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Stability of volcanic conduits during explosive eruptions and the influence of conduit geometry

Álvaro Aravena[1], Mattia de' Michieli Vitturi[2], Raffaello Cioni[1] and Augusto Neri[2].

1 Universita degli studi di Firenze, Firenze, Italia.
2 Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Pisa, Pisa, Italia.

Motivation and objectives

Erosion mechanisms in volcanic conduits [*Macedonio et al.*, 1994]:

- Impact of pyroclasts.
- Fluid shear stress.
- Conduit wall collapse.
- Volcanic tremor.

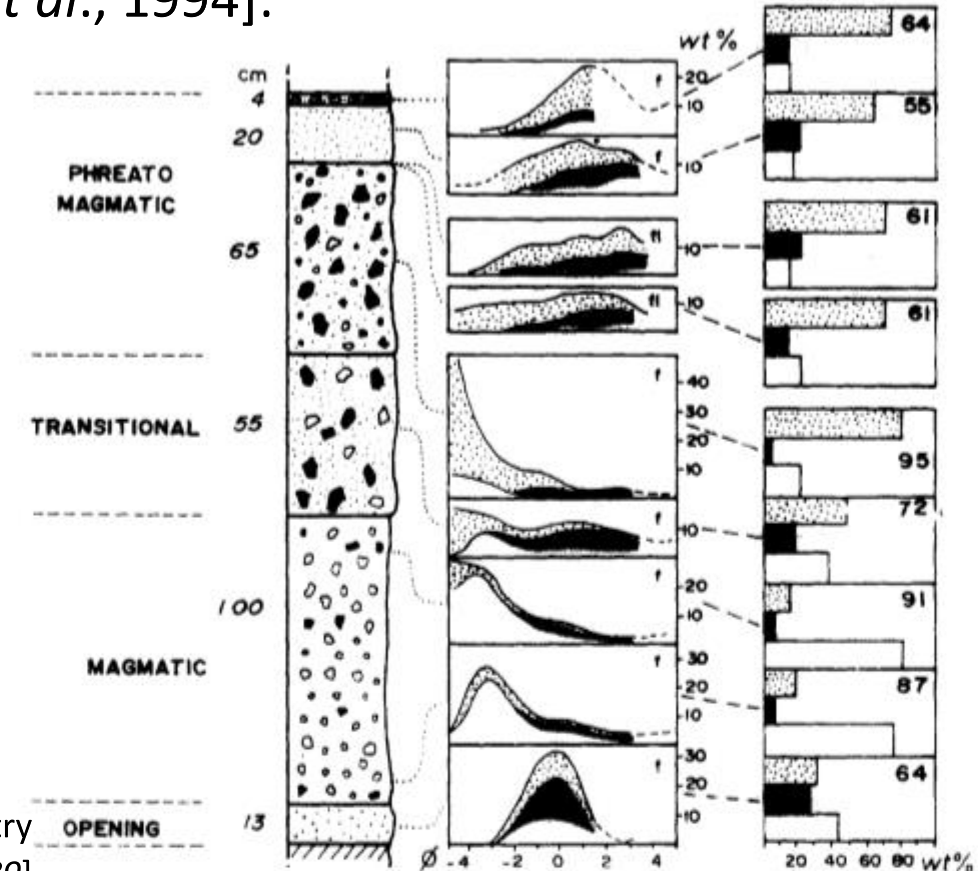


Fig. 1. 1631 eruption of Vesuvius. Eruptive phases, grain-size distribution and componentry (juvenile: unornamented; crystals: solid; lithics: stippled) [*Barberi et al., 1989*]

Motivation and objectives

- Conduit widening processes are still misunderstood. They have never been addressed numerically.
- Here we evaluate the mechanical stability of volcanic conduits, using:
  - A 1D-steady state model of volcanic conduits.
  - Mogi-Coulomb collapse criterion.

1D – Steady State Model

Our model considers:

- Crystallization.
- Rheological changes.
- Magma fragmentation.
- Drag forces.
- Outgassing.
- Degassing.

