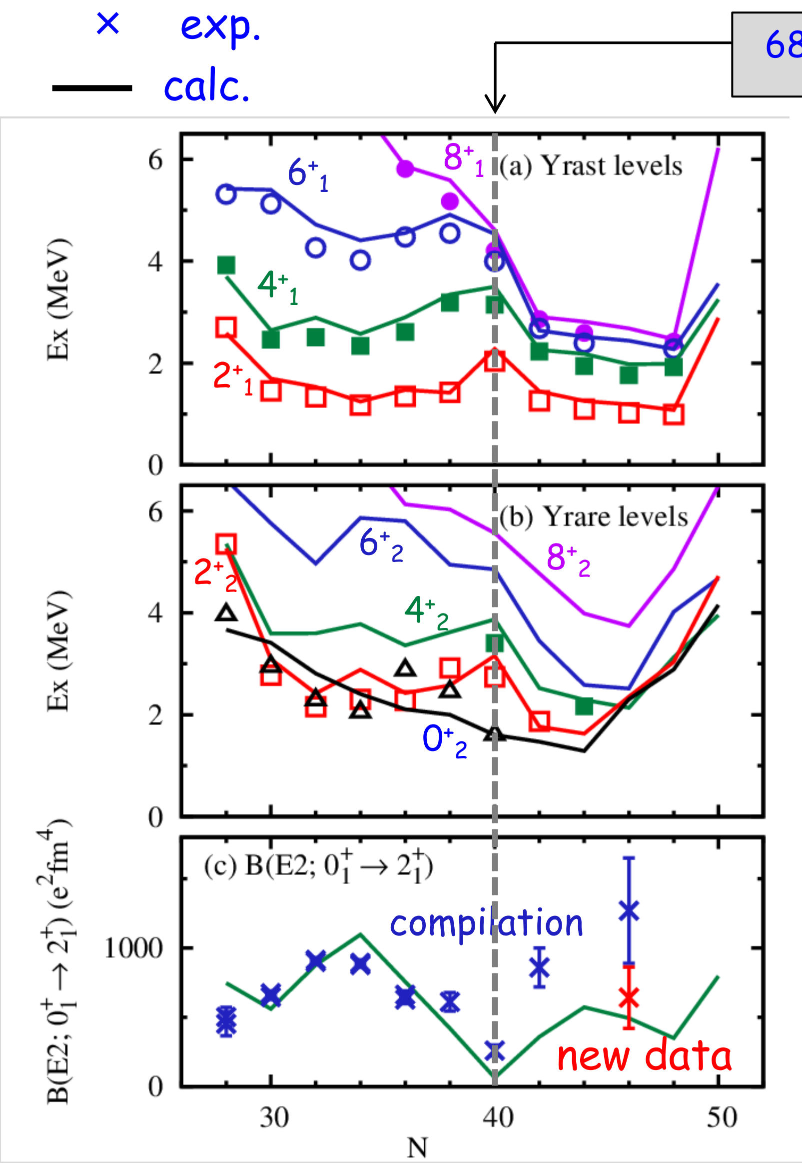
This model space is wide enough to discuss how magic numbers 28, 50 and semi-magic number 40 are visible or smeared out.

Interaction:
A3DA interaction is used with minor corrections

Energy levels and B(E2) values of Ni isotopes

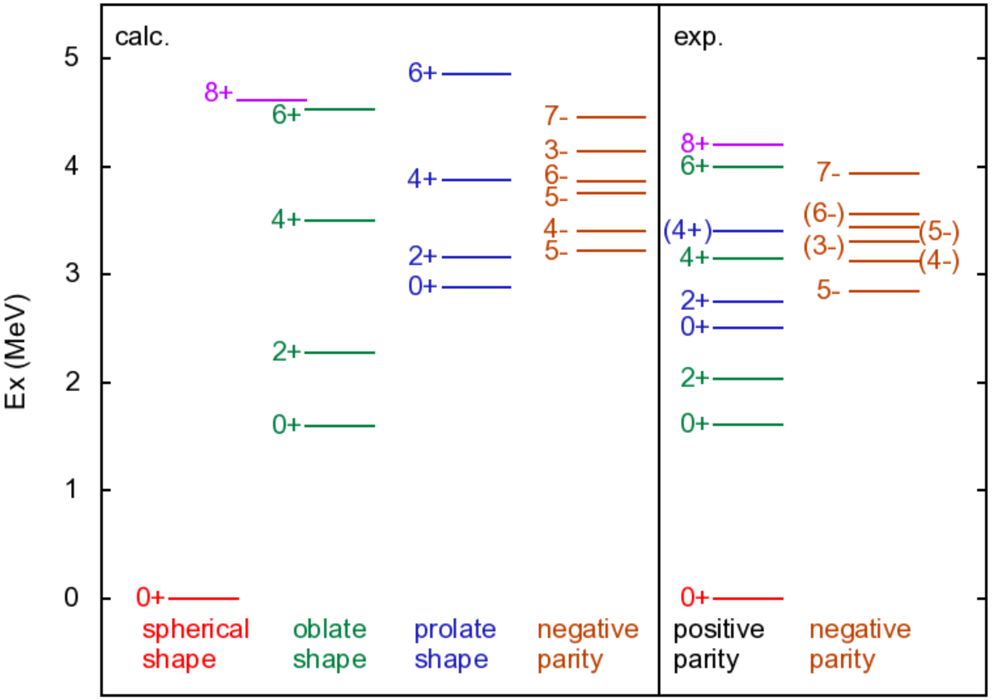
Description by the same Hamiltonian

Shape coexistence in ^{68}Ni

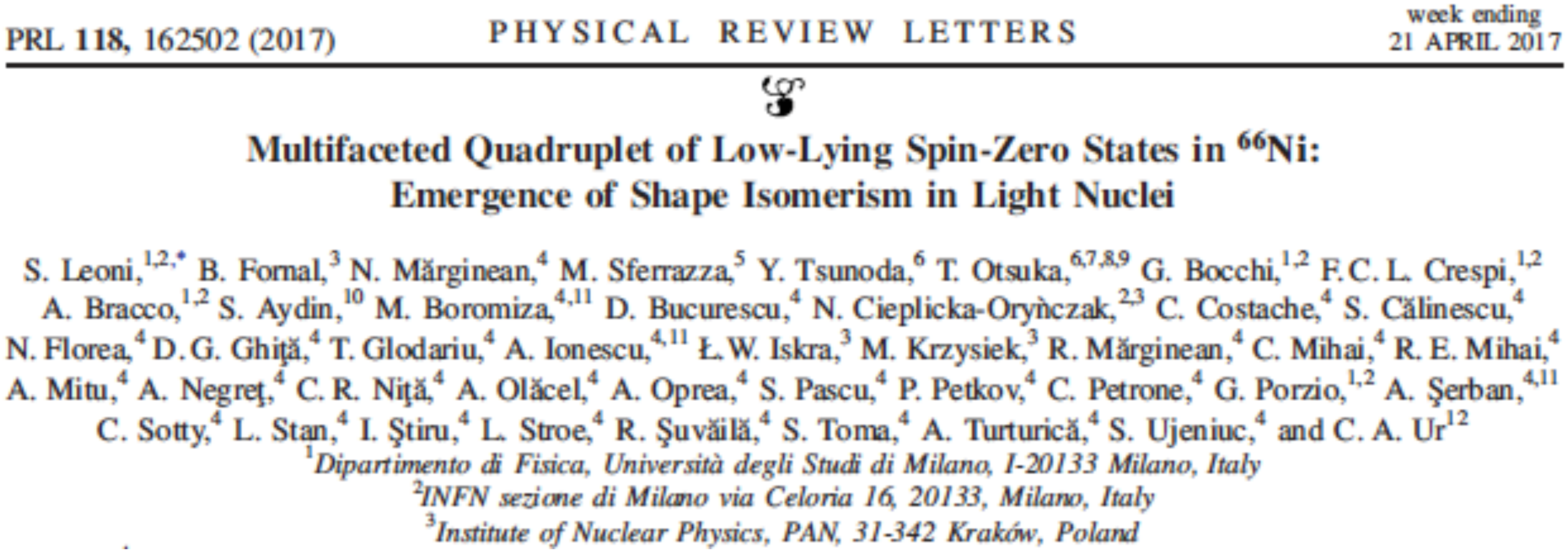


^{68}Ni

calc. exp.



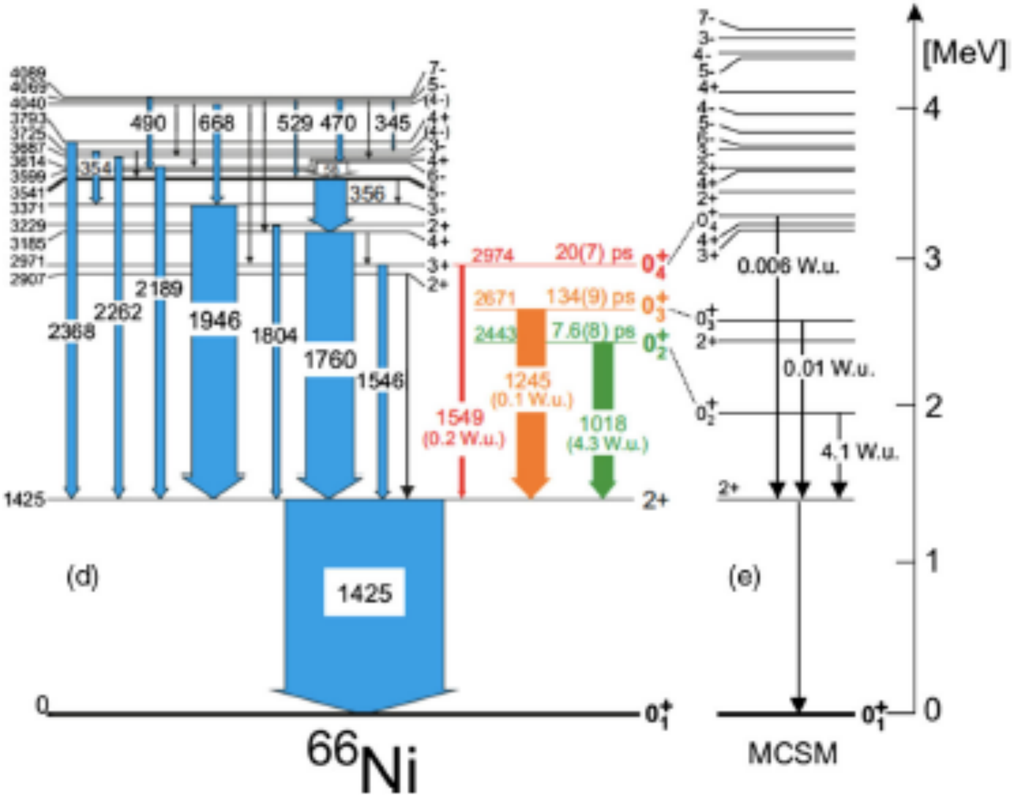
Y. Tsunoda, TO, Shimizu, Honma and Utsuno, PRC 89, 031301 (R) (2014)

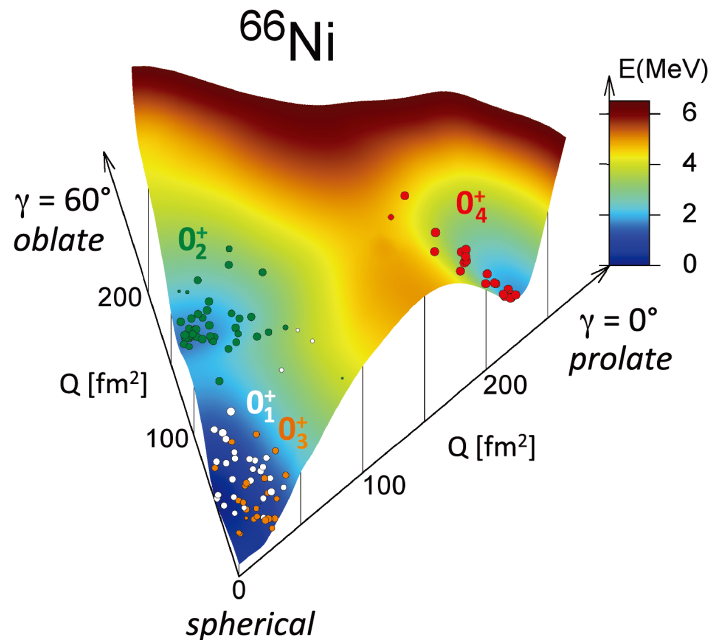


PRL 118, 162502 (2017) PHYSICAL REVIEW LETTERS week ending 21 APRIL 2017

Multifaceted Quadruplet of Low-Lying Spin-Zero States in ^{66}Ni: Emergence of Shape Isomerism in Light Nuclei

S. Leoni,[1,2,*] B. Fornal,[3] N. Mărginean,[4] M. Sferrazza,[5] Y. Tsunoda,[6] T. Otsuka,[6,7,8,9] G. Bocchi,[1,2] F. C. L. Crespi,[1,2]
A. Bracco,[1,2] S. Aydin,[10] M. Boromiza,[4,11] D. Bucurescu,[4] N. Cieplicka-Oryńczak,[2,3] C. Costache,[4] S. Călinescu,[4]
N. Florea,[4] D. G. Ghiţă,[4] T. Glodariu,[4] A. Ionescu,[4,11] Ł. W. Iskra,[3] M. Krzysiek,[3] R. Mărginean,[4] C. Mihai,[4] R. E. Mihai,[4]
A. Mitu,[4] A. Negreţ,[4] C. R. Niţă,[4] A. Olăcel,[4] A. Oprea,[4] S. Pascu,[4] P. Petkov,[4] C. Petrone,[4] G. Porzio,[1,2] A. Şerban,[4,11]
C. Sotty,[4] L. Stan,[4] I. Ştiru,[4] L. Stroe,[4] R. Şuvăilă,[4] S. Toma,[4] A. Turturică,[4] S. Ujeniuc,[4] and C. A. Ur[12]
[1]*Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy*
[2]*INFN sezione di Milano via Celoria 16, 20133, Milano, Italy*
[3]*Institute of Nuclear Physics, PAN, 31-342 Kraków, Poland*





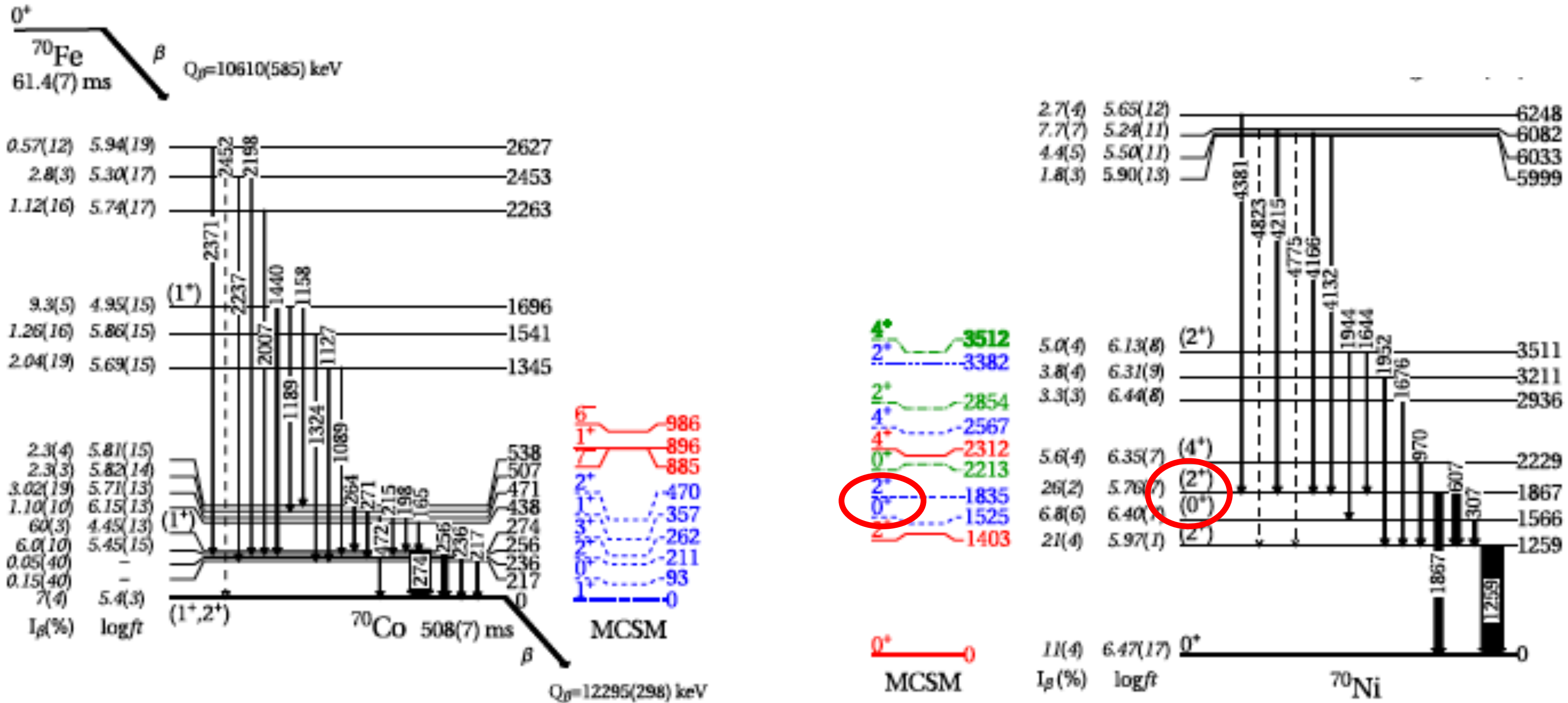
Physics Letters B 765 (2017) 328–333

What is this ?

Type II shell evolution in $A = 70$ isobars from the $N \geq 40$ island of inversion

A.I. Morales[a,b,*], G. Benzoni[a], H. Watanabe[c,d], Y. Tsunoda[e], T. Otsuka[f,g,h], S. Nishimura[d], F. Browne[i,d], R. Daido[j], P. Doornenbal[d], Y. Fang[j], G. Lorusso[d], Z. Patel[k,d], S. Rice[k,d], L. Sinclair[l,d], P.-A. Söderström[d], T. Sumikama[m], J. Wu[d], Z.Y. Xu[f,d], A. Yagi[j], R. Yokoyama[f], H. Baba[d], R. Avigo[a,b], F.L. Bello Garrote[n], N. Blasi[a], A. Bracco[a,b], F. Camera[a,b], S. Ceruti[a,b], F.C.L. Crespi[a,b], G. de Angelis[o], M.-C. Delattre[p], Zs. Dombradi[q], A. Gottardo[o], T. Isobe[d], I. Kojouharov[r], N. Kurz[r], I. Kuti[q], K. Matsui[f], B. Melon[s], D. Mengoni[t,u], T. Miyazaki[f], V. Modamio-Hoybjor[o], S. Momiyama[f], D.R. Napoli[o], M. Niikura[f], R. Orlandi[h,v], H. Sakurai[d,f], E. Sahin[n], D. Sohler[q], H. Schaffner[r], R. Taniuchi[f], J. Taprogge[w,x], Zs. Vajta[q], J.J. Valiente-Dobón[o], O. Wieland[a], M. Yalcinkaya[y]

[a] Istituto Nazionale di Fisica Nucleare, Sezione di Milano, Via Celoria 16, 20133 Milano, Italy
[b] Dipartimento di Fisica, Università degli Studi di Milano, Via Celoria 16, 20133 Milano, Italy



End of the 1st lecture

Int. School of Physics “Enrico Fermi”- Course 201
Nuclear Physics with Stable and Radioactive Ion Beams
Varenna
July 14-19, 2017

Recent developments in shell model studies of atomic nuclei

Takaharu Otsuka

2nd lecture

JICFuS
RIKEN NiSHiNA CENTER
東京大学 THE UNIVERSITY OF TOKYO
KU LEUVEN
MICHIGAN STATE UNIVERSITY Founded 1855

This work has been supported by MEXT and JICFuS as a priority issue (Elucidation of the fundamental laws and evolution of the universe) to be tackled by using Post ‘K’ Computer

Three pillars combined for future

In order to understand the physics behind these calculations, let us jump to

computation

Monte Carlo Shell Model (MCSM)

(almost) unlimited dimensionality

massive parallel computers

Hamiltonian

pf
pfg9d5 (A3DA) (Ni)
8+8 on ^{56}Ni core (Zr)
8+8 on ^{80}Zr core (Sn)
8+10 on ^{132}Sn core (Sm)
…
island of stability
+
χEFT based multi-shell int.

many-body dynamics

Shell evolution (Type I & II)

Quantum Phase Transition

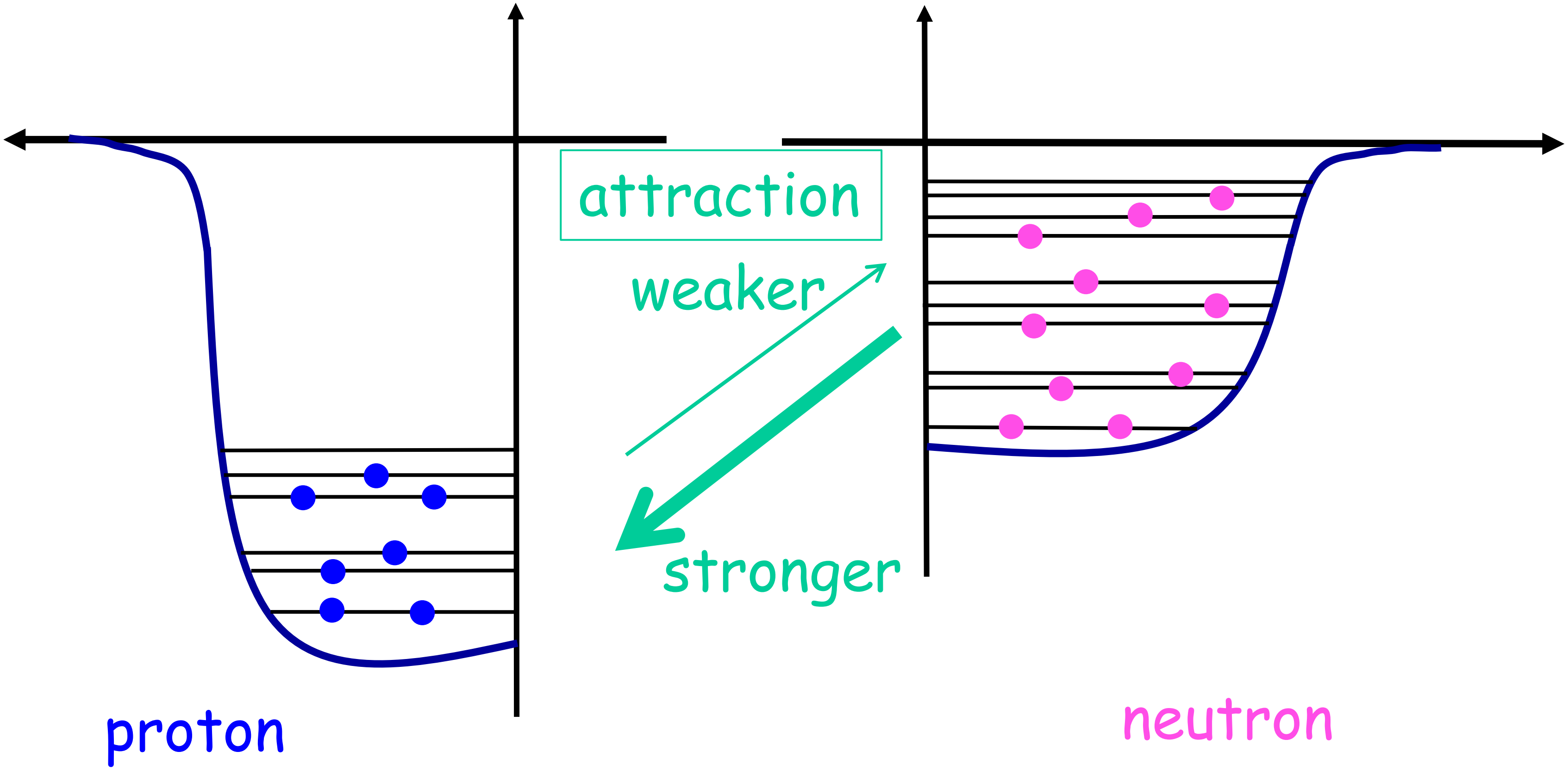
Shape coexistence

Quantum Self-organization

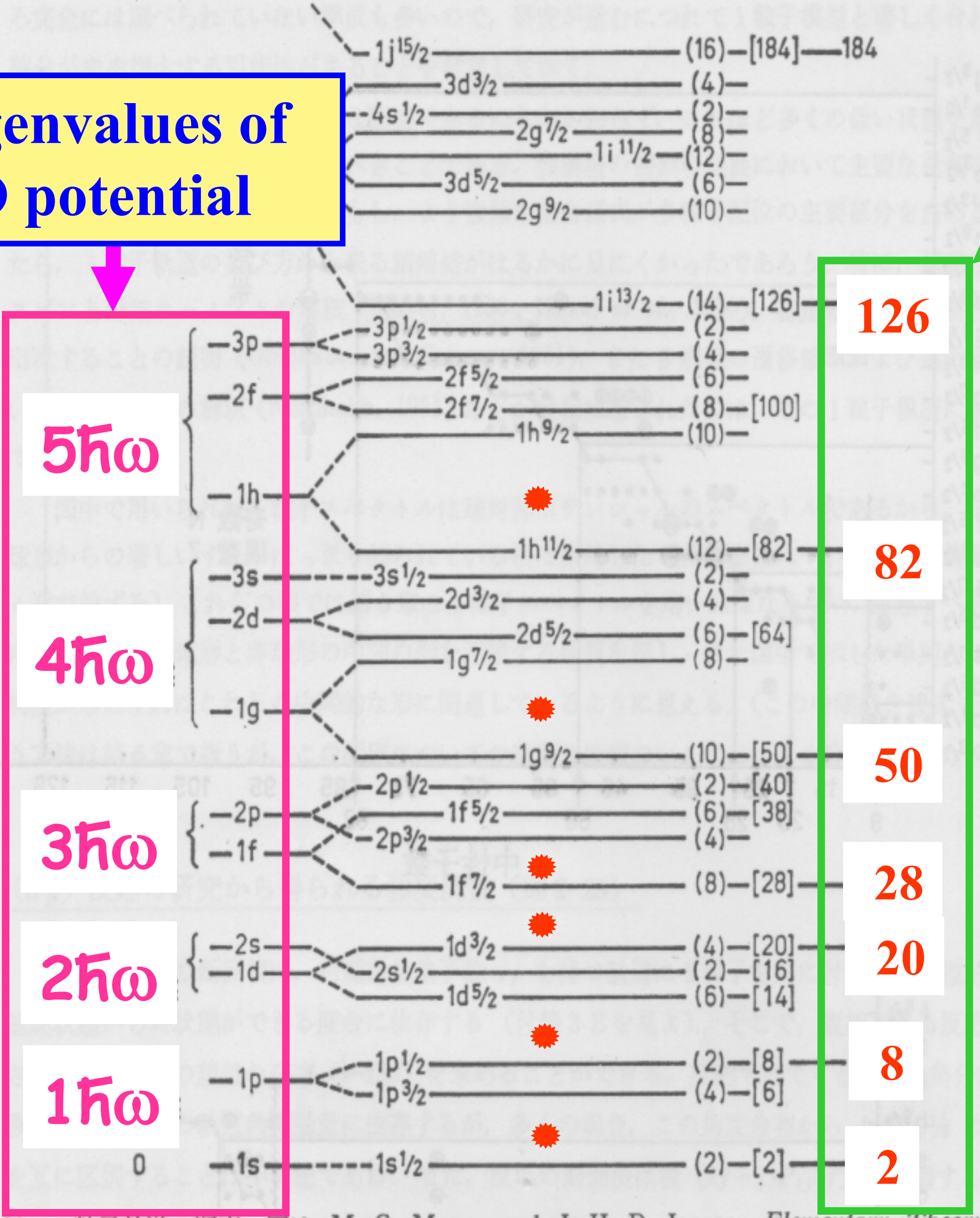
proton-neutron interaction
>> proton-proton or neutron-neutron

If Z << N, protons are more bound.

Relative relations among orbits are preserved, because basically only the depths change. *Can we assume this ?*



Eigenvalues of HO potential



5ħω

4ħω

3ħω

2ħω

1ħω

126

82

50

28

20

8

2

図2-23　1粒子軌道の順序. 図は M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955 からとった.

Magic numbers by Mayer and Jensen (1949)

From simple but general properties as,

density saturation
+ short-range *NN* interaction
+ spin-orbit splitting

→ **Mayer-Jensen's magic number**
with rather constant gaps
(except for gradual A dependence)

robust feature -> no way out ???

This question leads us to one of the major developments of recent nuclear structure studies.

Let's see what occurs
in the shell structure of exotic nuclei.

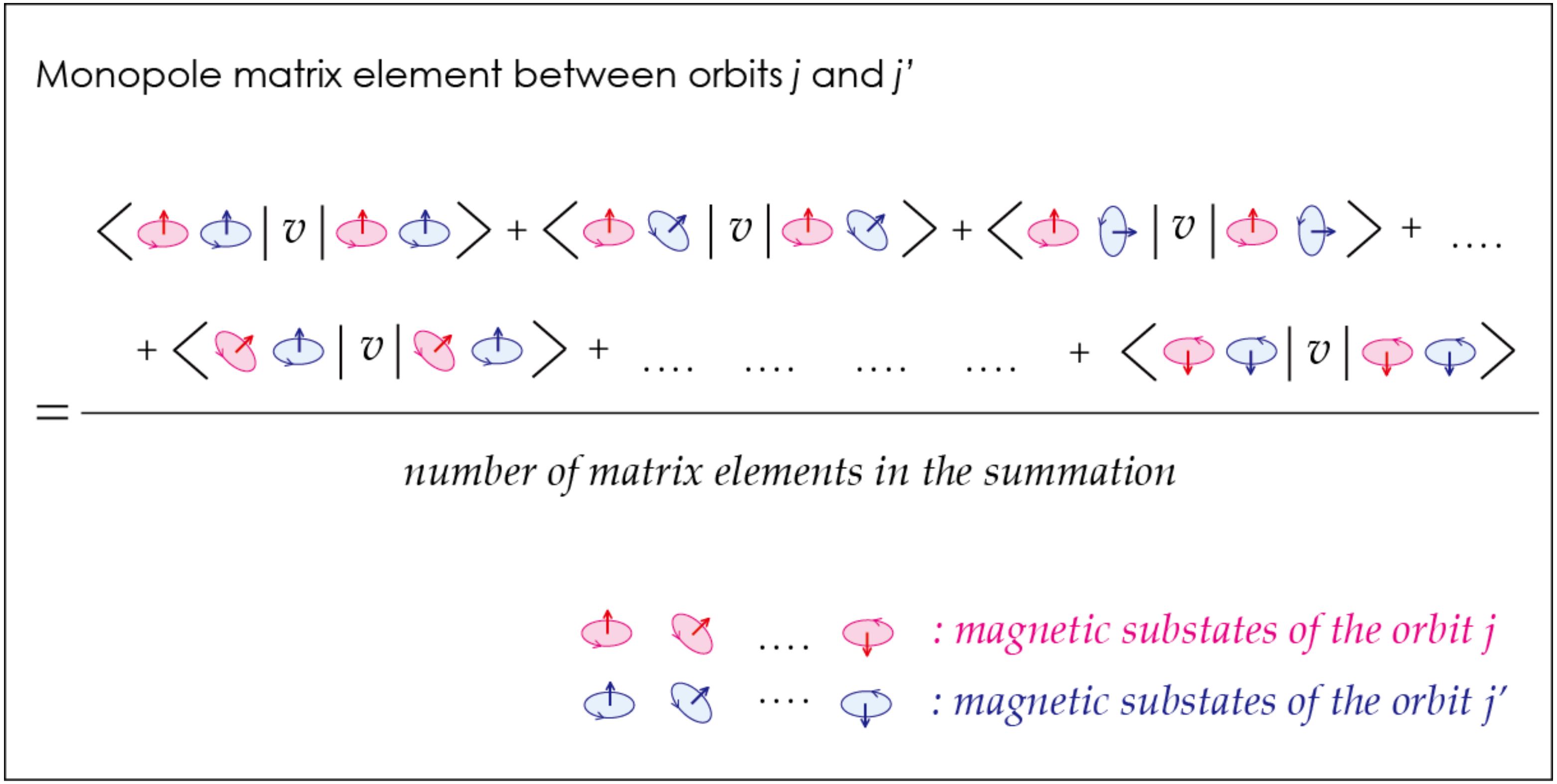
In other words, let's discuss whether
the shell evolution,
the change of the shell structure,
occurs or not.
If it occurs, how ?

The key tool is
the monopole interaction.

