Out-of-equilibrium density dynamics of a quenched fermionic system

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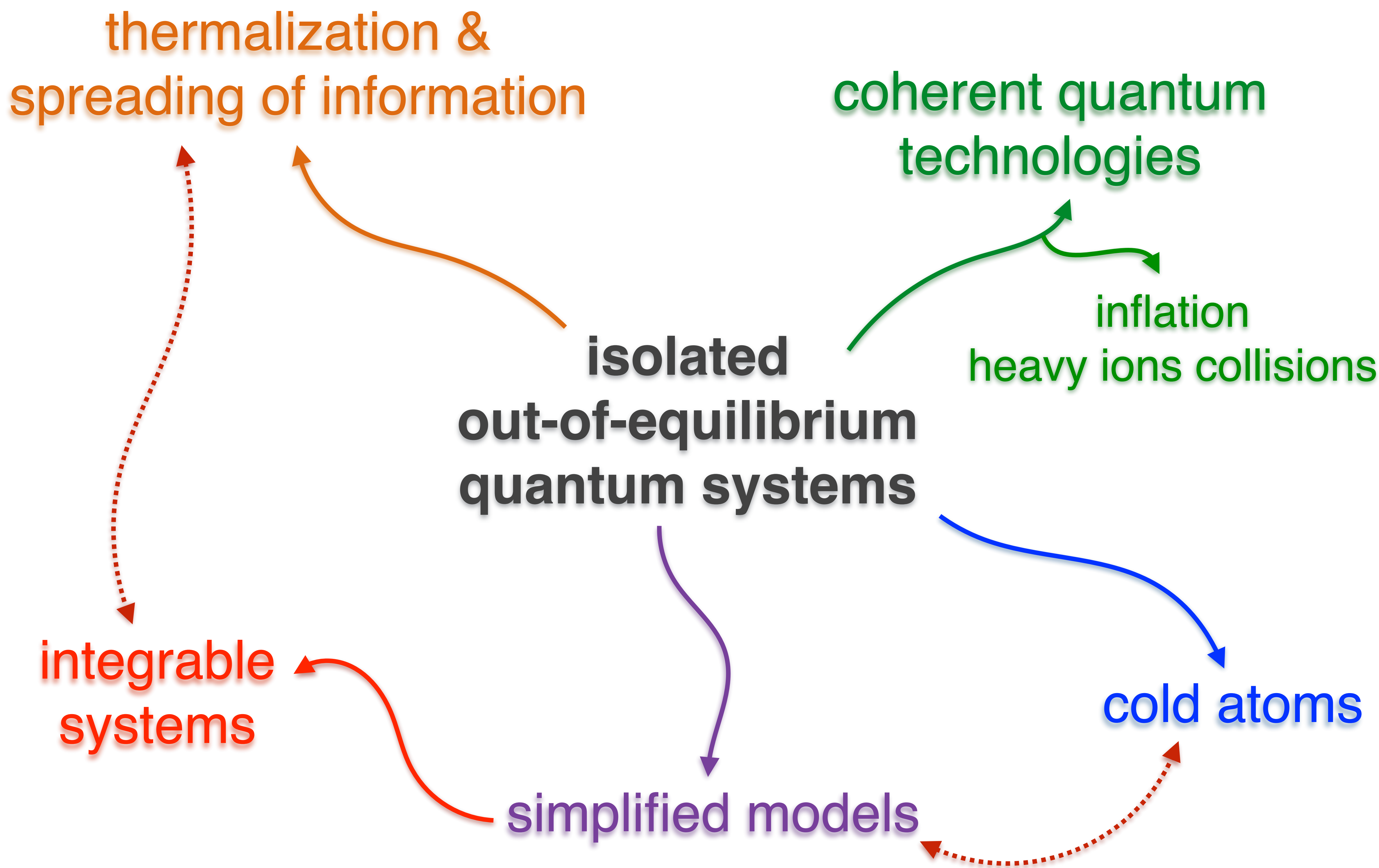
Introduction



Quantum quenches

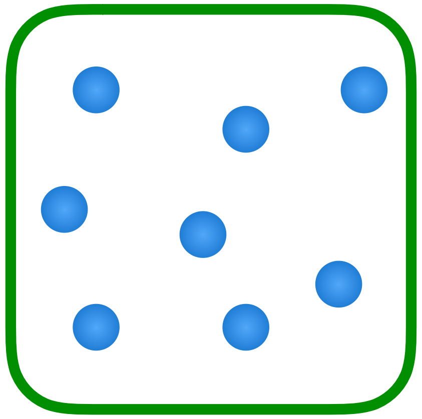
Definition: a change in time of the parameter(s) that governs the dynamics of an isolated quantum system.

- sudden, adiabatic, or more general: linear ramps,…
- parameter(s): coupling constants, external fields, confinement,…
- isolated quantum system: no coupling with the environment → unitary time-evolution

$|\Psi_0^i\rangle$

Quench!

$$H(t) = H(\{\boldsymbol{g}(t)\})$$

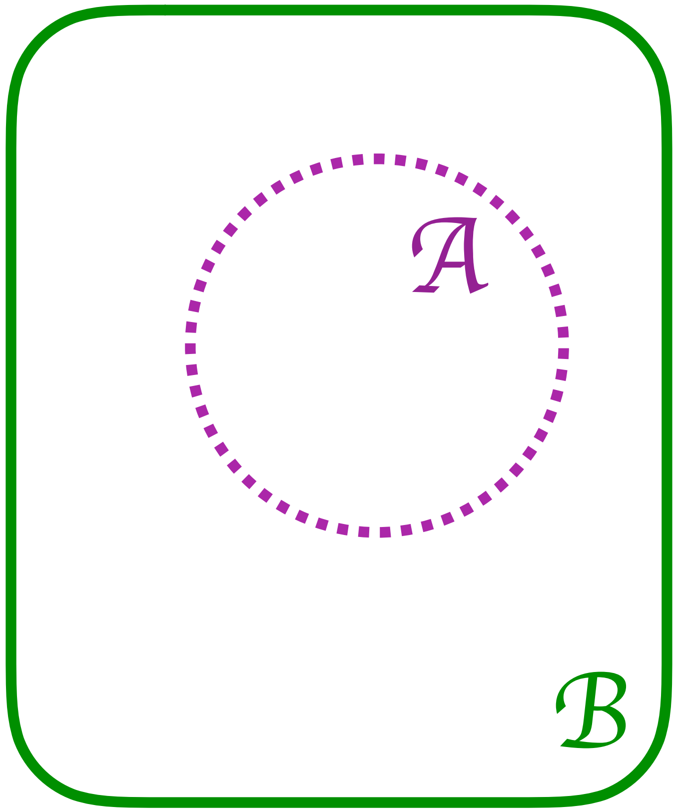


$$H_i = H(\{\boldsymbol{g}_i\})$$

$$H_f = H(\{\boldsymbol{g}_f\})$$

Thermalization in isolated quantum systems

Does an isolated quantum system thermalize?



- thermal density matrix $\rho^{\mathrm{th}} = \dfrac{e^{-\beta H}}{Z}$, with β fixed by $\langle \Psi_0^i | H_f | \Psi_0^i \rangle = \mathrm{Tr}[\rho^{\mathrm{th}} H_f]$
- reduced density matrix $\rho_A(\infty) = \lim\limits_{t \to \infty} \mathrm{Tr}_B \, \rho(t)$
- thermalization if $\rho_A(\infty) = \rho_A^{\mathrm{th}}$

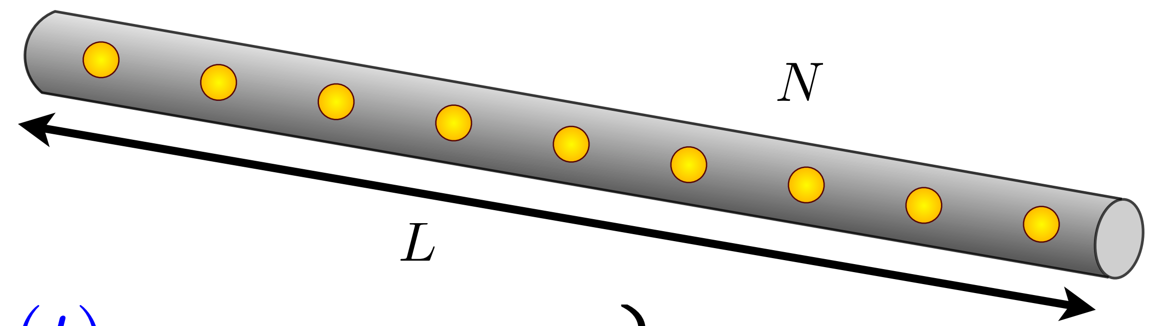
Polkovnikov et al., RMP (2011)

What happens in integrable systems?