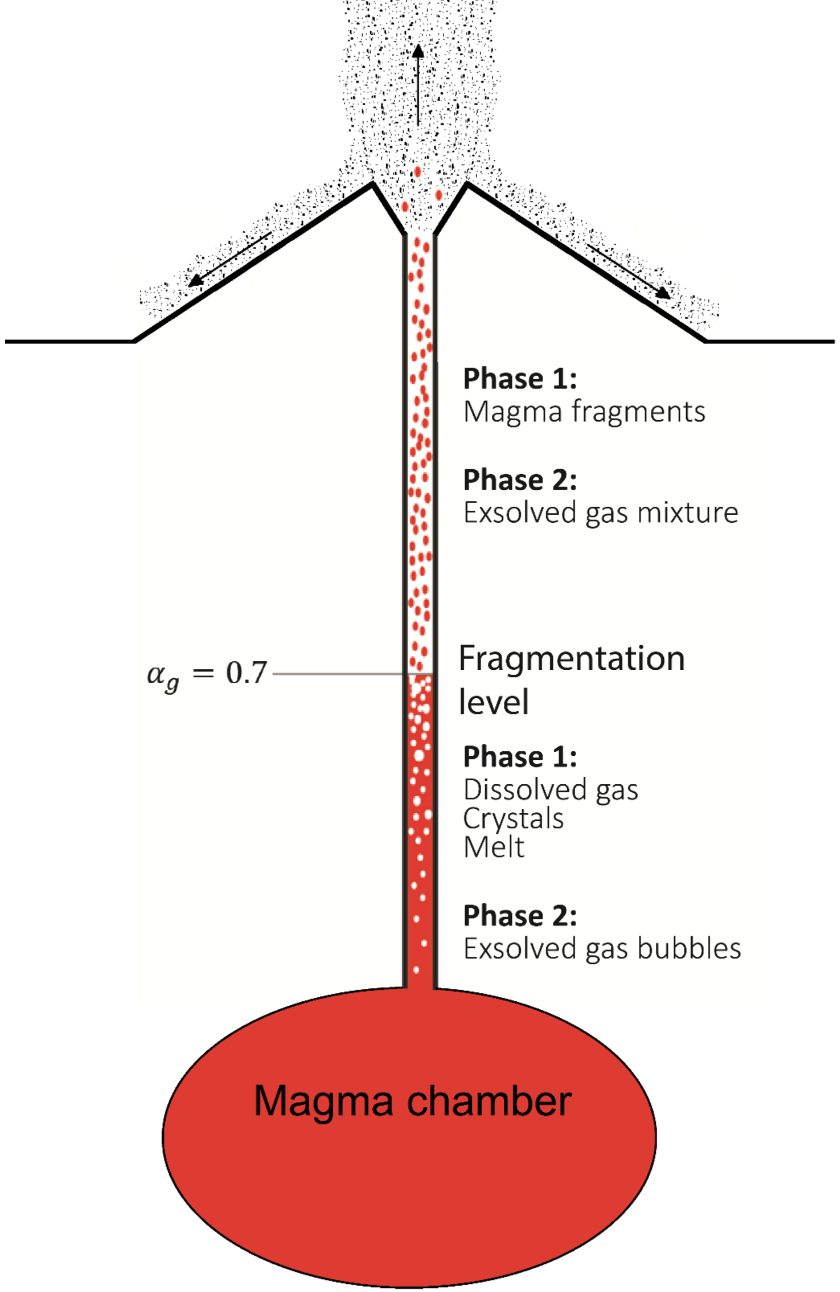


Fig. 2. Scheme of 1D-steady state model.

System of equations

(1) $$\frac{\partial}{\partial z}(\rho u) = 0$$

(2) $$\frac{\partial}{\partial z}(\alpha_1\rho_1 u_1^2 + \alpha_2\rho_2 u_2^2 + \alpha_1\rho_1 + \alpha_2\rho_2) = -\rho g - \frac{8\mu_{mix} u}{R^2}$$

(3) $$\frac{\partial}{\partial z}\left[\alpha_1\rho_1 u_1\left(e_1 + \frac{p_1}{\rho_1} + \frac{u_1^2}{2}\right) + \alpha_2\rho_2 u_2\left(e_2 + \frac{p_2}{\rho_2} + \frac{u_2^2}{2}\right) - \rho x_1 x_2 (u_1 - u_2)(s_1 - s_2)T\right] = -\rho g u - \frac{4\mu_{mix} u^2}{R^2}$$

where z is the vertical coordinate; ρ is the mixture density, u is the mixture velocity; α_i, ρ_i, u_i, e_i, p_i, x_i and s_i are the volume fraction, density, velocity, internal energy, pressure, mass fraction and entropy of the phase i, respectively; g is the acceleration of gravity, μ_{mix} is the mixture viscosity, R is the conduit radius and T is the mixture temperature.

- ✓ Pressure and temperature are derived from the internal energy.
- ✓ Closure equations related to:

- Liquid volume fraction.
- Solubility model.
- Outgassing model.
- Mass balance of crystals, phase 1 and dissolved gas.
- Relative velocity between both phases.
- Equations of state.
- Magma viscosity.
- Crystallization model.