Monopole matrix element between orbits *j* and *j'*

$$V^{m}_{nn}(j,j') = \frac{\sum_{(m,m')} \langle j,m\,;\,j',m'|\hat{v}_{nn}|j,m\,;\,j',m'\rangle}{\sum_{(m,m')} 1},$$

v_{nn} is interaction; *m, m'* are magnetic substates



Monopole matrix elements can be written equivalently by usual TBMEs as

$$V_T^m(j,\, j') = \frac{\sum_J (2J+1) \langle j, j'; J, T | \hat{v} | j, j'; J, T \rangle}{\sum_J (2J+1)}.$$

$$\text{for } \; T = 0 \text{ and } 1,$$

Monopole interaction for n-n (or p-p) interaction is defined as

$$\hat{v}_{nn}^m(j, j') = \left\{ \boxed{\begin{array}{ll} V_{nn}^m(j,j) \, \frac{1}{2} \, \hat{n}_j \, (\hat{n}_j - 1) & \text{for } j = j' \\ & \\ V_{nn}^m(j,j') \, \hat{n}_j \, \hat{n}_{j'} & \text{for } j \neq j' \end{array}} \right.$$

T=1 part of the p-n monopole interaction is given as

$$
\begin{aligned}
\hat{v}_{pn,mono,T=1} = & \sum_{j,j'} V^m_{T=1}(j,j')\,\frac{1}{2}\,\hat{n}^p_j\,\hat{n}^n_{j'} \\
& + \sum_{j<j'} V^m_{T=1}(j,j')\,\frac{1}{2}\left\{\hat{\tau}^+_j\hat{\tau}^-_{j'} + \hat{\tau}^-_j\hat{\tau}^+_{j'}\right\} \\
& + \sum_{j} V^m_{T=1}(j,j)\,\frac{1}{2} : \hat{\tau}^+_j\hat{\tau}^-_j : .
\end{aligned}
$$

Finally, we get the full expression for the p-n interaction

$$
\begin{aligned}
\hat{v}_{pn,mono} = & \sum_{j,j'} \frac{1}{2}\left\{V^m_{T=0}(j,j') + V^m_{T=1}(j,j')\right\} \boxed{\hat{n}^p_j\,\hat{n}^n_{j'}} \\
& - \sum_{j<j'} \frac{1}{2}\left\{V^m_{T=0}(j,j') - V^m_{T=1}(j,j')\right\} \\
& \qquad \left\{\hat{\tau}^+_j\hat{\tau}^-_{j'} + \hat{\tau}^-_j\hat{\tau}^+_{j'}\right\} \\
& - \sum_{j} \frac{1}{2}\left\{V^m_{T=0}(j,j) - V^m_{T=1}(j,j)\right\} : \hat{\tau}^+_j\hat{\tau}^-_j : .
\end{aligned}
$$

This is consistent with the final form of the monopole interaction of Poves and Zuker (Phys. Rep. 70, 235 (1981)) besides different formulation.

$$
\begin{aligned}
\hat{v}_{pn,mono} = & \sum_{j,j'} \frac{1}{2}\left\{V^{m}_{T=0}(j,j') + V^{m}_{T=1}(j,j')\right\} \hat{n}^{p}_{j}\,\hat{n}^{n}_{j'} \\
& - \sum_{j<j'} \frac{1}{2}\left\{V^{m}_{T=0}(j,j') - V^{m}_{T=1}(j,j')\right\} \\
& \qquad \left\{\hat{\tau}^{+}_{j}\hat{\tau}^{-}_{j'} + \hat{\tau}^{-}_{j}\hat{\tau}^{+}_{j'}\right\} \\
& - \sum_{j} \frac{1}{2}\left\{V^{m}_{T=0}(j,j) - V^{m}_{T=1}(j,j)\right\} :\hat{\tau}^{+}_{j}\hat{\tau}^{-}_{j}:
\end{aligned}
$$

The second and third terms can be visualized as shown in the figure.

They are still monopole, but proton and neutron must exchange their states !

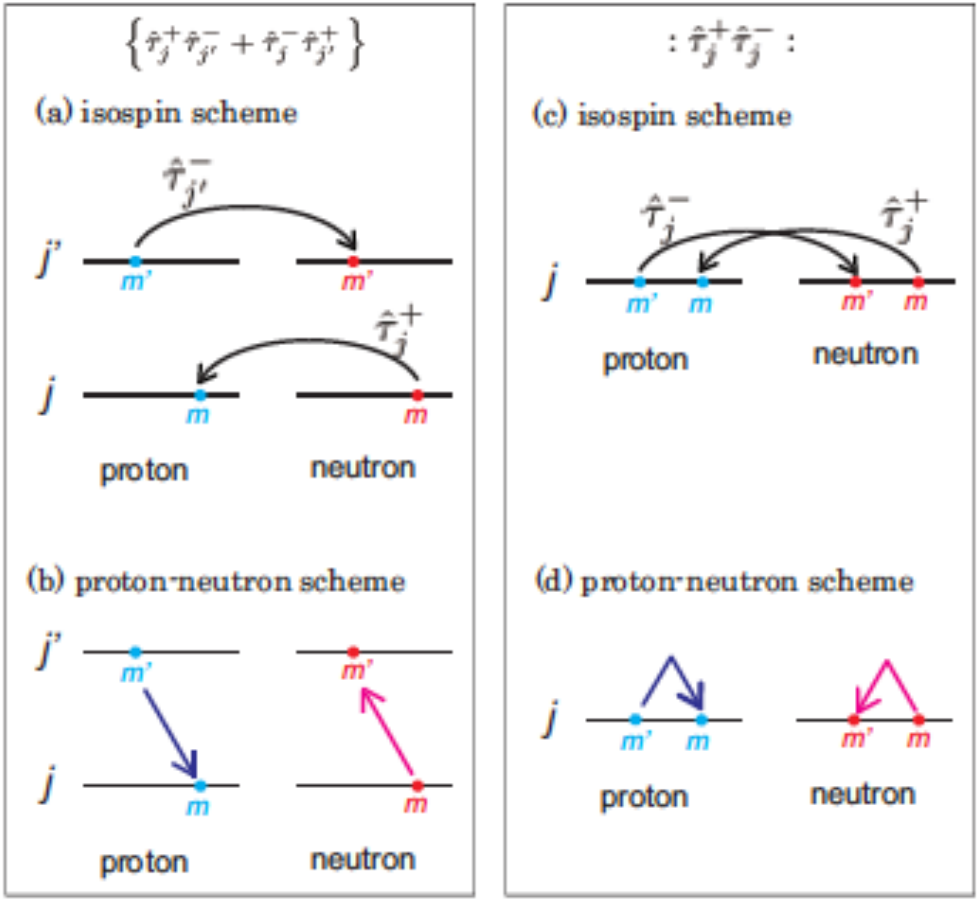


Figure 16 Implication of $\hat{\tau}^{+}_{j}\hat{\tau}^{-}_{j'}$ terms. Panels (a) and (c) are for the $\{\hat{\tau}^{+}_{j}\hat{\tau}^{-}_{j'} + \hat{\tau}^{-}_{j}\hat{\tau}^{+}_{j'}\}$ and $:\hat{\tau}^{+}_{j}\hat{\tau}^{-}_{j}:$ cases in the isospin scheme, respectively. Panels (b) and (d) are similar to (a) and (c), respectively, in the proton-neutron scheme. The magnetic substates are indicated by m and m'.

Effective single-particle energy (ESPE)

$$\epsilon_j = \epsilon_j^0 + \hat{\epsilon}_j \; .$$

Contribution from valence nucleons through the monopole interaction

contribution from the inert core (closed shell) ; constant within a given nucleus

Changes of ESPEs between nuclei

for protons

$$\Delta\hat{\epsilon}_j^p = \sum_{j'} \boxed{V^m_{T=1}(j,j')}\,\Delta\hat{n}^p_{j'} + \sum_{j'} \boxed{V^m_{pn}(j,j')}\,\boxed{\Delta\hat{n}^n_{j'}},$$

for neutrons

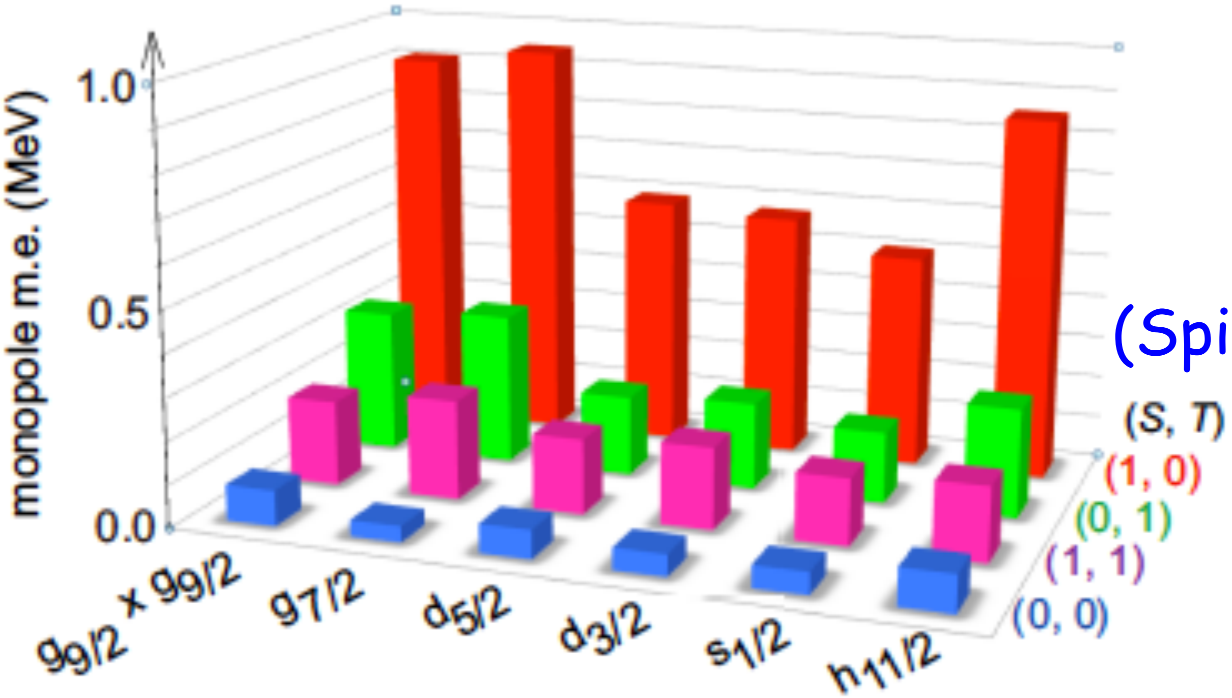
$$\Delta\hat{\epsilon}_j^n = \sum_{j'} \boxed{V^m_{T=1}(j,j')}\,\boxed{\Delta\hat{n}^n_{j'}} + \sum_{j'} \boxed{V^m_{pn}(j',j)}\,\Delta\hat{n}^p_{j'}$$

For a chain of isotopes, $\boxed{\Delta\hat{n}^n_{j'}}$ denotes the change of the neutron number in the orbit *j* '.

If monopole matrix elements, $V^m_{T=1}(j,j')$ and $V^m_{pn}(j,j')$, are uniform, nothing happens.

However, the shell evolution occurs, if $V^m_{T=1}(j,j')$ or $V^m_{pn}(j,j')$ change significantly depending on *j* and *j* '.

Monopole matrix element of the central force with a Gaussian dependence on the distance.



(Spin, Isospin)

Figure 24 Monopole matrix elements of central gaussian interactions for (S,T) channels with an equal strength parameter (see the text). One of the orbit is $1g_{9/2}$, and the other is shown.

Monopole matrix element from a central force : A=70

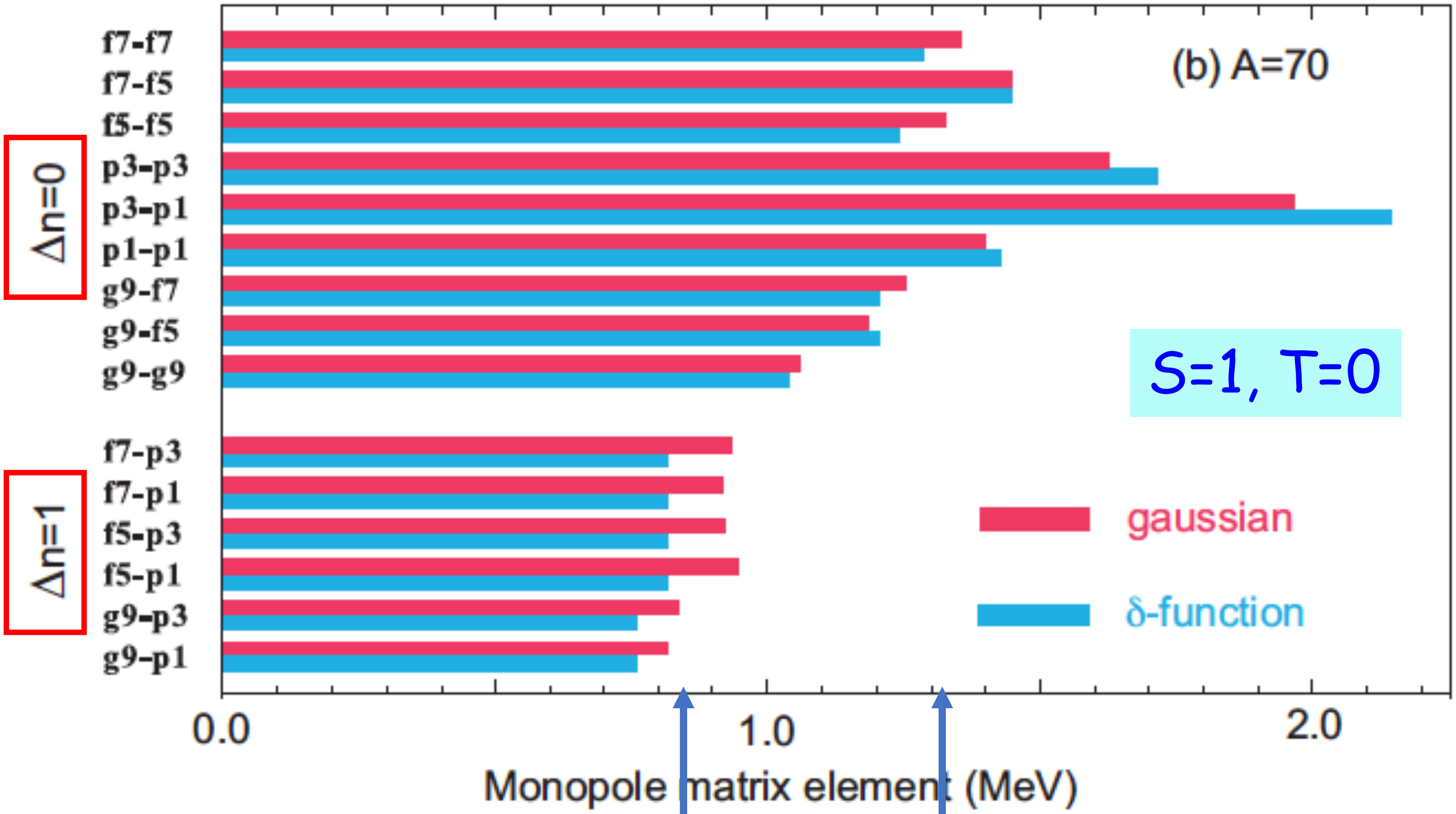


Figure 26 Monopole matrix elements of central gaussian and delta interactions for $(S = 1, T = 0)$ channel. The orbit labeling is abbreviated like g9 for $1g_{9/2}$, *etc.* The orbits are from valence shell for (a) $A = 100$ and (b) $A = 70$.

mean values ~0.8 MeV ~1.3 MeV

variations ~0.1 MeV ~0.3 MeV

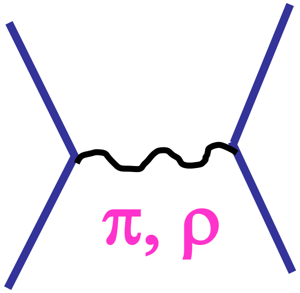
difference
~ <0.5 MeV

Besides central corce,
another important contribution
comes from the tensor force

Tensor Force

π meson : primary source

ρ meson (~ π+π) : minor (~1/4) cancellation



Ref: Osterfeld, Rev. Mod. Phys. 64, 491 (92)

Multiple pion exchanges
→ strong effective central forces in *NN* interaction
(as represented by σ meson, *etc*.)
→ nuclear binding

Presently : First-order tensor-force effect
(at medium and long ranges)

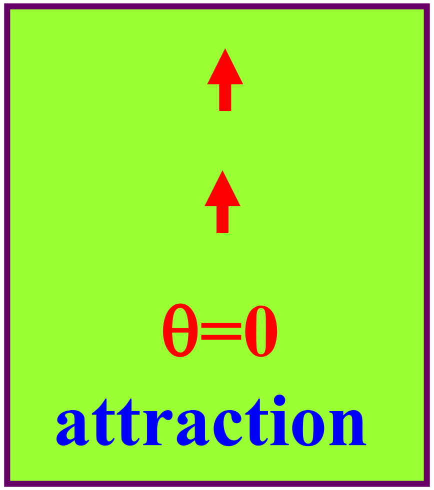
One pion exchange → Tensor force

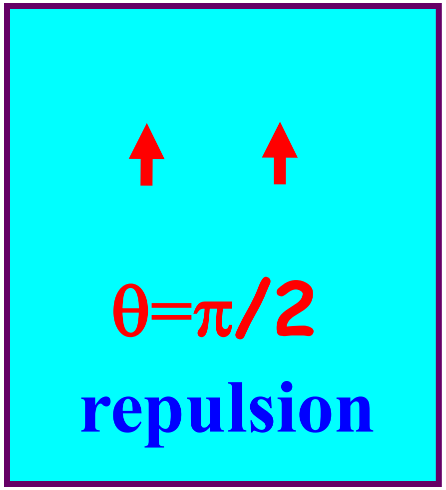
How does the tensor force work ?

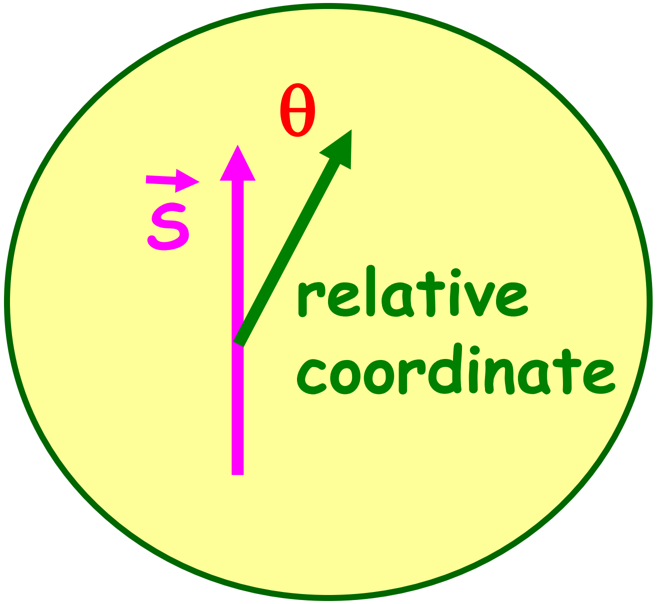
Spin of each nucleon ↑ is parallel, because the total spin must be S=1

The potential has the following dependence on the angle θ with respect to the total spin $\vec{S}$.

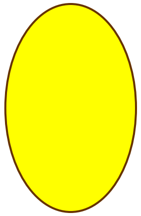
$$V \sim Y_{2,0} \sim 1 - 3\cos^2\theta$$







Monopole effects due to the tensor force

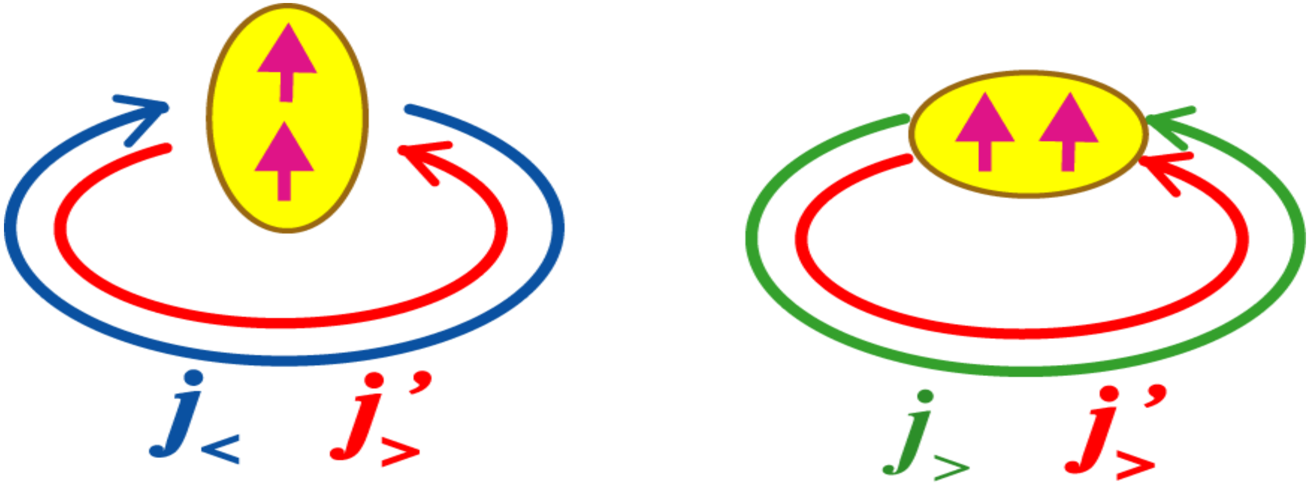


wave function of relative motion

↑↑ spin of nucleon

large relative momentum

small relative momentum



$j_<$ $j'_>$

$j_>$ $j'_>$

attractive

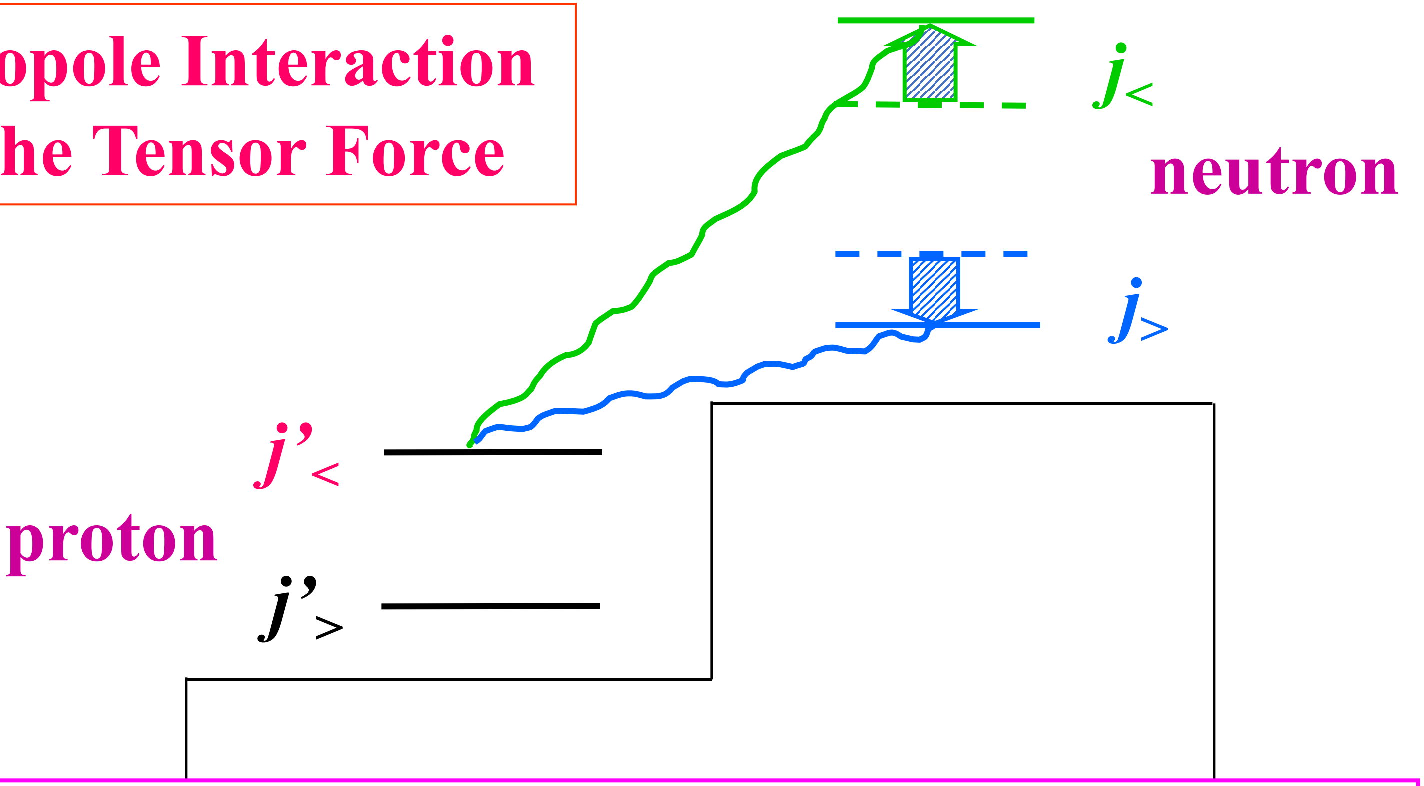
repulsive

$j_> = l + \frac{1}{2}$, $j_< = l - \frac{1}{2}$

TO *et al.*, Phys. Rev. Lett. 95, 232502 (2005)

T. Otsuka et al., Phys. Rev. Lett. 95, 232502 (2005)

Monopole Interaction of the Tensor Force



Identity for tensor monopole interaction

$$(2j_> +1)\; v_{m,T}^{(j' j_>)} + (2j_< +1)\; v_{m,T}^{(j' j_<)} = 0$$

$v_{m,T}$: monopole strength for isospin T

Variation of monopole matrix element from tensor force : A=70

variations 0.5 ~ 1 MeV

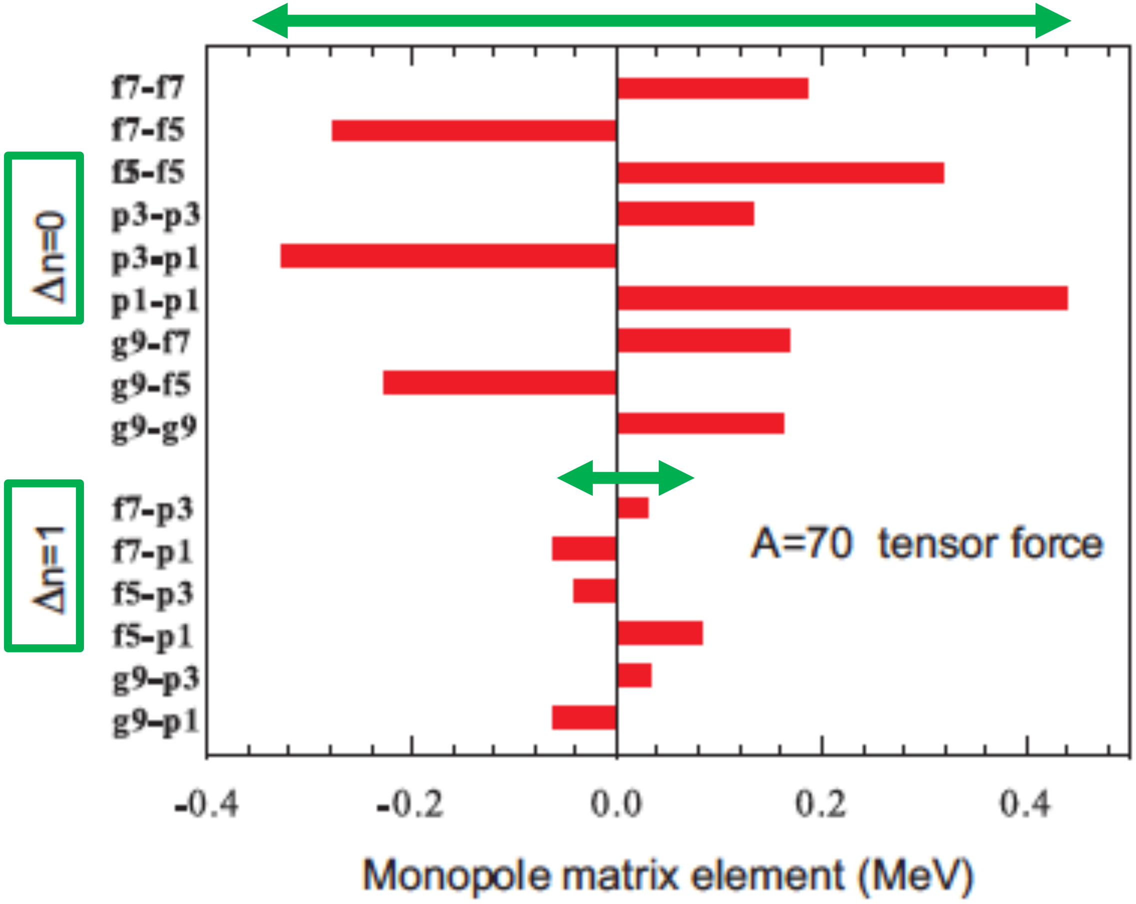
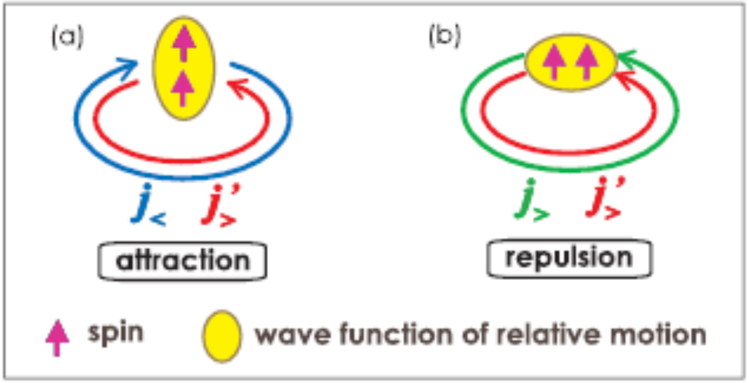


Figure 34 Monopole matrix elements of the tensor force in the T=0 channel. The orbit labeling is abbreviated like f7 for $1f_{7/2}$, *etc.* The orbits are from valence shell for $A = 70$.



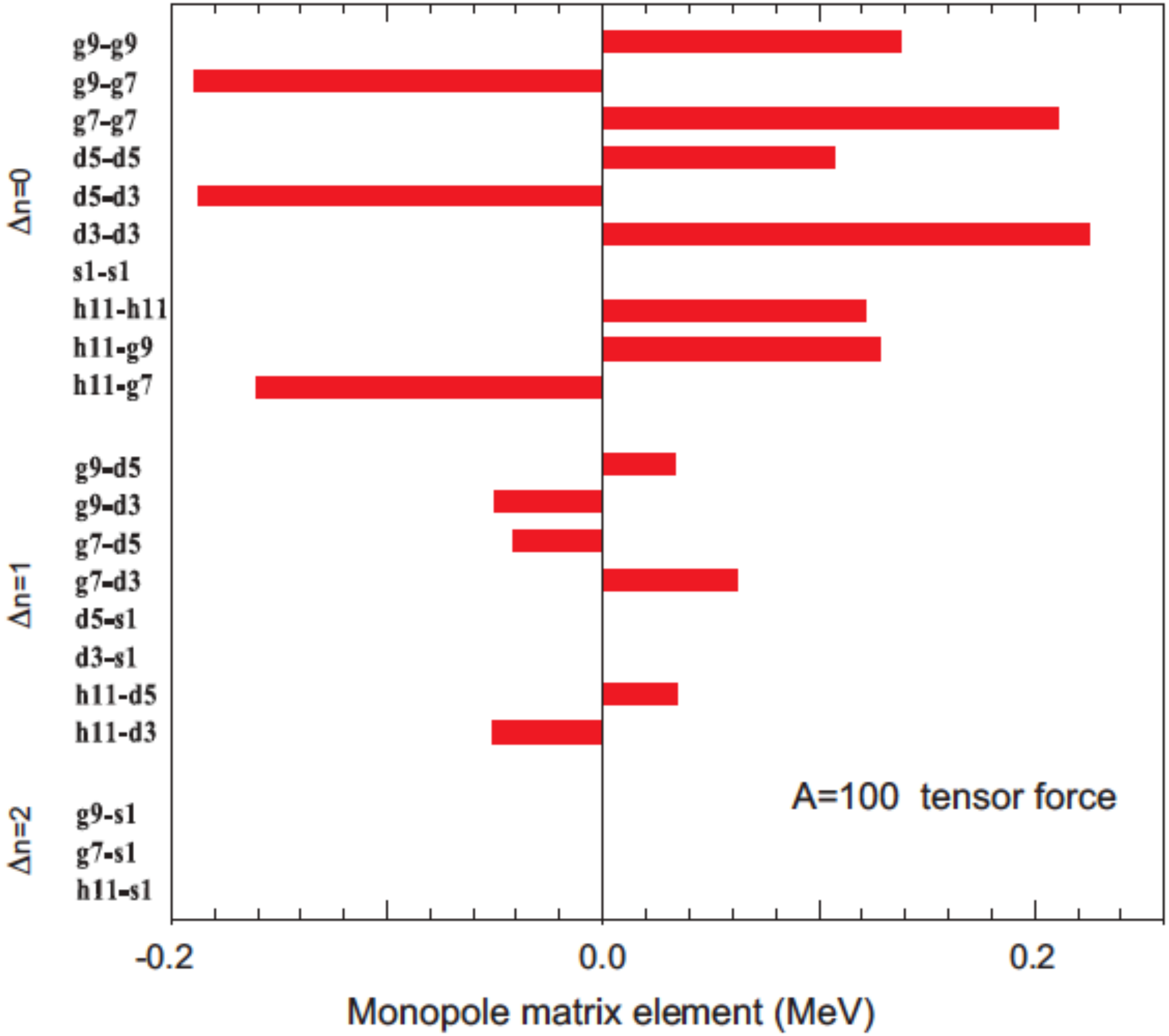
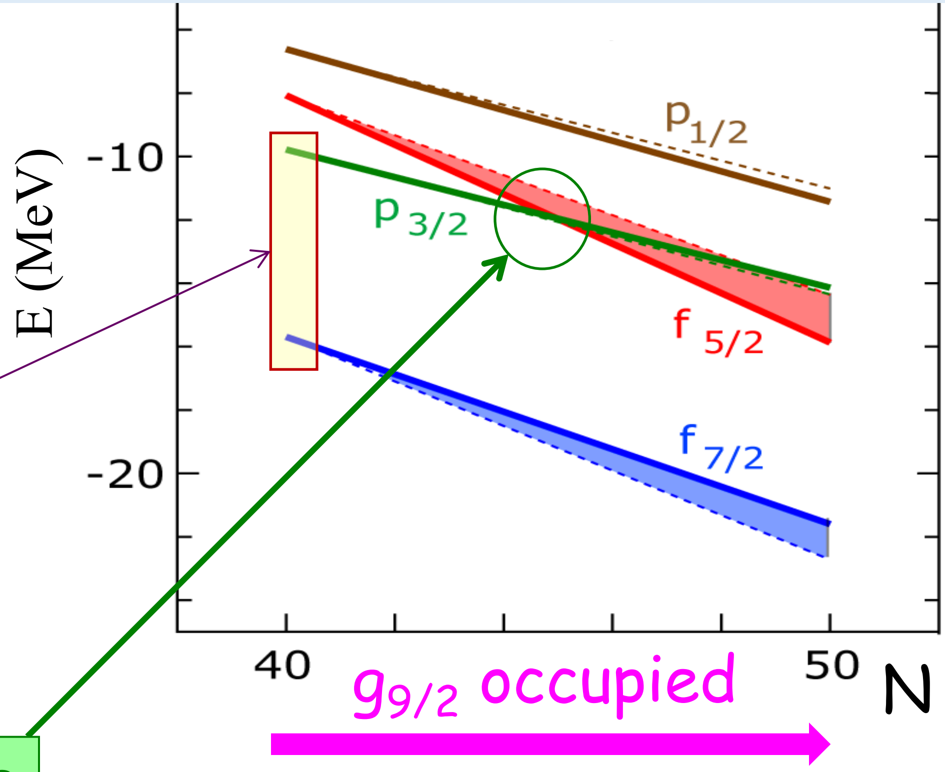


Figure 33 Monopole matrix elements of the tensor force in the T=0 channel. The orbit labeling is abbreviated like g9 for $1g_{9/2}$, *etc.* The orbits are from valence shell for A = 100.