- Generalized Gibbs Ensemble (GGE) $\rho^{\mathrm{GGE}} = \dfrac{e^{-\sum_m \lambda_m I_m}}{Z^{\mathrm{GGE}}}$, with λ_m fixed by $\langle \Psi^i(t) | I_m | \Psi^i(t) \rangle = \langle \Psi_0^i | I_m | \Psi_0^i \rangle = \mathrm{Tr}[\rho^{\mathrm{GGE}} I_m]$
- GGE “thermalization” $\rho_A(\infty) = \rho_A^{\mathrm{GGE}}$

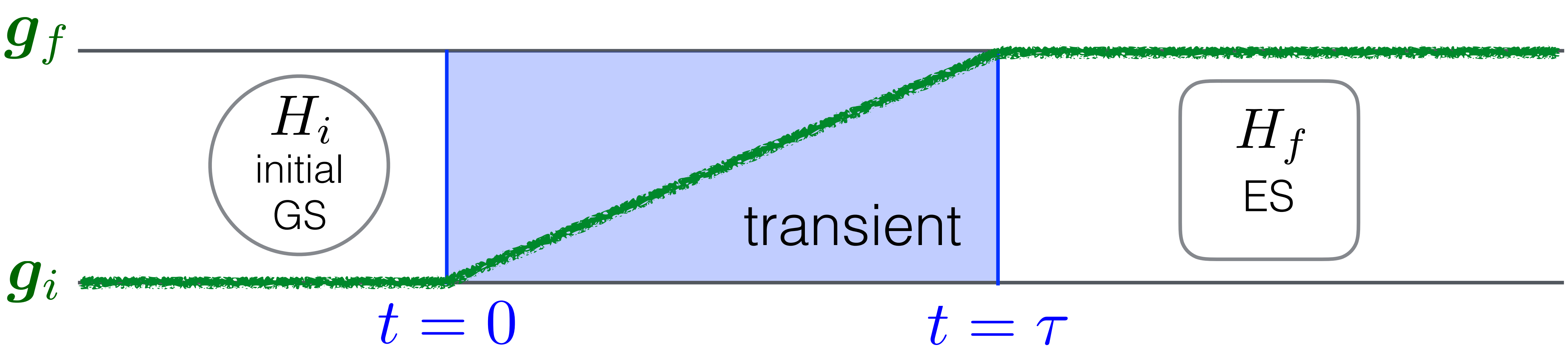
Rigol et al., PRL (2007)

Quench in a open boundaries LL



- (bosonized) Hamiltonian

$$H(t) = \sum_{q>0} q \left\{ [v_F + g_4(t)]\, b_q^\dagger b_q - \frac{g_2(t)}{2} \left(b_q b_q + b_q^\dagger b_q^\dagger \right) \right\} + \mathcal{E}(N)$$



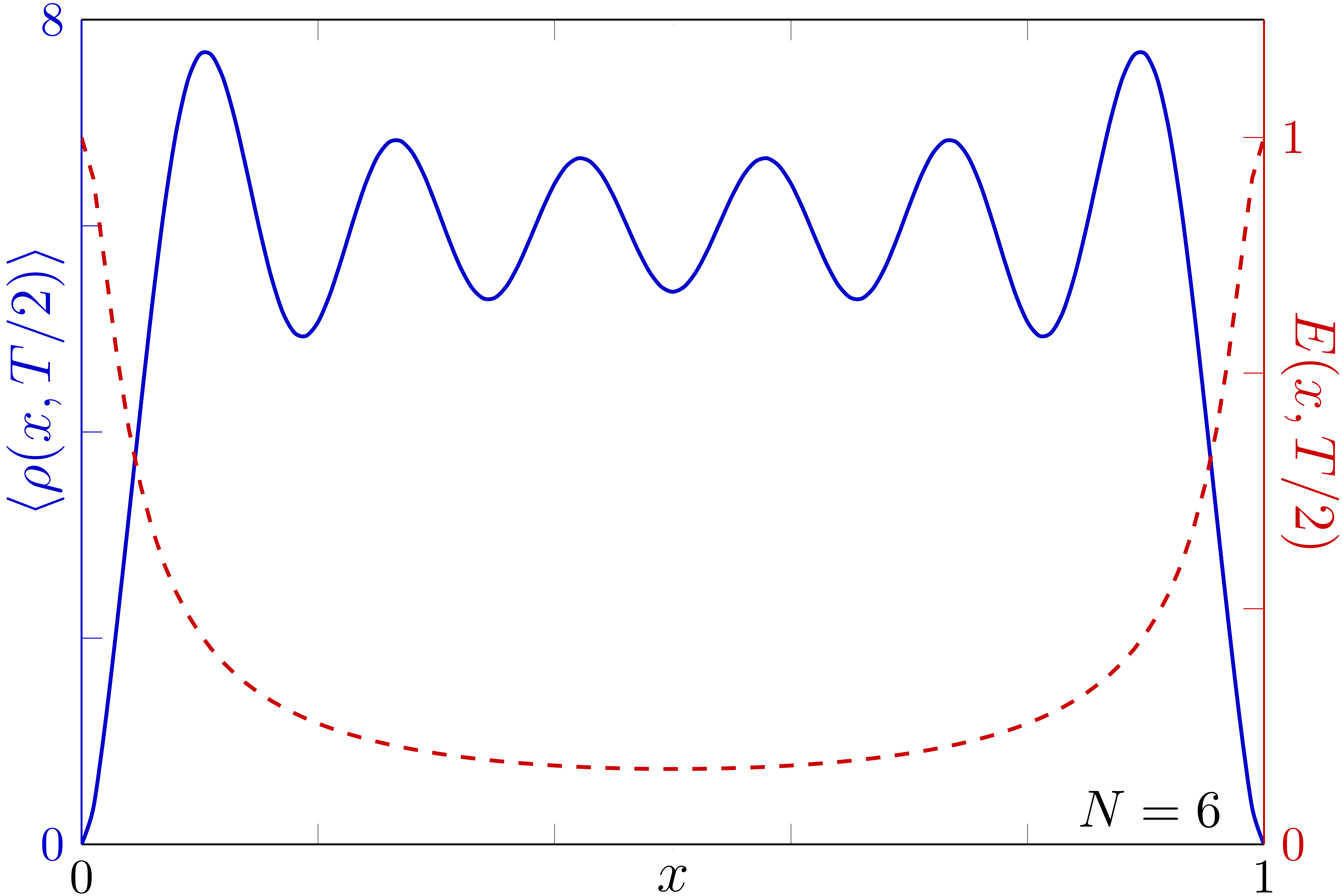
$$|\Psi\rangle = |GS_i\rangle \longrightarrow |\Psi(t)\rangle = U(t,0)|\Psi\rangle \longrightarrow |\Psi(t)\rangle = e^{-iH_f(t-\tau)}|\Psi(\tau)\rangle$$

Cazalilla, PRL (2006); Dóra et. al, PRL (2011)