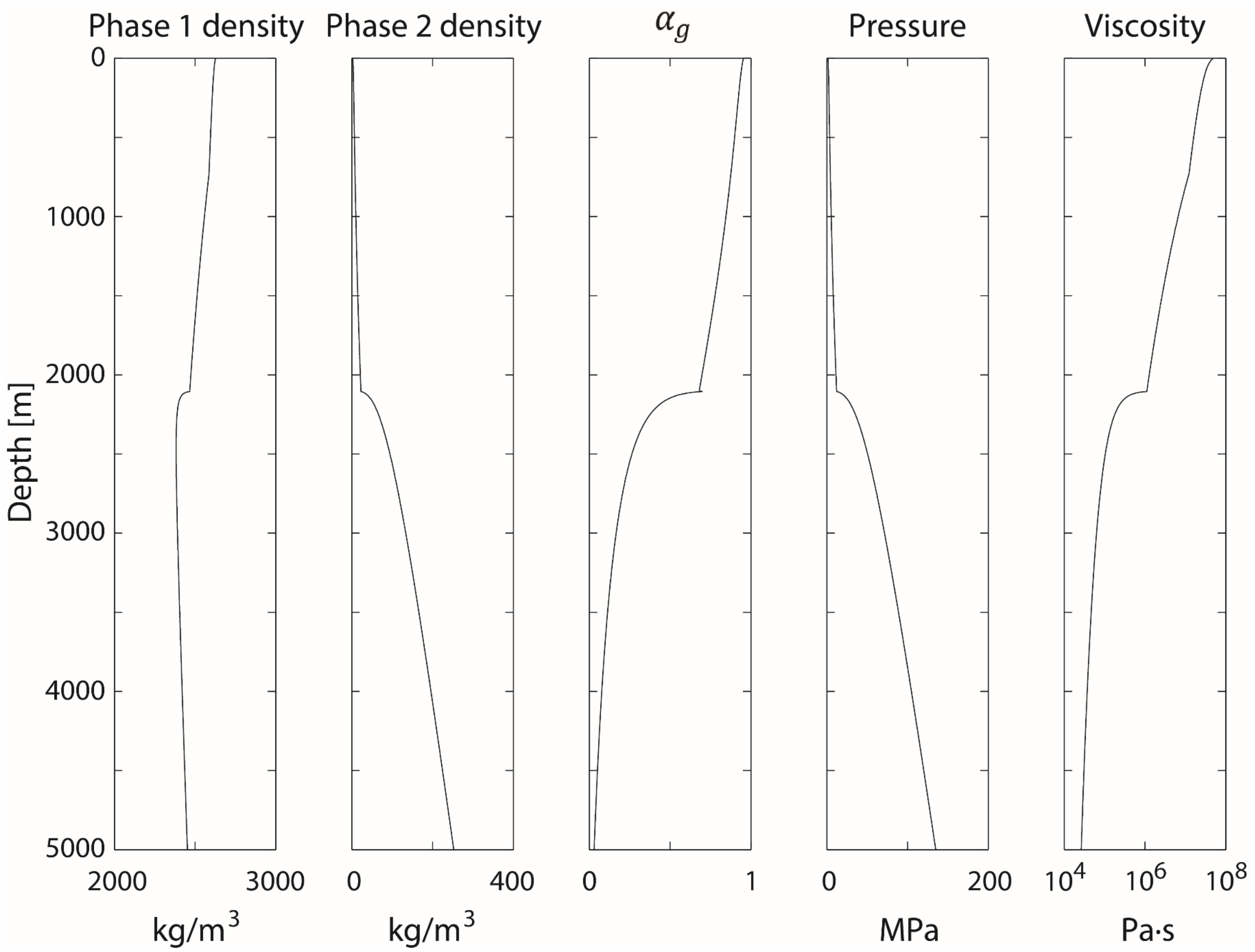


Fig. 3. Profiles along the conduit of some physical parameters. They are results of a specific simulation (radius of 35 m, water content of 6.5%, inlet pressure of 125 MPa, rhyolitic magma).

Mogi – Coulomb collapse criterion [Al-Ajmi and Zimmerman, 2006]

Case	$\sigma_1 \geq \sigma_2 \geq \sigma_3$	Failure occurs if $P(z) \leq P_{collapse}(z)$
1	$\sigma_z \geq \sigma_\theta \geq \sigma_r$	$P_{collapse} = \frac{1}{6-2b'^2} \cdot \left[(3A+2b'K) - \sqrt{H+12(K^2+b'AK)}\right]$
2	$\sigma_\theta \geq \sigma_z \geq \sigma_r$	$P_{collapse} = \frac{A}{2} - \frac{1}{6}\sqrt{12[a'+b'(A-2P_0)]^2 - 3(A-2B)^2}$
3	$\sigma_\theta \geq \sigma_r \geq \sigma_z$	$P_{collapse} = \frac{1}{6-2b'^2} \cdot \left[(3A-2b'G) - \sqrt{H+12(G^2+b'AG)}\right]$

$P_{collapse}$: Minimum pressure for inhibiting conduit collapse. σ_z: Vertical stress. σ_r: Radial stress. σ_θ: Tangential stress.