Proton single-particles levels of Ni isotopes



Central Gaussian + Tensor

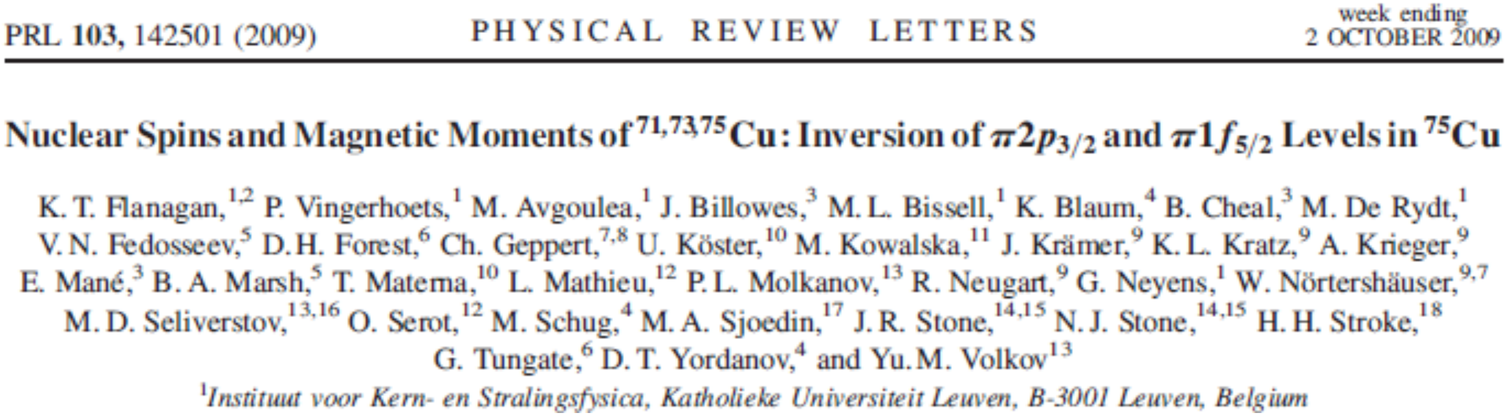
solid line:
full effect

dotted line:
central only

shaded area :
effect of
tensor force

From Grawe, EPJA25, 357

Crossing here is consistent with exp. on Cu isotopes



PRL 103, 142501 (2009) PHYSICAL REVIEW LETTERS week ending 2 OCTOBER 2009

Nuclear Spins and Magnetic Moments of ^{71,73,75}Cu: Inversion of $\pi 2p_{3/2}$ and $\pi 1f_{5/2}$ Levels in ^{75}Cu

K. T. Flanagan,[1,2] P. Vingerhoets,[1] M. Avgoulea,[1] J. Billowes,[3] M. L. Bissell,[1] K. Blaum,[4] B. Cheal,[3] M. De Rydt,[1] V. N. Fedosseev,[5] D. H. Forest,[6] Ch. Geppert,[7,8] U. Köster,[10] M. Kowalska,[11] J. Krämer,[9] K. L. Kratz,[9] A. Krieger,[9] E. Mané,[3] B. A. Marsh,[5] T. Materna,[10] L. Mathieu,[12] P. L. Molkanov,[13] R. Neugart,[9] G. Neyens,[1] W. Nörtershäuser,[9,7] M. D. Seliverstov,[13,16] O. Serot,[12] M. Schug,[4] M. A. Sjoedin,[17] J. R. Stone,[14,15] N. J. Stone,[14,15] H. H. Stroke,[18] G. Tungate,[6] D. T. Yordanov,[4] and Yu. M. Volkov[13]

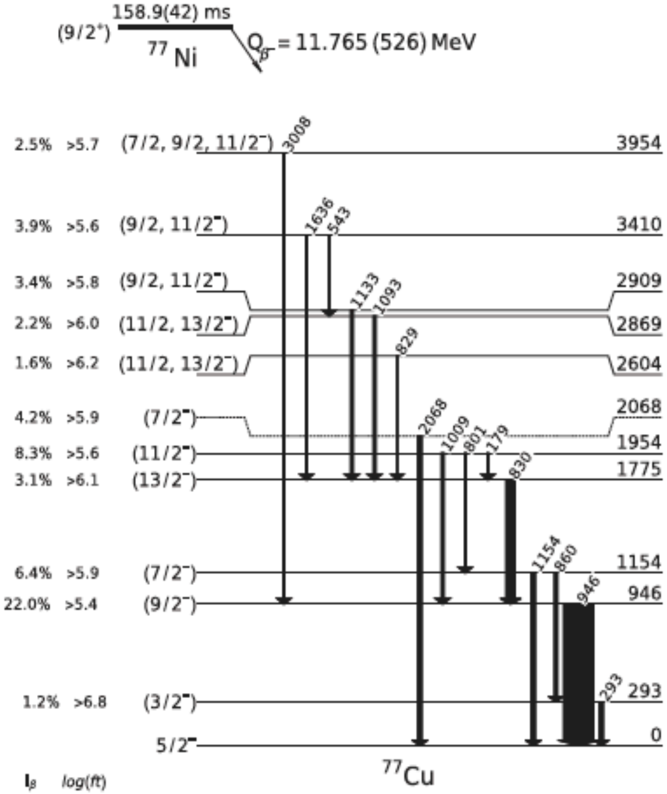
[1]*Instituut voor Kern- en Stralingsfysica, Katholieke Universiteit Leuven, B-3001 Leuven, Belgium*

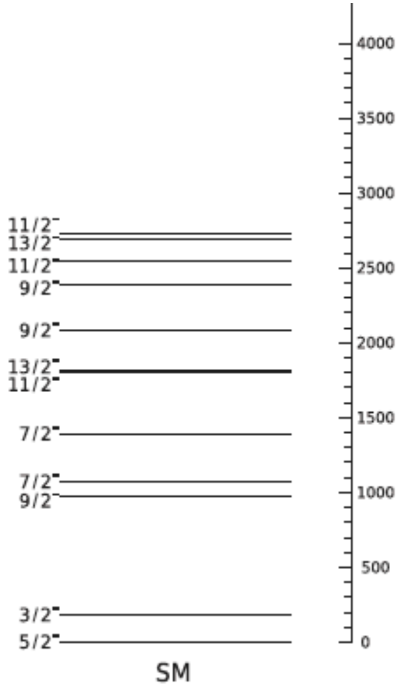
PRL **118,** 242502 (2017) PHYSICAL REVIEW LETTERS week ending 16 JUNE 2017

Shell Evolution towards ^{78}Ni: Low-Lying States in ^{77}Cu

E. Sahin,[1,*] F. L. Bello Garrote,[1] Y. Tsunoda,[2] T. Otsuka,[2,3,4,5] G. de Angelis,[6] A. Görgen,[1] M. Niikura,[3] S. Nishimura,[7]

above ~1 MeV. A rather large $Z = 28$ gap is consistent with this, while the gap decreases modestly by ~2 MeV from $N = 40$ to 50.





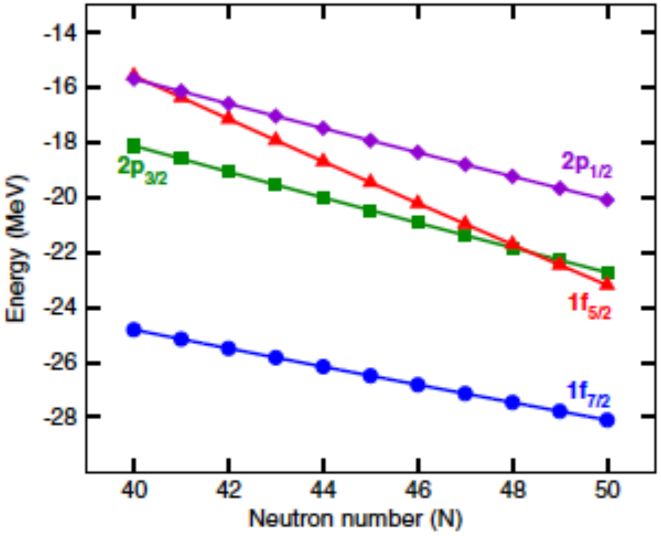
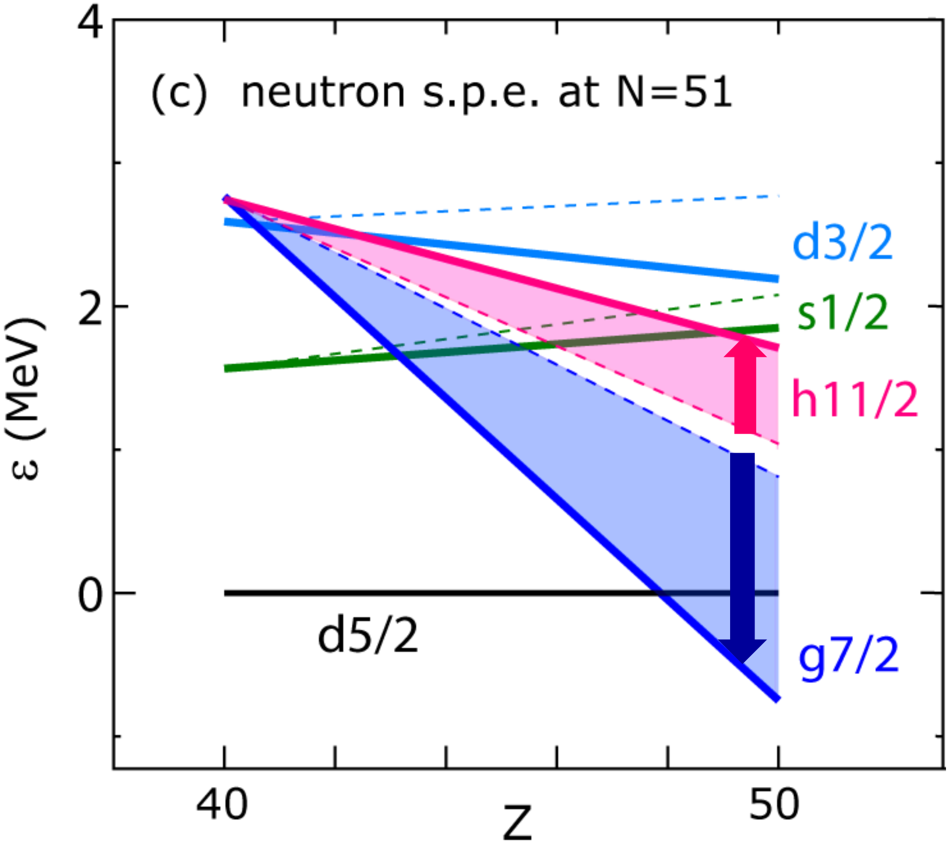


FIG. 4. Calculated proton ESPEs for the Ni chain.

Shell structure of a key nucleus ^{100}Sn



solid line : full
(central + tensor)

dashed line : central only

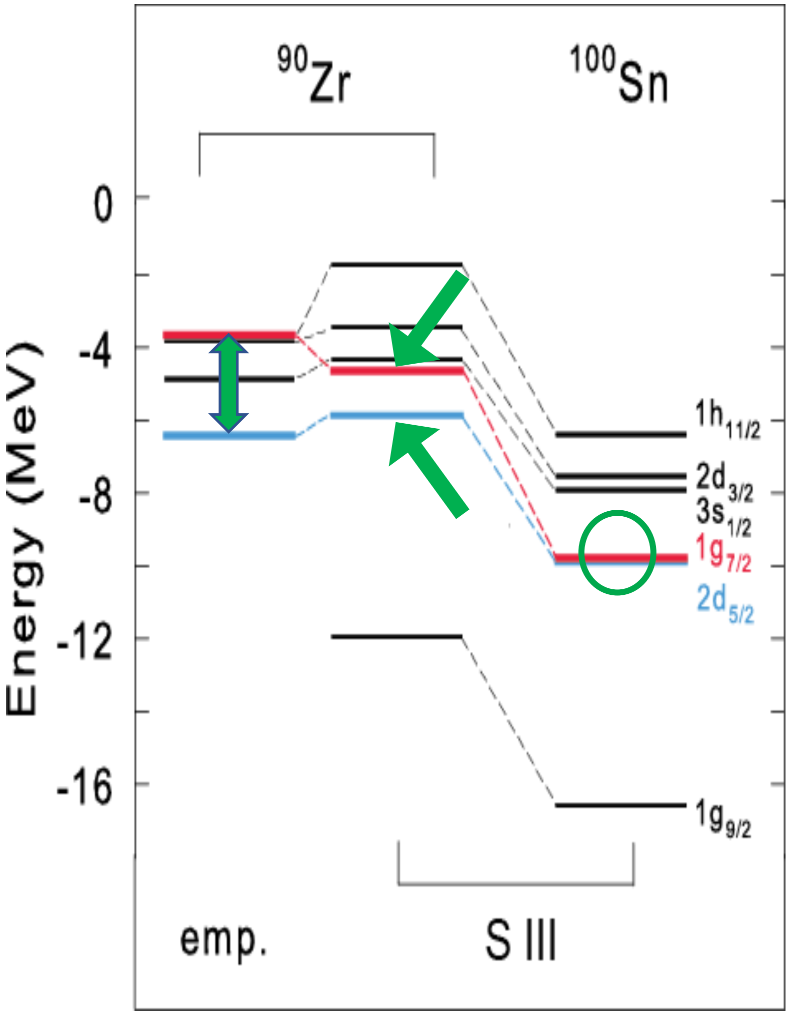
Fedderman-Pittel (1977)

shaded area :
effect of tensor force

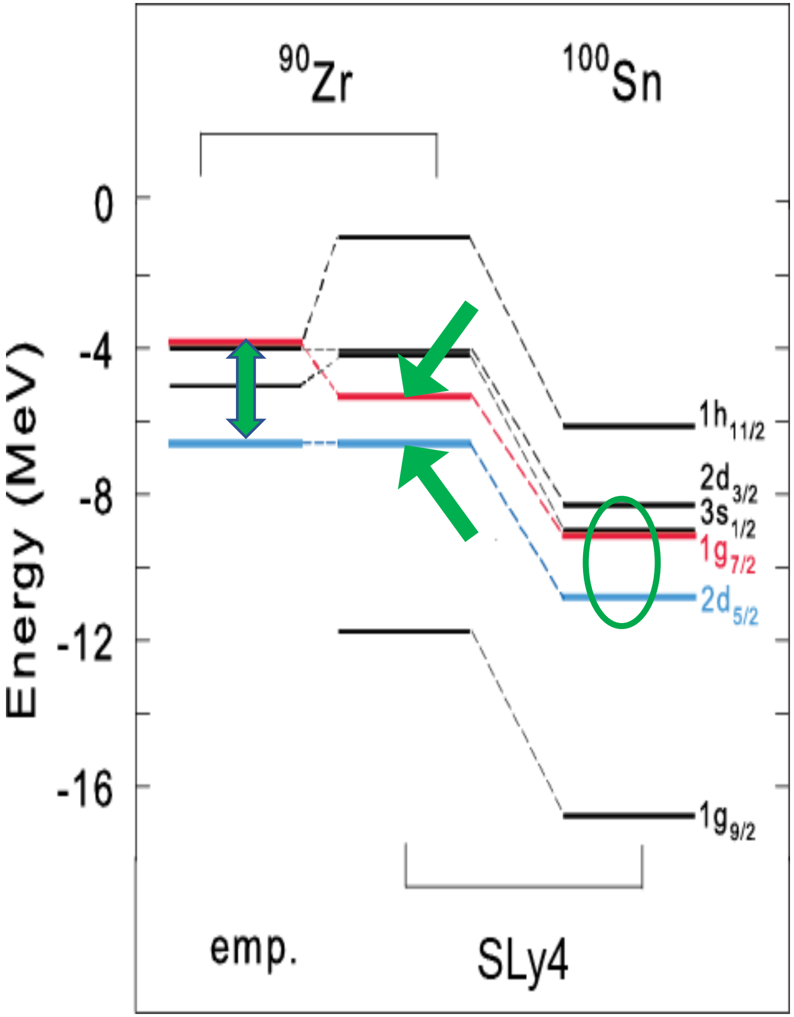
Exp. d5/2 and g7/2 should be close
Seweryniak et al.
Phys. Rev. Lett. 99, 022504 (2007)
Gryzywacz et al.

Predictions by Skyrme model (HF calculation)

$g_{7/2}$ lowered relative to $d_{5/2}$ by ~ 2 MeV (←→ previous page)



$g_{7/2}$ raised relative to $d_{5/2}$ by ~ 2 MeV (opposite)

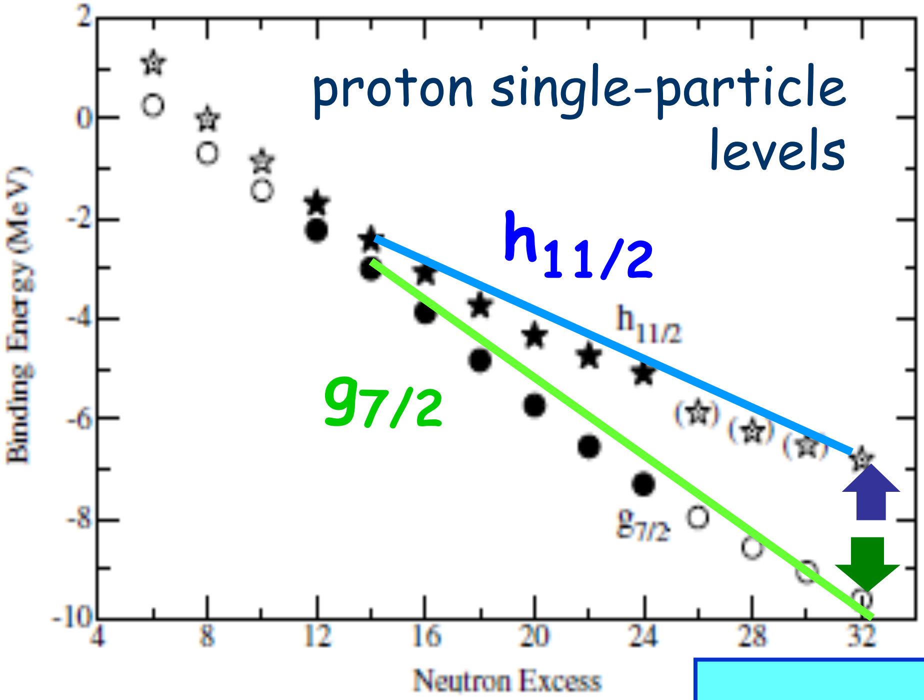


An example with $_{51}$Sb isotopes

VOLUME 92, NUMBER 16 PHYSICAL REVIEW LETTERS week ending 23 APRIL 2004

Is the Nuclear Spin-Orbit Interaction Changing with Neutron Excess?

J. P. Schiffer,[1] S. J. Freeman,[1,2] J. A. Caggiano,[3] C. Deibel,[3] A. Heinz,[3] C.-L. Jiang,[1] R. Lewis,[3] A. Parikh,[3] P. D. Parker,[3] K. E. Rehm,[1] S. Sinha,[1] and J. S. Thomas[4]



Z=51 isotopes

change driven by neutrons in 1$h_{11/2}$

$h_{11/2}$ - $h_{11/2}$ repulsive ⬆

$h_{11/2}$ - $g_{7/2}$ attractive ⬇

π + ρ meson exchange tensor force

(splitting increased by ~ 2 MeV)

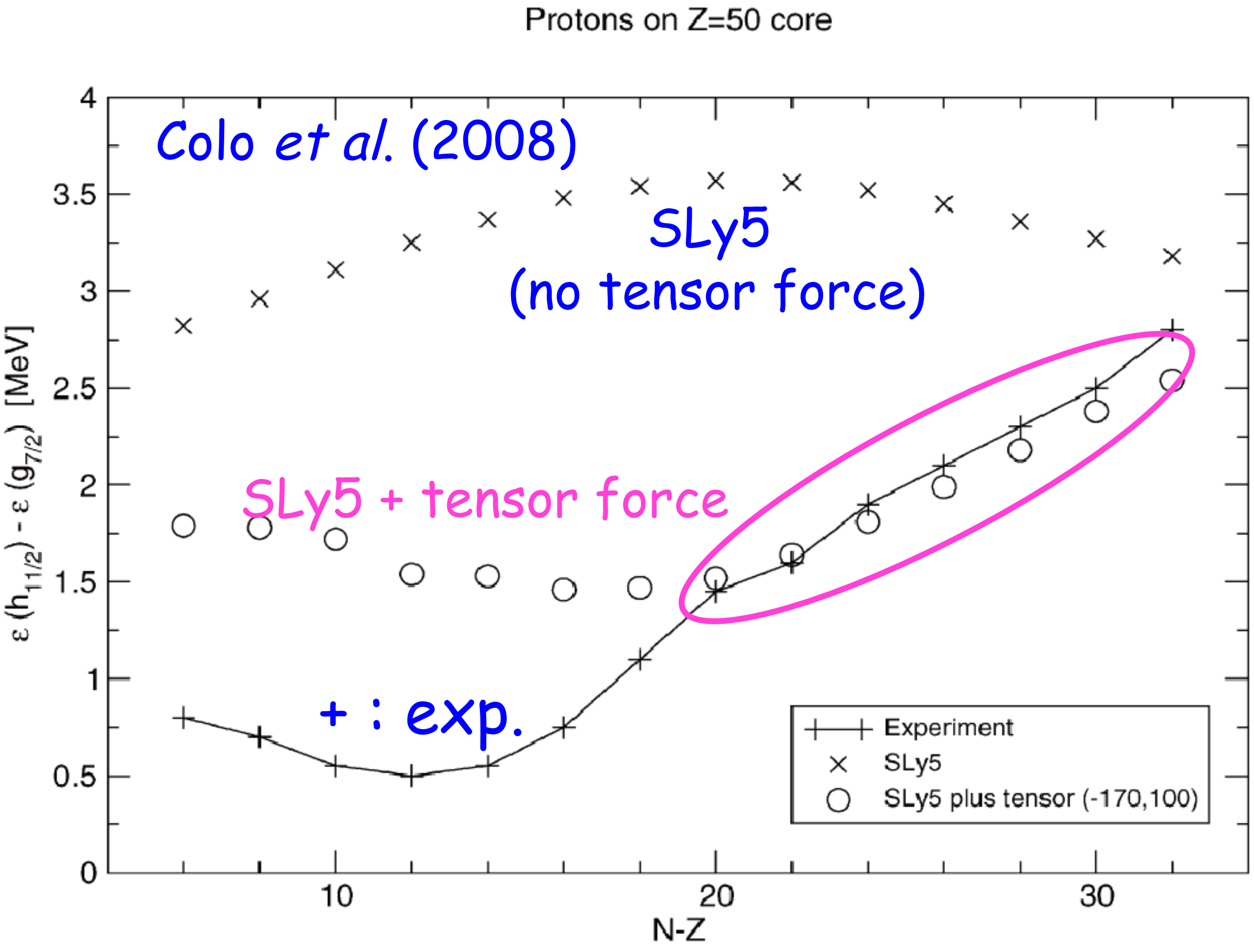
No mean field theory, (Skyrme, Gogny, RMF) explained this before.

TO *et al.*, PRL 95, 232502 (2005)

Tensor force effect added to the Skyrme (maen-field) model :

Proton $g_{7/2}$ and $h_{11/2}$ single-particle orbits

on the Z=50 core with N=64-82



New magic number ?

Eigenvalues of HO potential

Magic numbers Mayer and Jensen (1949)

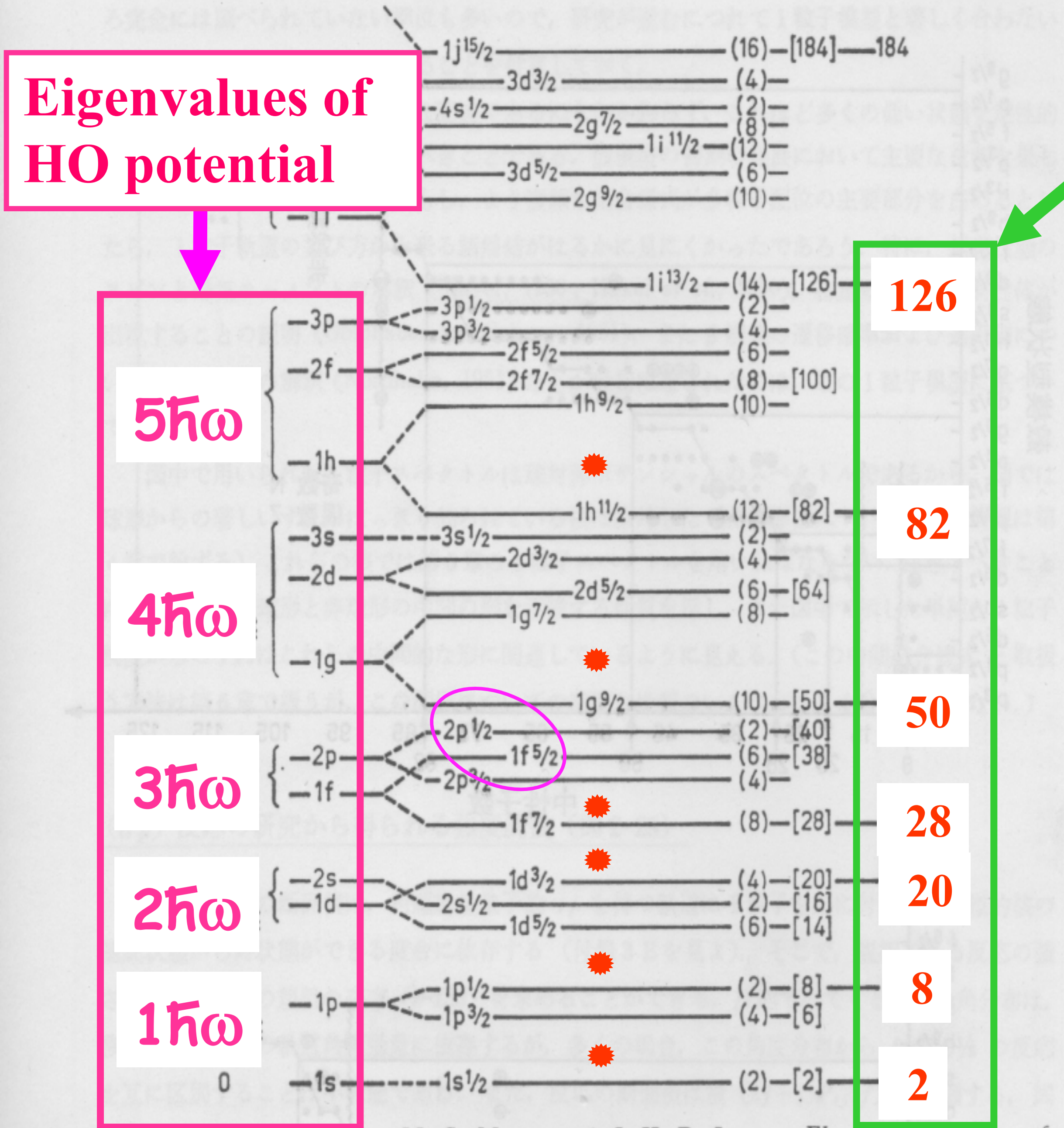
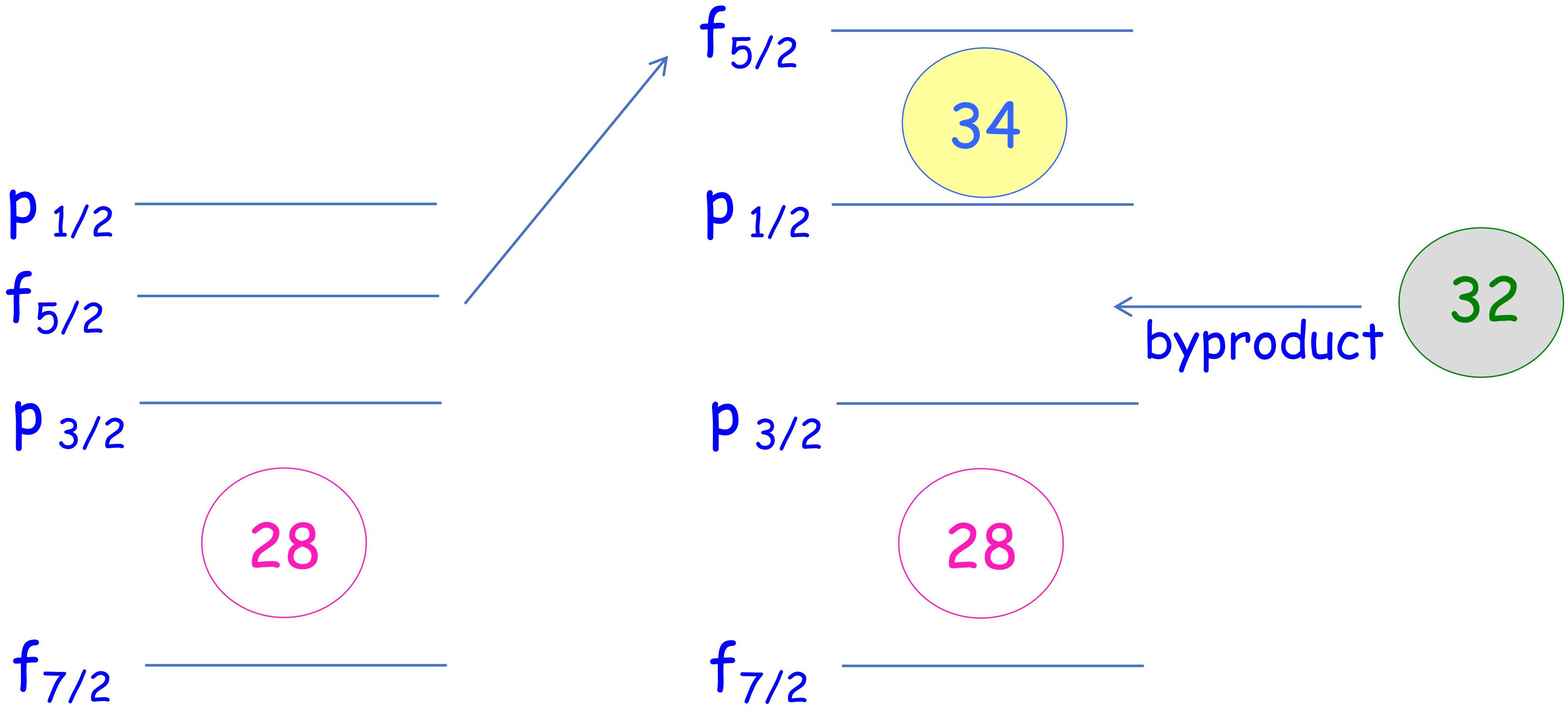


図2-23　1粒子軌道の順序．図は M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure*, p. 58, Wiley, New York, 1955 からとった．

Basic picture

shell structure
for neutrons
in Ni isotopes
($f_{7/2}$ fully occupied)

N=34 magic number may appear
if proton $f_{7/2}$ becomes vacant (Ca)
($f_{5/2}$ becomes less bound)



$p_{1/2}$

$f_{5/2}$

$p_{3/2}$

28

$f_{7/2}$

$f_{5/2}$

34

$p_{1/2}$

byproduct

32

$p_{3/2}$

28

$f_{7/2}$

Appearance of new magic number N=34

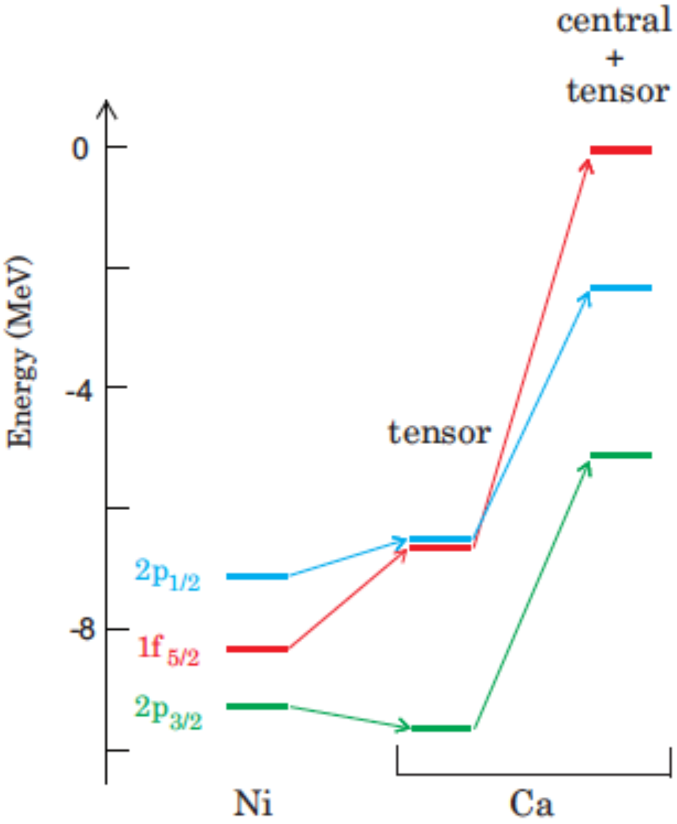


Figure 41 Change of single-particle energies from ^{56}Ni to ^{48}Ca.