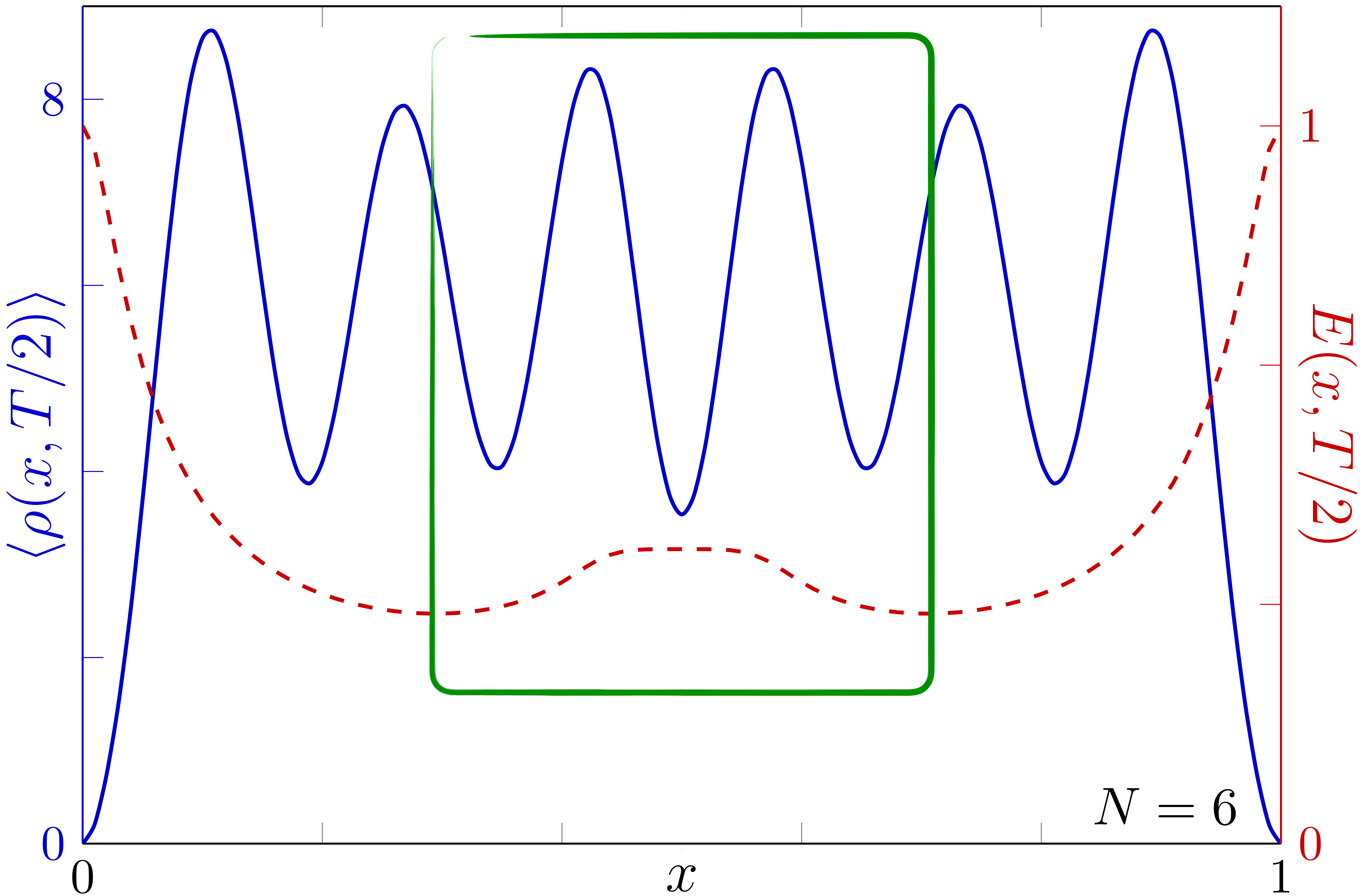
- Average particle density ($\hat{\rho}(x,t) = \hat{\Psi}^\dagger(x,t)\hat{\Psi}(x,t)$)

$$\langle \rho(x,t) \rangle_i = \frac{N}{L} \left\{ 1 - E(x,t) \cos\left[2k_F x - 2f(x) \right] \right\}$$

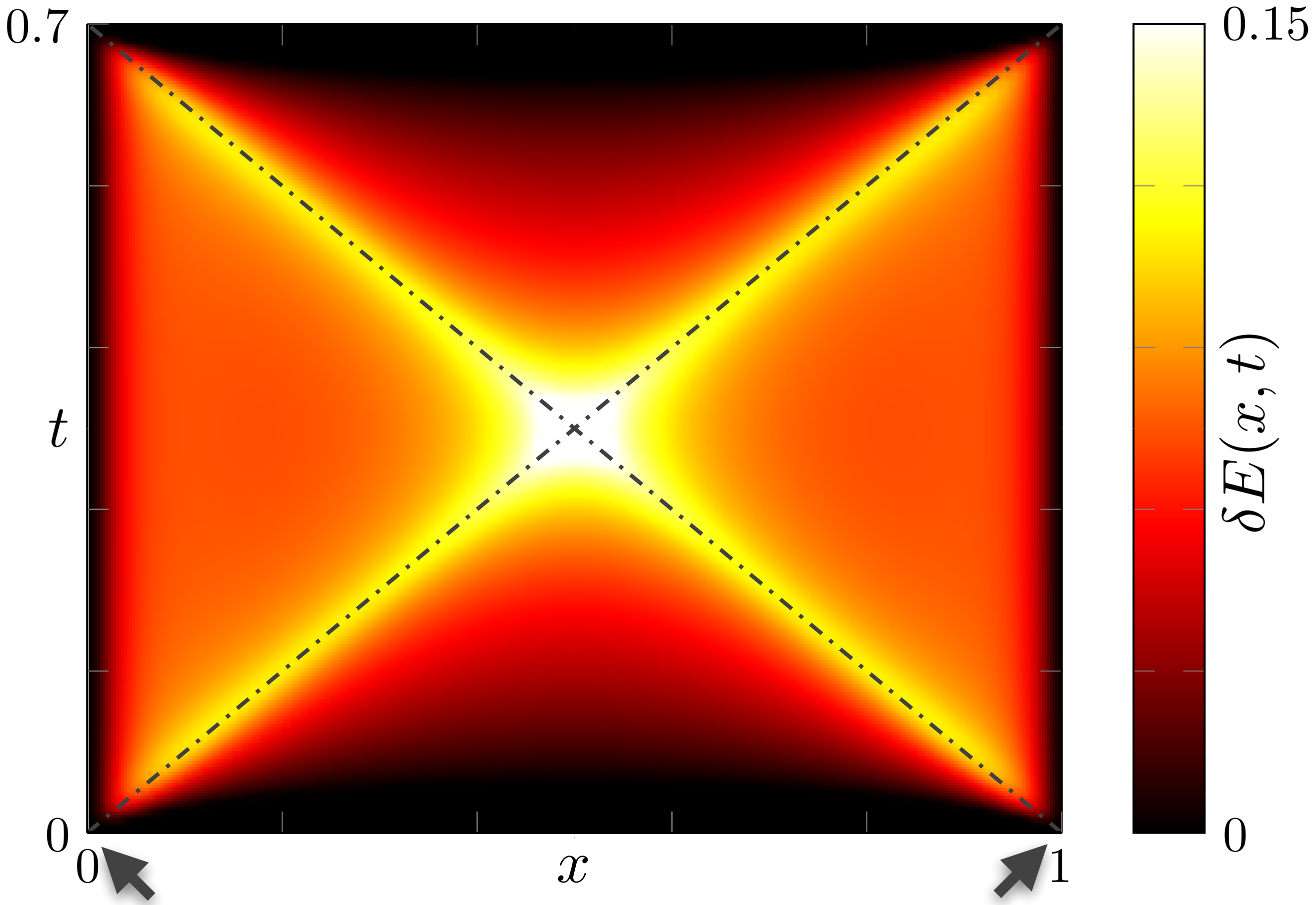
Density in a standard 1D LL with OBC



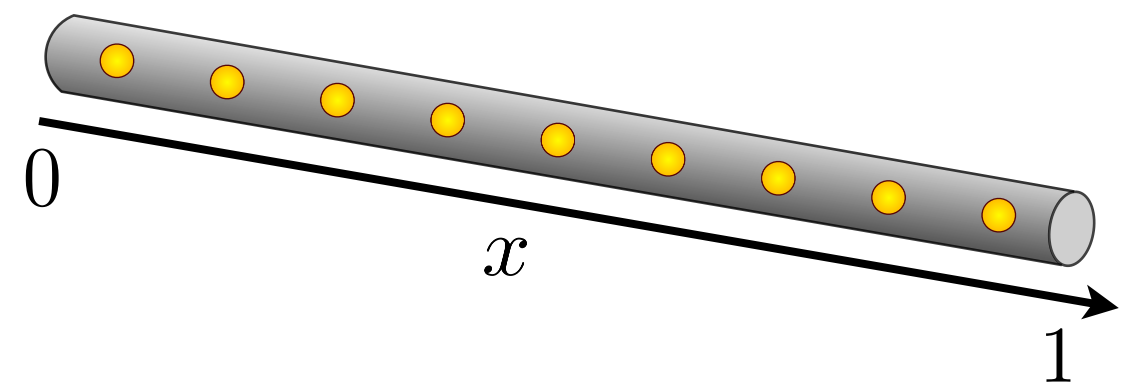
Density in a quenched 1D LL with OBC



Light cones dynamics — sudden quench



- Focus on
  $\delta E(x,t) = E(x,t) - E(x,0)$
  over one period.