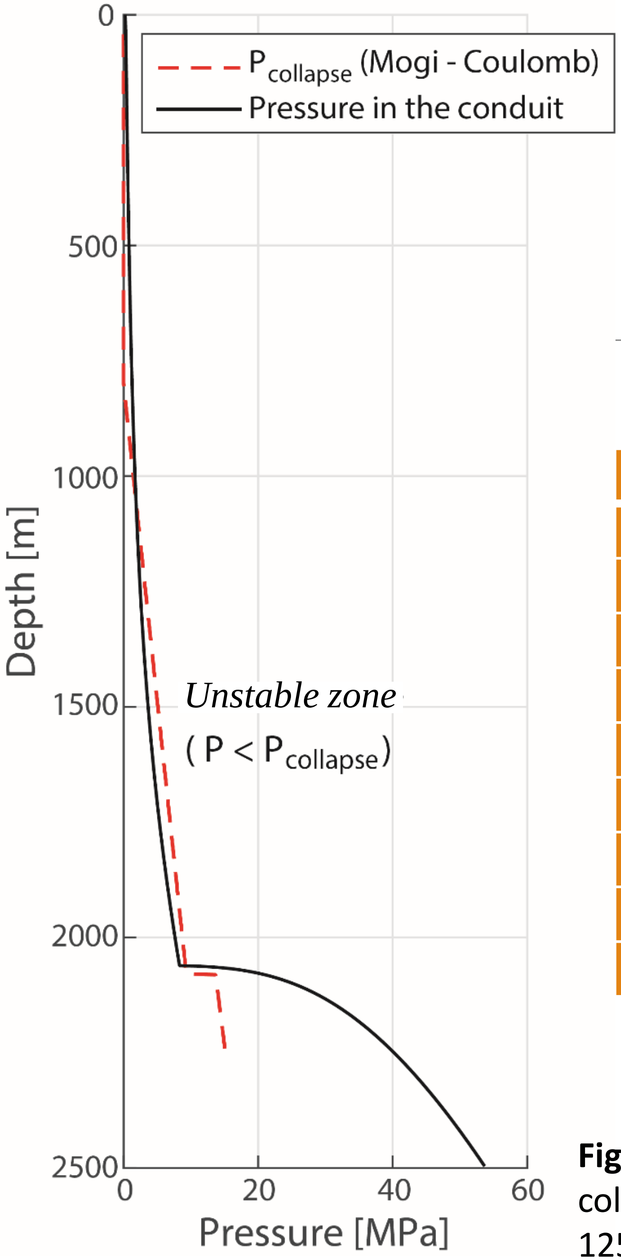
$b' = \sin(\phi)$. ϕ: Angle of internal friction.

$A = 3\sigma_H - \sigma_h$. σ_H: Maximum horizontal stress ($\max(\sigma_r, \sigma_\theta)$). σ_h: Minimum horizontal stress ($\min(\sigma_r, \sigma_\theta)$).

$K = a' + b'(B - 2P_0)$. $a' = 2c \cdot \cos(\phi)$. c: Cohesion. $B = \sigma_v + 2\upsilon(\sigma_H - \sigma_h)$. σ_v: Vertical stress. υ: Poisson ratio. P_0: Pore pressure.

$H = A^2(4b'^2 - 3) + (B^2 - AB)(4b'^2 - 12)$. $G = K + b'A$.

Instability index: $\max(P_{collapse}(z) - P(z))$



Input data and eruptive conditions

Property	Trachytic magma	Rhyolitic magma
Temperature	900ºC	850ºC
Viscosity model	Romano et al. (2003)	Hess and Dingwell (1996)
Crystallinity model	de‘ Michieli Vitturi et al. (2010)	de‘ Michieli Vitturi et al. (2010)
Solubility model	Henri’s law	Henri’s law
Solubility constant (σ)	$7.6 \cdot 10^{-7}$ (1)	$4.1 \cdot 10^{-6}$ (2)
Solubility constant (ϵ)	0.6 (1)	0.5 (2)
Inlet pressure	115 – 135 MPa	115 – 135 MPa
Water content	3.5 – 6.5 wt.%	4.5 – 6.5 wt.%
Conduit radius	5 - 40 m	25 - 115 m

Fig. 4. Profiles of conduit pressure and pressure needed to avoid conduit collapse, predicted by Mogi – Coulomb collapse criterion. They are results of a specific simulation (radius of 35 m, water content of 6.5%, inlet pressure of 125 MPa, rhyolitic magma).