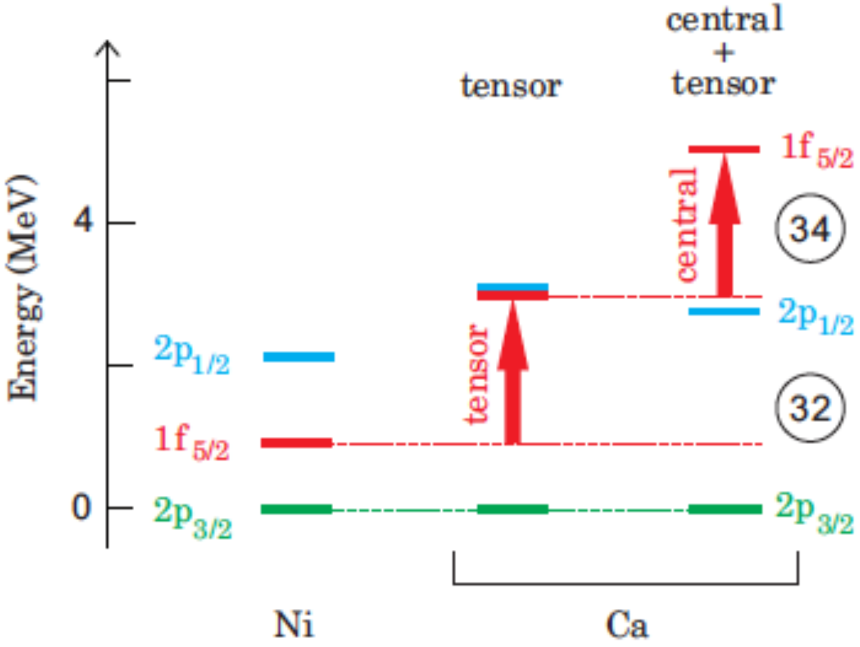
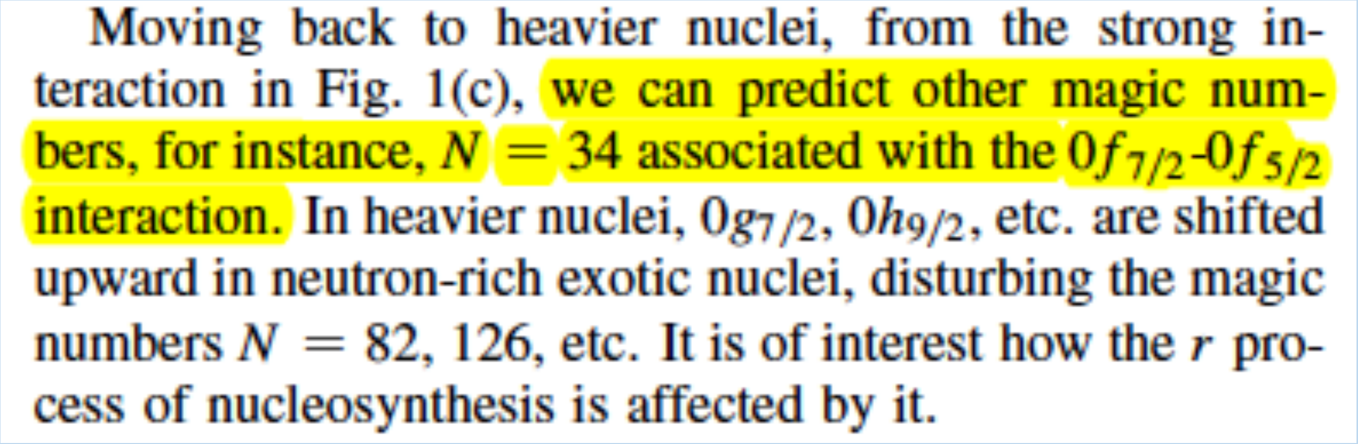


Figure 42 Change of single-particle energies from ^{56}Ni to ^{48}Ca relative to the $2p_{3/2}$ orbit. The red arrows indicate the change of the $1f_{5/2}$ ESPE. The arising magic numbers, N=32 and 34, are shown in black circles.

Is there N=34 magic number ?

In comparison to N=32 magic number known experimentally for nearly 30 years.



Moving back to heavier nuclei, from the strong interaction in Fig. 1(c), we can predict other magic numbers, for instance, $N = 34$ associated with the $0f_{7/2}$-$0f_{5/2}$ interaction. In heavier nuclei, $0g_{7/2}$, $0h_{9/2}$, etc. are shifted upward in neutron-rich exotic nuclei, disturbing the magic numbers $N = 82$, 126, etc. It is of interest how the r process of nucleosynthesis is affected by it.

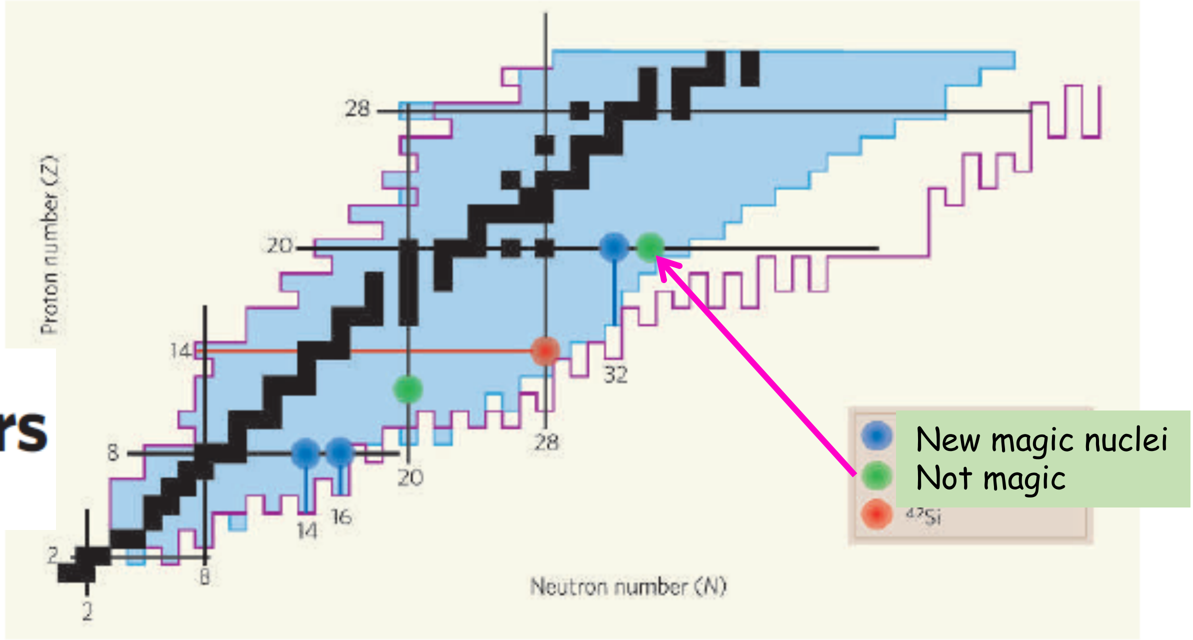
TO et al.
PRL 87 (2001)

NATURE|Vol 435|16 June 2005

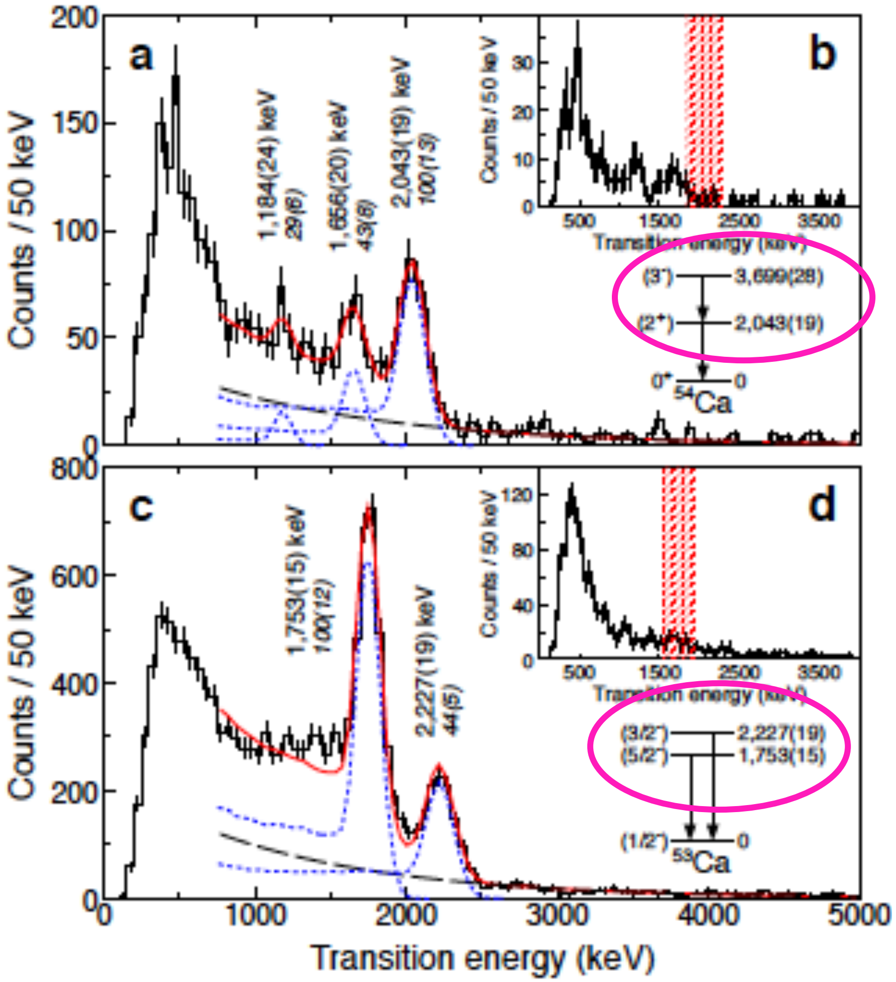
NUCLEAR PHYSICS

Elusive magic numbers

Robert V. F. Janssens



Experiment @ RIBF → Finally confirmed



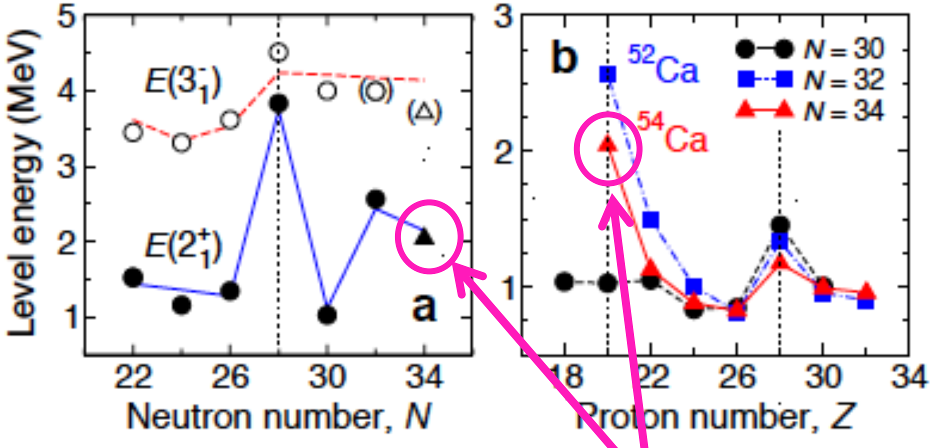


FIG. 4: **Systematics of excited-state e even-even Ca isotopes and neighbouring n** ergies of first 2^+ (closed symbols) and 3^- (open els for even-even $^{42-54}$Ca isotopes [28]. The res resent study are indicated by triangular marker l and dashed lines are shell-model predictions of (3_1^-), respectively (see text for details). Tentat ty assignments are enclosed by parentheses. **b,** $E(2_1^+)$ along the $N = 30$, 32 and 34 isotonic chains. The solid and dashed lines are intended to guide the eye. Vertical dotted lines represent the traditional magic numbers in both plots.

new RIBF data

er-corrected γ-ray energy spectra. De-excitation γ rays measured in coinci-

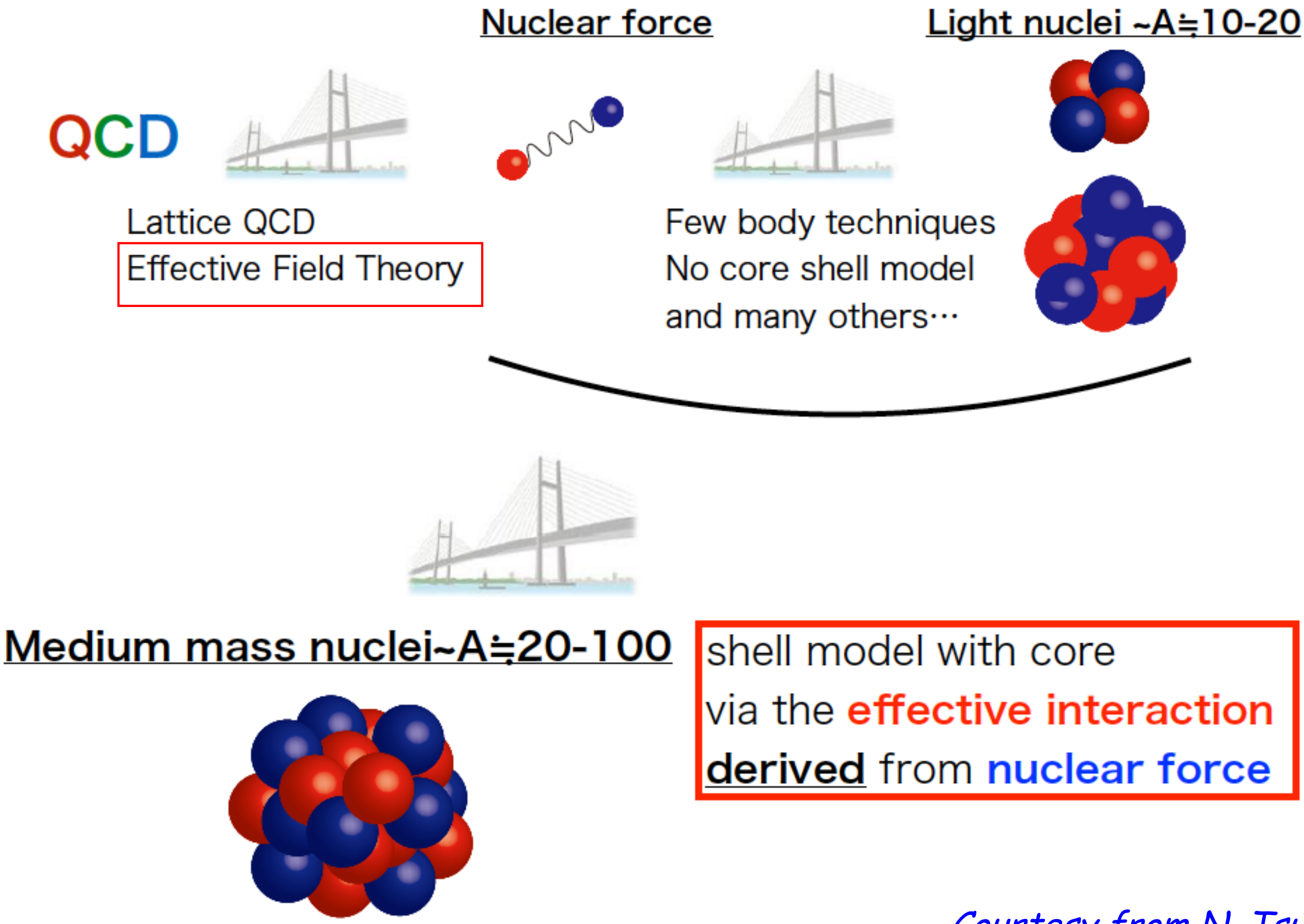
^{4}Ca and **c,** ^{53}Ca reaction products. Peaks a

Steppenbeck *et al.* Nature, 502, 207 (2013)

ve intensities are indicated by italic fonts. The short-blue and long-black dashed

Shell evolution with the modern nuclear forces

shell model powered by modern nuclear forces



Nuclear force

Light nuclei ~A≒10-20

QCD

Lattice QCD

Effective Field Theory

Few body techniques
No core shell model
and many others…

Medium mass nuclei~A≒20-100

shell model with core
via the **effective interaction**
derived from **nuclear force**

Courtesy from N. Tsunoda

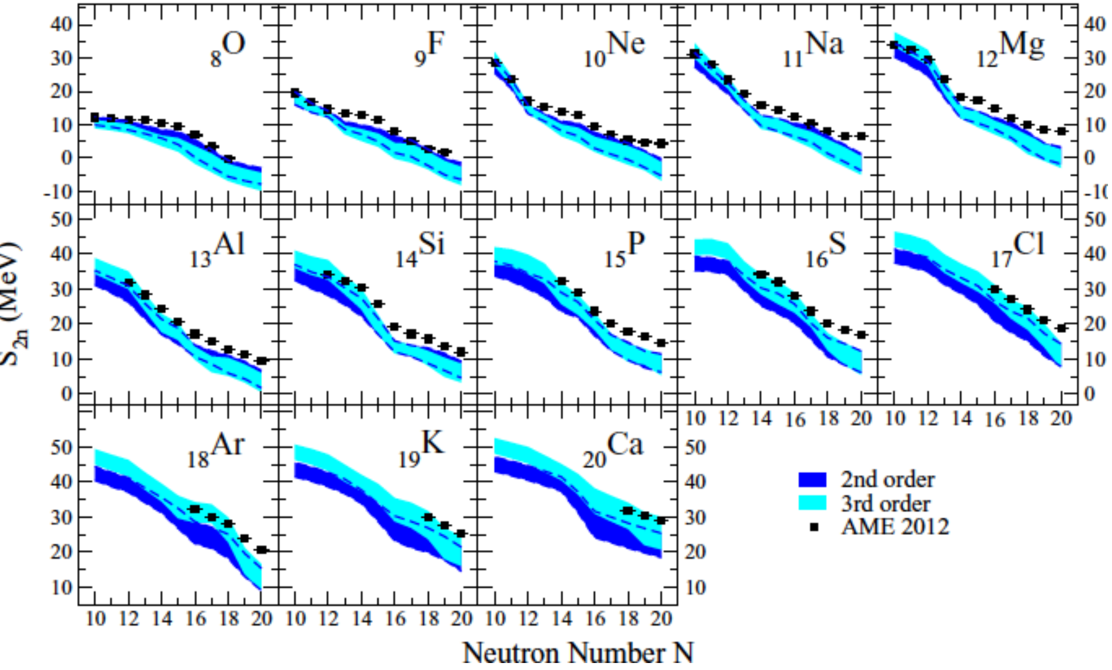
Examples of structure calculations starting from chiral EFT forces

NN force: N3LO + 3N force: N2LO

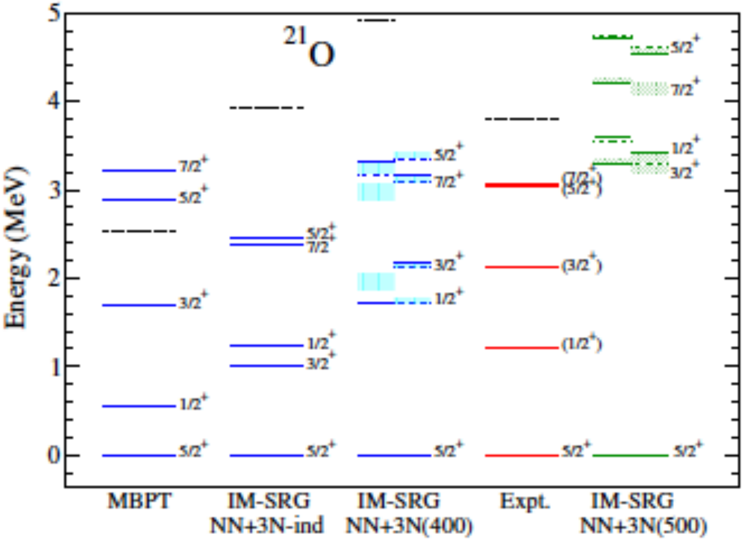
-> valence shell interaction

MBPT
(Many-body Perturbation Theory)
applicable to one major shell

IM-SRG
(In-Medium Similarity
Renormalization Group)



Simonis et al. PRC 93, 011302(R) (2016)



Bogner et al. PRL 113, 142501 (2014)

+ Coupled Cluster calculation + N2LOsat potential + ….

A recent development starting from ***chiral EFT + 3NF***

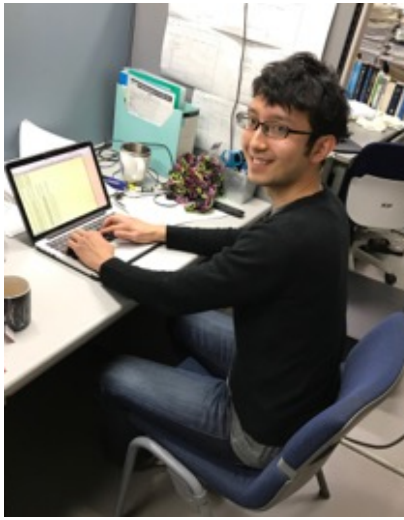
EKK method to handle consistently two (or more) major shells

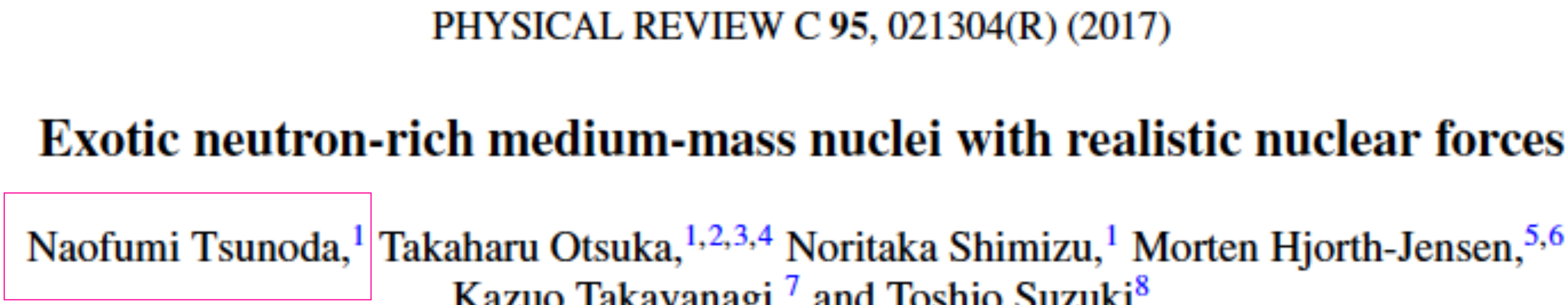
-> Effective shell-model interaction

(i) without fit of two-body m. e.,

(ii) applicable to broken magicity, or fusion of two shells,

both are crucial for exotic nuclei.

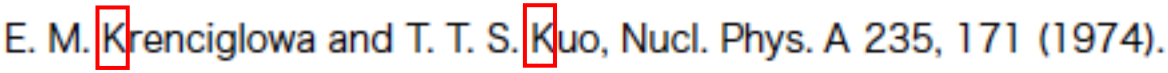




PHYSICAL REVIEW C **95**, 021304(R) (2017)

Exotic neutron-rich medium-mass nuclei with realistic nuclear forces

Naofumi Tsunoda,[1] Takaharu Otsuka,[1,2,3,4] Noritaka Shimizu,[1] Morten Hjorth-Jensen,[5,6]
Kazuo Takayanagi,[7] and Toshio Suzuki[8]



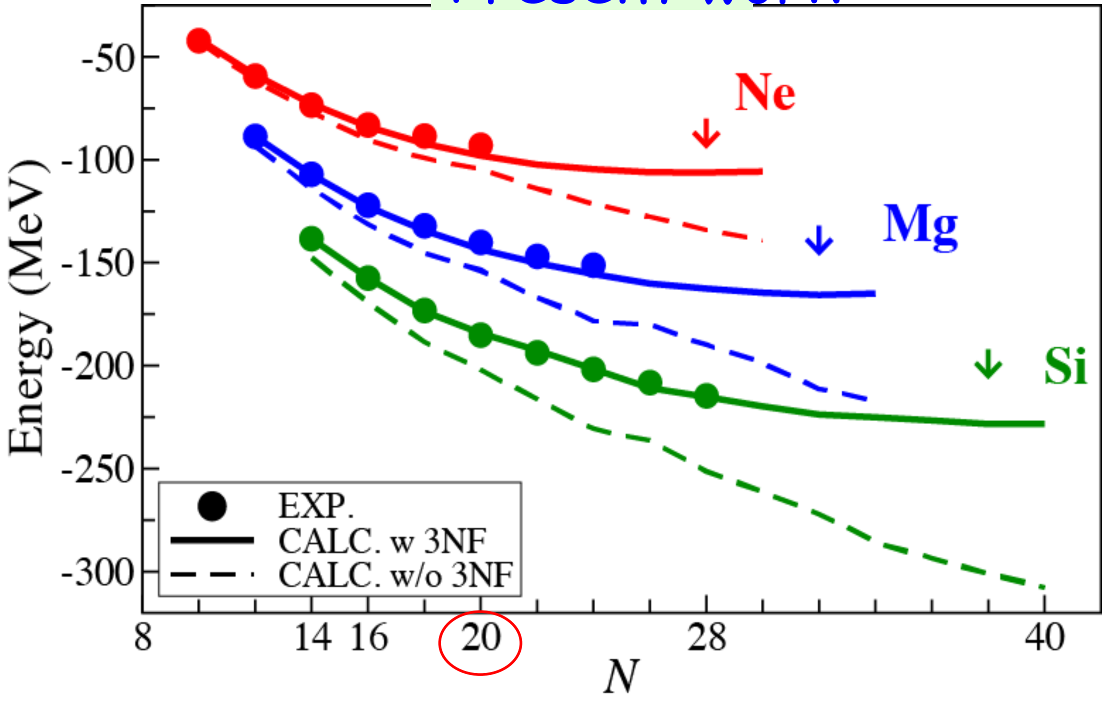
*E. M. Krenciglowa and T. T. S. Kuo, Nucl. Phys. A 235, 171 (1974).

Re-visit to the "Island of Inversion" with ab initio TBMEs

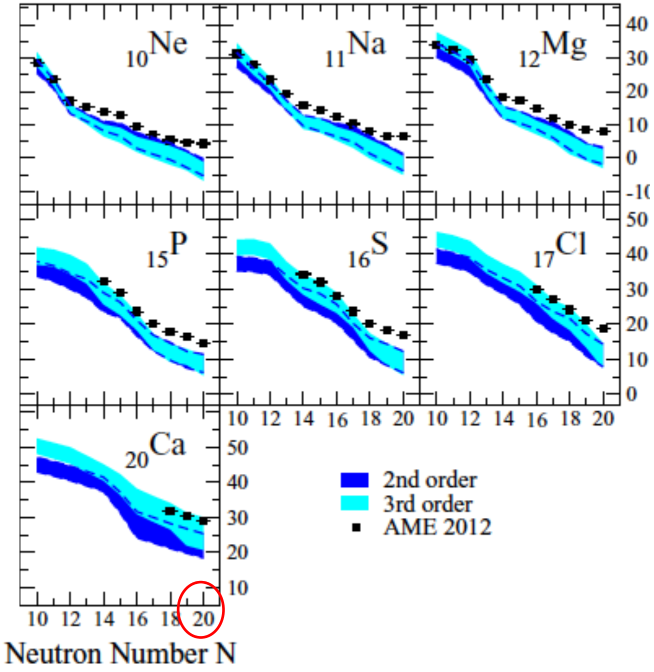
Calculations *with full sd + pf* shell

ground-state energies

Present work



Earlier work

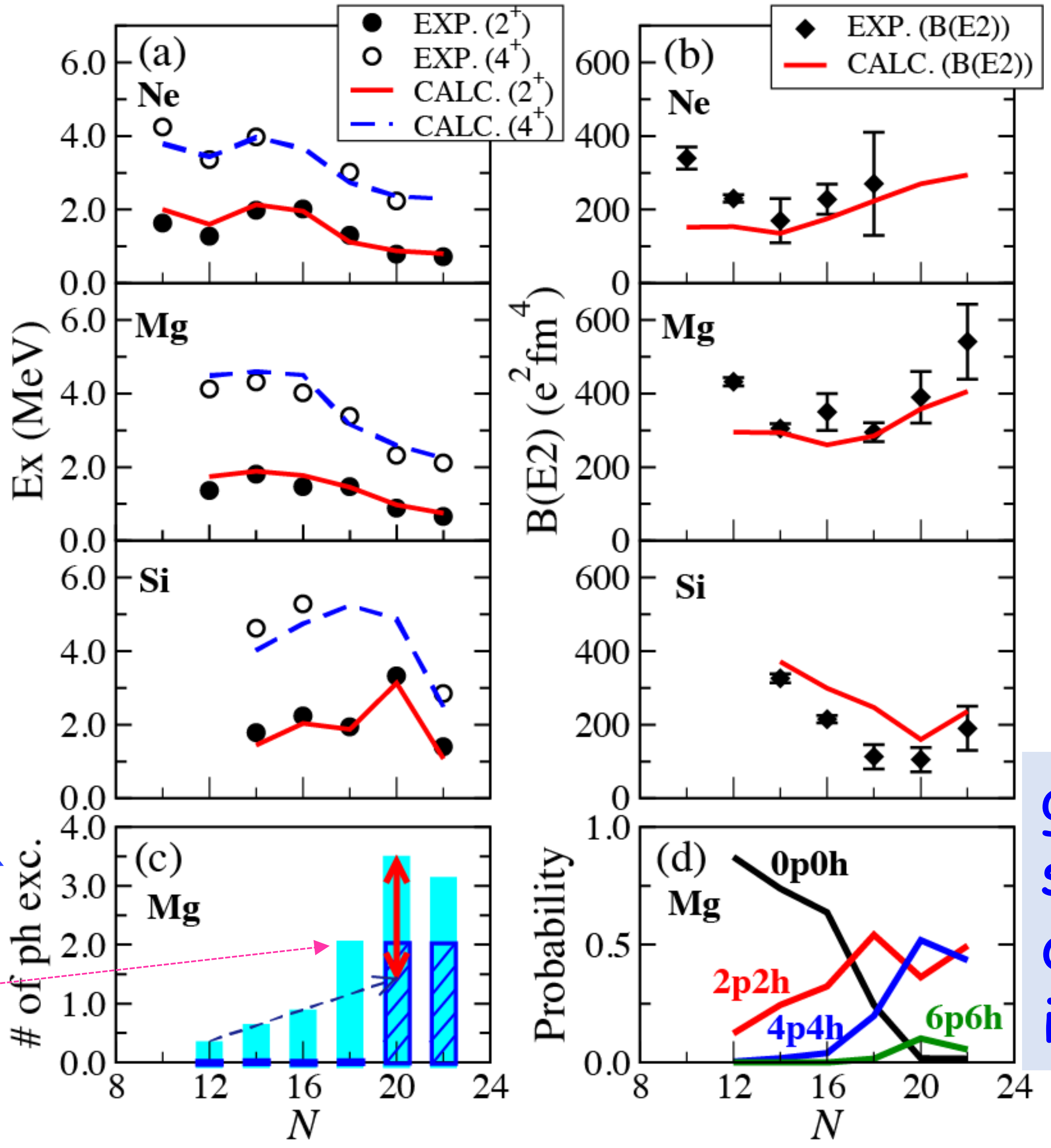


Simonis et al. PRC 93, 011302(R) (2016)

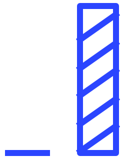
Ne-Mg-Si

2^+ & 4^+ levels and B(E2) values

# of particle-hole excitations across N=20 gap :
(modest) steady increase
+
abrupt increase after N=18

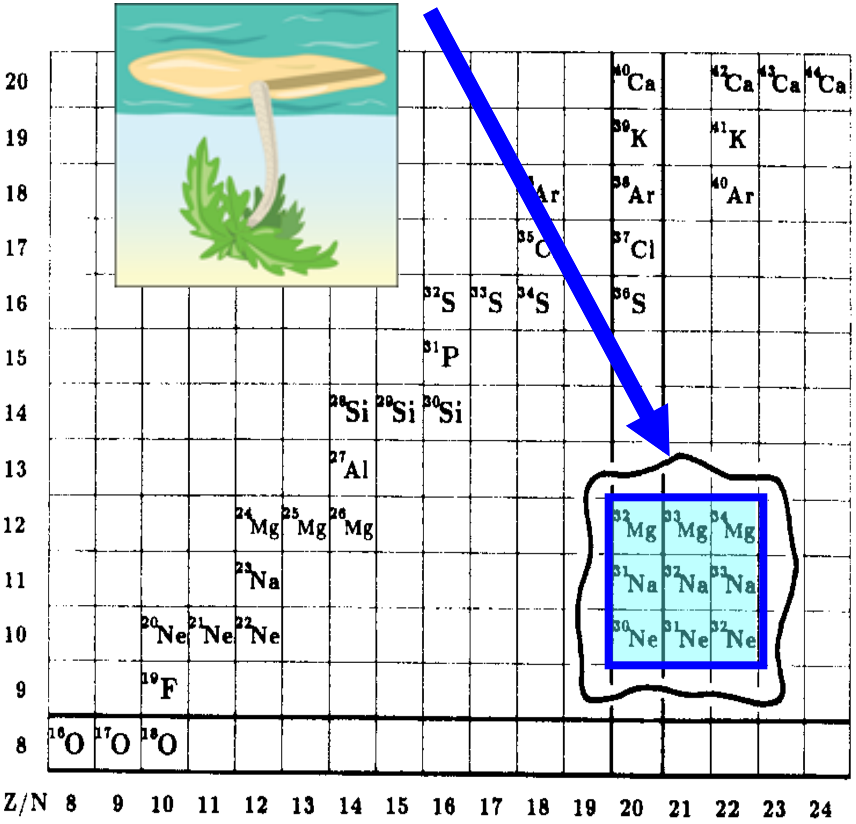


ground states of Mg isotopes



Early idea of the Island of Inversion (WBB)
0p-0h or 2p-2h (discrete)