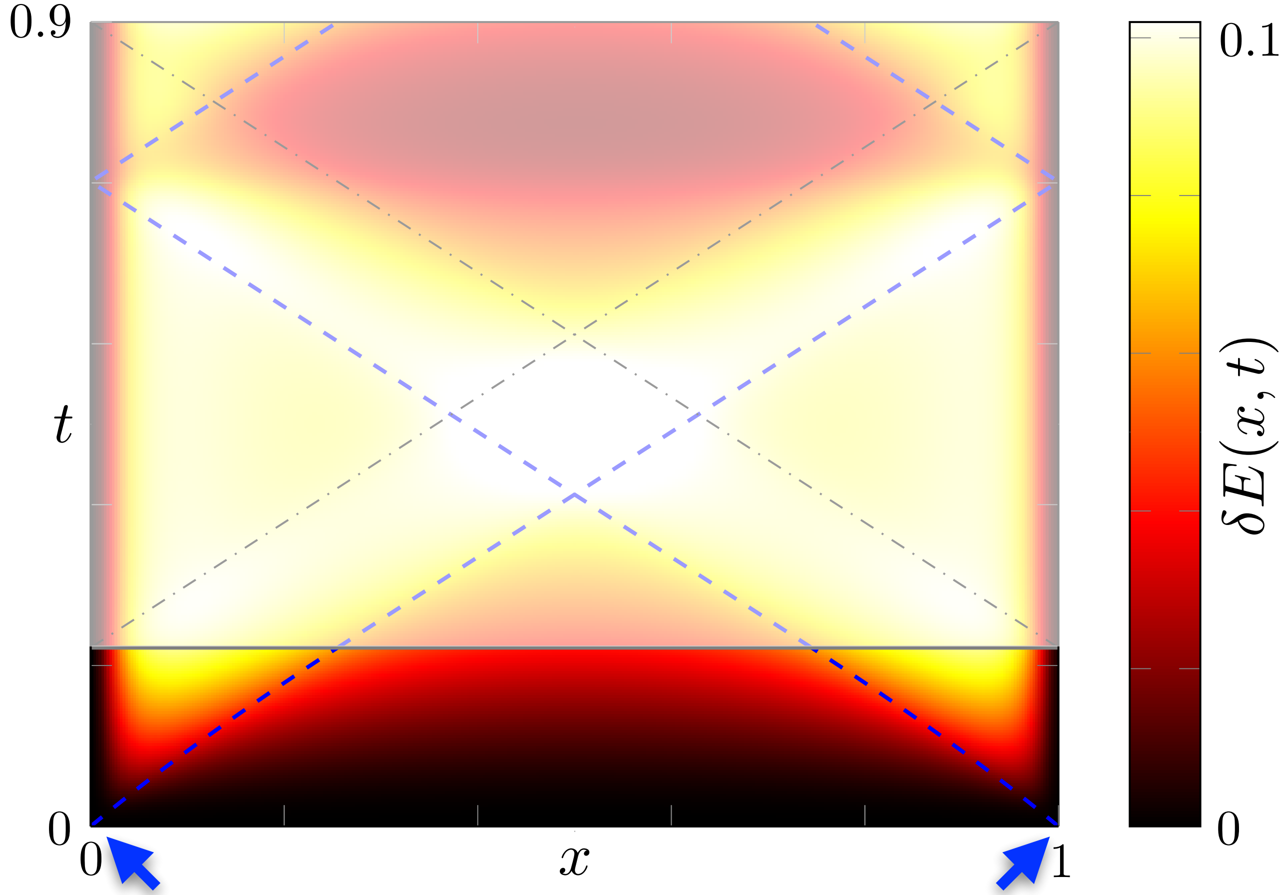
- Sudden quench: a **LC** perturbation emerges at the boundaries and moves *ballistically* at velocity v_f

$$H_f = \sum_{q>0} q v_f \beta^\dagger_{q,f} \beta_{q,f}$$

- Analytically $E(x,t) = E^{(\mathrm{GGE})}_{\mathrm{sq}}(x) f_{\mathrm{sq}}(t) \mathcal{C}_{\mathrm{sq}}(x, v_f t)$

$$\mathcal{C}_{\mathrm{sq}}(x,y) = \mathcal{C}^R_{\mathrm{sq}}(x-y)\mathcal{C}^L_{\mathrm{sq}}(x+y)$$

Light cones dynamics — finite duration quench



- Adiabatic quench
  $(\tau \gg \tau_{\text{ad}} \sim L|\eta|/v_i)$

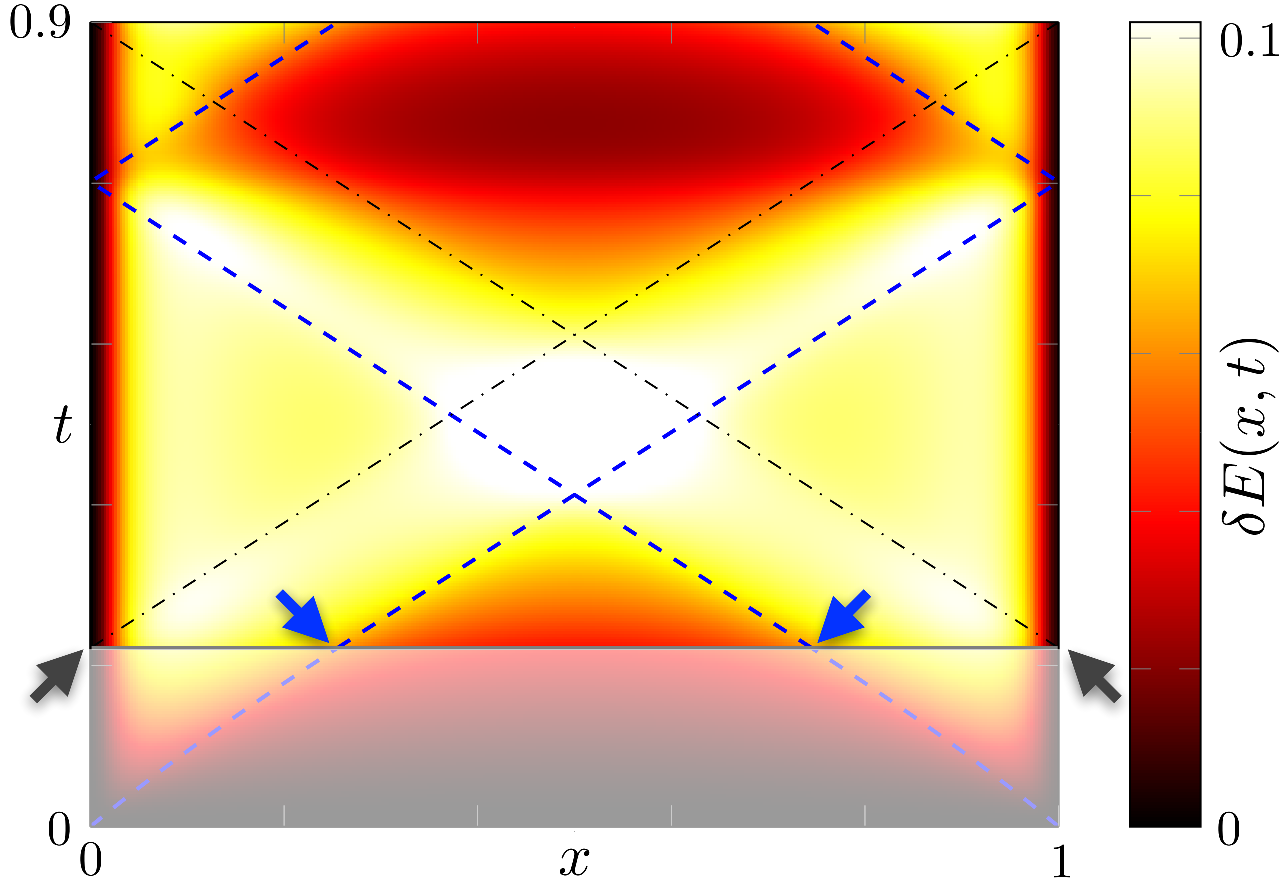
$$E(x,t) = G(x,t)\mathcal{C}_{\text{ad}}(x, \ell(t))$$

$$\mathcal{C}_{\text{ad}}(x,y) = \mathcal{C}^R_{\text{ad}}(x-y) + \mathcal{C}^L_{\text{ad}}(x+y)$$