Conduit radius vs. Instability Index

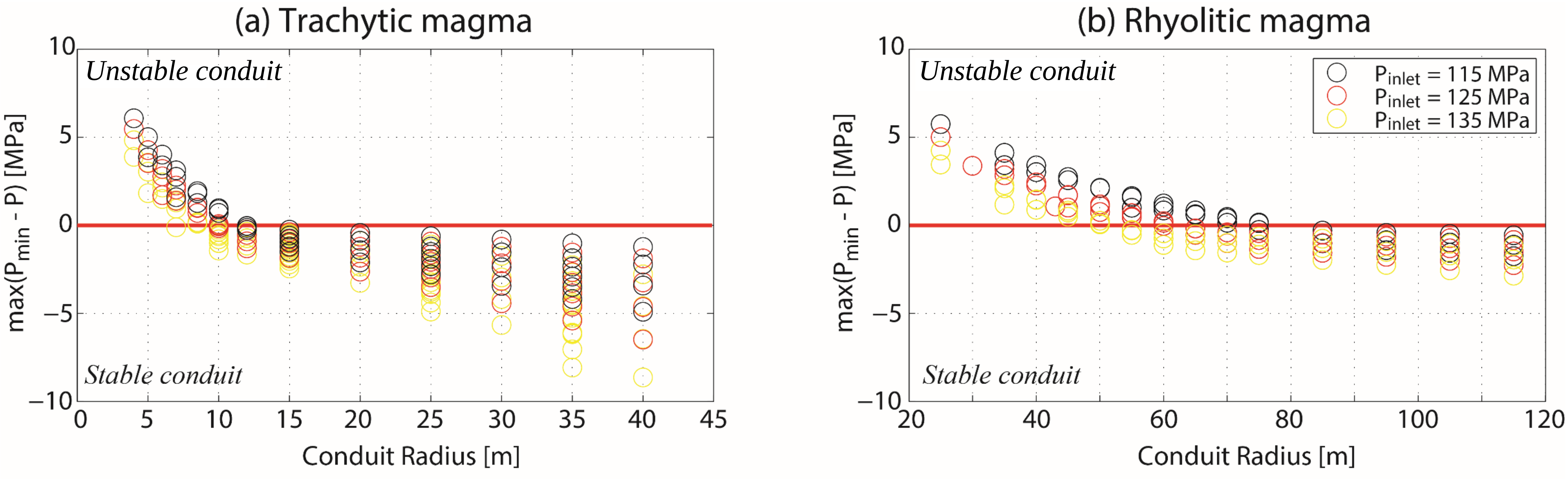
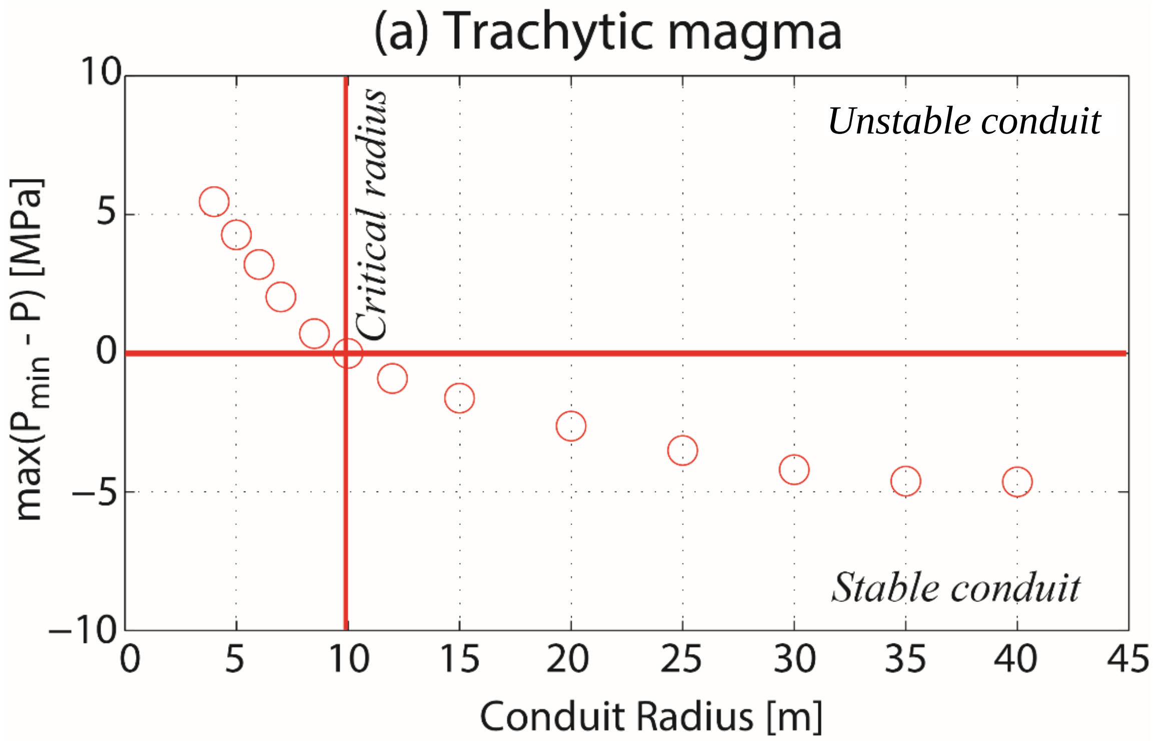


Fig. 5. (a-b) Instability index versus conduit radius, for simulations performed with trachytic and rhyolitic magma, respectively. We present here the results related to simulations with variable values of water content (3.5 - 6.5 wt. % for trachytic magma; 4.5 – 6.5 wt. % for rhyolitic magma) and inlet pressure.

Conduit radius vs. Instability Index



- For given values of inlet pressure and water content, the instability index is a monotonous function of the conduit radius.
- **Critical radius** is defined as the minimum radius for avoiding conduit collapse, for given values of inlet pressure and water content.
- It is also dependent of the magma rheology.

$$\text{Critical radius} = f(\text{P}_{\text{inlet}}, \text{H}_2\text{O}, \text{Rheology})$$

Fig. 6. Instability index versus conduit radius, for simulations performed with trachytic magma. We present here the results related to simulations with constant values of water content and inlet pressure.