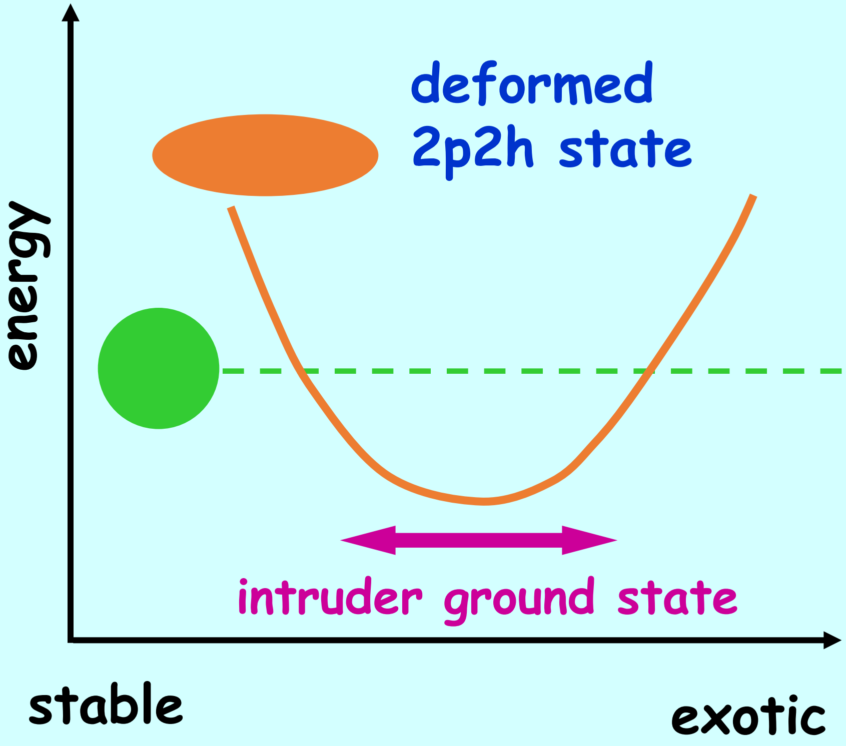
Island of Inversion



9 nuclei:
Ne, Na, Mg with N=20-22

Phys. Rev. C 41, 1147 (1990),
Warburton, Becker and
Brown

Basic picture was



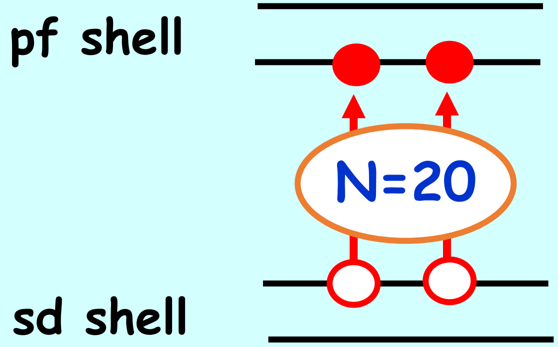
deformed
2p2h state

energy

intruder ground state

stable

exotic



pf shell

N=20

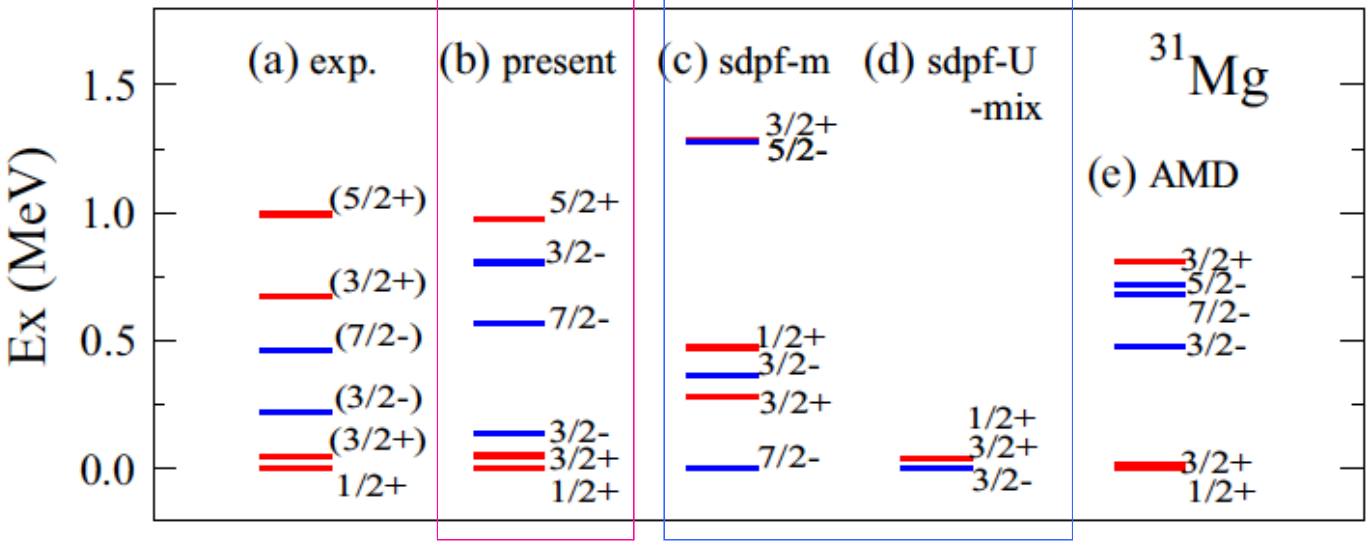
sd shell

gap ~
constant

$^{31}_{12}\mathrm{Mg}_{19}$: *very difficult to fit* *by the shell model*

$1/2^+$: not a simple $s_{1/2}$ single-particle state

mag. moment
-0.88 exp
-0.50 th
-1.91 Schmidt



N3LO + 3NF by EKK

conventional interactions

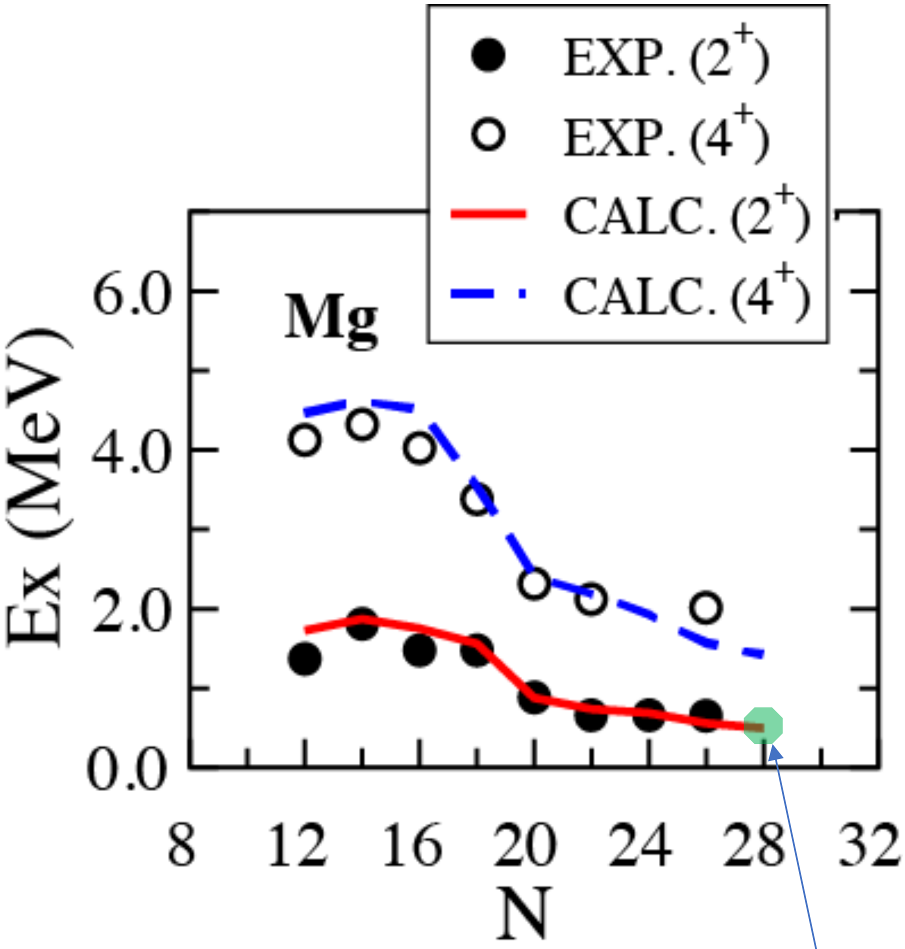
FIG. 4. Energy levels of ^{31}Mg. (a) experimental values, (b) present work, (c) sdpf-m [16], (d) sdpf-U-mix [17], and (e) AMD+ calculation [52].

exp. by laser spectroscopy : PRL 94, 022501 (2005), G. Neyens, *et al.*

Mixing between sd and pf shells is crucial
Fit of relevant TBME's is infeasible : too many TBME's but too few data

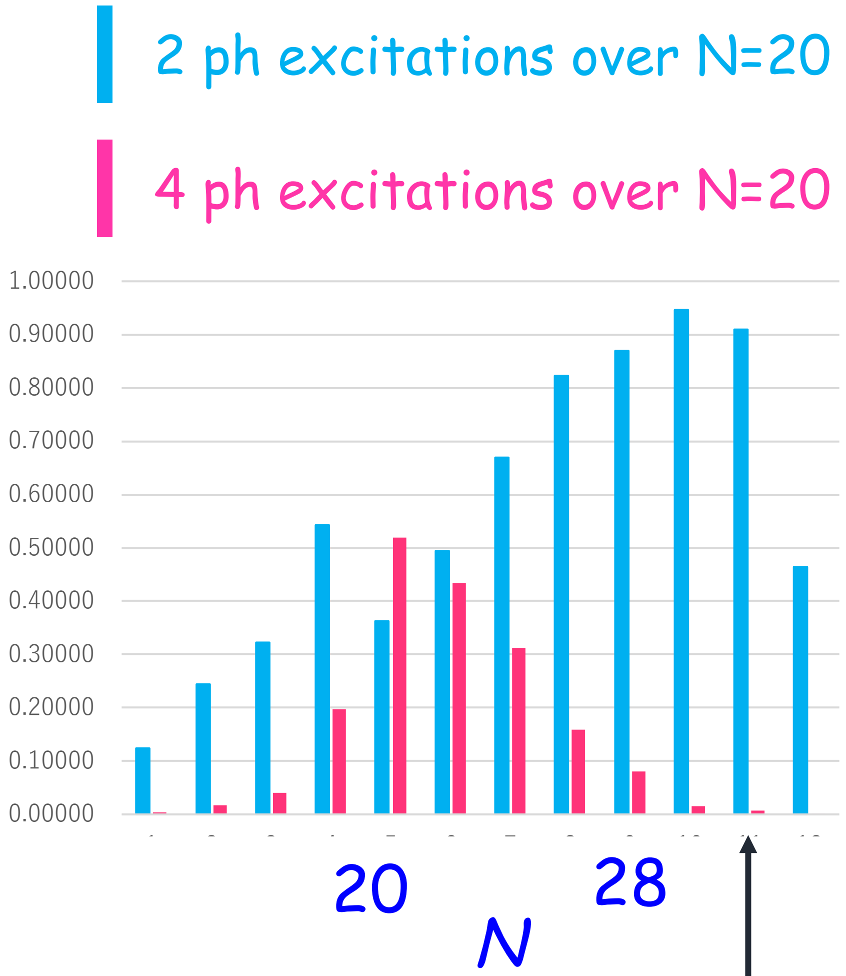
[16] Y. Utsuno, T. Otsuka, T. Mizusaki, and M. Honma, Phys. Rev. C 60, 054315 (1999).
[17] E. Caurier, F. Nowacki, and A. Poves, Phys. Rev. C 90, 014302 (2014).
[52] M. Kimura, Phys. Rev. C 75, 041302 (2007).

Mg up to N=28 2^+ & 4^+ levels



Recent data
Crawford *et al.*
(ARIS 2017 + priv. com.)

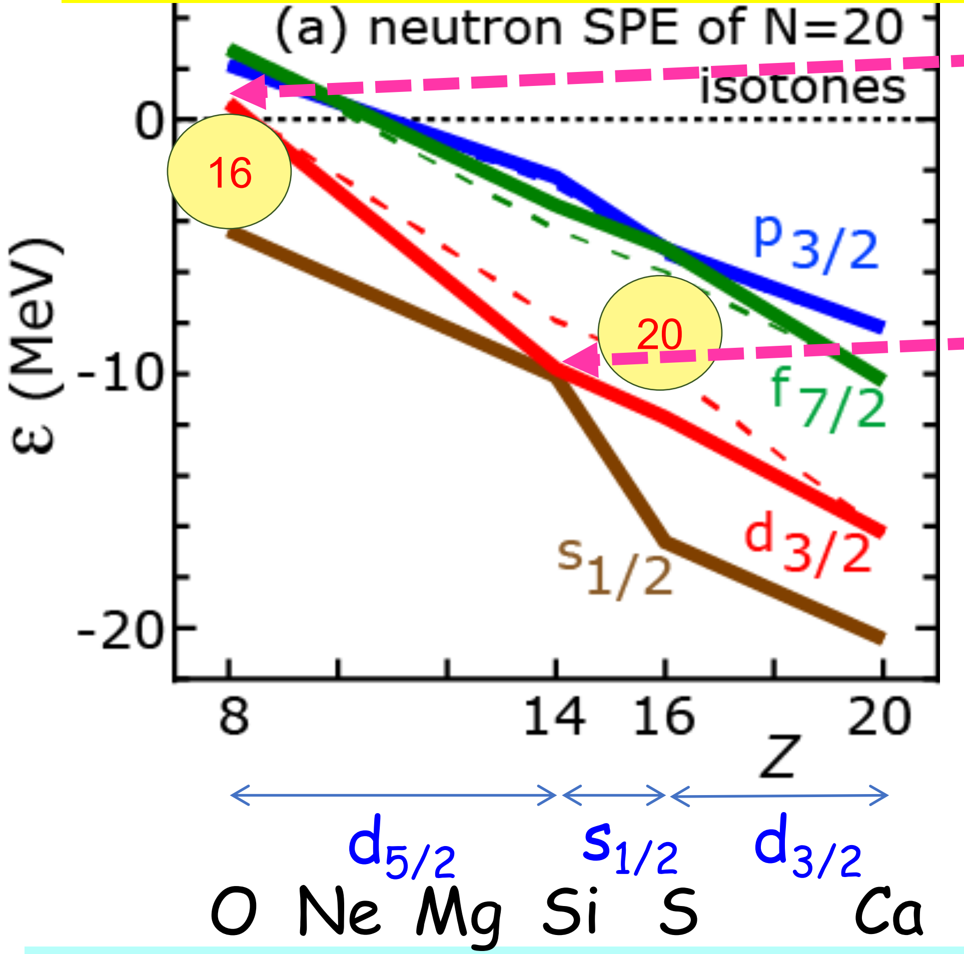
Probability of ph configurations over Z=N=20 (ground state)

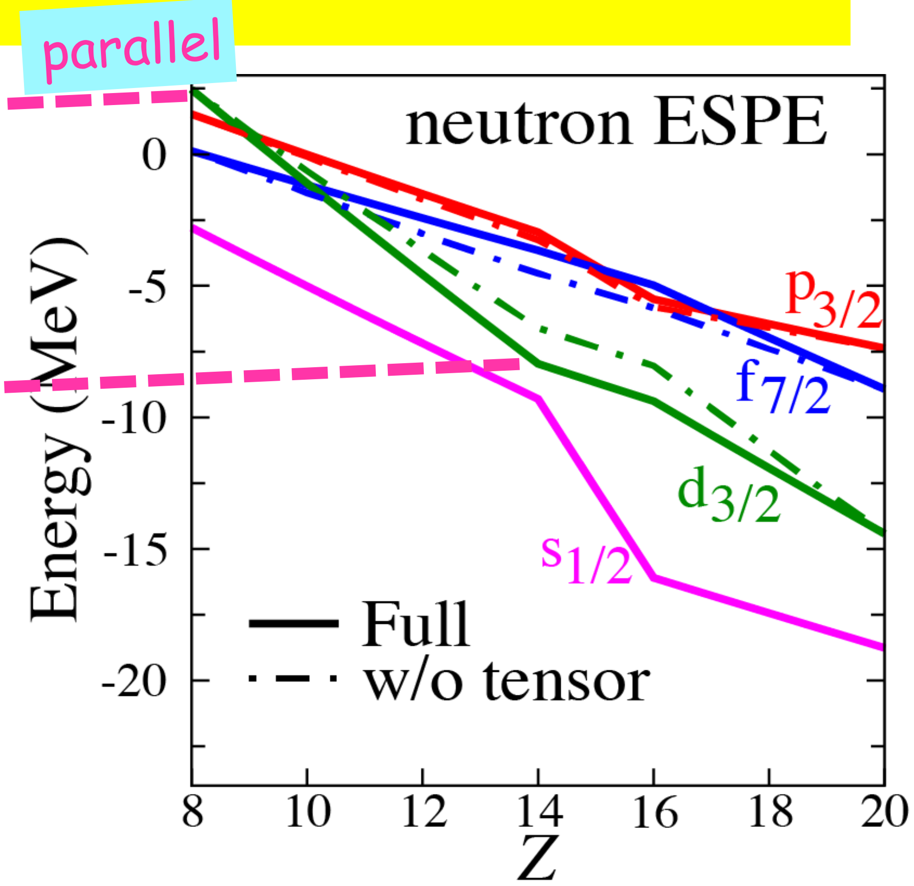


our
drip line

Neutron single-particle energy (SPE) at N=20

Tensor and central force effects are almost identical between the two schemes.





Obtained from VMU interaction : phenomenological central (Gaussian) and π + ρ meson-exchange tensor interaction

Obtained by EKK method from chiral EFT *NN* interaction

The traditional picture such as the Island of Inversion is being re-visited and re-examined, leading to a renewed picture !

The shell structure can be indeed changed in many cases !

The same physics drives the shape coexistence and in the quantum phase transition.   Tomorrow …

Int. School of Physics "Enrico Fermi"- Course 201
Nuclear Physics with Stable and Radioactive Ion Beams
Varenna
July 14-19, 2017

Recent developments in shell model studies of atomic nuclei

Takaharu Otsuka

3rd lecture

This work has been supported by MEXT and JICFuS as a priority issue (Elucidation of the fundamental laws and evolution of the universe) to be tackled by using Post 'K' Computer

Summary of shell evolution

p-n monopole interaction between orbits j and j'

particularly strong, if Δn=0 (n : # of nodes of radial wave func.)

central : attractive

tensor : attractive ($j_>$ -$j'_<$) or repulsive ($j_>$ -$j'_>$)

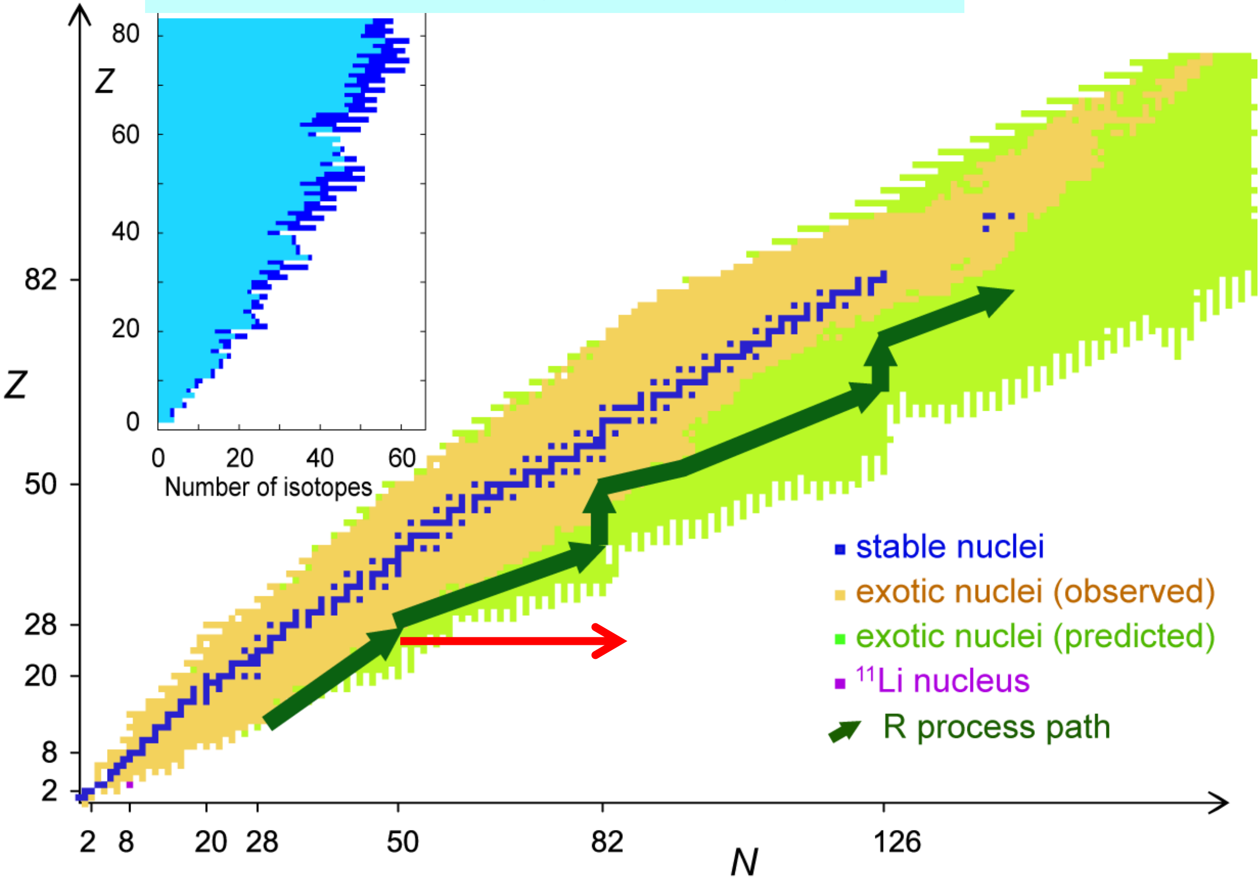
example : $g_{9/2}$-$f_{5/2}$: coherent $g_{9/2}$-$f_{7/2}$: cancellation

monopole interaction changes ESPE (effective single-particle energy) with linear dependence on the occupation number $n_{j'}$
→ effect can be magnified → shell evolution

change of spin-orbit splitting
change of shell gap
change of the ordering of orbits
new magic numbers, disappearance of usual magic numbers

Neutron drip line is quite far, and there are many well-bound exotic nuclei.

Nuclear forces can play leading roles !



# of well-bound isotopes (S_{2n} > 2MeV) as a function of *Z* (vertical line)

Effective single-particle energy

This is not uniform as a function of j and j'.

$$\Delta\hat{\epsilon}_j^p = \sum_{j'} V^m_{T=1}(j,j')\,\Delta\hat{n}^p_{j'} + \sum_{j'} \boxed{V^m_{pn}(j,j')}\,\Delta\hat{n}^n_{j'},$$

This can be large.

Three pillars combined for future

computation

Monte Carlo
Shell Model
(MCSM)

(almost)
unlimited
dimensionality

massive
parallel
computers

Hamiltonian

pf
pfg9d5 (A3DA) (Ni)
8+8 on ^{56}Ni core (Zr)
8+8 on ^{80}Zr core (Sn)
8+10 on ^{132}Sn core
(Sm)
…
island of stability
+
χEFT based
multi-shell int.

many-body dynamics

Shell evolution
(Type I & II)

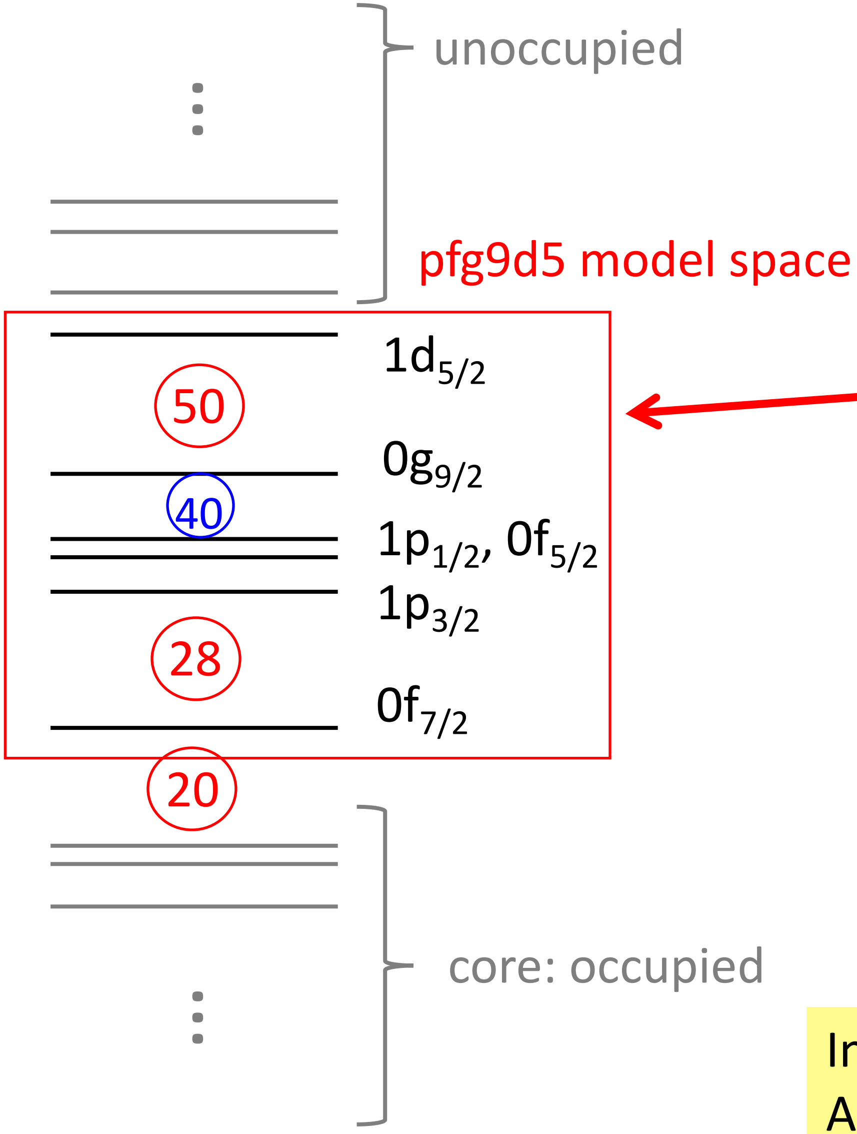
Quantum Phase
Transition

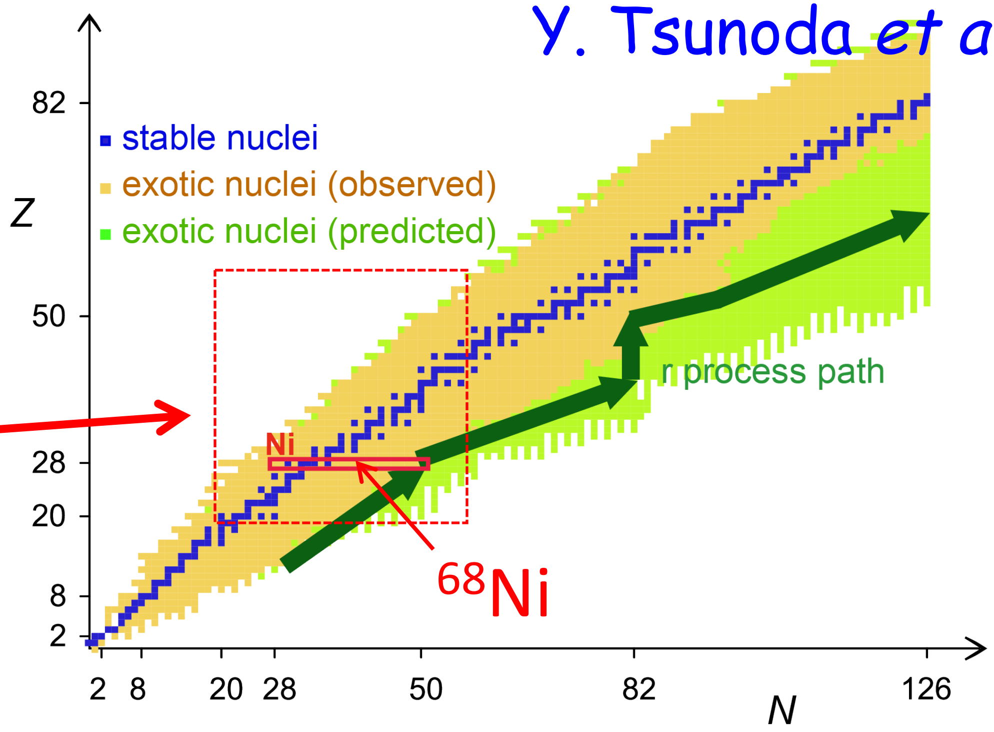
Shape coexistence

Quantum
Self-organization

Monte Carlo Shell Model (MCSM) calculation on Ni isotopes

Y. Tsunoda *et al.*





This model space is wide enough to discuss how magic numbers 28, 50 and semi-magic number 40 are visible or smeared out.

Interaction:
A3DA interaction is used with minor corrections

Energy levels and B(E2) values of Ni isotopes

Description by the same Hamiltonian

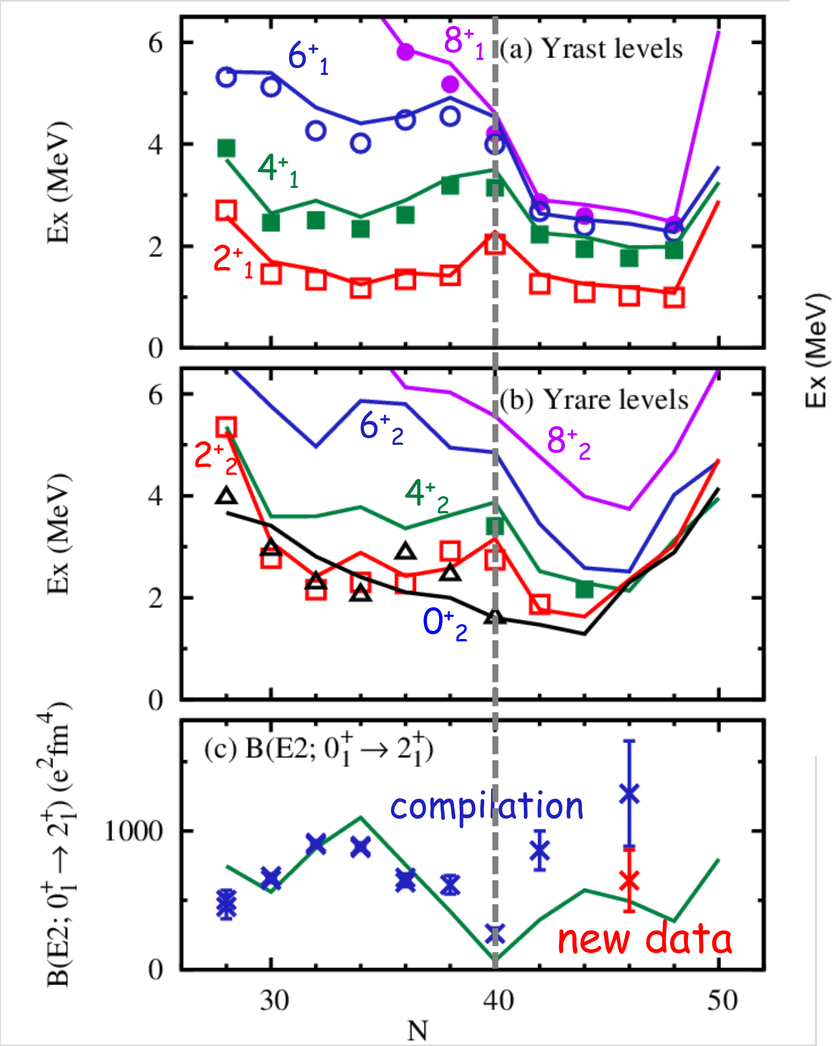
Shape coexistence in ^{68}Ni

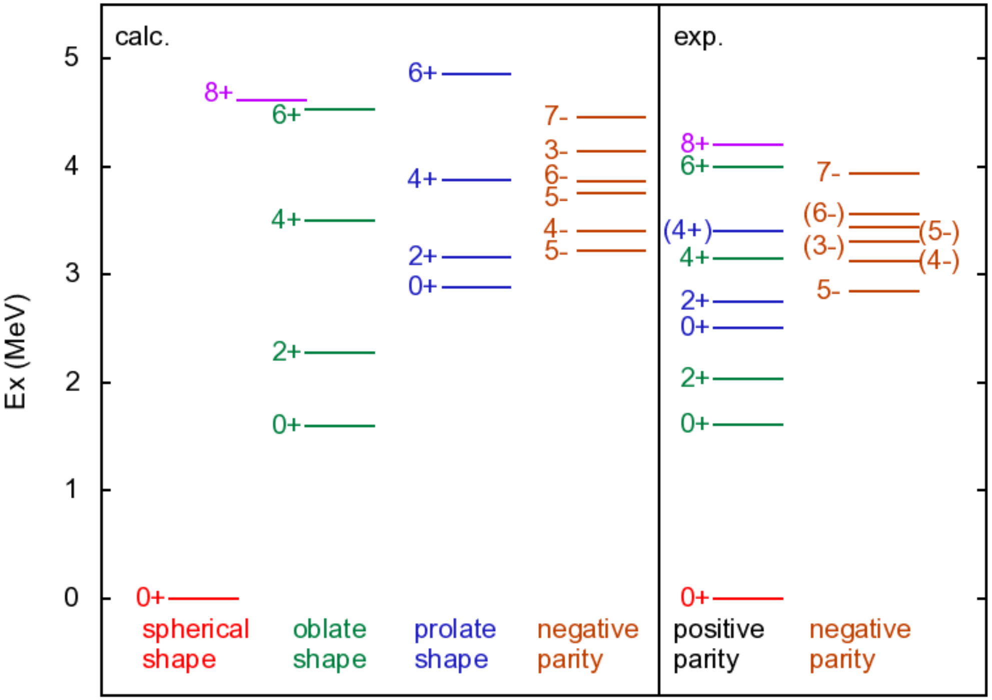
× exp.

— calc.