- On-ramp: **LC1** emerges at the boundaries

$$H(\bar{t}) = \sum_{q>0} q\bar{v}(\bar{t})\beta^\dagger_{q,\bar{t}}\beta_{q,\bar{t}} \longrightarrow \bar{v}(t) = v_i\sqrt{1+\frac{\eta t}{\tau}}$$

Light cones dynamics — finite duration quench



- Adiabatic quench
  $(\tau \gg \tau_{\rm ad} \sim L|\eta|/v_i)$

$$E(x,t) = E_{\rm ad}^{\rm (GGE)}(x)\,[f_{\rm ad}(t) + A_1 \mathcal{C}_{\rm ad}(x, v_f(t-\tau)+d) - A_2 \mathcal{C}_{\rm ad}(x, v_f(t-\tau))]$$

$$\mathcal{C}_{\rm ad}(x,y) = \mathcal{C}^R_{\rm ad}(x-y) + \mathcal{C}^L_{\rm ad}(x+y)$$

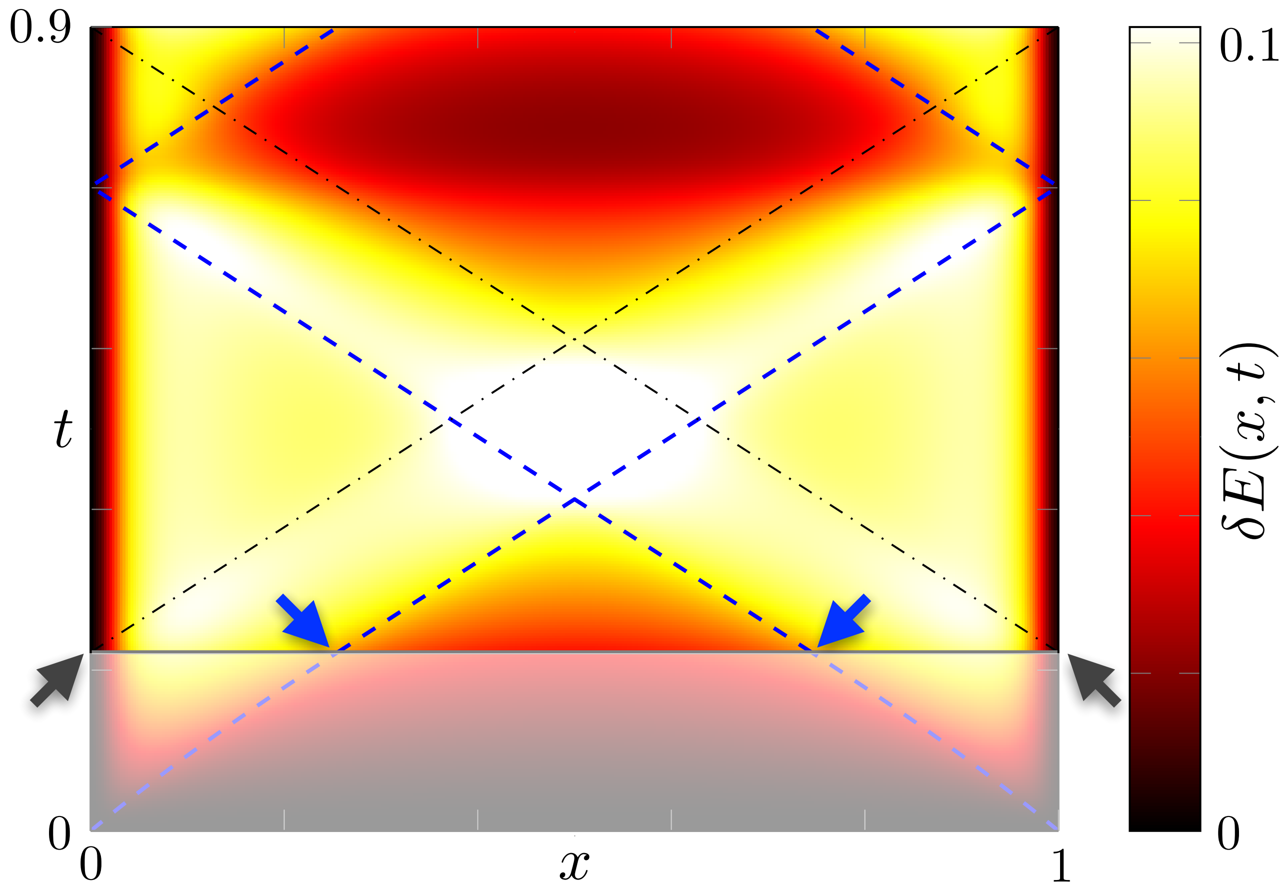
- On-ramp: **LC1** emerges at the boundaries

$$H(\bar{t}) = \sum_{q>0} q\bar{v}(\bar{t})\beta^\dagger_{q,\bar{t}}\beta_{q,\bar{t}} \longrightarrow \bar{v}(t) = v_i\sqrt{1+\frac{\eta t}{\tau}}$$

- Post-quench: **LC2** emerges at the boundaries, **LC1** goes on with velocity v_f

$$H_f = H(t \geq \tau) = \sum_{q>0} q v_f \beta^\dagger_{q,f}\beta_{q,f}$$

Light cones dynamics — finite duration quench



- Adiabatic quench
  ($\tau \gg \tau_{\rm ad} \sim L|\eta|/v_i$)

$$E(x,t) = E_{\rm ad}^{\rm (GGE)}(x)\,[f_{\rm ad}(t) + A_1 \mathcal{C}_{\rm ad}(x, v_f(t-\tau)+d) - A_2 \mathcal{C}_{\rm ad}(x, v_f(t-\tau))]$$

$$\mathcal{C}_{\rm ad}(x,y) = \mathcal{C}_{\rm ad}^R(x-y) + \mathcal{C}_{\rm ad}^L(x+y)$$

- On-ramp: **LC1** emerges at the boundaries

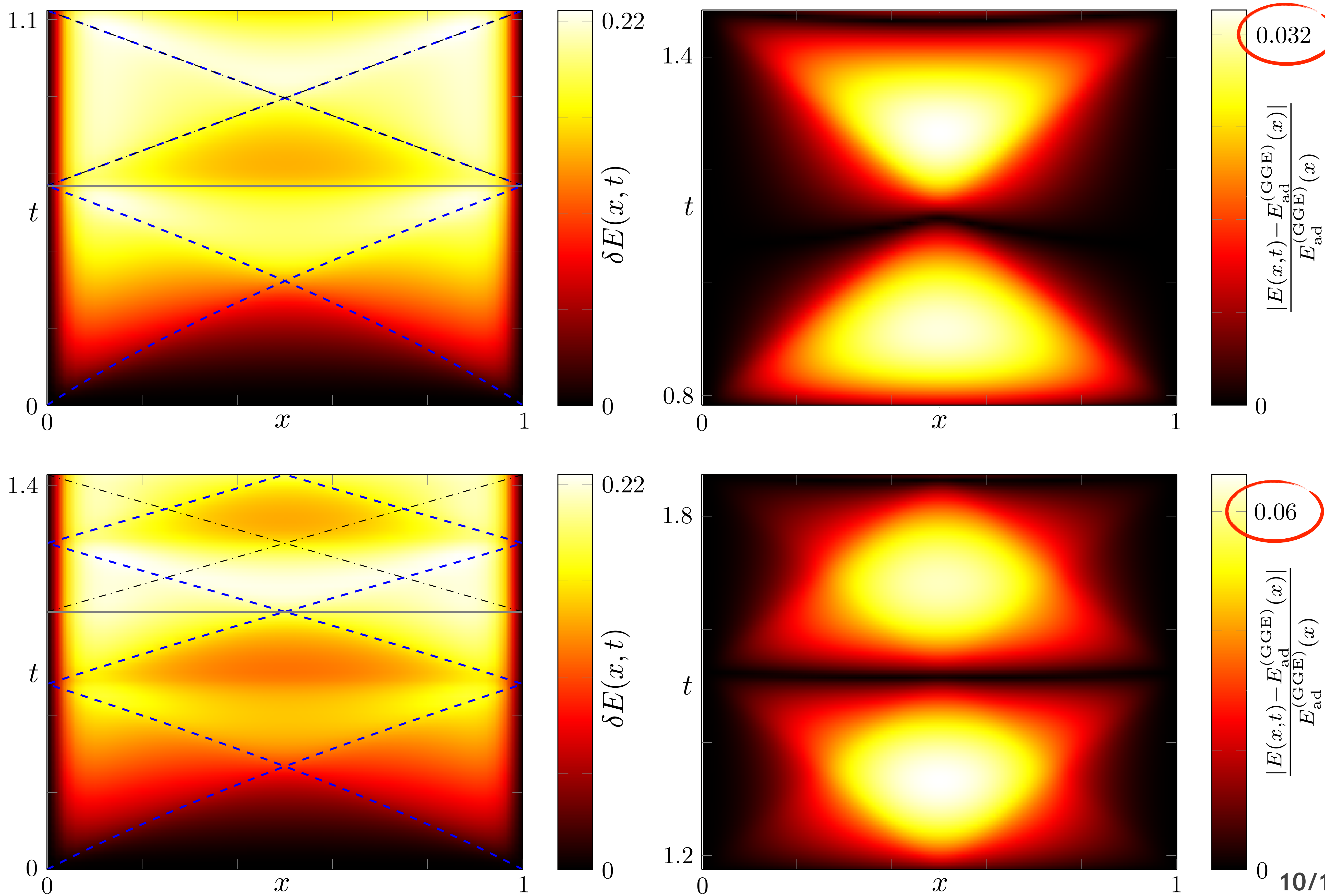
$$H(\bar{t}) = \sum_{q>0} q\bar{v}(\bar{t})\beta^\dagger_{q,\bar{t}}\beta_{q,\bar{t}} \longrightarrow \bar{v}(t) = v_i\sqrt{1+\frac{\eta t}{\tau}}$$