Critical radius

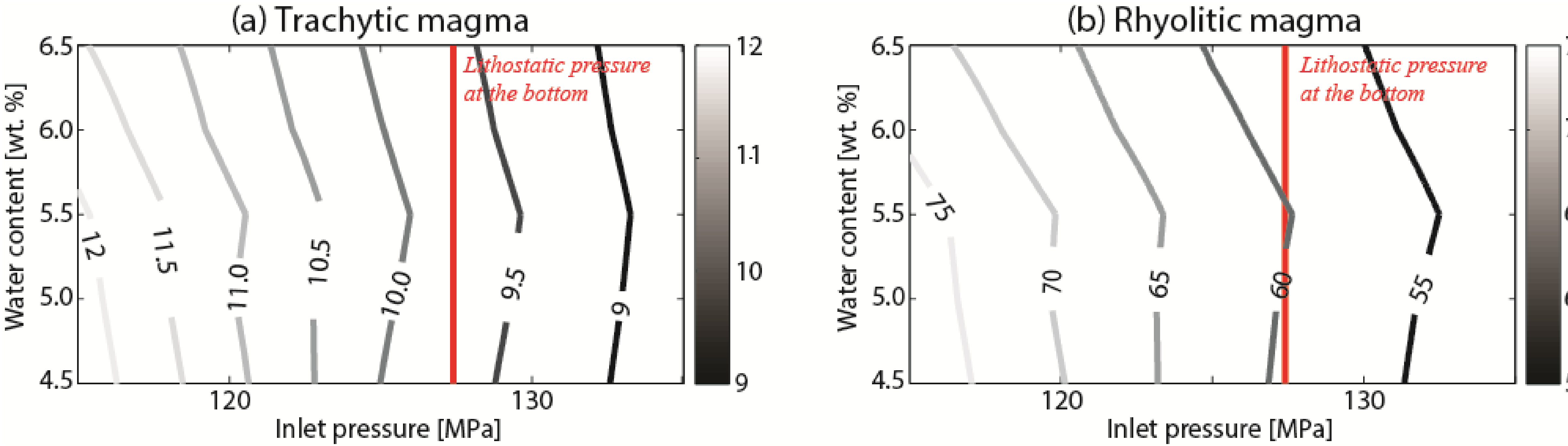


Fig. 7. (a-b) Critical radius (i.e. minimum radius for a stable conduit) for different conditions of inlet pressure and water content. Magma composition is indicated in titles.

Mass discharge rate vs. Instability Index

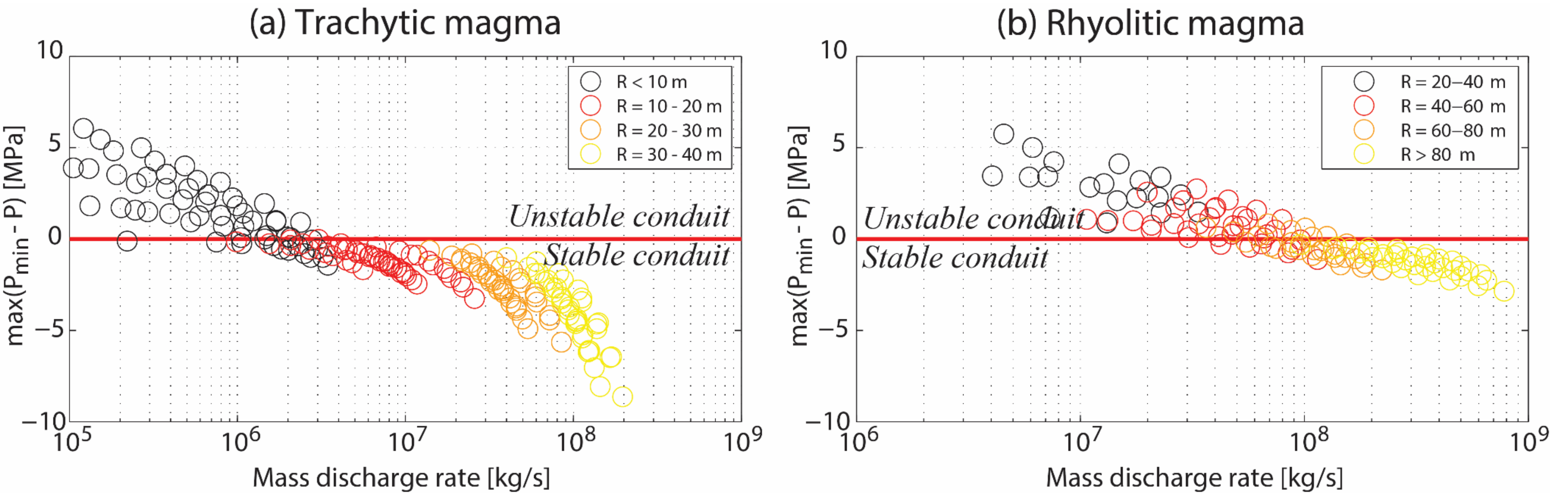


Fig. 8. (a-b) Instability index versus mass discharge rate, for simulations performed with trachytic and rhyolitic magma, respectively. We present results related to a set of simulations with variable values of inlet pressure (115 – 135 MPa) and water content (3.5 - 6.5 wt. % for trachytic magma; 4.5 – 6.5 wt. % for rhyolitic magma).