^{68}Ni

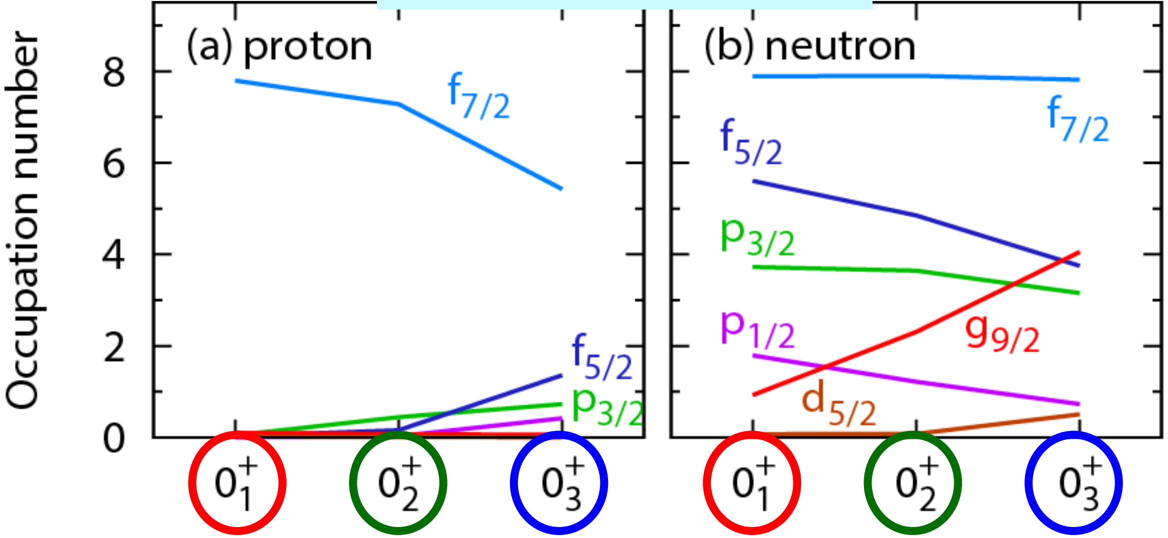
calc. exp.

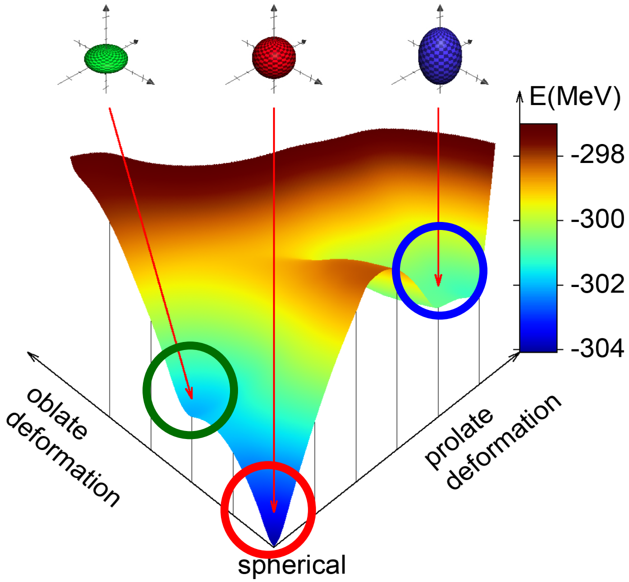




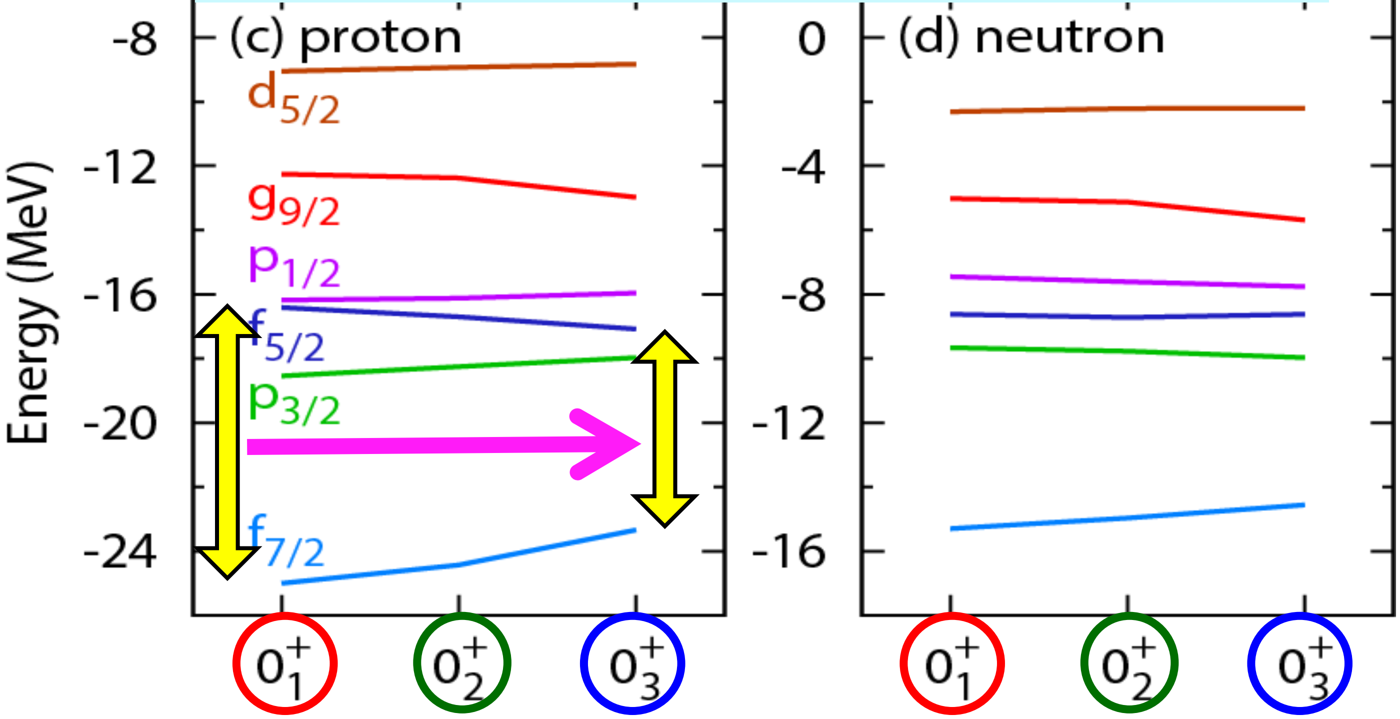
Y. Tsunoda, TO, Shimizu, Honma and Utsuno, PRC 89, 031301 (R) (2014)

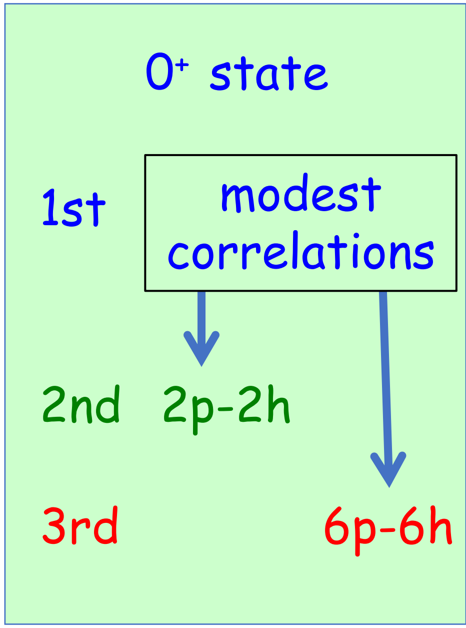
Occupation numbers





Effective s.p.e. by actual occupation numbers





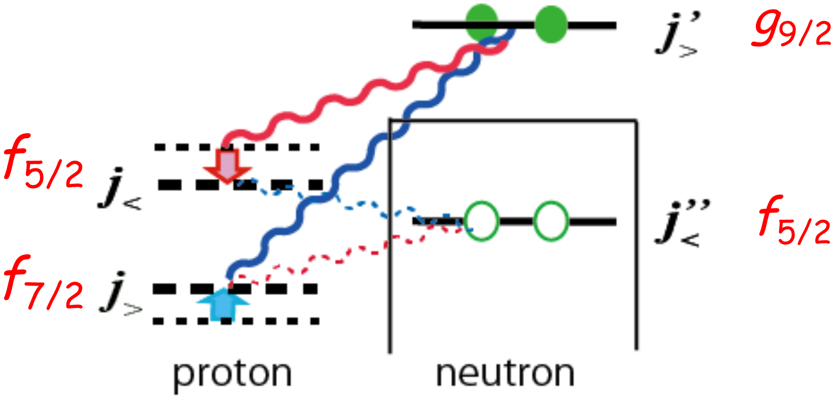
0^+ state

1st modest correlations

2nd 2p-2h

3rd 6p-6h

Type II Shell Evolution in ^{68}Ni (Z=28, N=40)



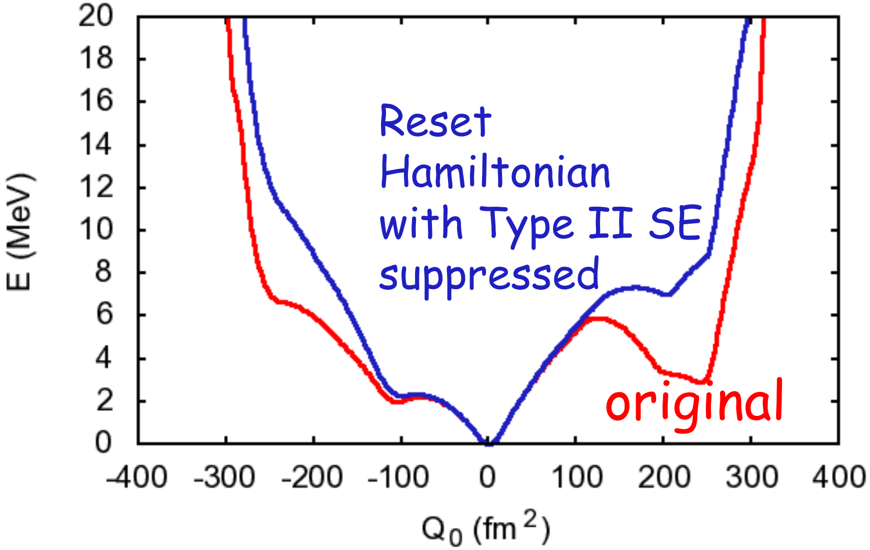
Spin-orbit splitting works against quadrupole deformation (*cf. Elliott's SU(3)*).

weakening of spin-orbit splitting

Type II shell evolution

stronger deformation of protons
→ more neutron p-h excitation

PES along axially symmetric shape



Type II shell evolution is suppressed by resetting monopole interactions as

$\pi f_{7/2} - \nu g_{9/2} = \pi f_{5/2} - \nu g_{9/2}$

$\pi f_{7/2} - \nu f_{5/2} = \pi f_{5/2} - \nu f_{5/2}$

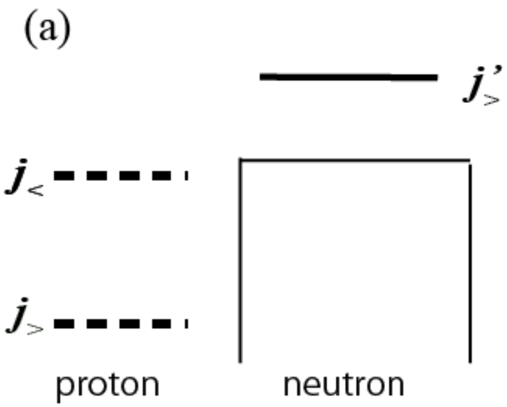
The local minima become much less pronounced.

Shape coexistence is enhanced by type II shell evolution because the same quadrupole interaction can work more efficiently.

Underlying mechanism of the appearance of low-lying deformed states : Type II Shell Evolution

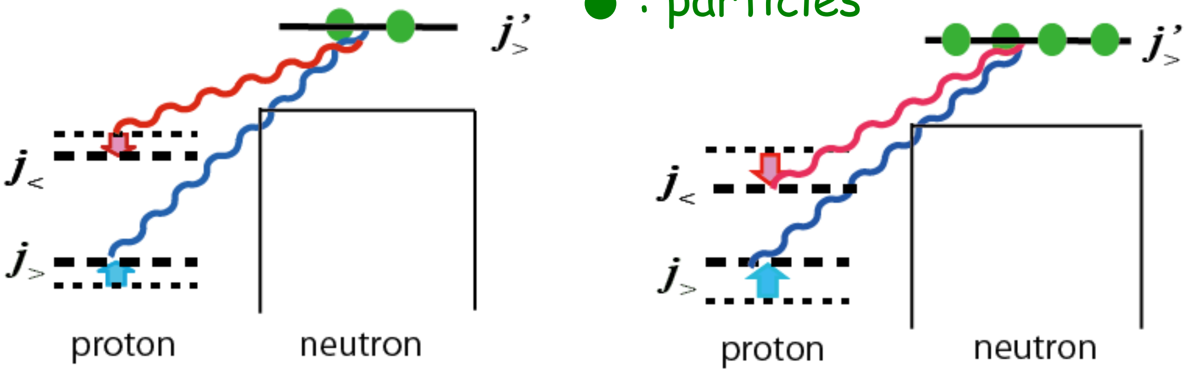
TO and Y. Tsunoda, J. Phys. G: Nucl. Part. Phys. 43 (2016) 024009

Monopole effects on the shell structure from the tensor interaction



Type I Shell Evolution : different isotopes

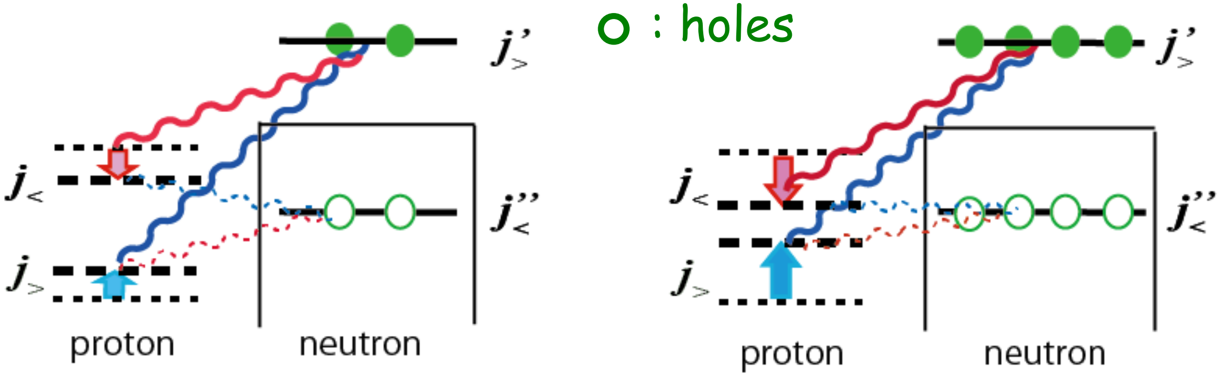
● : particles



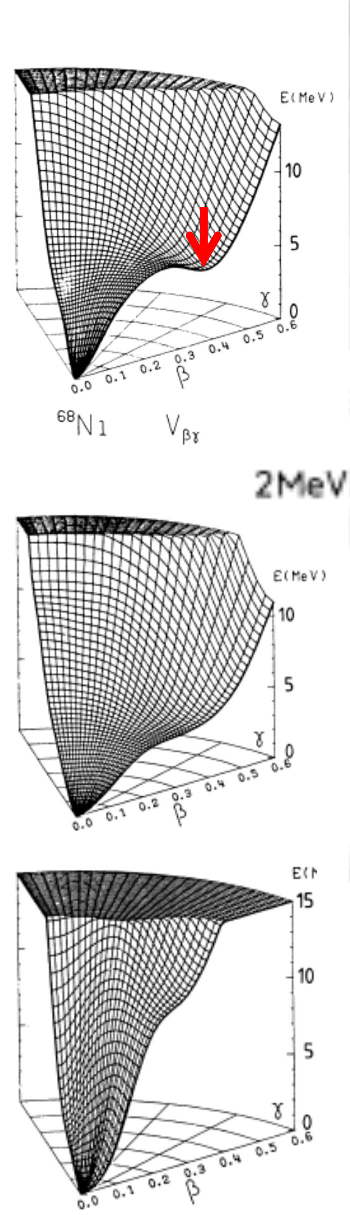
Type II Shell Evolution : within the same nucleus

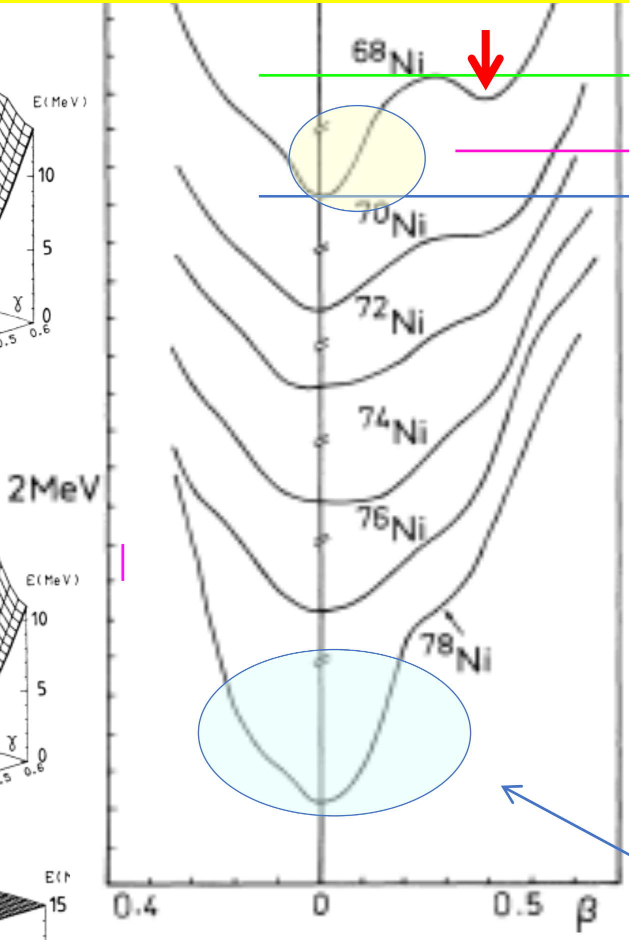
● : particles

○ : holes



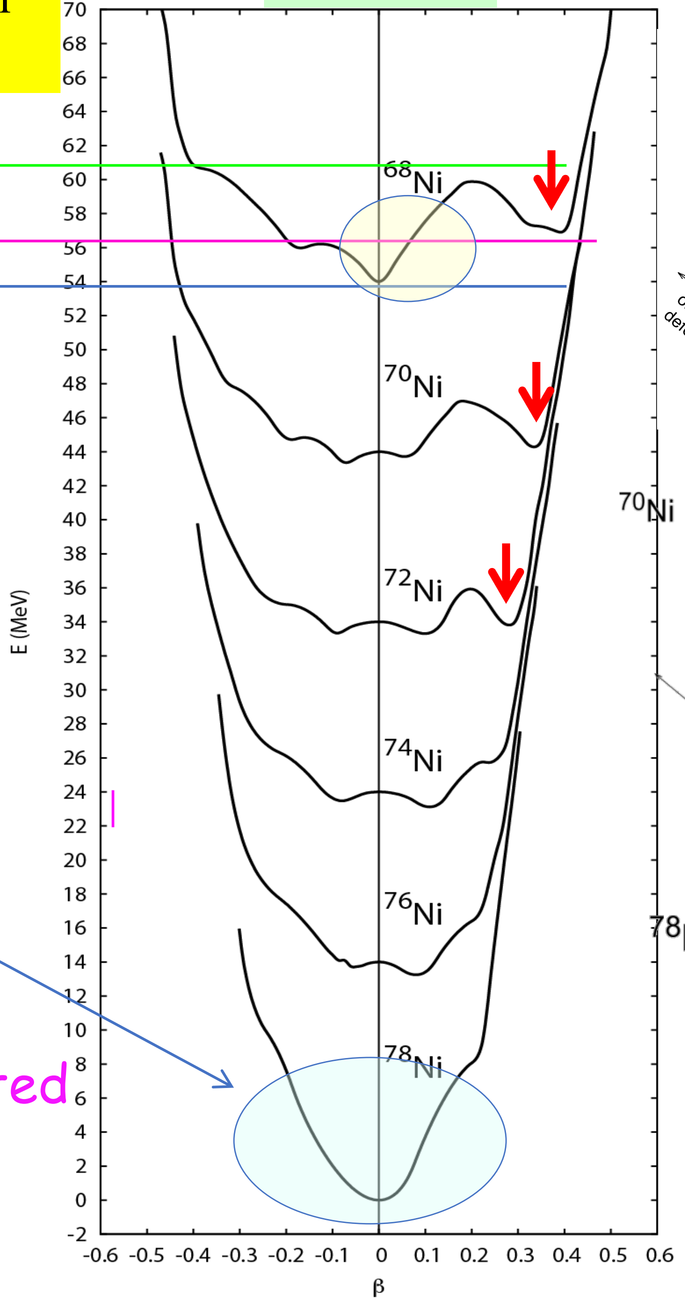
Bohr-model calc. by HFB with **Gogny** force, Girod, Dessagne, Bernes, Langevin, Pougheon and Roussel, PRC 37,2600 (1988)

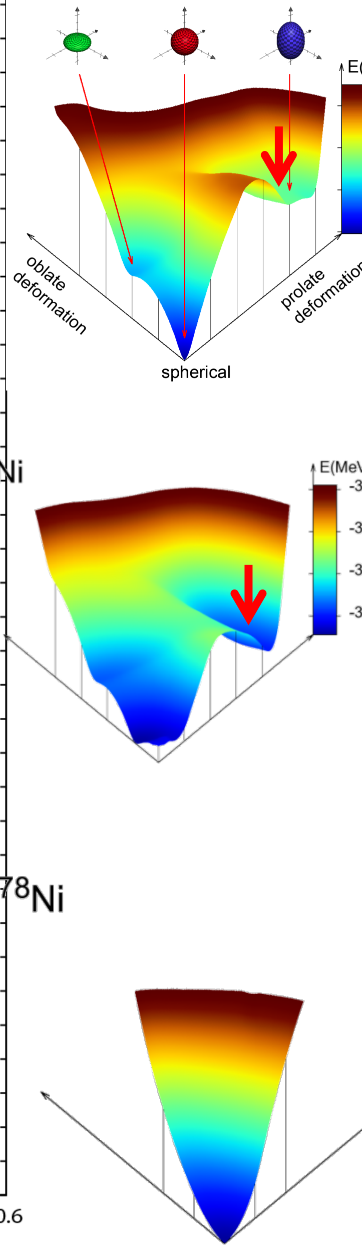




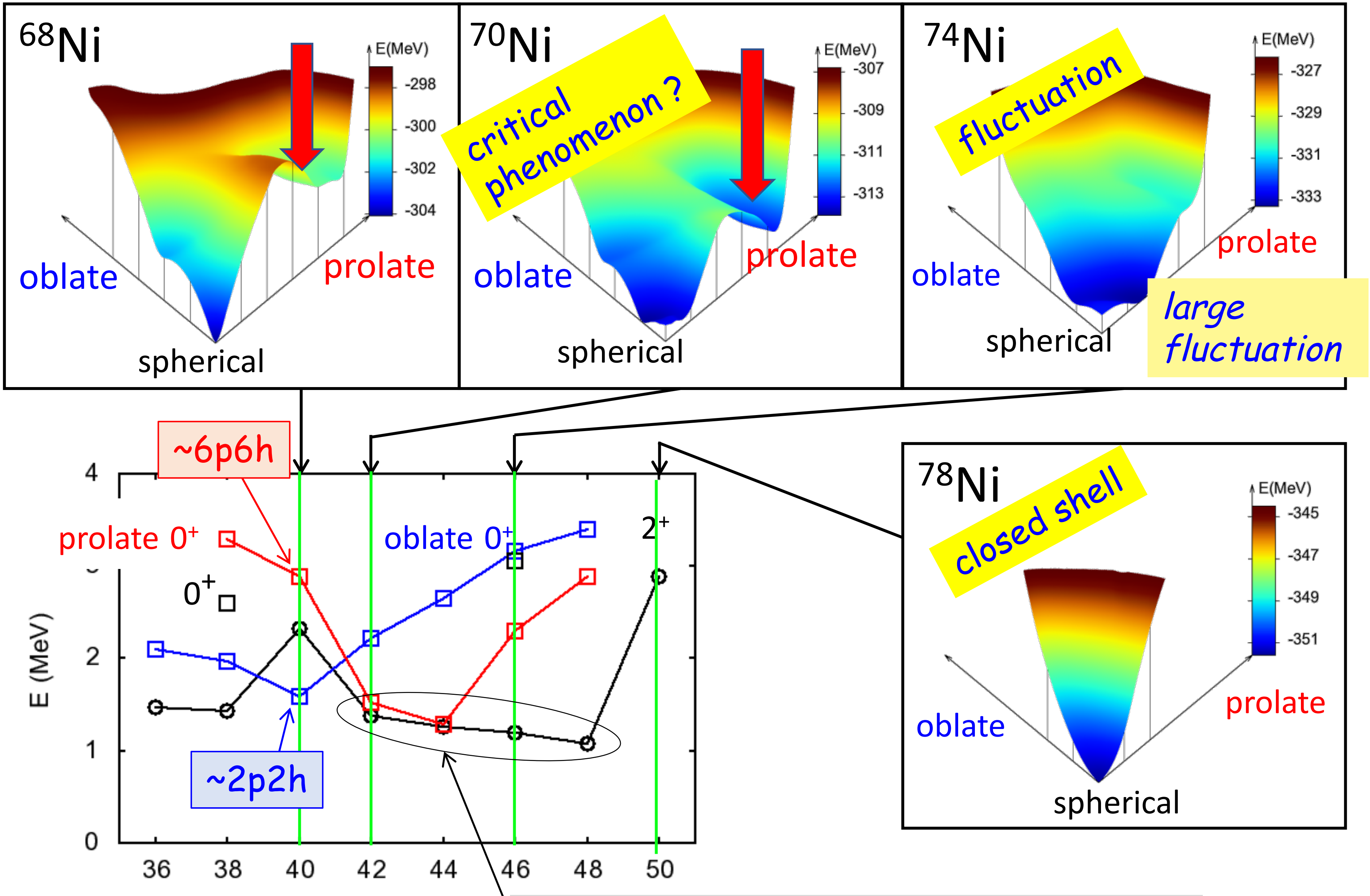
scale adjusted

present





Shape or structure evolution of Ni isotopes



⇔ 2^+ states in the $g_{9/2}$ seniority scheme

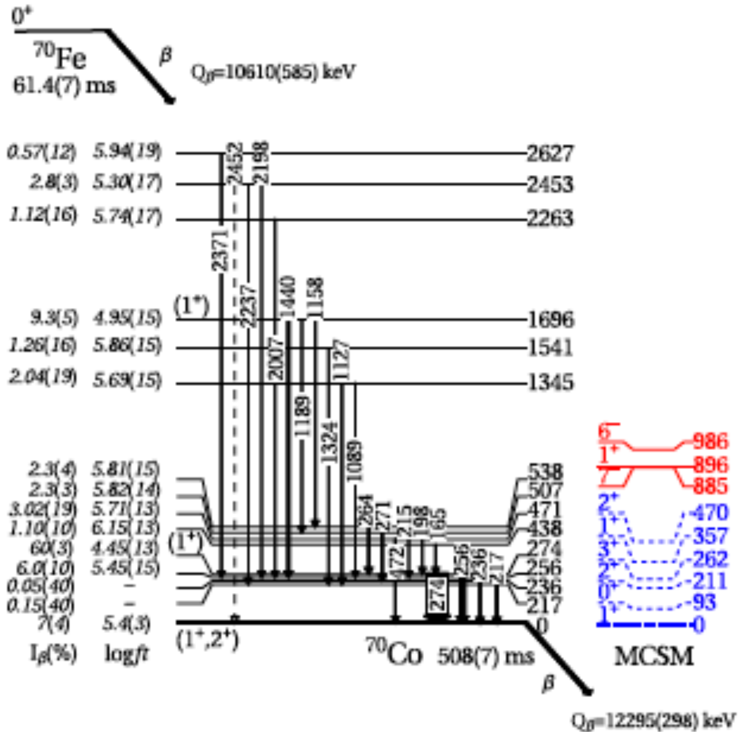
Physics Letters B 765 (2017) 328–333

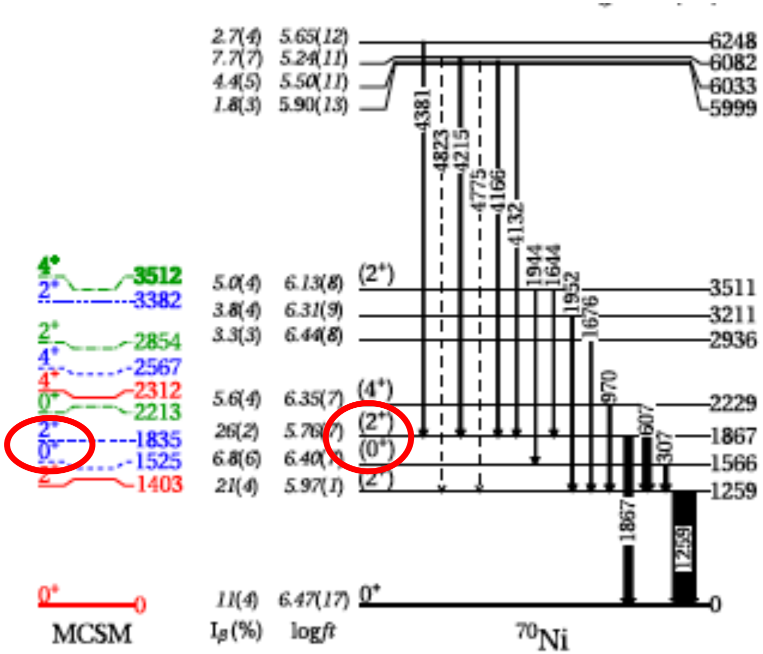
What is this ?

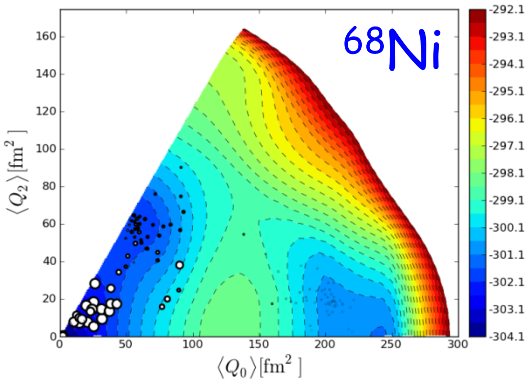
Type II shell evolution in $A = 70$ isobars from the $N \geq 40$ island of inversion

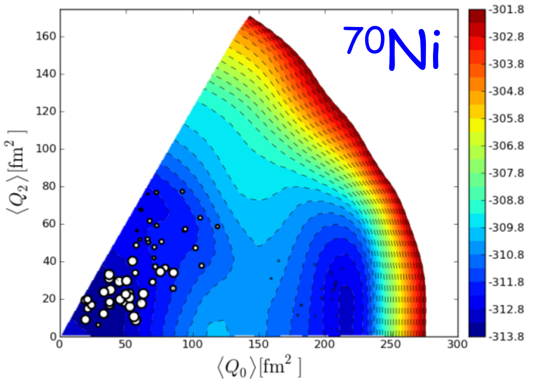
A.I. Morales[a,b,*], G. Benzoni[a], H. Watanabe[c,d], Y. Tsunoda[e], T. Otsuka[f,g,h], S. Nishimura[d], F. Browne[i,d], R. Daido[j], P. Doornenbal[d], Y. Fang[j], G. Lorusso[d], Z. Patel[k,d], S. Rice[k,d], L. Sinclair[l,d], P.-A. Söderström[d], T. Sumikama[m], J. Wu[d], Z.Y. Xu[f,d], A. Yagi[j], R. Yokoyama[f], H. Baba[d], R. Avigo[a,b], F.L. Bello Garrote[n], N. Blasi[a], A. Bracco[a,b], F. Camera[a,b], S. Ceruti[a,b], F.C.L. Crespi[a,b], G. de Angelis[o], M.-C. Delattre[p], Zs. Dombradi[q], A. Gottardo[o], T. Isobe[d], I. Kojouharov[r], N. Kurz[r], I. Kuti[q], K. Matsui[f], B. Melon[s], D. Mengoni[t,u], T. Miyazaki[f], V. Modamio-Hoybjor[o], S. Momiyama[f], D.R. Napoli[o], M. Niikura[f], R. Orlandi[h,v], H. Sakurai[d,f], E. Sahin[n], D. Sohler[q], H. Schaffner[r], R. Taniuchi[f], J. Taprogge[w,x], Zs. Vajta[q], J.J. Valiente-Dobón[o], O. Wieland[a], M. Yalcinkaya[y]

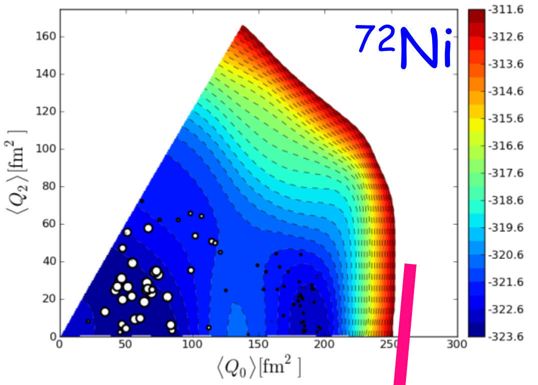
[a] Istituto Nazionale di Fisica Nucleare, Sezione di Milano, Via Celoria 16, 20133 Milano, Italy
[b] Dipartimento di Fisica, Università degli Studi di Milano, Via Celoria 16, 20133 Milano, Italy









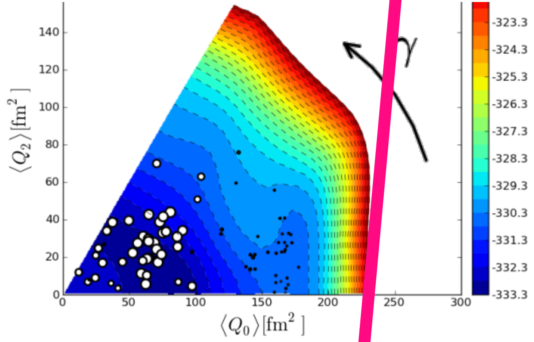


Energy of prolate state comes down.
Barrier becomes low.