- Post-quench: **LC2** emerges at the boundaries, **LC1** goes on with velocity v_f

$$H_f = H(t \geq \tau) = \sum_{q>0} q v_f \beta^\dagger_{q,f}\beta_{q,f}$$

LCs move with the *instantaneous* bosonic velocity

Interference between light cones



Conclusions

Density in a quenched LL with OBC

LC perturbations

ballistic propagation

"destructive interference"

recurrence time

no relaxation to final GS

no thermalization

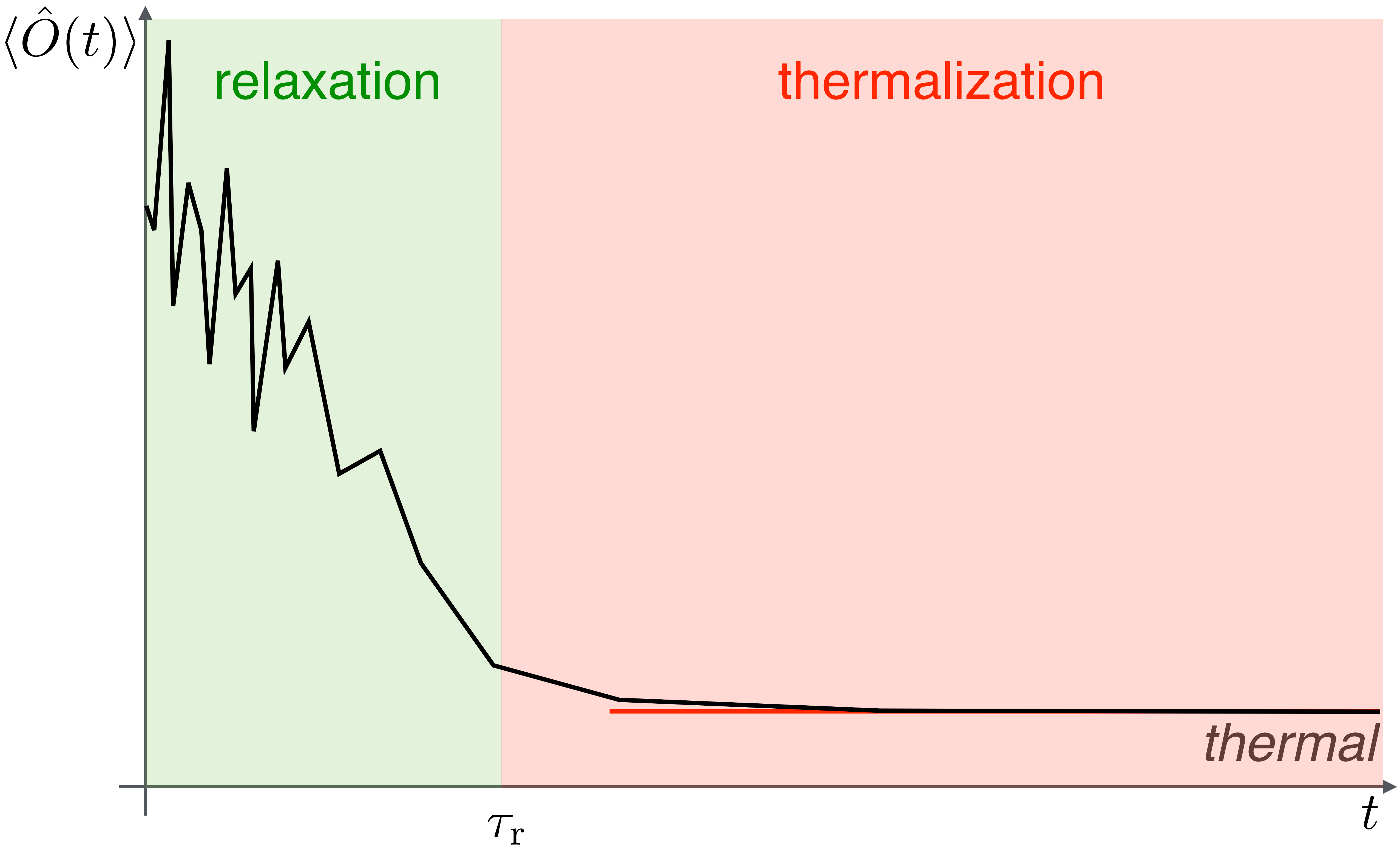
GGE as "zeroth order"