Mass discharge rate vs. Instability Index

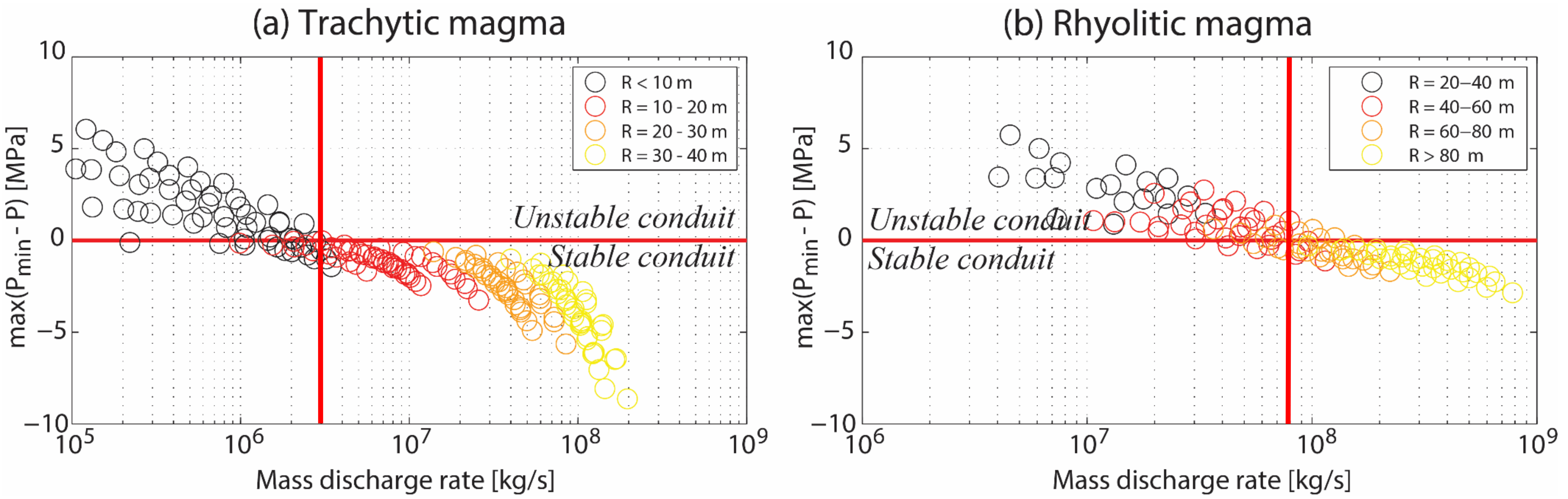


Fig. 8. (a-b) Instability index versus mass discharge rate, for simulations performed with trachytic and rhyolitic magma, respectively. We present results related to a set of simulations with variable values of inlet pressure (115 – 135 MPa) and water content (3.5 - 6.5 wt. % for trachytic magma; 4.5 – 6.5 wt. % for rhyolitic magma).