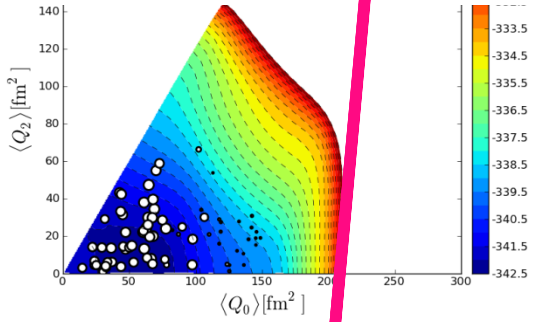
T-plot
for 0^+_1 states of $^{68-78}$Ni

The ground state is always like seniority-zero (BCS-type) spherical state.

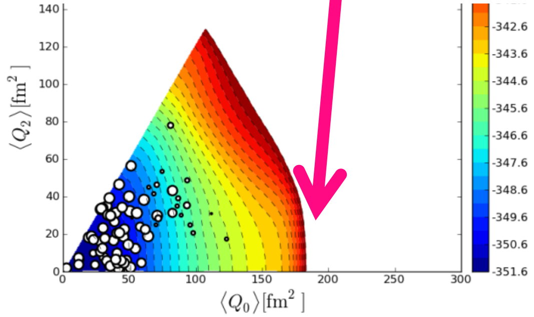
^{74}Ni



^{76}Ni



^{78}Ni



γ-soft deformation
(strong fluctuation in the γ direction)

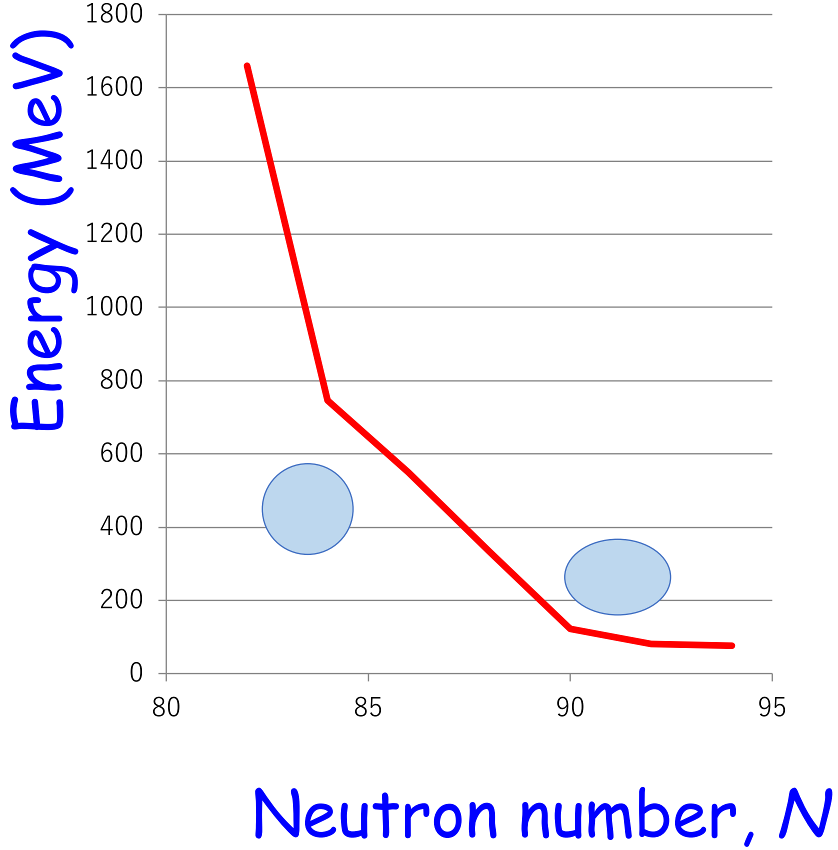
Shape transition
and
quantum phase transition

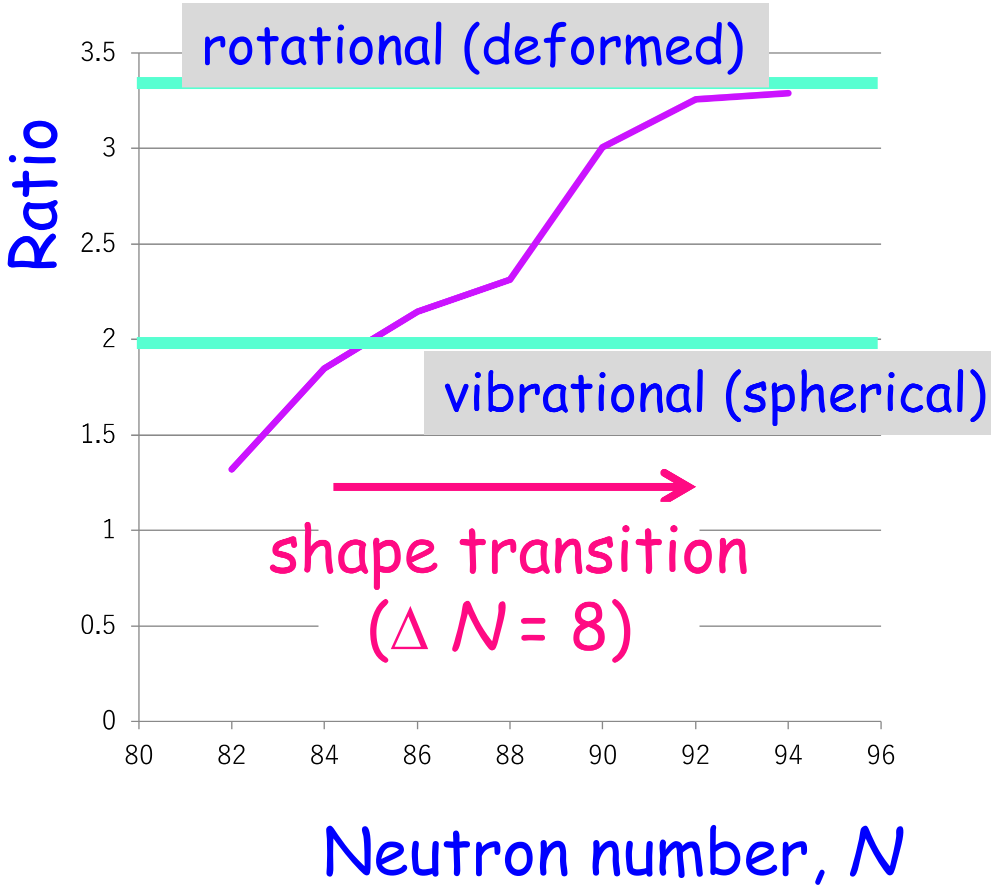
Shape change as a function of N (or Z)

2^+ and 4^+ level properties of Sm isotopes

Ex (2^+) :
excitation energy of first 2^+ state

$R_{4/2}$ = Ex (4^+) / Ex(2^+)

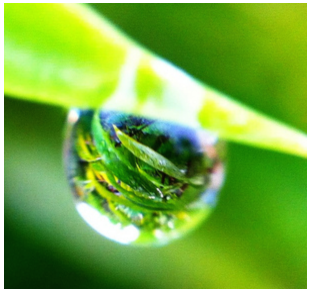




Can this be a kind of *Phase Transition* ?

Can the shape transition in nuclei be a “Phase Transition” ?

ice



water

Phase Transition :

A macroscopic system can change qualitatively from a stable state (*e.g.* ice for H_2O) to another stable state (*e.g.*, water for H_2O) as a function of a certain parameter (*e.g.*, temperature).

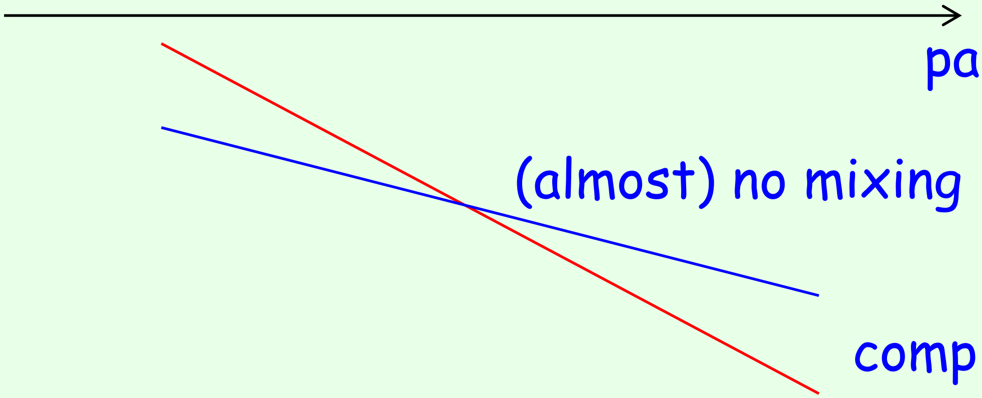
The phase transition implies this kind of phenomena of macroscopic systems consisting of almost infinite number of molecules.

Can the shape transition be a "Quantum Phase Transition" ?

The shape transition occurs rather gradually.

Quantum Phase Transition :
an abrupt change in the ground state of a many-body system
by varying a physical parameter at zero temperature.
(*cf., Wikipedia*)

possible scenario



Sizable mixing occurs usually in finite quantum systems.

Quest for Quantum Phase Transition: Shapes of Zr isotopes by Monte Carlo Shell Model

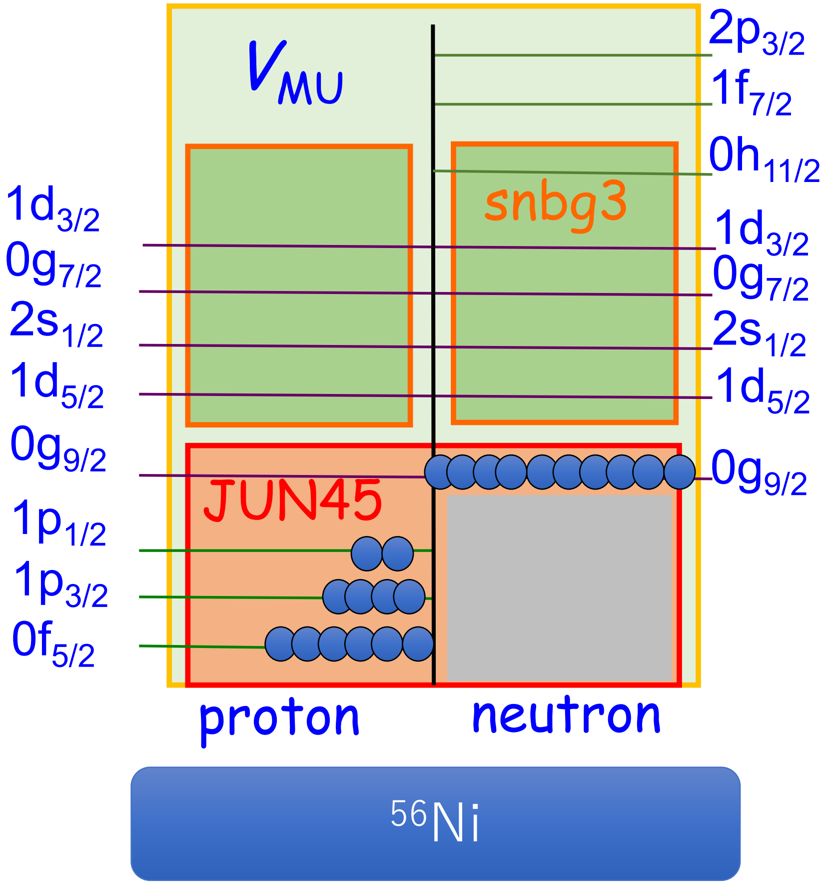
- Effective interaction:
  JUN45 + snbg3 + V_{MU}

  known effective interactions

  \+ minor fit for a part of
  T=1 TBME's

  Nucleons are excited fully within this model space (no truncation)

We performed Monte Carlo Shell Model (MCSM) calculations, where the largest case corresponds to the diagonalization of 3.7 x 10 ^{23} dimension matrix.



Togashi, Tsunoda, TO *et al.* PRL 117, 172502 (2016)

From earlier shell-model works …

PHYSICAL REVIEW C VOLUME 20, NUMBER 2 AUGUST 1979

Unified shell-model description of nuclear deformation

P. Federman
Instituto de Fisica, Universidad Nacional Autonoma de Mexico, Apartado Postal 20-364, Mexico 20, D. F.

S. Pittel

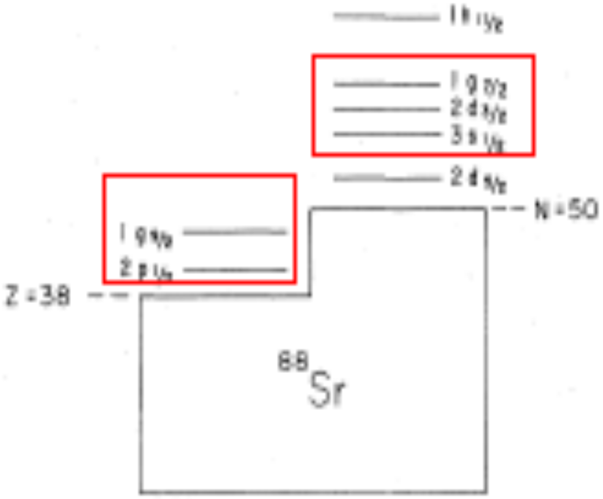


FIG. 3. Single-particle levels appropriate to a description of nuclei in the Zr-Mo region. An ^{88}Sr core is assumed.

PHYSICAL REVIEW C **79**, 064310 (2009)

Shell model description of zirconium isotopes

K. Sieja,[1,2] F. Nowacki,[3] K. Langanke,[2,4] and G. Martínez-Pinedo[1]

In this paper, we perform for the first time a SM study of Zr isotopes in an extended model space ($1f_{5/2}$, $2p_{1/2}$, $2p_{3/2}$, $1g_{9/2}$) for protons and ($2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, $1g_{7/2}$, $1h_{11/2}$) for neutrons, dubbed hereafter $\pi(r3-g)$, $\nu(r4-h)$.

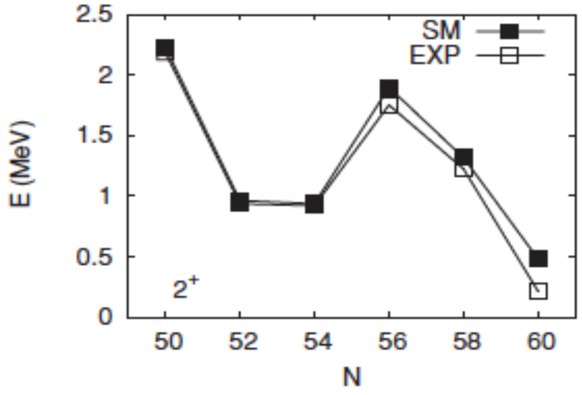
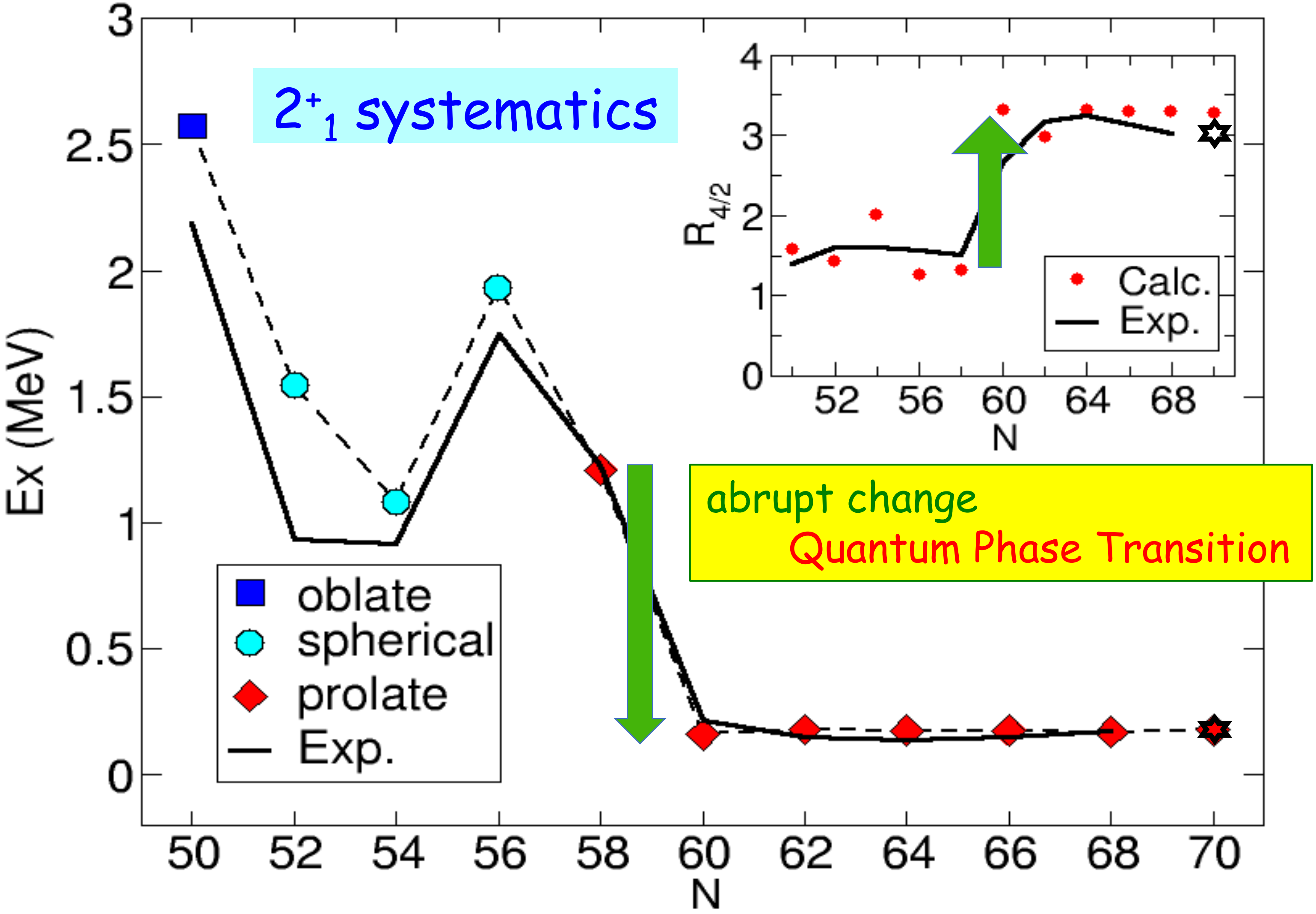


FIG. 12. Systematics of the experimental and theoretical first excited 2^+ states along the zirconium chain.

PRL **117,** 172502 (2016) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2016

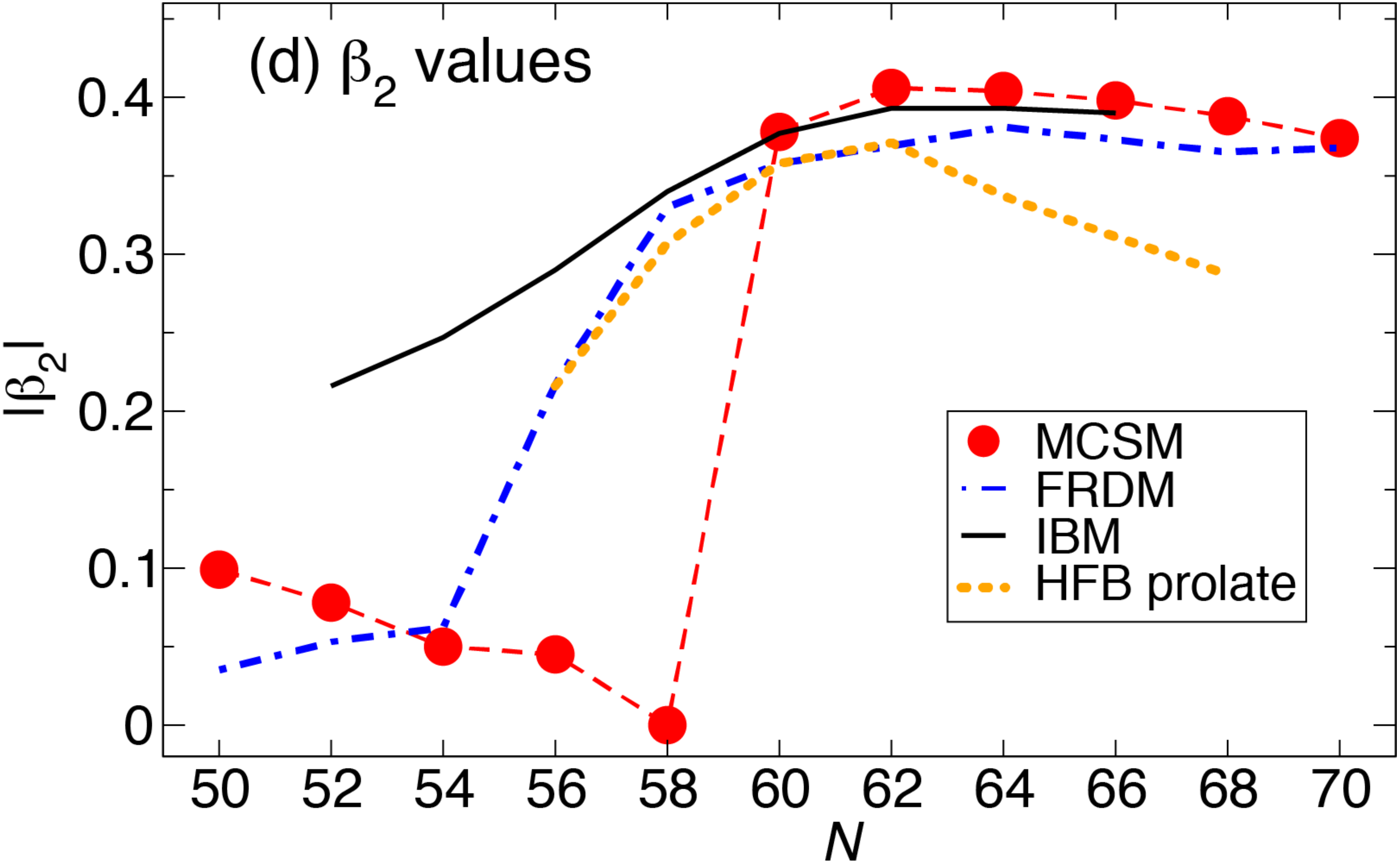
Quantum Phase Transition in the Shape of Zr isotopes

Tomoaki Togashi,[1] Yusuke Tsunoda,[1] Takaharu Otsuka,[1,2,3,4] and Noritaka Shimizu[1]



2^+_1 systematics

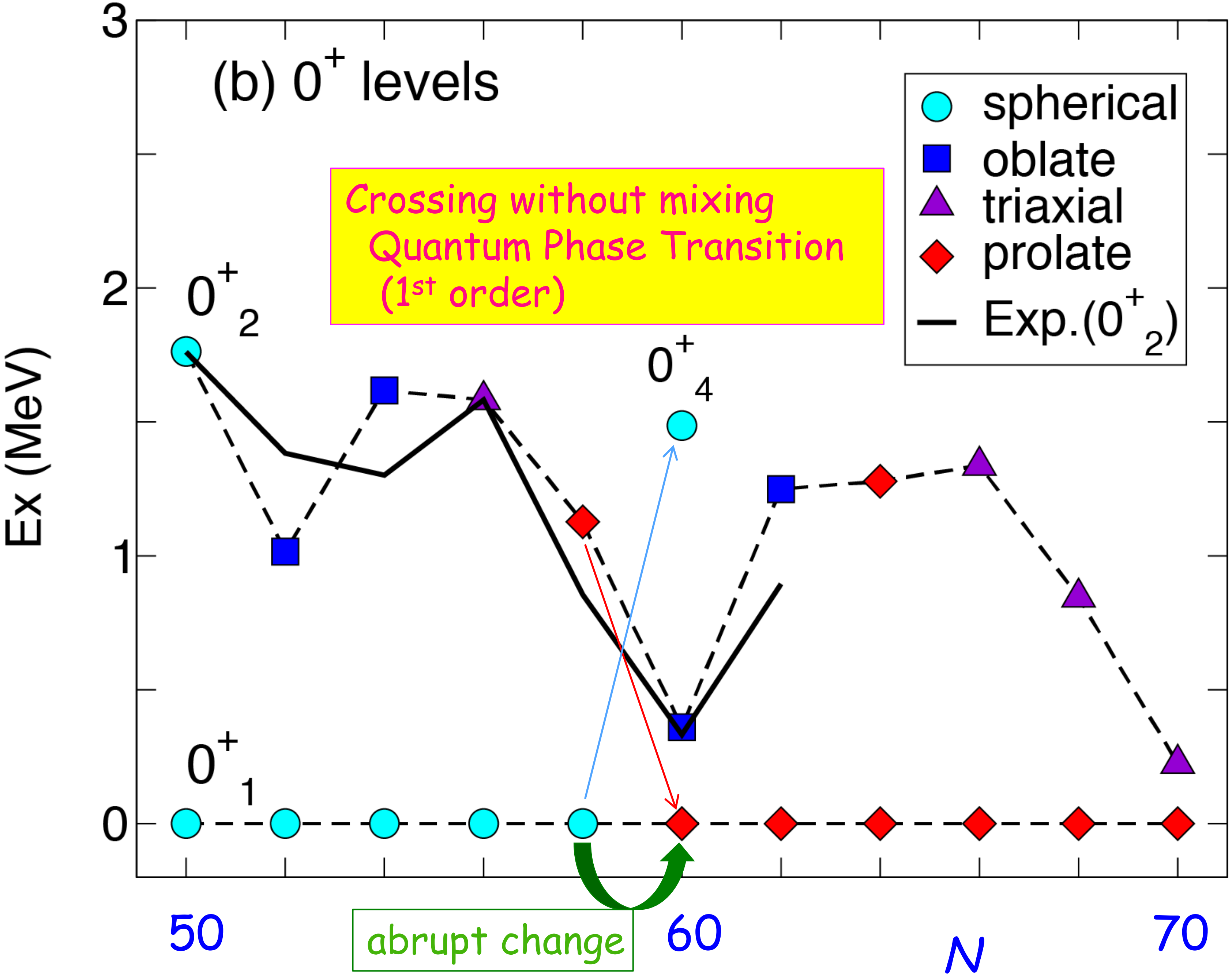
abrupt change
Quantum Phase Transition



FRDM: S. Moeller et al. At. Data Nucl. Data Tables 59, 185 (1995).

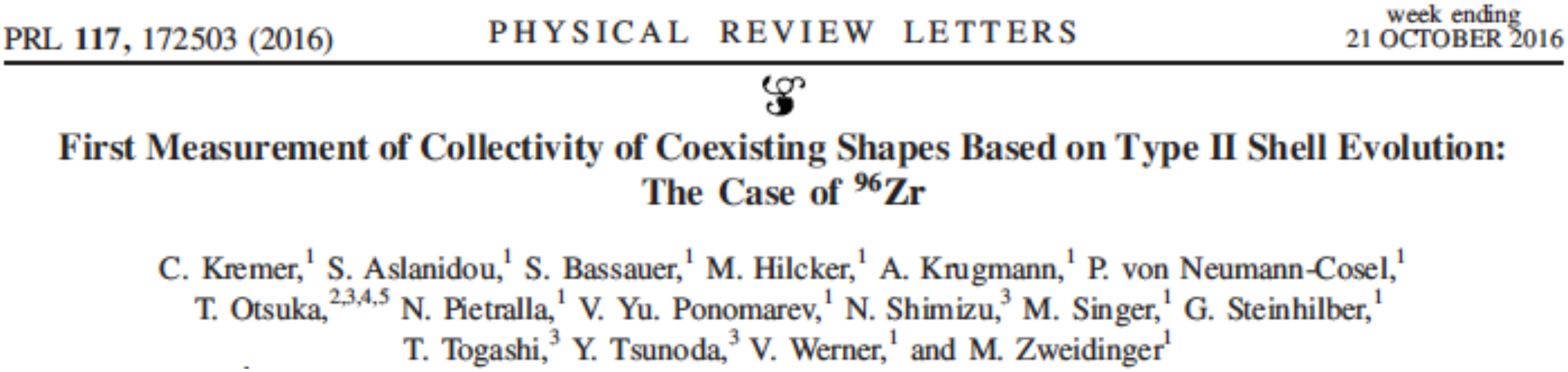
IBM: M. Boyukata et al. J. Phys. G 37, 105102 (2010).

HFB: R. Rodriuez-Guzman et al. Phys. Lett. B 691, 202 (2010).



Crossing without mixing
Quantum Phase Transition
(1st order)

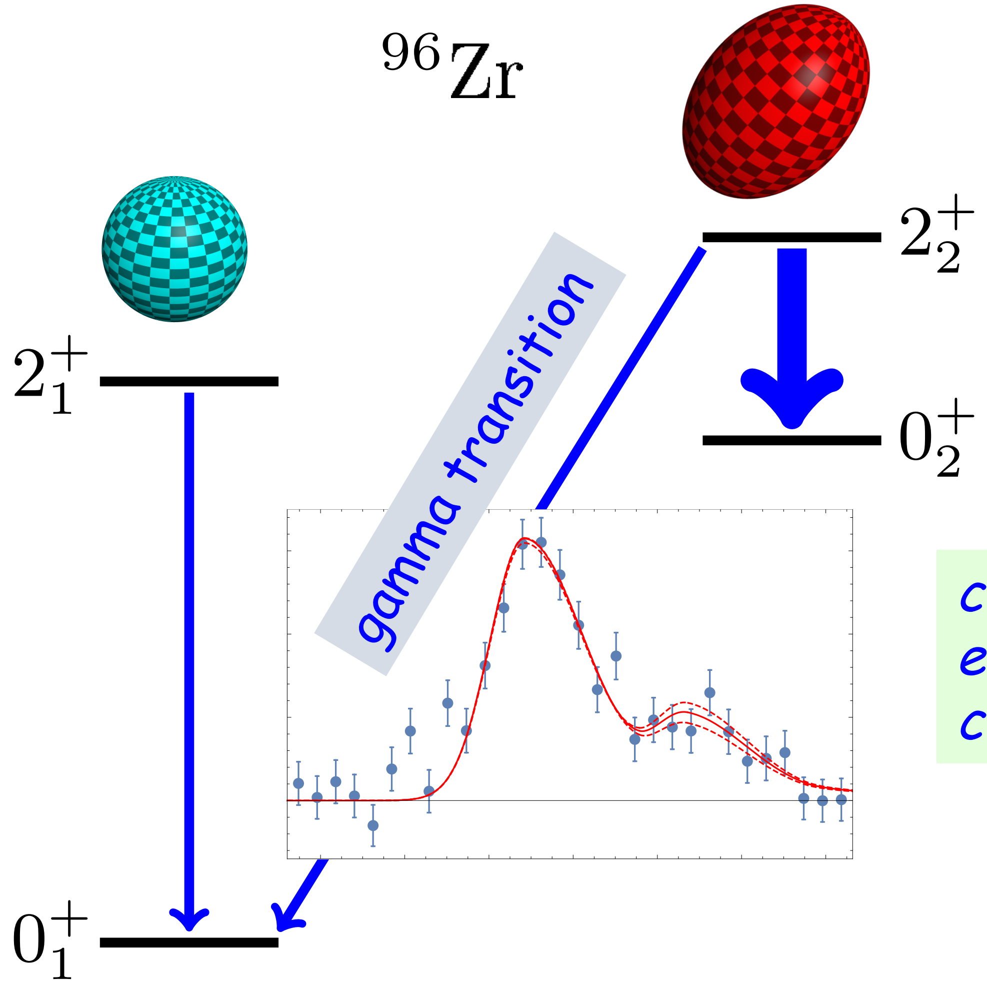
abrupt change



PRL 117, 172503 (2016) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2016

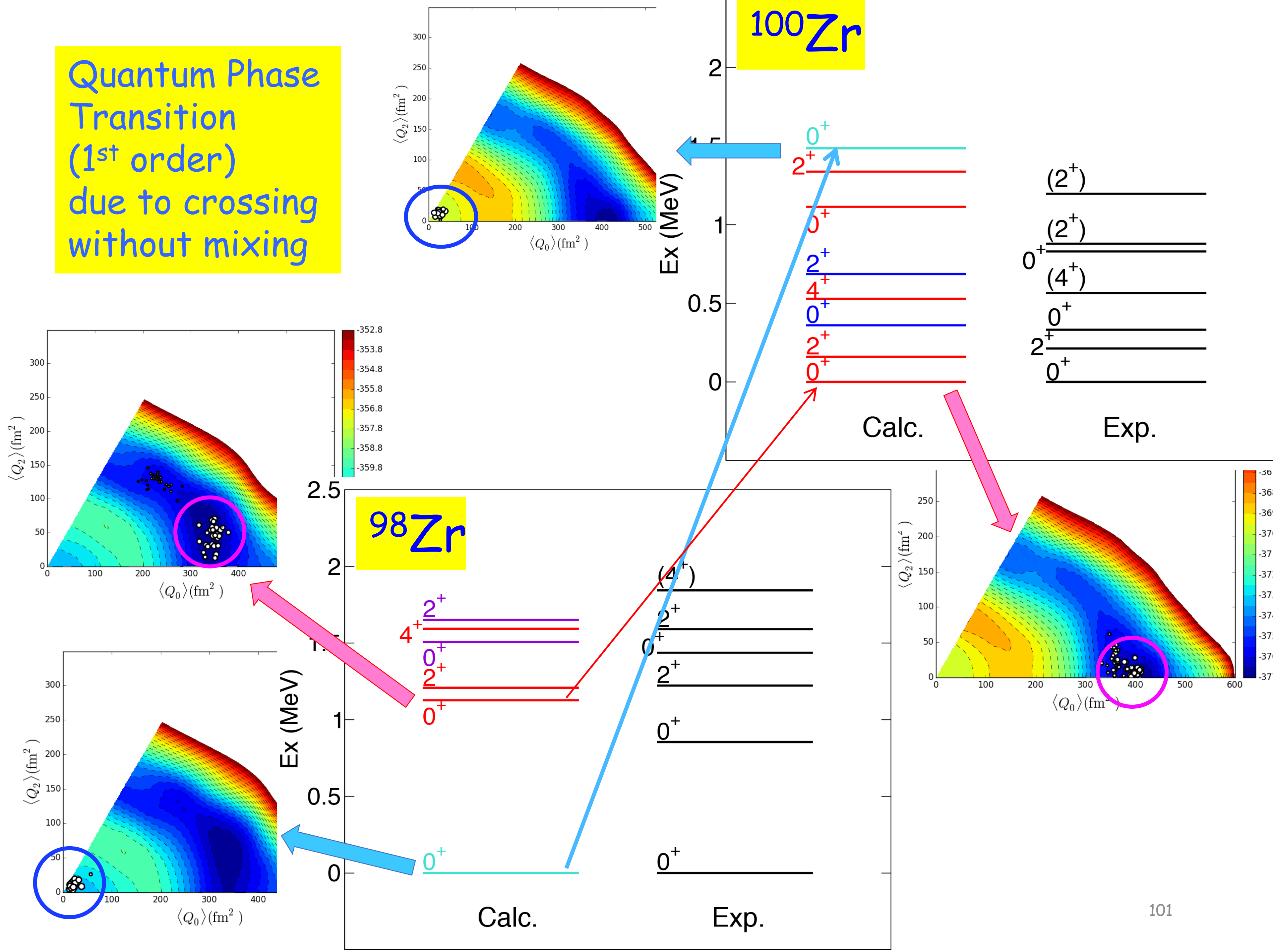
First Measurement of Collectivity of Coexisting Shapes Based on Type II Shell Evolution: The Case of ^{96}Zr