faster approach

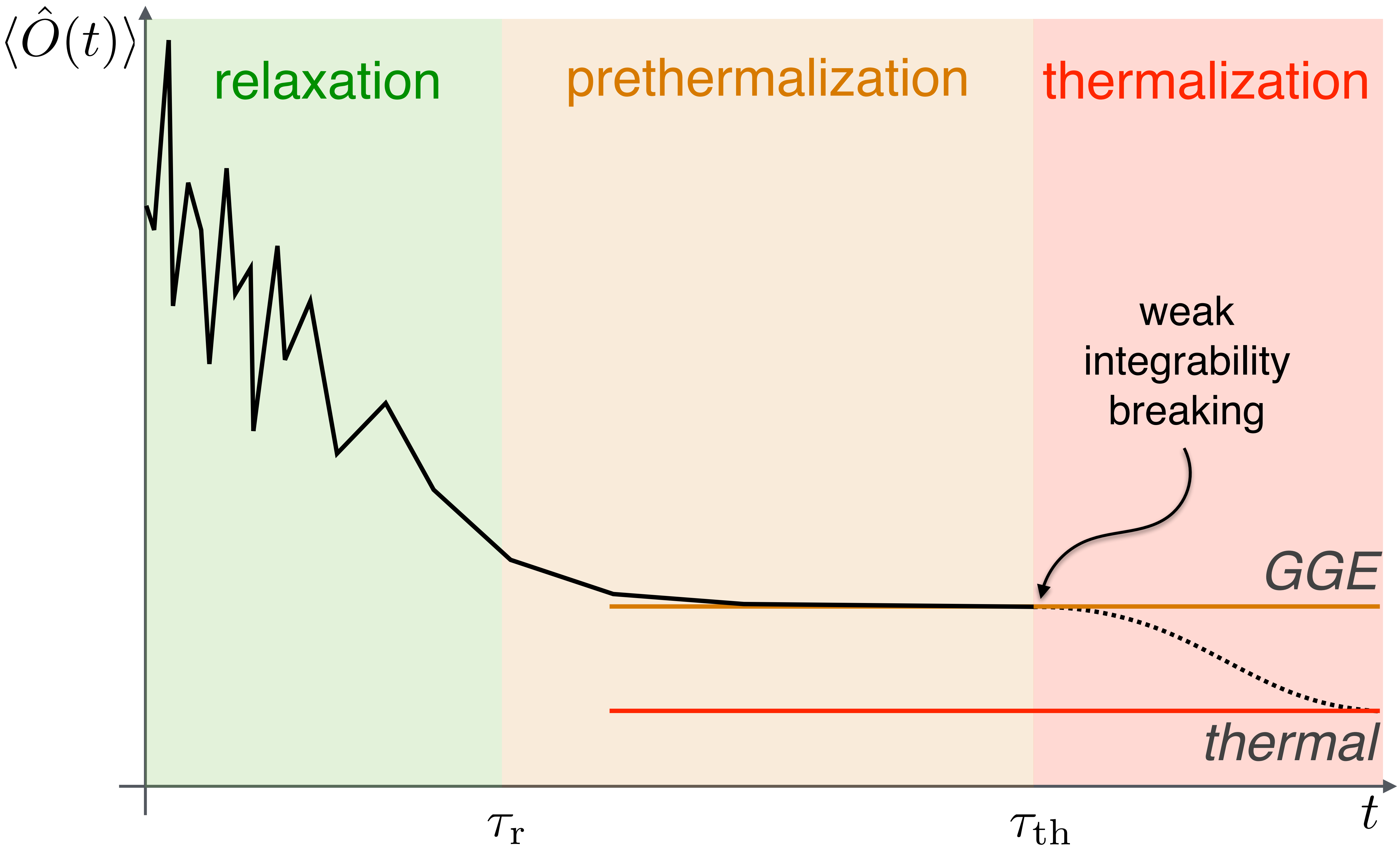
adiabatic quenches

collapse on the final GS

Thermalization



Prethermalization

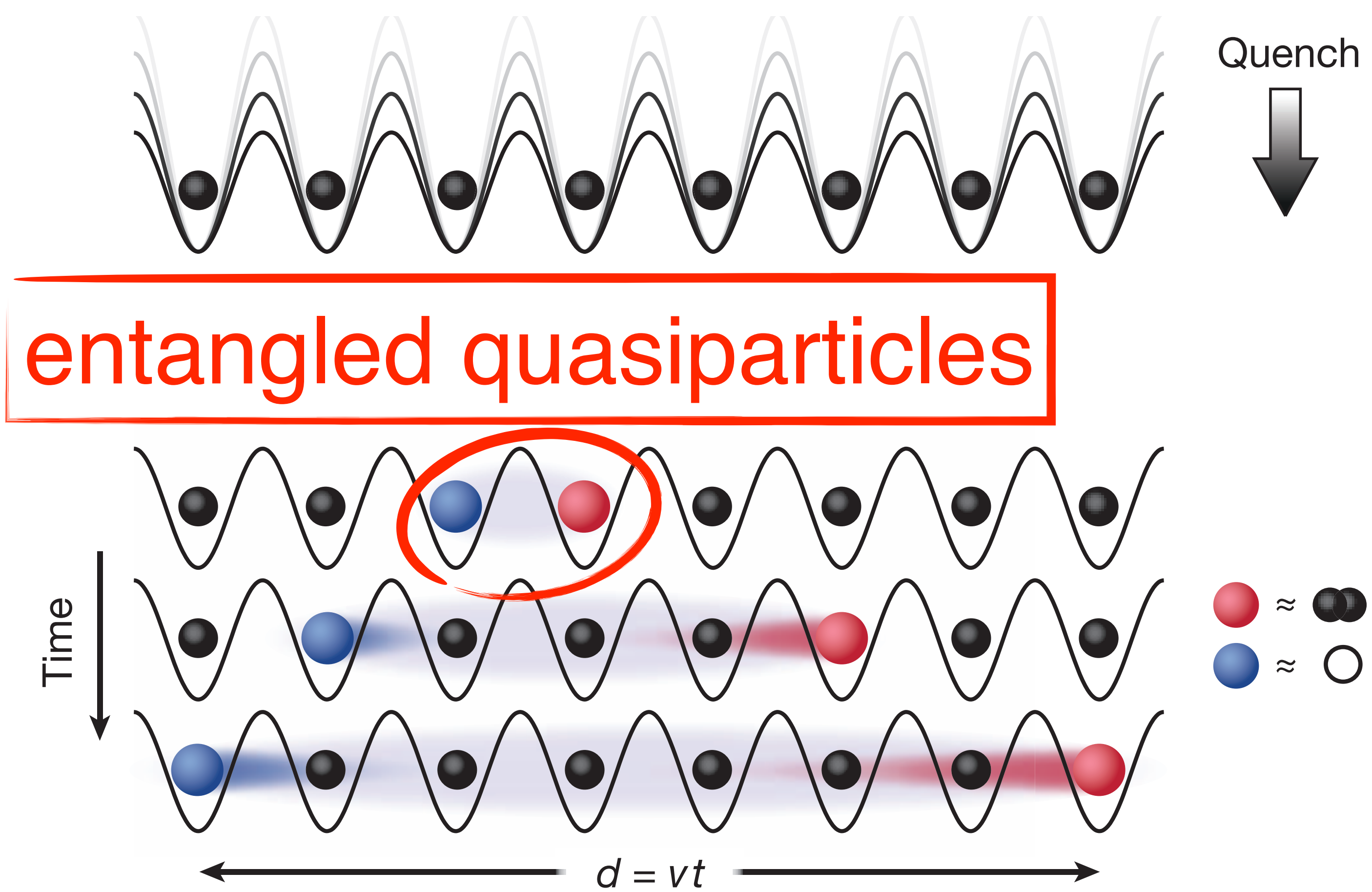


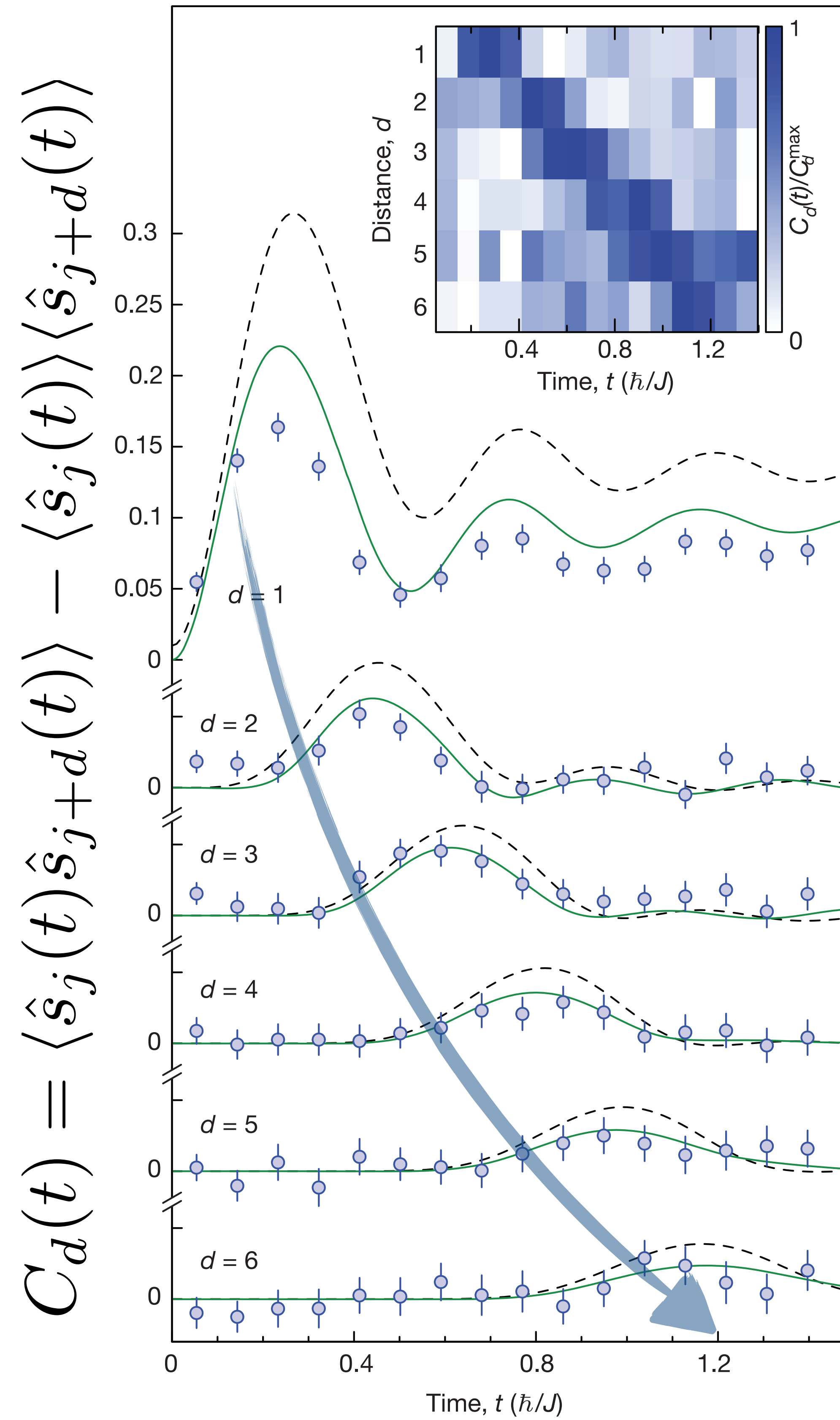
Light-cone spreading of correlations

- Rel. QFT → information propagation bounded by c → **light-cone effect**

How fast “information” can spread in CM systems?