Mass discharge rate and rheology

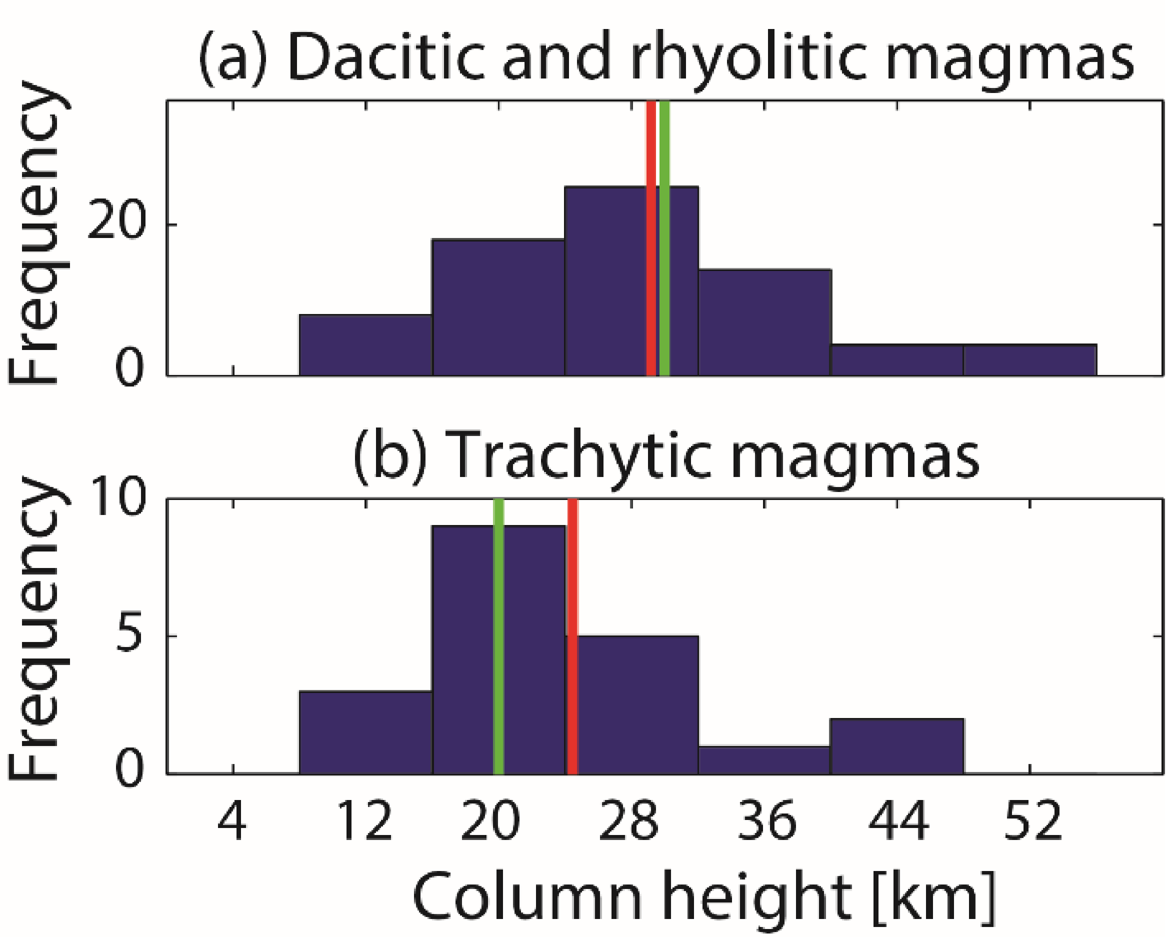


Fig. 9. Figure S1. Histograms of eruptive column height of a set of documented explosive volcanic eruptions, for different magma compositions. Data have been extracted from the database LaMEVE [*Crosweller et al.*, 2012].

Summary

- Cylindrical conduits represent a mechanically stable geometry only for large radiuses.
- The minimum conduit radius needed to avoid conduit collapse (i.e. **critical radius**) is mainly controlled by magma rheology and inlet overpressure.
- Rhyolitic magmas need conduits several times wider than trachytic magmas for developing mechanically stable conduits.
- It is possible to calculate a minimum mass discharge rate for a mechanically stable conduit, which is controlled by magma rheology ($\sim 3 \cdot 10^6$ kg/s for trachytic magmas and $\sim 8 \cdot 10^7$ kg/s for rhyolitic magmas).
- **Our procedure is easily applicable to specific case studies.**

Grazie

References

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Supplementary material

(3) $p_i = \rho_i^2 \frac{\partial e_i}{\partial \rho_i}$

(4) $T_i = \frac{\partial e_i}{\partial s_i}$

where T_i is the temperature of the phase i.

(5) $\frac{\partial}{\partial z}(\rho u \alpha_1) = -\frac{1}{\tau^{(p)}}(p_2 - p_1)$

(6) $\frac{\partial}{\partial z}(\alpha_1 \rho_1 u_1) = \frac{1}{\tau^{(d)}}(x_d - x_d^{eq})\alpha_1(\rho_1 - \beta\rho_c)$

(7) $\frac{\partial}{\partial z}\left(\frac{u_1^2}{2} - \frac{u_2^2}{2} + e_1 + \frac{p_1}{\rho_1} - e_2 - \frac{p_2}{\rho_2} - (s_1 - s_2)T\right) = -\rho g - \chi_j \frac{8\mu_{mix} u}{\alpha_j \rho_j R^2} - \frac{\rho}{\alpha_1 \rho_1 \alpha_2 \rho_2}\delta(u_1 - u_2)$

(8) $\frac{\partial}{\partial z}(\rho_c \alpha_l \beta u_l) = \frac{1}{\tau^{(c)}}\alpha_l \rho_c(\beta - \beta^{eq})$

(9) $\frac{\partial}{\partial z}[x_d \alpha_1(\rho_1 - \beta\rho_c)u_1] = -\frac{1}{\tau^{(d)}}(x_d - x_d^{eq})\alpha_1(\rho_1 - \beta\rho_c)$

where $\tau^{(p)}$ is the characteristic time which controls the pressure difference between both phases, $\tau^{(d)}$ is the characteristic time which controls the gas exsolution rate, x_d is the mass fraction of the dissolved gas, x_d^{eq} is the equilibrium value of the same parameter, β is the volume fraction of crystals, ρ_c is the density of crystals, δ is a drag factor, j is the continuous phase index (1 or 2), χ_j controls the direction of the viscous term contribution (1 or -1), $\tau^{(c)}$ is the characteristic time of the crystallization process, β^{eq} is the equilibrium volume fraction of crystals and the subscript l refers to the liquid component of the system.