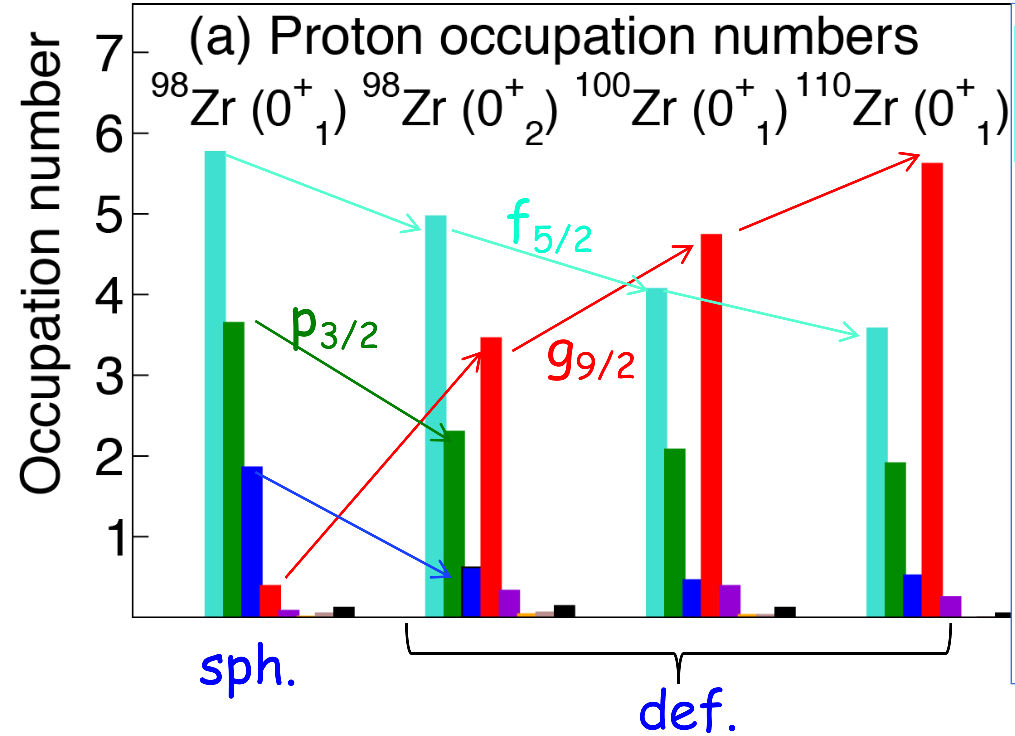
C. Kremer,[1] S. Aslanidou,[1] S. Bassauer,[1] M. Hilcker,[1] A. Krugmann,[1] P. von Neumann-Cosel,[1]
T. Otsuka,[2,3,4,5] N. Pietralla,[1] V. Yu. Ponomarev,[1] N. Shimizu,[3] M. Singer,[1] G. Steinhilber,[1]
T. Togashi,[3] Y. Tsunoda,[3] V. Werner,[1] and M. Zweidinger[1]



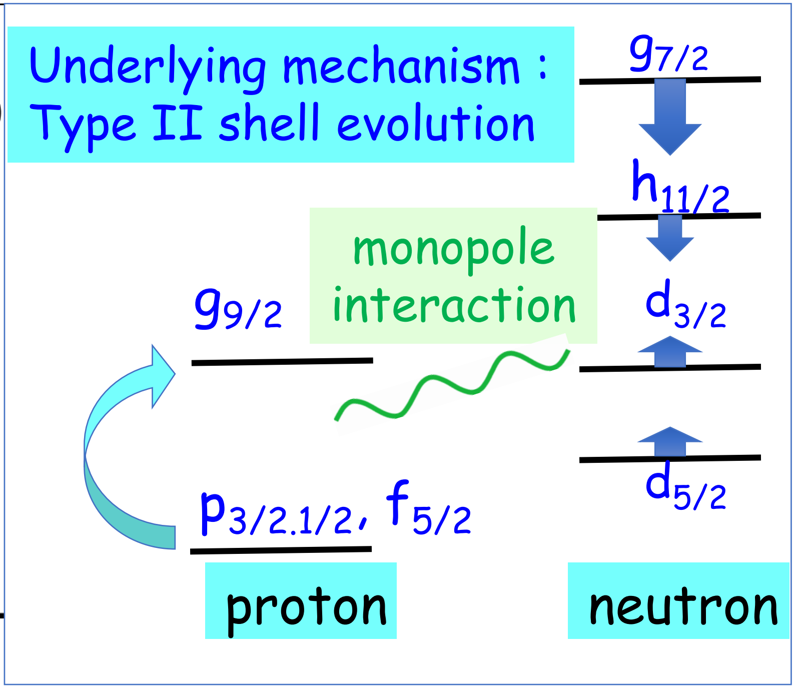
co-existence experimentally confirmed

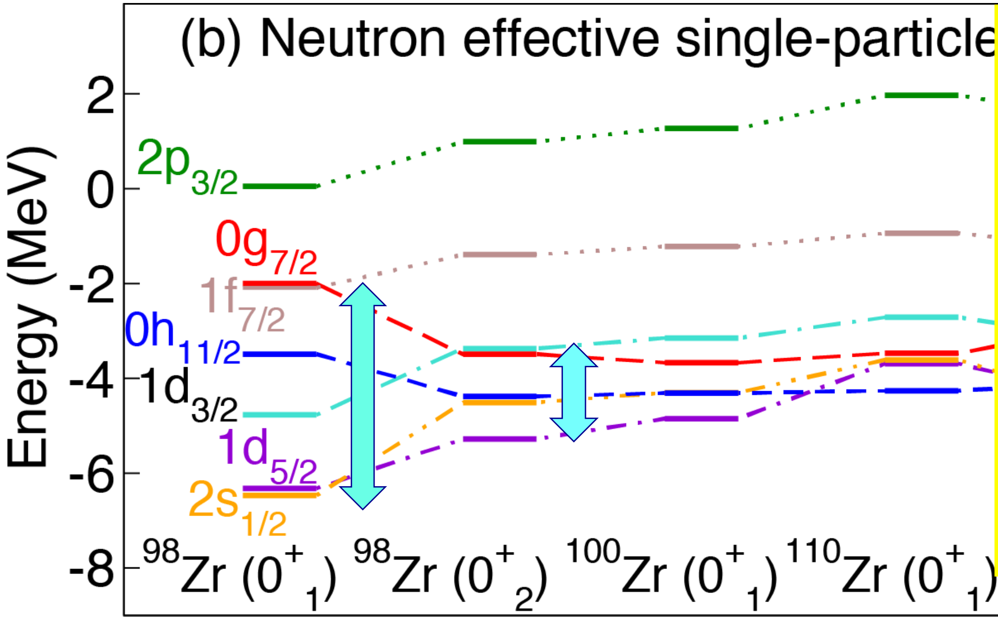
Quantum Phase Transition (1st order) due to crossing without mixing





Underlying mechanism : Type II shell evolution





Relevant neutron single-particle levels get closer as a combined effect of nuclear forces (tensor and central) and particular configurations.

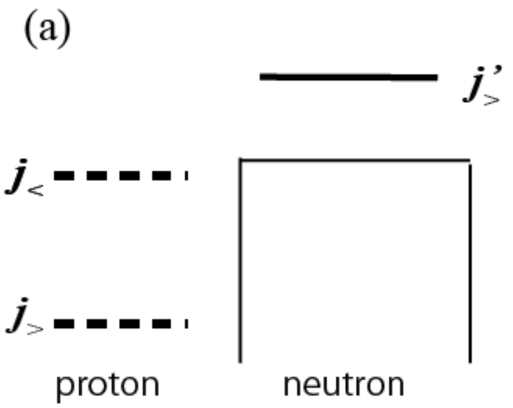
The resistance power against deformation is then reduced.

Large difference in ESPEs and configurations ➔ crossing w/o mixing

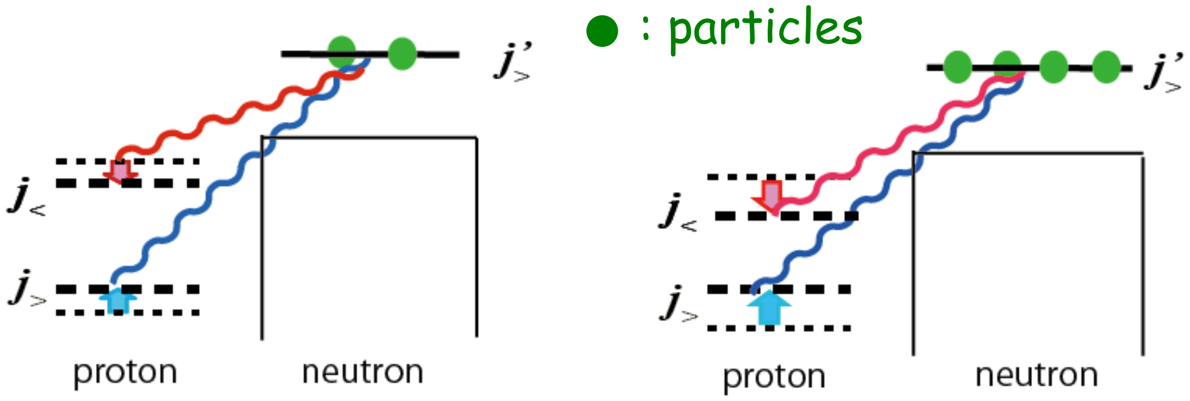
Underlying mechanism of the appearance of low-lying deformed states : Type II Shell Evolution

TO and Y. Tsunoda, J. Phys. G: Nucl. Part. Phys. 43 (2016) 024009

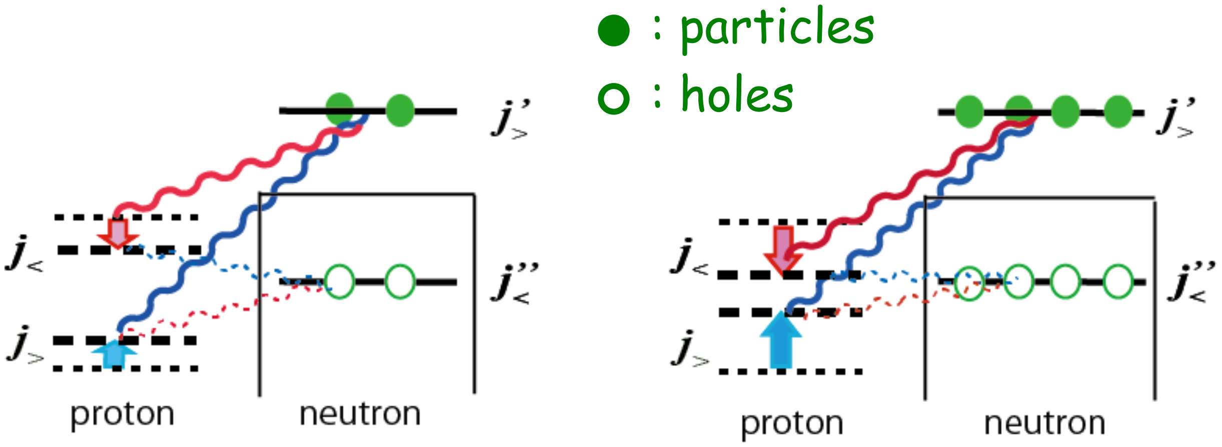
Monopole effects on the shell structure from the tensor interaction



Type I Shell Evolution : different isotopes



Type II Shell Evolution : within the same nucleus



Type II shell evolution is a simplest and visible case of

Quantum Self Organization

$$\text{deformation} = \frac{\text{quadrupole force}}{\text{resistance power}}$$

resistance power ← pairing force

↖ single-particle energies

Atomic nuclei can "organize" their single-particle energies by taking particular configurations of protons and neutrons optimized for each eigenstate, thanks to orbit-dependences of monopole components of nuclear forces (*e.g.*, tensor force).

→ an enhancement of Jahn-Teller effect.

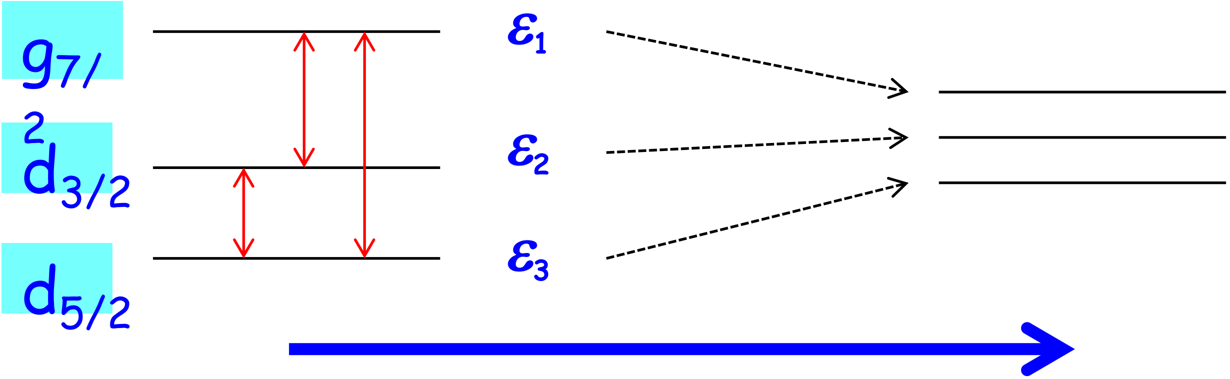
Reminder : Jahn -Teller effect for nuclear deformation

(Self-consistent) quadrupole deformed field $\propto Y_{2,0}(\theta,\phi)$
mixes the orbits below

$$\Psi\,(J_z=1/2) = c_1\,|g_{7/2};\, j_z=1/2\rangle + c_2\,|d_{3/2};\, j_z=1/2\rangle + c_3\,|d_{5/2};\, j_z=1/2\rangle$$

stronger mixing = larger quadrupole deformation

Mixing depends not only on the strength of the $Y_{2,0}(\theta,\phi)$ field,
but also the spherical single-particle energies ε_1, ε_2, ε_3, etc.



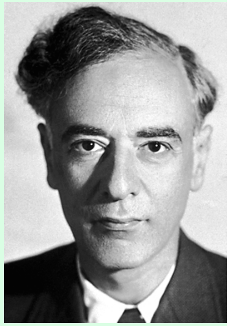
closer to degeneracy

larger deformation for the same deformed field

Let's shed light on an old problem

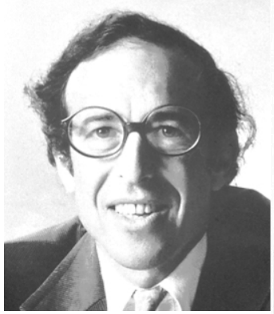
Atomic nucleus is a quantum Fermi liquid :

The nucleus is composed of almost free nucleons interacting weakly via residual forces in a (solid) (mean) potential like a solid vase.



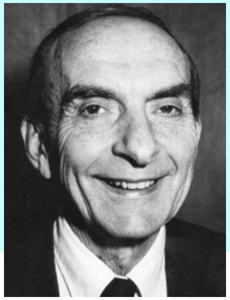
Surface deformation produced by additional deformed mean field → *Nilsson model*

(deformed "vase", monopole effect missing)



Sven Gösta Nilsson

"how single-particle states can coexist with collective modes" *conceived* also *by Gerry Brown* as an open unresolved problem (T. Schaefer, Fermi Liquid theory: A brief survey in memory of Gerald E. Brown, NPA 2014)



Renewed picture:

Single-particle levels can be re-organized to enhance collective modes.

SP states and coll. mode are not enemies but friends !

Nilsson model Hamiltonian

"*Nuclear structure II*" by Bohr and Mottelson

deformed nuclei, is obtained by a simple modification of the harmonic oscillator (Nilsson, 1955; Gustafson *et al.*, 1967),

$$H = \frac{\mathbf{p}^2}{2M} + \underbrace{\tfrac{1}{2}M\left(\omega_3^2 x_3^2 + \omega_\perp^2\left(x_1^2 + x_2^2\right)\right)}_{\text{quadrupole deformed field}} + \underbrace{v_{ll}\hbar\omega_0\left(\mathbf{l}^2 - \langle \mathbf{l}^2 \rangle_N\right) + v_{ls}\hbar\omega_0(\mathbf{l}\cdot\mathbf{s})}_{\text{spherical field}} \qquad (5\text{-}10)$$

$$\langle \mathbf{l}^2 \rangle_N = \tfrac{1}{2}N(N+3)$$

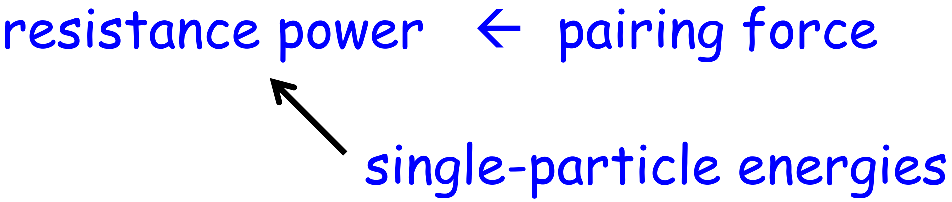
Figure	Region	$-v_{ls}$	$-v_{ll}$
5-1	N and $Z < 20$	0.16	0
5-2	$50 < Z < 82$	0.127	0.0382
5-3	$82 < N < 126$	0.127	0.0268
5-4	$82 < Z < 126$	0.115	0.0375
5-5	$126 < N$	0.127	0.0206

Table 5-1 Parameters used in the single-particle potentials of Figs. 5-1 to 5-5.

Spin-orbit force

$$\left.\begin{array}{ll} A=\ 68 & 1.28 \\ A=100 & 1.12 \\ A=186 & 0.91 \end{array}\right\} (\mathbf{l}\cdot\mathbf{s})$$

$$\text{deformation} = \frac{\text{quadrupole force}}{\text{resistance power}}$$


resistance power ← pairing force

single-particle energies

Analogy to electric current,

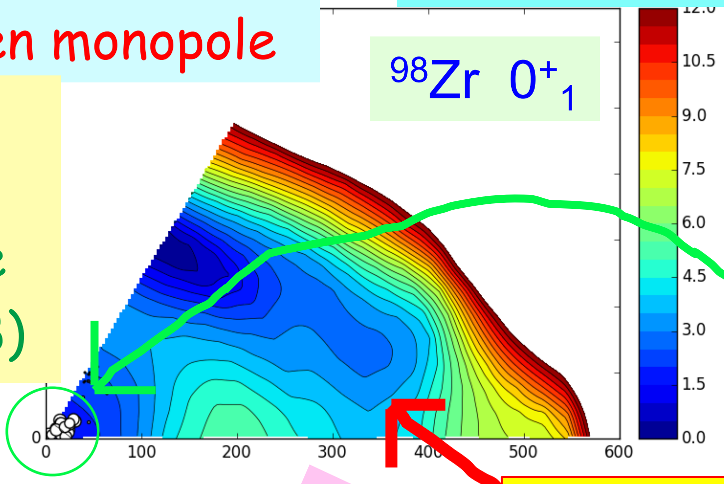
$$\text{current} = \frac{\text{voltage}}{\text{resistance}}$$

Anatomy of this effect : ^{98}Zr spherical 0^+_1 and deformed 0^+_2

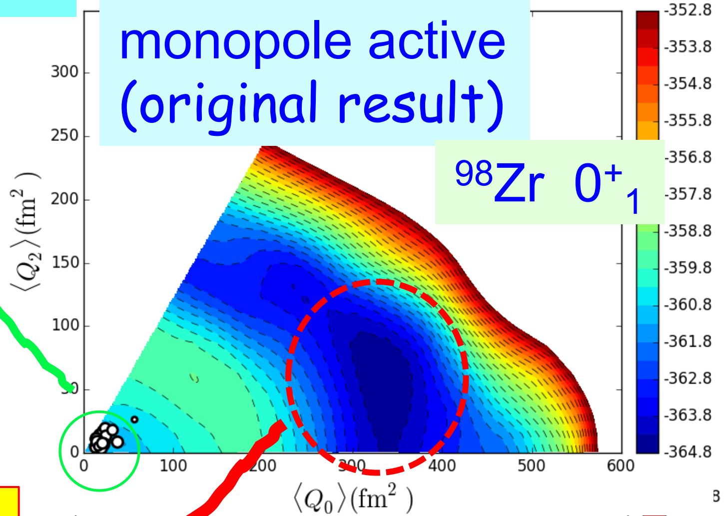
PES with T-plot

Frozen monopole

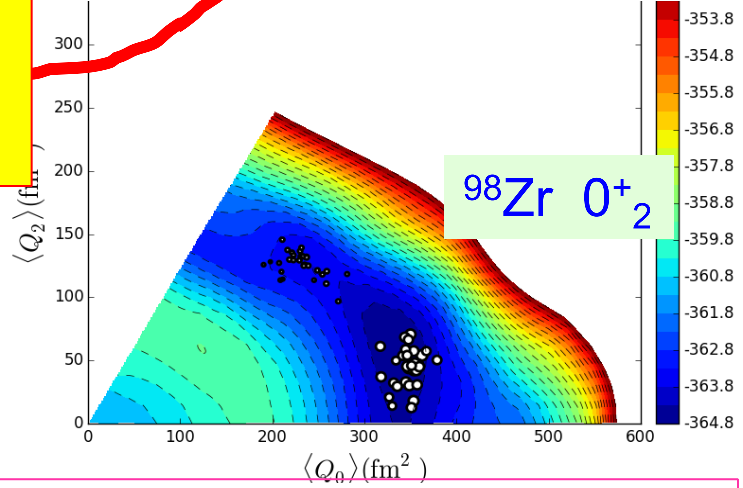
spherical ground state do not change (overlap~0.98)



monopole active (original result)

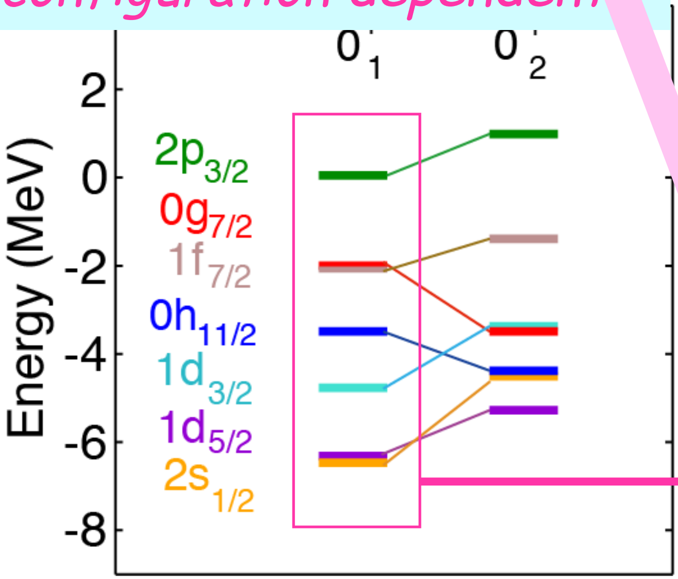


prolate minimum is gone



Effective SPE

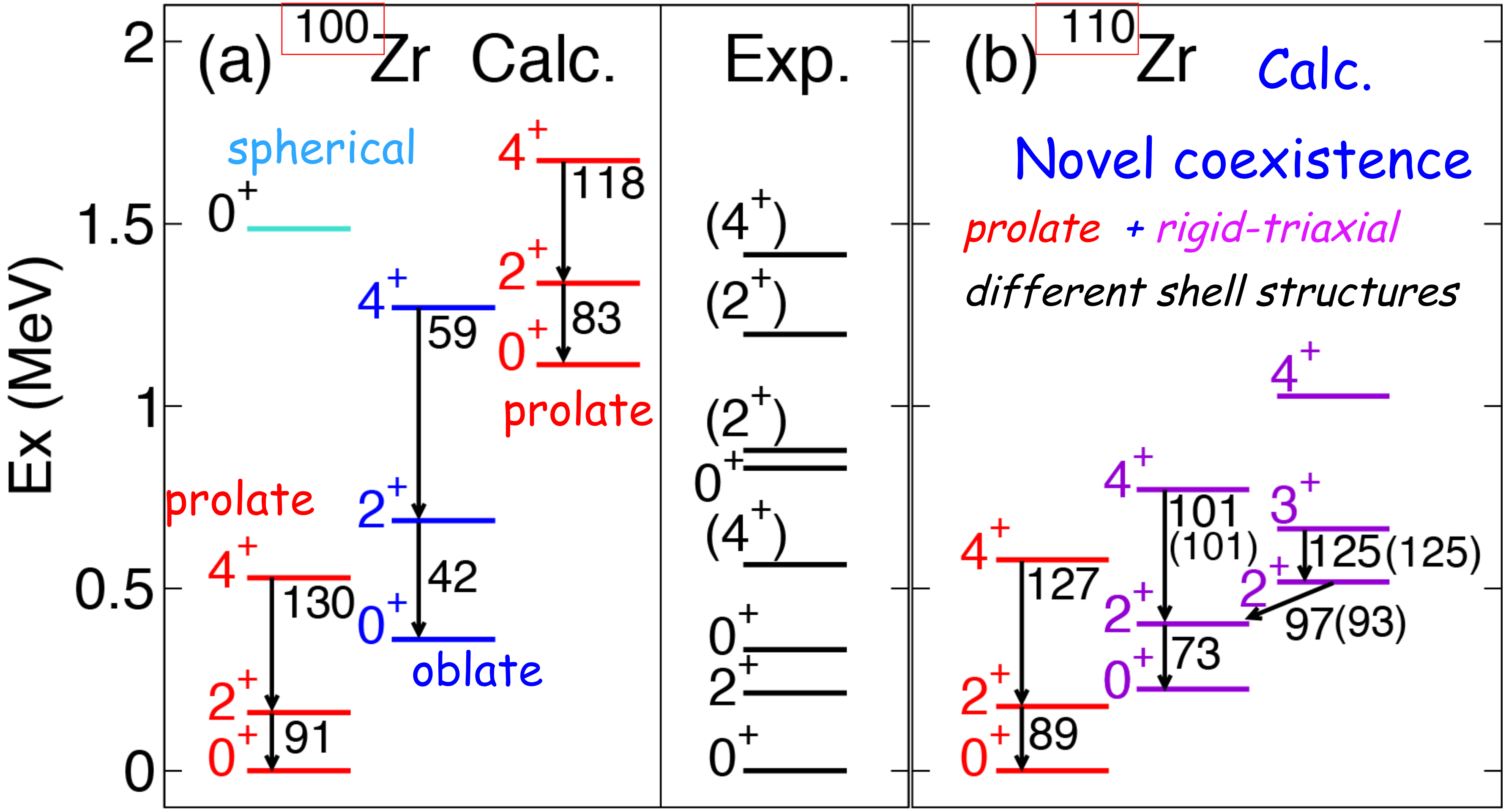
- *configuration dependent*



Use them as constant SPEs *independent of configurations, putting monopole int. aside*

➔ Frozen monopole treatment

Prolate – rigid-triaxial shape coexistence



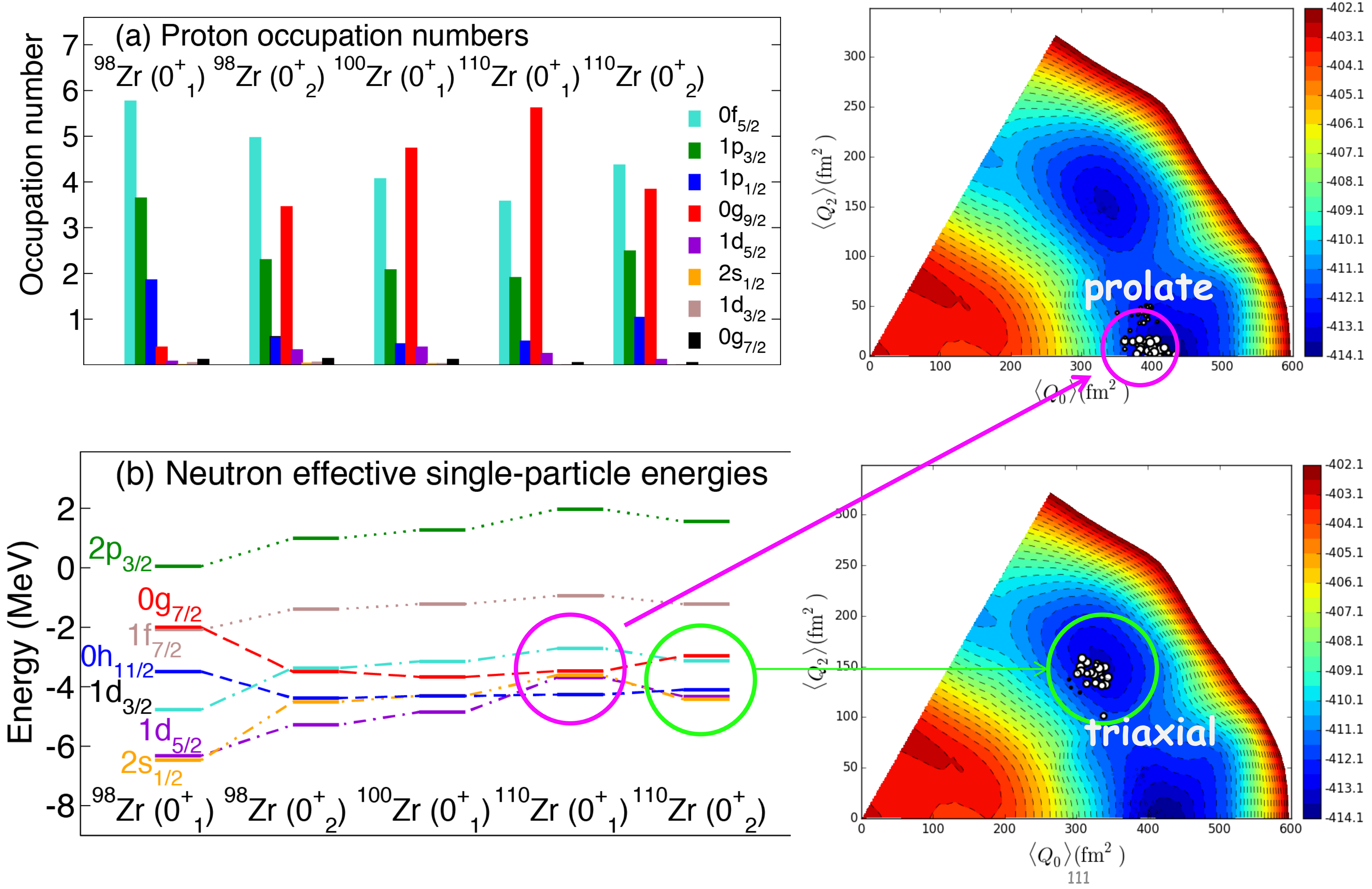
Novel coexistence

prolate + rigid-triaxial

different shell structures

( ) : Rigid-triaxial rotor with gamma=28 degrees
normalized at 2^+_2 -> 0^+_2

different shell structures ~ like "different nuclei"



After mastering the shell model, three (possible) pillars combined for future

computation

Monte Carlo
Shell Model
(MCSM)

(almost)
unlimited
dimensionality

massive
parallel
computers

Hamiltonian

pf
pfg9d5 (A3DA) (Ni)
8+8 on ^{56}Ni core (Zr)
8+8 on ^{80}Zr core (Sn)
8+10 on ^{132}Sn core
(Sm)
...
island of stability

+

χEFT based
(multi-)shell int.

many-body dynamics

Shell evolution
(Type I & II)

Quantum Phase
Transition

Shape coexistence

Quantum
Self-organization

Remark on Fermi liquid picture of nuclei

Naïve Fermi liquid picture (a la Landau) is revised, as *atomic nuclei are not necessarily like simple solid vases containing almost free nucleons*.

Nuclear forces are rich enough to optimize single-particle energies for each eigenstate (especially in the cases of collective-mode states), as referred to as quantum self-organization.

The quantum self-organization produces sizable effects with
(i) two quantum fluids (protons and neutrons),
(ii) two major forces : *e.g.*, quadrupole interaction to drive collective mode
monopole interaction to control resistance

Type II shell evolution is one of the most visible cases of the quantum self-organization, with massive p-h excitations across the shell gap. Quantum phase transition, shape coexistence, various deformation, fission, … are releted to the quantum self-organization.

The beauty of the collective modes is enhanced. Time-dependent version for reactions is of great interest (beyond thermalisation *etc*.).

The microscopic foundation of the IBM (Interacting Boson Model) is also related, for instance, regarding the origin of the Majorana interaction.

Be ambitious and deepen nuclear physics !

ご清聴
ありがとう
ございました