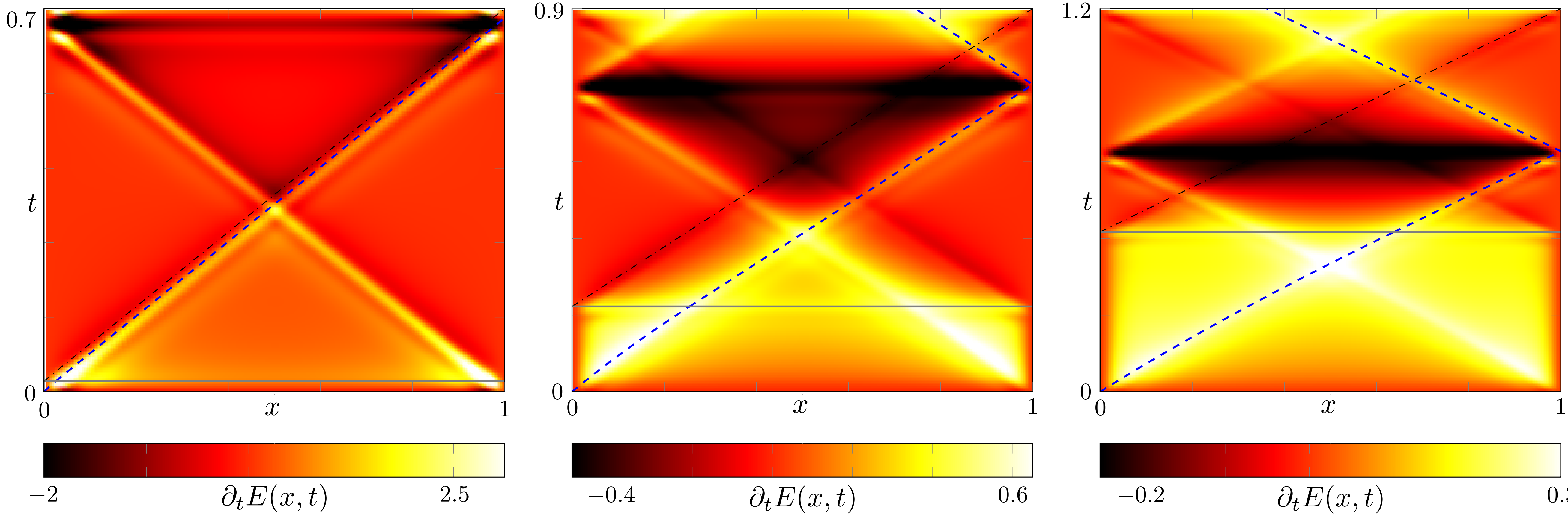
- Short-range interactions → **Lieb-Robinson bound**
- Quench in ultracold atomic gases





Cheneau et al., Nature (2012); Calabrese et al., PRL (2006)

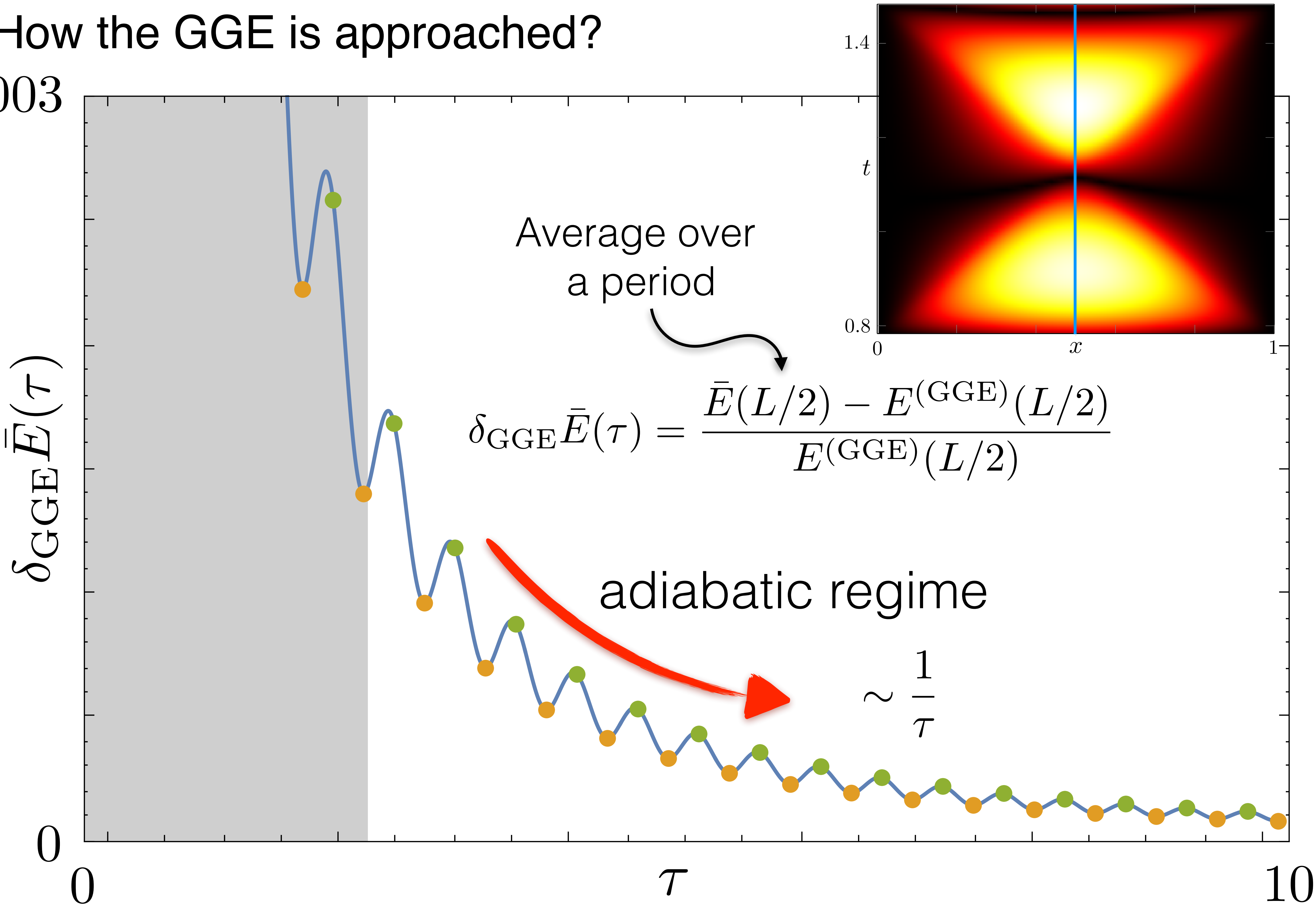
Where are the LCs?



- **Sudden quench** $E(x,t) = E_{\mathrm{sq}}^{(\mathrm{GGE})}(x) f_{\mathrm{sq}}(t) \mathcal{C}_{\mathrm{sq}}(x, v_f t)$
- Adiabatic quench ($\tau \gg \tau_{\mathrm{ad}} \sim L|\eta|/v_i$)
  - transient $E(x,t) = G(x,t) \mathcal{C}_{\mathrm{ad}}(x, \ell(t))$
  - post-quench $E(x,t) = E_{\mathrm{ad}}^{(\mathrm{GGE})}(x) \left[f_{\mathrm{ad}}(t) + A_1 \mathcal{C}_{\mathrm{ad}}(x, v_f(t-\tau) + d) \right.$ $\left. + A_2 \mathcal{C}_{\mathrm{ad}}(x, v_f(t-\tau)) \right]$

Approach to GGE

- How the GGE is approached?



$$\delta_{\mathrm{GGE}}\bar{E}(\tau) = \frac{\bar{E}(L/2) - E^{(\mathrm{GGE})}(L/2)}{E^{(\mathrm{GGE})}(L/2)}$$

Approach to final envelope

- How the GGE is approached?
- Is the final envelope approached by the GGE envelope?